African Educational Research Journal
Vol. 4(4), pp. 160-170, December 2016
ISSN: 2354-2160
Full Length Research Paper

Compound subtraction in non-decimal bases: Relative effectiveness of base-complement additions and decomposition algorithms

Asiedu Lloyd Owuba and Churcher Kwesi Amanyi*

Department of Mathematics, Faculty of Mathematical Sciences, University for Development Studies, Ghana.

Accepted 17 October, 2013

ABSTRACT

The purpose of the study was to determine whether there was any significant difference in speed, accuracy, retention and transferability between the Decomposition (DEC) and the Base-Complement Additions (BCA) algorithms for performing compound subtraction in non-decimal bases. Fifty-nine students with a mean age of approximately 15 years from two Agona Swedru Junior High Schools in the Agona District participated in the study. The study employed the pretest-posttests non-equivalent design. The two schools were randomly assigned to the treatment groups. The data collected on the four achievement tests namely Pretest, Posttest, a Third test and Retention test administered were then studied and analysed by employing the t-test at 0.05 level of significance. From the study it was found out that the mean performance of the BCA group was significantly higher than the DEC group on measures of accuracy. There was no relationship between the BCA and the DEC groups on the measure of speed. The BCA group produced significantly better computational accuracy than the DEC group. The mean score of the BCA group in the retention measures were significantly higher than that of the DEC group. The third test which was the test for understanding and application of the two methods showed that the BCA group performed better than the DEC group though the difference was slightly significant. Based on the findings, it was established that BCA method has substantial gains over the DEC method and as such recommended that the BCA should thus be included in the Junior Hugh mathematics curriculum as alternative method of solving compound subtraction in non-decimal bases.

Keywords: Decomposition, base-complement additions, compound subtraction.

*Corresponding author. E-mail: laowuba@yahoo.com.

INTRODUCTION

The method of compound subtraction has a long-standing history right from the 1920s to the present time. Opinions on the most efficient and suitable method for teaching and learning compound subtraction are since then divided and inconclusive. To compound the problem was the introduction of the non-decimal bases in the late 1960s and its adoption by the West African Examinations Council in the 70s into the Ghana Modern Mathematics syllabus. Since then all the revised versions of the syllabus retains the topic. A research conducted on the WAEC's Basic Education Certificate Examination Questions revealed that test items on subtraction under number bases (non-decimal) are hardly set. This shows that the transfer of the DEC method which has been in practice since then to this new topic will need a critical attention. Hence there is a need for an alternative method.

The Third Millennium is an age of computer and technology. Many of our problems of this millennium need experts in mathematics. According to Jerold (1970) mathematics is man's finest creation for the investigation of nature. The major concepts, broad methods and even specific theorems and axioms were derived from nature. Mathematics is valuable, largely because of its contributions to the understanding of the physical world.

Dainton as cited in Eshun (1985), gave the following specific objectives for the teaching and learning mathematics: (a) means of communicating quantifiable ideas, (b) means of training for discipline of thought and for logical reasoning, (c) tool in activities arising from the developing needs of many fields, and (d) study in itself, where development of new techniques and concepts can have economic consequences like those flowing from scientific research development.

In spite of all these, conditions are not as good as one would have liked. Parents and the general public have expressed much concern with the results that their wards do not perform well in mathematics examinations. This is a pointer to the fact that there is something wrong with the computational skills of pupils in mathematics in general and compound subtraction in particular. The concern of lack of computational skills in children and even adults has been worldwide. For instance, in 1978, the then British Prime Minister James Calaghan appointed Sir Cockroft to chair the committee which was called for a probe in September, the same year, to investigate the circumstance leading to the falling standards in simple computation by pupils. The committee was charged with the responsibility of looking at the teaching of mathematics in primary and secondary schools in England and Wales, with particular reference to the mathematics required in further and higher education, employment and adults' life generally and subsequently make appropriate recommendation. As reported on the research study Mathematics Count in 1978, many adults in British have the greatest difficulty with even such apparently simple matter as checking their change in shops. Some of the findings of the committee chaired by Cockcroft are that:

1. Study of mathematics is regarded by most people as being essential in the sense that it would be very difficult to live a normal life in many parts of the world in the twentieth century without making use of mathematics of some kind.
2. Usefulness of mathematics is perceived in different ways. For many it is seen in terms of the arithmetical skills which are needed for use at home or in the office or workshop; some see mathematics as the basis of scientific development and modern technology.

Sometimes the situations in which people find themselves doing subtraction oblige them to do it not only accurately but also fast. One striking daily example is where a conductor of a bus has to give change to about a dozen passengers who want to alight at the next bus stop. Another example is the newspaper vendor and other such vendors who are found selling to people in moving vehicles in traffic. In such situations it is just not conducive to use machines or electronic calculators to determine the change to give or receive.

Generally, reports from Ghana and elsewhere on pupils' classroom achievement in mathematics indicate that pupils lack basic arithmetical skills. It was stated emphatically in the 2010 West African Examination Council (WAEC) Chief Examiner's Report on mathematics paper two in the West African Senior School Certificate Examination (WASSCE) that candidates seemed to have neglected the basics. Cummings (1988) report of declining competitiveness of Americans students in solving subtraction problems as compared to world performance. This is carried to non-decimal bases when subtraction is considered.

