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Effects of Mathematics Content Knowledge on Mathematics Pedagogical Content Knowledge[1]

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Abstract
The related literature emphasizes that mathematics content knowledge (MCK) itself is not sufficient for effective teaching; teachers also need to have deep and wide mathematics pedagogical content knowledge (MPCK). In this study, we worked with 12 pre-service primary teachers through Teaching Practice course in order to examine how their MCK affects their MPCK. Observation, interview and field notes were used as data collecting tools and data gathered were analyzed with induction and deduction methods. The results showed that pre-service teachers' lack of MCK made it difficult for them to use MPCK effectively. Their weaknesses in MCK were apparent in their evaluation and interpretation of the students' responses and explanations, as well as in giving feedback to these comments. On the other hand, in cases where pre-service teachers' MCK was good, but their MPCK was insufficient, the lessons were also ineffective. In these instances, they had difficulties in reducing the level of instruction to the level of the students, and they used some knowledge which had not been mastered by the students.

Key words: Content knowledge, pedagogical content knowledge, pre-service teacher

Introduction

Content knowledge (CK), includes knowledge of the subject and its organizing structures (Shulman, 1986; Wilson et al, 1987). Pedagogical content knowledge (PCK) includes an understanding of how a student learns a subject, as well as how to design and manage the learning process (Shulman, 1986; Magnussan et al, 1999). A significant factor affecting teaching and learning process is teachers' content knowledge (Ahn & Choi, 2004). However, CK itself is not sufficient for effective instruction and teachers should also have deep and broad-ranging PCK (An et al, 2004; Ball et al, 2008; Kahan et al, 2003). Accordingly, researchers have pointed out that mathematics pedagogical content knowledge (MPCK) is directly correlated with mathematics content knowledge (MCK), and teachers should thoroughly master the mathematical meaning of the concepts so that they can form conceptually correct representations and explanations (Borko & Putnam, 1996; Ma, 1999; McDiarmid et al, 1989). Although teacher training and professional practices have important effects on teachers' profes-

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sional development, experience alone is not sufficient to improve both CK and PCK (Kleickman et al, 2013). This means that pre-service teacher training is of great significance in the process of acquisition of MPCK. Therefore, preservice teachers should know the nature of MPCK to be ready for the early years of their teaching careers and should practice it in the mathematics teaching and applied courses.

There are two well-known research projects that aimed to examine pre-service teachers' MCK and MPCK. For example in the COACTIV project, a quantitative research was carried out to determine the professional competence of secondary school mathematics teachers in terms of MCK and MPCK (Krauss et al., 2008). As a result, deep connection between both knowledge categories was found. The second project, TEDS-M conducted as a wide-scale study in 2008 aiming to determine the degree of MCK and MPCK of pre-service teachers. The study provided a medium to compare the state of MCK and MPCK in various countries (Blömeke & Delaney, 2012) and showed that those who have sufficient MCK have also enough MPCK. MCK and MPCK are strongly correlated in most countries that took part in TEDS-M.

Both projects provided quantitative data indicating that MCK affects MPCK. Recently, researchers in the field have recommended carrying out qualitative studies to investigate undergraduate courses and field experiences of pre-service teachers in terms of MCK and MPCK (Schmidt et al, 2011; Youngs & Qian, 2013). Ding, He, Shing and Leung (2014) investigated the relationship between pre-service mathematics teachers' MCK and MPCK on teaching the topic of ratio. They used both video-based interviews and task-based interviews to elaborate six pre-service mathematics teachers' conceptual understanding of ratio and their MPCK on teaching the topic. They found that pre-service teachers who possessed multiple understanding of the concept tend to be more flexible when choosing different representations. On the other hand pre-service who had an unstable and inconsistent understanding of ratio appeared to teach a more procedural way of simplifying ratios. This result illustrated that insufficient MCK might be one factor for undeveloped MPCK. This means that limited MCK leads to immature MPCK.