A research conducted on the West African Examinations Council's BECE questions reveal that items on subtraction under number bases (non-decimal) is hardly set. This is because examiners have learned from experience that students find them very difficult so they feel inhibited in setting them. Statistics show that from the inception of the Junior Secondary School (JSS) now Junior High School (JHS) programme in 1990 to 211 out of the 28 number bases objective items set only one involved subtraction – making 3.5%. In the essay items section a 0.0% was recorded since none of the six questions set fell under subtraction.

Similarly, from 1993 to 2005 it is shown that out of the 22 SSSCE objectives on number bases items set, only 4 of them involved subtraction constituting 18.2%. Again there were no subtraction items among the 6 non-decimal essay questions. However, technology of today calls for the binary numbers, as it is the root of the computer language test items.

The 2011 report of the Primary Education Project (PREP) reveals that:

i) Just over one percent of the pupils in primary six involved in the Criterion Referenced Test (CRT) could achieve fifty percent pass in mathematics.
ii) A little over 60% of pupils in primary six involved in the CRT could answer compound subtraction problems involving two digits numbers correctly as against (70%) for addition. Here it is clear that more pupils could do addition with regrouping than compound subtraction. This significant difference cannot be blamed on poor teaching because the same teachers taught both addition and subtraction in the schools with regrouping. Again information on the CRT showed that between 1992 and 1996, with mastery scores at fifty-five percent for mathematics, the percentages of pupils scoring above the level had consistently been between one and two percent.

Situations outside Ghana are no different. Results of the National Assessment of Education Progress (NAEP) Mathematics assessment (Kouba, 1998) revealed that from the six whole number subtraction items administered to three-grade levels–third, seventh and eleventh – about 85% of the third-grade students correctly solved the items involving two-digit subtraction with no regrouping (borrowing). Performance for third-grade students fell 15 and 20 percentage points on two-digit subtraction items involving regrouping presented in the vertical and horizontal modes respectively. Fifty percent of the third-grade students could correctly solve three-digit compound subtraction problems without blocking zero and only 45% could solve similar items involving blocking zero. Performance of students in grade seven and eleven was above ninety percent on the two-digit simple subtraction items and about 85 to 90% on the three-digit compound subtraction items. From their performance, it appeared that students have difficulty in solving compound subtraction.

In arithmetic, for example, it is one thing to comprehend the mathematical principles governing decomposition in subtraction (something that can come from a single insightful experience) and another to be able to subtract quickly and accurately. The strategies for addition and subtraction require at least implicit knowledge of properties of operations (commutativity and associativity).

Multi-digit subtraction seems to be more difficult for children than multi-digit addition. Some difficulties at this point seem to be inherent, and some may result from particular aspects of classroom activities, such as an emphasis on a take-away meaning. Children also may incorrectly generalize attributes of addition methods to subtraction; this may be exacerbated if addition is experienced for a long time before subtraction. How many of these difficulties could be reduced by changes in classroom activities is an important issue for future research (Fuson, 1997).

It could, therefore, be realised that the role and importance of subtraction in real life situation is paramount. Each time we wish to effect payment in return for services rendered or goods bought, we are invariably doing subtraction. Both newspaper vendors and assistant drivers' usage of subtraction cannot be overemphasized. Accountants employ subtraction in finding their balances.

It seems that the situations where pupils find it difficult to do compound subtraction are due to the methods employed in teaching subtraction with regrouping. According to Ballard (1959) everybody grumbles at pupils' inability to subtract with that ease and accuracy which ordinary life demands. To Ballard (1959) subtraction is weak and that the root of the weakness lies in the method (that is, Decomposition) employed. The weakness of the Decomposition method lies in the fact that it fails to give a reasonable measure of accuracy.

It should be noted that before Ballard's claim above Brownell and Moser (1949) and others had recommended the use of the Decomposition (DEC) in schools. Prior to the work of Brownell and Moser (1949) the Equal-addition (EA) algorithm was the one practised in most American (USA) schools. The shift from EA to DEC was seen as a gateway to pupils' competency in solving compound subtraction task.

Statement of the problem

Research on compound subtraction has a long-standing history right from the 1920s to the present time. Researchers like Armar and Brown (1971), Brownell and Moser (1949), Carpenter (1981), Gyening (1993), Johnson (1938), Appiah (2001), Winch (1920), and a host of others have been at the forefront in finding the most efficient way of doing compound subtraction.

In response to the outcry of the public about lack of innovations in mathematics education in general and compound subtraction in particular these innovators took up the challenge. The need to bring about innovations in mathematics education in general has been their major concern. Notwithstanding the public furore about pupil's lack of basic computational skills, opinions on the most efficient and suitable method for teaching and learning compound subtraction are divided and inconclusive. Armar and Brown (1971), Ballard (1928) and others advocate the EA method.

Those who advocate the DEC include such personalities like Brueckner and Grossnickle (1953), Rheins and Rheins (1955) and Suydam and Weaver (1977). These differing opinions have engaged the minds of many a researcher. Ghana, like most Anglophone countries of the West African sub-region uses the DEC method at all levels of educational structure. It has been revealed that there is uneasiness on the part of pupils and students as well as teachers when subtraction with regrouping is encountered (Potter, 1961; Brueckner and Grossnickle, 1953; Carpenter et al., 1975). This DEC method is what is employed in solving compound subtraction in other bases besides the decimal (JHS Maths book 2). To ease this difficulty the potentiality of the BCA method was investigated and recommended by Armar and Brown (1971), Byrkit (1988) and Gyening (1993).