Finally, above mentioned quantitative studies quantitatively proved the connection between MCK and MPCK. However these studies could not show us how both kind of knowledge affect each other in teaching process. In our study we conducted a study based on a qualitative method to examine the connection between MCK and MPCK. Our research may contribute to the field as it offers data regarding how the MCK of pre-service teachers affects MPCK's subcomponents. For instance, how pre-service teachers' MCK affect their lesson organization and presentation performance within real classroom situations. Carrying out qualitative research in real classroom environments that require teaching practices may provide more comprehensive data about the settings where both MPCK and MCK are utilized at the same time. We worked with pre-service primary teachers throughout their teaching practice and qualitatively analyzed that how their MCK affect their MPCK knowledge. Hence, our research question was: How might pre-service teacher' MCK affect their MPCK?

Theoretical Framework

Shulman (1986) initiated the studies in this field by categorizing the knowledge teachers possess as content knowledge, pedagogical content knowledge and curricular knowledge. According to Shulman, knowing a subject requires more than knowing its phenomena and concepts. Teachers need to understand why something is so and on what foundations it is built; more than whether it is so or not. MCK includes mathematical definitions, concepts, algorithms and procedures.

PCK includes, among others, an understanding of how a student learns a subject, as well as how to design and manage the learning process. Thus, the teacher must be aware of the most useful presentation techniques, the strongest analogies and demonstrations, examples, explanations, and how best to present and formulize the subject (Ball et al., 2008; Magnussan et al, 1999; Shulman, 1986; Wood & Geddis, 1997). Researchers have recognized a variety of components in PCK framework inaugurated by Shulman (1986) and the common components they all agree on are the knowledge of students, instructional strategies and representation.

Knowledge of students refers to matters such as students' prior knowledge of specific topics, students' misconception, and students' difficulties. Teachers should be aware of students' prior knowledge, should understand what topics the students might have difficulty understanding, and should design the learning process accordingly (An et al., 2004; Shulman, 1986; Magnusson et al., 1999; Park & Oliver, 2008). Instructional strategies and representation should be treated in detail. Ball et al. (2008) explains this issue as to what example the teacher should use while starting the lesson, what example would help the students understand the topic better, and how the teacher would be able to notice the useful and useless aspects of the materials used in the teaching process. Marks (1990) defines this type of knowledge as the teaching process. He also emphasizes determining the teaching activities, asking questions to the students, getting feedback from them and determining teachings instructions. Referring to Shulman (1986), the following facets were distinguished in TEDS-M: MCK, MPCK, including curricular knowledge, knowledge of lesson planning and interactive knowledge applied to teaching situations and general pedagogical knowledge (Blömeke & Delaney, 2012). In brief, we can see PCK as the convenient integration of CK and PCK in the phases of lesson comprehension and enactment. In our study, we considered the subcomponents of the MPCK knowledge under the following three subtitles: knowledge of students, organizing lesson and presenting the lesson.

Method

Context of the study and participants

The current study is designed as a case study in a qualitative research nature. The first researcher engaged in the process from the perspective of both teacher educator and researcher. This paper, which is part of a larger research project (Baki, 2012), was conducted with 12 pre-service primary teachers in Teaching Practice I–II courses. In the four year undergraduate program for pre-service primary teachers, after passing Teaching Mathematics I-II in the third year, pre-service teachers are sent to schools for teaching practice in real classroom environments as a requirement of the Teaching Practicum, undertaken in the final year of their program. Their participation in the current study was on an entirely voluntary basis, and they were informed about the nature and purpose of the study.

Data sources

Observation (video-recorded), interviews and field notes were used as data collecting tools. Each pre-service teacher was observed for 3-4 hours through two semesters (42 hours in total). During the observation, multifaceted field notes were taken; and the failures of teacher STs' enactment depending on their MPCK and their difficulties in reaching out to the students during teaching practices were noted. The candidates were interviewed about the instruction after each lesson. The interview was conducted in order for the teacher to evaluate his/her performance during the lesson and to find out the reasons of the instructional behaviors based on the notes of the observation.