Gyening (1993) account of pupils' uneasiness to contend with subtraction with regrouping has been taken up by Appiah (2001) and Essel (2000) on the primary level. Esson (1999) compared the two methods of teaching compound subtraction in different number bases in Anomabo in the central region of Ghana at the JHS 1 level. According to their findings, Appiah's investigation showed significant difference in transferability, speed, accuracy and retention between the two algorithms in favour of the BCA. On the part of Essel, there was no significant difference between the two methods. Esson's report on the JSS level was rather in favour of the DEC method when the mean scores on the various tests were considered though there was no significant difference between the two algorithms. Based on the limitations and the recommendations on their findings, requiring a further investigation in different settings and with a different research design, it was thus evident that the problem is still open and calls for more research.

The question is if the BCA method is proving good in the primary level, can it not be tried again in the Junior High School level (JHS) and even better still transfer the related skills to other bases besides the decimal? It is against this background that the present study is geared to addressing the problems associated with the teaching and learning compound subtraction in non-decimal bases among JHS 2 pupils. Therefore, the purpose of the study was to find out how students could be helped to solve compound subtraction in non-decimal bases so that the topic would be included in the examinable ones. Thus the study compares the relative effectiveness of the Base-Complement Additions and the Decomposition method on Compound subtraction in non-decimal bases on the measures of speed, accuracy, retention and understanding.

Hypotheses

The following hypotheses were formulated to be tested at 0.05 level of significance.

1. There is no significant difference between the mean speed time of the DEC and the BCA groups on the measure of time per score per item on the posttests conducted after the teaching episode.
2. There is no significant difference between the mean scores of the DEC and the BCA groups on the posttest administered on the first day after the teaching episode.
3. There is no significant difference between the mean scores of the DEC and the BCA groups on the retention test.
4. There is no significant difference between the mean scores of the DEC and the BCA groups on the third test which is the test for understanding and application of the related skills administered after the teaching episode.

Delimitation

This study covered only two schools. This was partly due to difficulty of means of mobility and limited resources (for example teaching materials like square grid boards and spike abacus) at the disposal of the investigator. The study was restricted to JHS 2 children of Methodist Junior High School and Agona District Assembly ‘E’ Junior High School all in Agona Swedru. These schools were purposely chosen because previous works on similar topics had been carried out with subjects within the rural setting. The schools were also chosen because of their proximity to the investigator’s residence and the healthy relationship between most of the teachers in these schools and the investigator.

Limitation

The study was limited to 59 children in JHS 2 from two schools in the Agona District in the Central Region of Ghana. The two groups used in the study were kept intact and thus could place a group at an advantage over the other with respect to classroom achievement or prerequisite knowledge. All the children in the DEC group and the BCA groups had already learnt compound subtraction using ”borrowing” and have all along used it in their computations. The mathematics textbooks in use at present also stressed compound subtractions by the method of Decomposition. This situation clearly put the DEC group at advantage over the BCA group who have the dual task of unlearning the Decomposition algorithm and accommodating the BCA method.

Assumptions

The following assumptions were made; that all the items in each test were of equal difficulty, it was assumed that the children receiving instruction in specific method of doing subtraction would perform differently depending on their cognitive understanding of the respective algorithm, the age difference of subjects in the selected schools was not significant, students in each treatment group had equal mathematical abilities, students in the respective treatment group were equally motivated in their study of mathematics, that the researcher could teach the BCA algorithm as effectively as the DCE method, it was assumed that the group had been using the Decomposition method at least seven years, and finally it was assumed that the subjects were almost at the same level of cognitive development and therefore the DEC and the BCA groups were not different in terms of rate of work, accuracy, retention and understanding.

Significance of study

The concern expressed by parents and the general public about pupils’ deficiency in simple computations is a pointer to the fact that there is something wrong with the computational skills of pupil’s in schools and employees at work places. This implies that there is something wrong with the teaching and learning of arithmetic, particularly compound subtraction in general and compound subtraction in non-decimal bases in particular that must be corrected.

The study therefore, looked at the aspect of compound subtraction involving number bases. This is because the new textbooks for the JHS have limited the topic, number bases to the addition and multiplication only. The current Mathematics for Junior High Schools Pupils’ Book 2 treats Number Bases under the conversion of a base to the decimal only (Mathan and Wilmot, 2005).

First the current study reveals the superiority of the BCA algorithm over the DEC. The method would help to improve upon the falling standards in pupils’ subtraction skills. With these conventional methods especially the

DEC the child or learner is confronted with problem of having to master one hundred subtraction facts (Mueller, 1964). Hopefully the new method (BCA) would demand knowledge of only seventy-three subtraction facts ensuring that minimum memory requirement is achieved. Further, if the Base-Complement Additions algorithm were proved to be significantly superior to the Decomposition method it would be recommended as a viable algorithm in the JHS Mathematics syllabus. This would address the issue of falling standards because fewer errors would be made by the use of this method. Consequently it would enhance the prospects for further study in a greater dimension with the ultimate adoption of the new method by curriculum developers, mathematics educators and teachers at large. Teachers currently under training in Colleges of Education and teachers in the field could be exposed to the new method when found to be significantly superior to the DEC method.

Since the present study is geared to addressing the relative effectiveness of both methods, it might bring to bear the strengths and weakness existing in them.

LITERATURE REVIEW

Theoretical overview of time allotment

Teaching children to subtract has been considered a problem in mathematics pedagogy. This is because there are several approaches to the tackling of subtraction problems. As a result considerable progress has been made in understanding the topic. Grossnickle and Brueckner (1953) asserted that subtraction had attracted more investigations than any topic in arithmetic. Research has provided a sophisticated grasp of the mathematical and linguistic structure of the subtraction problems and many of the factors that affect their difficulties. The question why compound subtraction has several approaches has been the concern of not only the investigator, but many mathematics educators as well. Winch (1920) made the following remarks: "No methods give more trouble and are less successful than those of teaching subtraction" (p. 207). Thorndike (1921) believed that the controversy of how children should be taught to subtract centered on the argument of whether to use the "subtractive" method or the "additive" method, a direct reference to the DEC versus the Equal-Additions algorithms. In his "History of mathematics" published in 1925, Smith pointed out that the terminology of subtraction had "varied greatly and is not settled even now".

In considering, at least the above prelude and quotations it would be appreciated that doing compound subtraction has got a long-standing history to offer. The first mathematical education thesis submitted to a British university was a B.Ed. thesis on 'Different Methods of Subtraction' presented in 1910 to Edinburgh University. In 1919 came the first Master's thesis in England and in 1930 the first for a PhD (Hinkle, 1988). Johnson (1938) had indicated that by 1975 many textbooks in the US were using the Decomposition approach.

The two most outstanding methods that stood the test of time are the DEC algorithm sometimes referred to as the borrowing or regrouping method and the EA algorithm, also referred to as" borrow and repay" method, or the method of compensation. This DEC method date back to 1140 AD and was first introduced to America in 1822 (Smith, 1925) and the EA algorithm dates back to the writings of Fibonacci in 1202 AD. According to Johnson (1938), the Equal-Additions method prevailed in the US until 1850.

The methods used to approach compound subtraction are many but can be grouped under two main categories. These are the conventional and non-conventional methods. Under the conventional algorithms are basically the DEC and EA. Besides these two conventional methods are non-conventional ones like the Austrian, the Base-Complement Additions, the Complementary, the Colton (1980) and the Residue methods.

Decomposition algorithm

The application of the Decomposition method is not employed in situations where the subtraction problem is a simple one. Simple subtraction problems are where the place-values in the minuend are greater than or equal to the corresponding place-values in the subtrahend. For example in a case like 87 - 53, the answer readily comes out as 34, once the basic subtraction facts are known. There are, however, some situations when one comes across some subtraction tasks for which there is no entry in the set of basic subtraction facts. These situations are where the place-values in the minuend are less than the corresponding place-values in the subtrahend. Such subtraction problems are known as compound subtraction and many pupils are unable to solve. These simple and compound subtractions that children do learn may involve different number bases as well. Compound subtractions like the following examples are given to them:

(i) 82_{ten}	(ii) 623_{ten}	(iii) 432_{five}	(iv) 110_{two}
$-\underline{65}_{ten}$	$-\underline{149}_{ten}$	$-\underline{143}_{five}$	$-\underline{11}_{two}$

In such situations the Decomposition, which involves regrouping, is used as one of the conventional methods to solve such problems. In these examples the ones digits in the subtrahends are greater than the corresponding digits in the minuends. Muller (1964) refers to such situations as column impasse.

To solve the problems cited as examples, adequate knowledge of the place-value concept in the respective number base is required. Also because of the column impasse, regrouping should be done to solve the problems correctly. The process is illustrated in the steps below:

(i) 82 – 65 = (8 tens + 2 ones) – (6 tens + 5 ones)
 = (7 tens + 12 ones) – (6 tens + 5 ones)
 = 1 ten + 7 ones
 = 17

In this process, the subtrahend remains unchanged but the minuend is renamed or regrouped as 7 tens and 12 ones from which 6 tens and 5 ones are subtracted under the corresponding place-values to give the answer 17. Subsequently two subtractions are then performed to solve the example given:

(i) 12 ones – 5 ones = 7 ones
(ii) 7 tens – 6 tens = 1 ten

Thus the result is 1 ten and 7 ones which is 17. This mode of presentation, the horizontal form, is referred to as the expanded form. Another mode of presentation is the vertical form where "crutches" are employed to initially ease computation. Thus the approach looks like this:

$$\begin{array}{rr} 8 & 2 \\ -\underline{6} & \underline{5} \end{array} \quad \text{becomes} \longrightarrow \begin{array}{rr} {}^{7}8 & {}^{1}2 \\ -\underline{6} & \underline{5} \\ \underline{1} & \underline{7} \end{array}$$

Or it can as well be expressed in this form. (T for Tens and O for Ones)

$$\text{(i)}\ \begin{array}{rr} T & O \\ 8 & 2 \\ \underline{-6} & \underline{5} \end{array} = \begin{array}{rr} T & O \\ 7 & 12 \\ -\underline{6} & \underline{5} \\ \underline{1} & \underline{7} \end{array}$$

In another problem 623 – 149, 623 is renamed as 5 hundreds, 11 tens and 13 ones. Subsequently three subtractions are then performed to solve the problem:

i) 13 ones – 9 ones = 4 ones
ii) 11 tens – 4 tens = 7 tens
iii) 5 hundreds – 1 hundred = 4 hundreds

Thus the final result is 4 hundreds and 7 tens and 4 ones which is 474.
In the vertical mode (ii) would look like this:

$$\begin{array}{rrr} H & T & O \\ 6 & 2 & 3 \\ \underline{-1} & \underline{4} & \underline{9} \end{array} = \begin{array}{rrr} H & T & O \\ 6 & 2 & 3 \\ -\underline{1} & \underline{4} & \underline{9} \end{array} = \begin{array}{rrr} H & T & O \\ 6 & 2 & 13 \\ \underline{-1} & \underline{4} & \underline{9} \end{array} = \begin{array}{rrr} H & T & O \\ 5 & 11 & 13 \\ -\underline{1} & \underline{4} & \underline{9} \\ \underline{4} & \underline{7} & \underline{4} \end{array}$$