Data analysis

The inductive and deductive methods (Patton, 2002) were used in this content analysis through the triangulation of video records with interviews and field notes. The three components of MPCK were determined theoretically; some other sub-components emerged as a result of the data analysis (Table 1). By recurrently viewed the classroom applications of the teacher candidates in light of the main themes; their behaviors, as well as the difficulties and situations that challenged their practices were determined. Based on this analysis, common situations were found and sub-components became certain. To get data for this study, in-class video-records were viewed several times, and the situations in which the content knowledge of teacher candidates affected the classroom applications were determined. At that point, the notes of in-class observation supported the video analysis.

Table 1. Subcomponents of MPCK

Components	Sub-components	**Codes**
Knowledge of Student	Student's prior knowledge	Beginning the lesson by repeating the previous lesson Starting the lesson with daily life examples? Questions? Questioning students' knowledge about the new subject and related subjects
	Learning difficulties of student	Identifying the points where students will have difficulty in learning
Organizing the lesson	Selection and sequencing of activities	Taking into consideration prior knowledge of student Taking into consideration the parts where students will experience difficulty Sticking to the guide in the selection of the activities Number of activities Ranking of activities in an appropriate order Choosing activities according to students' level
	Being familiar with the purpose of the activity, informing the students of the purpose of the activities, being able to group the activities in line with their purposes	Being familiar with the purpose of the activity Informing student of the purpose of the activities Being able to group the activities in line with their purposes
Presenting the lesson	Student-centered approach	Keeping students mentally active Make the students grasp the knowledge by experience Listening to students' explanations and answers and giving them feedback
	Instructional explanations	Making instructional explanations in place Making explanations to sum up the core points of the lesson Using different forms of representation

Findings

The relationship between MPCK and MCK will be elaborated according to the three components and their subcomponents given in Table 1.

Knowledge of Student

The fact that the mathematical knowledge of some teacher candidates were good did not suffice to effectively use the mathematics pedagogical content knowledge. In order to teach the objective *"Determines arithmetic average",* Kutlu started with asking the students to calculate the average length of the bars given on a graph (Figure 1).

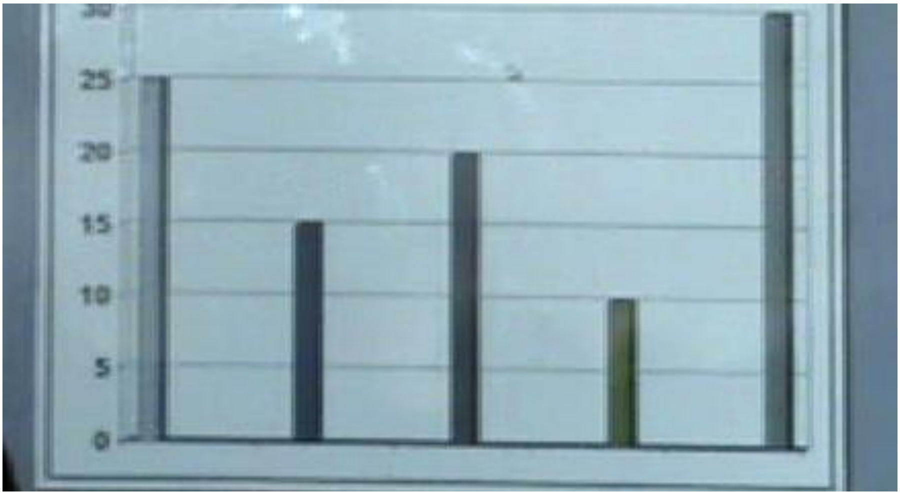

Figure 1. Graphic used in order to introduce arithmetic mean

As the question included arithmetic average of 5 variables, its level was too difficult for students who were exposed to the concept for the first time. This situation shows the teacher candidate's failure of determining the activities about the sub-components of the MPCK knowledge on student level.

The pre-service teacher conducted the lesson with questions about projection. The students completed the lesson without handling any concrete models, pencils or paper:

Kutlu: Children, we will learn a new subject with you today. Now, all of you look at the board. (Turned on the computer and projected a column graphic onthe board.) what do we see over there? ... Tell me.
Student: A graphic
Kutlu: It is a graphic. You have discussed the topic of graphics before. You have learnt it, right?
Students: Yes
Kutlu: You see 5 columns over there. Let them be our sticks. Any stick. They all have different heights, right?
Students: Yes
Kutlu: Alright, can we totalize the height of all of them?
Students: Yes
Kutlu: Now everyone calculate it and tell us.

Student: 100
Kutlu: The total amount is 100, right?
Students: Yes
Kutlu: We will do something with these sticks that we will totalize them again but the total amount will not change. The total height will not change; but the sticks will all have the same height. How can we do that?

Students equalized them all to twenty and then he asked the students to explain why. Without expecting answers from them Kutlu did all of the work himself.

Kutlu: We will equalize them all to twenty. How will we do it? As you see here, this is the longest one, right? (The teacher candidate showed the parts about which he asked questions on the column graphic he had projected on the board.)
Kutlu: There is one even shorter here. Shall we trim the longer one and add it to the shorter one, right?
Students: Yes

Kutlu equalized the height of sticks and made the following explanation to the students:

Kutlu: The total height has not changed and they are all equal now. So what have we done to the sticks now? We have calculated their average height, right? What do we call this operation? We call it arithmetic average. We have calculated the arithmetic average of the sticks.

Kutlu's explanations implied that he had sufficient knowledge of arithmetic average. His CK had appeared to helped him to prepare a good PowerPoint presentation, but the fact that his CK was not sufficient for him to carry out the requirements of MPCK. The students participated in the lesson as if they were an audience. They only gave short answers to the questions and approved what the teacher did and said.

Organizing the lesson

One of the components of MPCK is *organizing the lesson* of which a subcomponent is *being able to group the activities in line with their purposes.* CK affects the finalizing process of the activities. The result got at the end of the activities is not enough and the required mathematical knowledge was not presented. Rana was not able to interpret the student's answer and the mathematical result attained as a result of the activity remained superficial.

While teaching the learning objective *"Forms different structures with the same* *number of unit cubes"* Rana behave as follows: she grouped the students and gave each group six cubes, asking them to build different structures. She then asked the students to draw a conclusion from the activity:

> *Rana: Yes, we formed shapes with unit cubes. What else did we do? Let's conclude something from our game. You, what do you think?*
> *Student: They all have the same volume.*
> *Rana: Why do they have the same volume?*
> *Student: Because there are 6 cubes in each shape.*
> *Rana: Well done, so we can say that we can form different shapes with our 6 unit cubes, can't we? This was the purpose of our game.*

Then she asked the students to draw a conclusion from the activity. One of the students said: "These structures have the same volume because they have the same unit cube number." Rana gave her own explanation without interpreting the student's answer. She said "We can form different structures with 6 unit cubes" without mention the concept of volume. Her unfamiliarity with the concept of volume conservation affected the process of finalizing the activity and the process of evaluating the student's answer. As the teacher candidate's CK was insufficient she could not approve the answer given by the student though the student gave the right answer. Even though the student knew the concept of volume, the teacher candidate ended the activity without mentioning it. The teacher candidate had not determined such an objective while preparing the lesson and she pointed it out as followed during the interview:

> *Researcher: What was your aim here in the activity of forming different structures with the same number of unit cubes? How did you finalize it?*
> *Rana: What did you understand? What did we do?*
> *Researcher: Hı hı*
> *Rana: One student said: "Teacher, the volume did not change." And I asked why it did not change.*
> *Researcher: Hmm.*
> *Rana: He said: "Teacher, we used 6 unit cubes all the time."*
> *Researcher: Did you not guess that they would have that result? I mean, what did you think while drawing a conclusion?*
> *Rana: I had written my lesson plan. It kind of included a comment that we could form different structures with 6 unit cubes.*

As is seen, the objective of Rana's lesson plan was just to form different structures with 6 unit cubes. She never thought about the mathematical result that there could be different structures with the same volume. The student's response about volume conservation shocked her and she was not able to understand it. The fact that the CK of some teacher candidates were good did not suffice to effectively use the MPCK.