In likewise manner in solving example (iii), 432_{five} – 132_{five}, regrouping is done as:

432_{five} – 143_{five} = (4 five fives + 3 fives + 2 ones) – (1 five fives + 4 fives + 3 ones)
 = (3 five fives + 13 fives + 2 ones) – (1 five fives + 4 fives + 3 ones)
 = (3 five fives + 12 fives + 5 ones + 2 ones) – (1 five fives + 4 fives + 3 ones)
 = (3 five fives – 1 five fives) + (12 fives – 3 fives) + (7 ones – 3 ones)
 = 2 five fives + 3 fives + 4 ones
 = 234_{five}

In the vertical mode the method looks like this:

Five fives	Fives	Ones		Five fives	Fives	Ones		Five fives	Fives	Ones		Five fives	Fives	Ones
4	3	2	→	$^{3}4$	$^{5}3$	2	→	3	$^{7}8$	$^{5}2$	→	3	7	7
– 1	4	3		– 1	4	3		– 1	4	3		– 1	4	3
												2	3	4

In problem numbered (iv), $110_{two} – 11_{two}$ is renamed as 1 two twos and 1 two and 0 ones. Subsequently two subtractions are then performed to solve the problem:

1. 2 ones – 1 one = 1 one
2. 2 twos – 1 two = 1 two

Thus the result is 1 two and 1 one which is 11_{two}.
In the vertical mode the method looks like this:

Two twos	twos	Ones		Two twos	twos	Ones		Two twos	twos	Ones		Two twos	twos	Ones
1	1	0	→	$^{0}1$	$^{2}1$	0	→	0	$^{2}3$	$^{2}0$	→	0	2	2
–	1	1		–	1	1		–	1	1		–	1	1
													1	1

According to Underhill (1972), the horizontal form could be used in subtraction forms in situations where children can master all subtraction facts from 1 – 0 up to 18 – 9. In addition, the horizontal form is recommended in introductory work in subtraction of whole numbers. The most obvious drawback of the DEC algorithm appears evident when zeros, particularly successive zeros occur in the minuend.

Empirical evidence and studies related to compound subtraction under decomposition and base-complement additions methods

Base-complement additions method

The BCA is a modified form of the EA method (Gyening, 1993). As a variant of the EA method the BCA is still based on the principle of compensation. The BCA algorithm can be objectified. If the EA method was seen as a method that induces more accuracy and speed than the DEC method, Ballard (1928), then the BCA has the potential of inducing more accuracy and speed as well.

Gyening (1993) in a paper presented at a departmental seminar referred to the method of Equivalent Zero transformation now known as Base-Complement Additions. Incidentally he is not alone in thinking of the other much more improved methods of doing compound subtraction. Amar and Brown (1971) and also Byrkit (1988) have made mention of this method (Equivalent Zero Transformation or Base-Complement Additions) which is considered to be an improvement over the method of EA. The rationale of the BCA algorithm is to transform a given compound subtraction into a simple subtraction by adding the complement of ten and for that matter of any base, to both the minuend and the subtrahend where an impasse occurs. As Byrkit (1988) observed, with practice many are able to write down the answer without much intermediate work.

Some of the advantages derived from the BCA are that:

i) Concrete objects or materials can be used in teaching compound subtraction using the BCA method.
ii) It does not violate the normal place-value notion rule in our numeral system.
iii) It puts less cognitive load on the learner, as it requires fewer additional subtraction facts. That is the nine base ten complements.
iv) It is much easier to do compound subtraction using the BCA procedure when both the minuend and the subtrahend are in the horizontal mode. For example:

452 – 389 = 453 – 390 = 463 – 400 = 63

In solving the compound subtraction 82 – 64, we first find the base complement of 4 which is 6 and add it to 64 to get 70. We add the same 6 to 82 to make 88. In this wise the original problem is transformed into 88 – 70, which is now a simple subtraction problem and the answer 18, is obtained.
In the vertical mode the process is illustrated as:

8	2	→	82 + 6	→	8	8
– 6	4		– (64 + 6)		– 7	0

Similarly when working in a non-decimal base for example:
$412_{five} - 123_{five}$ the process becomes: To the ones column, 2 (that is, base five complement of 3 which is in the subtrahend) is added to both the subtrahend and minuend. The transformed problem is now $414_{five} - 130_{five}$. However, there is another column impasse in the fives column. To address this, 2 is added as before to arrive at the final transformation as $434_{five} - 200_{five}$ which is a complete simple subtraction resulting in 234_{five}.

In the vertical presentation the above problem is illustrated as follows:

Five fives	fives	ones
4	1	2^{+2}
– 1	2	3^{+2}

Five fives	fives	ones
4	1^{+2}	4
– 1	3^{+2}	0

Five fives	fives	ones
4	3	4
– 2	0	0
2	3	4

Much as the DEC algorithm can be taught meaningfully, the BCA method could also be taught rationally. The BCA algorithm could be taught right from the inactive stage through the iconic to the symbolic stages (Bruner, 1965).

Gyening (1993), for example, points out in particular how easy and efficient the BCA method lends itself to the use of concrete materials. He also demonstrates its few steps leading to accuracy of computation and speed, and above all its high potential for retention. Following Gyening's remarks about the BCA algorithm, quite a number of studies have been undertaken involving the BCA and the DEC algorithms on compound subtraction. Among these is Mccarthy's work.

McCarthy (1994) undertook the first experimental study on the comparison of the DEC and BCA methods using eight-year-old pupils from the University of Cape Coast Primary School in the Cape Coast Municipality. The investigator spent seven days in experimental teaching. The design used was the pretest-posttest comparative group model. The study was to investigate the two methods and to find whether the suspected inherent limitations associated with the DEC method account for pupil's difficulty to handle compound subtraction. The study also looked into the potentiality of the BCA method. There was a pretest at the beginning of the experiment followed by a three-week treatment duration after which the immediate posttest was administered. Two weeks later a retention test was conducted. The test carried on them yielded measures of accuracy and retention. The analysis of covariance was also used to determine the method, which was easier to learn and also to retain. These tests were done at 0.05 level of significance. It was clear that a tighter experimental design controlling for variations in individual teacher effectiveness was an appropriate direction for the research. The study utilized a sole implementer (the investigator) in an effort to control for varying teacher effectiveness. Results from this study showed that the mean score of the BCA group was higher than that of the DEC group. Statistical analysis showed that there was no significant difference between the mean scores of the two treatment groups on measures of accuracy. McCarthy (1994) wanted to determine efficiency, but unfortunately, did not include speed.

METHODOLOGY

The entire JSS 2 population in the Agona District constituted the target population for the study. The accessible population involved only JSS 2 students from Agona District Assembly 'E' Junior Secondary School and Swedru Methodist "A" Junior Secondary School. The investigation involved fifty-nine children enrolled in these schools. The average age of the children was 15.04 years old. These schools are not less than a kilometre apart from one another.

The samples involved in the investigation were chosen through purposive sampling technique. However, the groups involve in the study were randomly assigned to each of the two treatment groups. The schools were chosen due to their proximity to the investigator's place of residence and the existing healthy relationship between the staff and the investigator. Hence the investigator was able to visit the schools at agreed periods at the least cost. Thus both probability (random) and non-probability sampling methods were used to select the sample.

The design for the study was the pretest, posttest, transfer test and retention test comparison group design. The design was chosen because events of the study were in the natural setting of a Ghanaian JSS situation. In line with this, Christensen (1980) states that: In natural settings where planned or unplanned effects occur, one cannot randomly assign subjects to treatment conditions, nor is it possible to control for the influence of extraneous variables through other techniques. Therefore, Quasi-experimental design is needed to obtain some index of impact of the treatment condition (Christensen, 1980:198).

To ensure that the instrument measures what it is supposed to measure, the content and face validity of the items were ensured. A careful and critical examination of the test items as they relate to the specified content area was made. This was done to judge if the content and objectives measured by the test are representative of those that constitute the content domain. To determine whether the items in the test represent the course and objectives as stated in the curriculum guides, syllabuses, and JSS2 mathematics textbooks were consulted. In order to obtain an external evaluation of content validity experienced JSS 2 teachers and the research supervisors were asked to examine the test content systematically and evaluate its relevancy to the specified universe.

The students' scripts were scored dichotomously and the internal consistencies of the tests measured through Cronbach (1951) alpha-formula which revealed the following results 0.84, 0.89 and 0.78 for the three protests-accuracy, retention and the test of understanding respectively. The split-half reliability test for the data collected from all the four achievement tests gave the following correlation coefficients: pretest, 0.87 and 0.88; posttest, 0.74 and 0.79; third test (test for understanding and application of the related skills), 0.86 and 0.85 and the retention test 0.82 and 0.78.

Data analysis

Data were gathered and processed using frequency and percentage, computed t-value, and measures of variability. These statistical processes were computed using SPSS Version 10.05 for Windows software. Here the t-test, using the respective mean scores and standard deviations of the various four achievement tests, was the principal means used to analyze the data thus obtained. Since the two schools formed intact groups and these were randomly assigned to treatments, the various groups could differ. In order to control for this possible variation the t-test was used to analyze the pretest mean scores for both the speed test and the accuracy test in order to ascertain whether there was a significance differences between the means to justify any means of employing the analysis of covariance. The Split-half alpha method of reliability was used to establish the reliability of the test items.

FINDINGS AND DISCUSSIONS

The findings of the study with respect to finishing time or speed were that the BCA students were much faster and more accurate. When the finishing time was converted into speed (rate of completing the task successfully) the BCA group was slightly ahead of the DEC group. The mean scores for the pretest show that the DEC treatment group had the higher mark, 7.86 as against 7.68 for the BCA group. From the posttest results it was observed that these mean scores were doubled for the BCA group, 7.68 to 14.42 while the DEC group had just over 27% increment.

The DEC group produced twenty-five students (89%) above and three students (11%) below the mean speed time for the group. At the same time the BCA groups had thirty (96%) and one (4%) below the mean speed time for BCA group. And the speed values for two treatment groups-DEC and BCA- were 3.69 and 2.09min per score per item respectively. In the retention test, the median speed for both groups was found to be 1.80 min per score per item. From the findings the DEC group produced in percentages 71 and 29% students above and below the mean speed value of the group. At the same time in the BCA group saw 93% above and 7% below the mean speed value for the group. Thus with respect to speed these results contrast the assertion Gyening (1993) made. The findings are also inconsistent with Winch (1920), Johnson (1938), Datsomor (1997) and Appiah (2001). All these studies were done in the decimal system. The findings are consistent with Esson (1999) who researched into the compound subtraction in number bases other than the decimal.

Under the retention test it was noted that the BCA students' fell short in dealing effectively with 'blocking "N - 1" for any base N in the subtrahend. Martin (1992) in her study took some time to help the EA group to overcome the problem associated with 'blocking nines' (for base ten) in the subtrahend. Again in the Retention test 11% of the DEC students as against 52% of the BCA students respectively could answer test item number eight correctly. It was observed at the pretest level that 54% of the DEC students as against 45% of the BCA students scored 40% or more in the test. In the retention test, it was 67.8% for the DEC group and an average of 74% for BCA group.

The Third test which was used to test for understanding and application of the related skills, produced 67.8 and 97% for the DEC and the BCA groups respectively when the number of students attaining 40% and above of the test scores was considered. Furthermore, in each of the posttests, the percentage of the BCA students attaining 80% or more of the test scores was more than or equal to two and a half times as much as the percentage of the DEC students. It was that observed in the third test which was used to test for understanding and application of the related skills that the DEC group committed more errors than their BCA counterparts when new bases other than the three basic bases hammered on (during the treatment sessions) as well as increasing number of digits in the compound subtraction problems were given.

From the analysis of the data, the following findings were found: Posttest differences in speed or efficiency defined by the investigator as a ratio of finishing time to computational accuracy were tested, using the t- test. Consequently, the t-value of 1.76 at 57 degrees of freedom was not equal nor exceeded the critical t-value of 2.00 at 0.5 significance level. Thus there was no significant difference between the two groups. Posttest differences in computational accuracy, retention and transferability were also tested by the same test statistic. From the analysis of the data the following findings were made:

1. There was no significant difference between the mean speed time of the DEC and the BCA groups on the measure of time per score per item on the posttests conducted after the teaching episode.
2. There was significant difference between the mean scores of the DEC and the BCA groups on the measure of computational accuracy on the posttest administered on the first day after the teaching episode. ($t = -3.59$, $df = 57$, sig. = 0.001)
3. The t-test results revealed that there was significant difference between the DEC and the BCA groups on measures of retention. However the significance was not high. ($t = -2.4$, $df = 57$, sig. = 0.022)
4. There was significant difference between the mean scores of the DEC and the BCA groups on the measures of test for understanding and application of the related skills. ($t = -3.54$, $df = 57$, sig. = 0.001)

The above findings are consistent with the theoretical assumptions (Amar and Brown, 1971; Byrkit, 1988; Gyening, 1993), which was the basis of the present study. Findings indicate that the BCA algorithm is capable of helping students to give more accurate responses to compound subtraction problems than the DEC procedure.

Earlier researchers preferred the DEC method to the EA method on the basis that the EA method could not easily be rationalized and therefore difficult to teach it meaningfully (Brownelle and Moser, 1949; Grossnickle and Brueckner, 1953; Rheins and Rheins 1955).

As pointed out in the early chapters the BCA procedure is an improvement upon the EA method. Both the BCA and EA employ the same principle of compensation (that is, adding equal numbers to the minuend and the subtrahend but in a differing form). Since the BCA algorithm can be rationalized or taught meaningfully to children through the use of enactive, iconic and symbolic instructional activities, it corrects all deficiencies associated with EA approach and therefore the BCA is superior to the EA in terms of speed, accuracy and retention (Johnson, 1938; Ohlsson et al., 1992). No wonder that the results from the current study have proven that students using the BCA method are able to make both vertical and horizontal transfers better than their DEC counterparts.

Considering compound subtraction in non-decimal bases in the horizontal mode, it was observed that the BCA group outperformed their DEC counterpart. Another observation was that students in the BCA group did not have much difficulty in dealing effectively with "successive N - 1" in the subtrahend.

Conclusions

Based on the findings of the study, the following conclusions were made that:

i) From the study, it was found out that there was no significant difference between the BCA and the DEC groups on the measure of speed.
ii) The mean performance of the BCA algorithm group was significantly higher than the DEC method group on measures of accuracy, retention and test.
iii) Finally, the relatively better performances of students in the BCA group on measures of accuracy, retention and transferability suggest that the BCA algorithm is relatively more effective than the DEC algorithm.

RECOMMENDATIONS

i) In the light of the findings, the BCA algorithm for doing compound subtraction might have impact on today's mathematics curriculum. The present curricular choice in all mathematics texts in Ghana in particular and in most texts in the world at large is to introduce the BCA algorithm, to the neglect of other equally good approaches.
ii) The present state of mathematics on solutions of compound subtraction in non-decimal bases in junior secondary schools calls for more innovative and effective teaching and learning techniques and it has been demonstrated by the findings of the current study that the BCA method has substantial gains over the DEC method. Such method like the Base-Complement Additions should thus be included in the junior secondary mathematics curriculum as alternative method of solving compound subtraction in non-decimal bases.
iii) The introduction of the BCA method in the curriculum of the teacher training institutions and universities would also not be out of place since the newly train teachers would eventually replace the already serving.
iv) The relative effectiveness of the DEC and BCA methods of doing subtraction in non-decimal bases should be explored further.
v) The Mathematical Association of Ghana (MAG), Ghana Education service (GES), NGOs, and other interested should include workshops or in-service training in their programmes to expose the BCA algorithm of solving compound subtraction in non-decimal bases.
vi) Both examining and professional bodies that set the final JHS assessment questions should be exposed and encouraged to use these BCA algorithm in solving compound subtraction in non-decimal bases.

Suggestions for further research

The findings of the study suggest further research. This study is a pointer to the urgent need for GES, MOE, NGOs and Education researchers to conduct a nationwide study similar to it, to assess the efficacy of the BCA method and to give a broader picture of the findings for generalisation. The pedagogy of teaching the BCA algorithm is a virgin and ripe area for Educational researchers to delve into at the teacher training and university levels.

REFERENCES

Appiah, J. (**2001**). Comparison of the Decomposition and Base-complement Addition Algorithms for Teaching Compound Subtraction. Unpublished Thesis Submitted to the Department of Science Education, University of Cape Coast.

Armar, T. Q., and **Brown**, D. A. (**1971**). Modern Mathematics for Elementary Schools. Basingstoke: Macmillan and Co. Ltd.

Ballard P. B. (**1928**). Teaching the Essential of Arithmetic. London: University of London Press.

Ballard, D. P. (**1959**). Norms of performance in the fundamental processes of arithmetic. Journal of Experimental Pedagogy, 2:396 – 405.

Brown, J. S., and **Van Lehn**, K. (**1982**). Towards a Gemcrative Theory of 'Bugs'. In Carpenter, P. T. et al., Addition and Subtraction: Cognitive Perspective. Hillsdale, New Jersey: Lawrence Erlbaum Associates Inc.

Brownell, W. A., and **Moser**, H. E. (**1949**). Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction. Duke University Research Studies in Education, 8. Durham NC: Duke University Press.

Brueckner, L. J., and **Grossnickel**, F. E. (**1953**). Making Arithmetic Meaningful. Philadelphia: the John C. Winston Company.

Bruner, J. S. (**1965**). Learning and Thinking. In R. C. Anderson and D. P. Ausubel (Eds). Reading in the Psychology of Cognition, 12(2). Berkeley University of California Press.

Byrkit, D. R. (**1988**). Subtraction by restoration. Mathematics Teacher, 81(2):103.

Carpenter, T. P. (**1981**). National Assessment. In Fennema, E. (Ed), Mathematics Education Research: Implications for the 80s.

Association for Supervision and Curriculum Development in Co-operation with NCTM.

Carpenter, T. P., Coburn, T. G., Reys, R. E., and Wilson, J. G. (**1975**). National Assessment of Educational Progress, first Mathematics Assessment. 1972/ 73 NCTM Publications.

Christensen, L. B. (**1980**). Quasi-experimental Design. Experimental Methodology, (2nd ed), (pp. 198-210) Boston: Allyn and Bacon, Inc.

Colton, B. (**1980**). Subtraction without Borrowing. Mathematics Teacher, 73:196.

Cummings, D. D., Kintsch, W., Reusser, K., and Weimer, R. (**1988**). The role of understanding in solving word problems. Cognitive psychology, 20:405–438.

Datsomor, W. K. (**1997**). Effectiveness of two Algorithms for teaching Compound Subtraction: Decomposition versus Base-Complement Addition. Unpublished PGDE research work. Cape Coast: University of Cape Coast.

Eshun, B. A. (**1985**). Children's Adding and Subtracting Schemes. Doctorial Dissertation: University Of Athens, Georgia

Essel, K. M. A. (**2000**). A Comparison of two methods of teaching Compound Subtraction. Decomposition and Base-Complement Additions Algorithm. Published Thesis. Cape Coast: University of Cape Coast.

Esson, A. K. (**1999**). Comparison of two Methods of Teaching Compound Subtraction in Different Number Bases: Decomposition vs. Base-Complement Additions. Unpublished PGDE research work. Cape Coast: University of Cape Coast.

Gyening, J. (**1993**). Facilitating Compound Subtraction by Equivalent Zero Addition (EZA). Paper presented at a departmental seminar of the Science Education Department, University of Cape Coast.

Jerold, L. G. (**1970**). An Approach to Mathematics, A Study of Groups of Seven year-Old Children Exploring Mathematical Apparatus. London: A & C, Black Ltd.

Johnson, J. T. (**1938**). The Relative Merits of Three Methods of Subtraction. Contribution to Education, 738. New York: Bureau of Publications, Teachers College, Columbia University.

Mathan, M., and **Wilmot**, E. M. (**2005**). Mathematics for Junior High Schools Pupil's Book 2. Association of Ghana, Pearson Longman.

Martin, C. (**1992**). A comparison of decomposition and equal additions subtraction algorithms in second and third grades. Journal for Research in Mathematics Education, 28, 2.

McCarthy, P. (**1994**). The Relative Efficiency of two Methods of Teaching Compound Subtraction. Decomposition vs. Equivalent Zero Addition. Unpublished PGDE research work. Cape Coast: University of Cape Coast.

Ohlsson, S. Ernst, A. M., and Rees, E. (**1992**). The cognitive complexity of learning and doing arithmetic. Journal for Research in Mathematics Education, 23(5):441-467.

Potter, F. E. (**1961**). The Teaching of Arithmetic. London: Pitman Press.

Rheins, G. B., and **Rheins**, J. J. (**1955**). A comparison of two methods of teaching compound subtraction. The Arithmetic Teacher, 2:63-67.

Citation: Owuba, A. L., and Amanyi, C. K. (2016). Compound subtraction in non-decimal bases: Relative effectiveness of base-complement additions and decomposition algorithms. African Educational Research Journal, 4(4): 160-170.