Presenting the lesson

One of the components of MPCK is presenting the lesson of which a subcomponent is *student centered approach*. Even though teacher candidates tried to apply the student-centered approach properly, the lack of their CK negatively affected their class application. In such situations, they had difficulty answering the students' questions, making explanations to them or giving them feedback. Although the students' answers were correct, some teacher candidates (for example Hülya) said they were wrong.

Hülya tried to fulfill the categories related to the requirements of the student-centered approach of MPCK. She made an effort to include each student in the thinking process, to listen to students' explanations, to provide constructive feedback to the students when necessary, and to clarify when somebody gave an incorrect answer. However, the deficiency in her mathematical knowledge gave her some difficulty in the classroom. For example, she asked the students to find the height of the parallelogram in Figure 2.

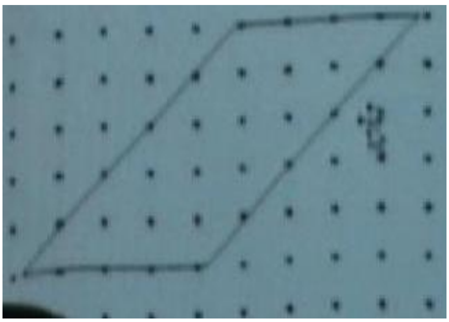

Figure 2. Parallelogram drawed by Hülya

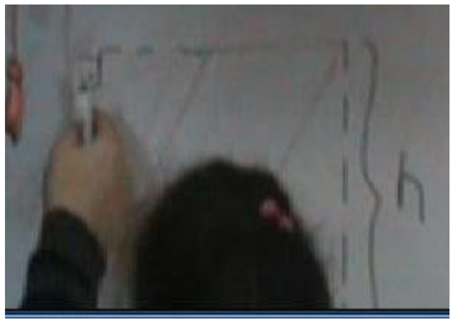

Figure 3. Student-teacher discussion

She realized that the students had difficulty in calculating the height of a parallelogram. And therefore started to discuss where the height was with the students about. Although one student drew the height correctly, Hülya evaluated the student's answer as incorrect(Figure3).

Hülya: Where do you think the height is?
Student: We can draw a height in the parallelogram. (Then student showed a height by drawing a line from the upper base to the lower base)(Hülya did not accept this height)
Hülya: Yes, it passes across the parallelogram, but then you go out of it. What you said is also possible, but which base does your height belong to?
Student: The upper base.

Hülya: Look, this goes to this base. So what? There will be a tangent line joining the lower and upper bases. Then the height you drew is the height of which base? The upper base.

Hülya said that the student drew a height belonging to another base. However, both heights belonged to the same base. As seen from the example, Hülya's lack of knowledge about the concept of height caused her to evaluate the student's response incorrectly.

The lack of CK of the teacher candidates put them in a difficult situation in interpreting students' answers and giving them feedback. For example Zuhal gave an activity designed to make students determine an arithmetic average. In the activity, she expected the students to calculate the arithmetic average of 8, 8, 4, and 4 by using unit cubes (Figure 4). The purpose of the activity was explained to students as how we can equalize the length of these towers? Through this question she tried to put her students into thinking process and ask them to apply their ideas. One student offered a solution which Zuhal could not understand (Figure 5). The following dialog is an example of such situation.

Figure 4. Activity used for arithmetic mean **Figure 5.** Student's suggestion

Student: Teacher! We need to join two of the cubes, I guess, to do this.
Zuhal: Then three of them will be equal. Yes, it is possible.
Student: We join these two (showing the cubes).
Zuhal : Come and equalize them all and show us. (The student puts one of the cubes onto another one).
Zuhal: They are equal now. Yes, ladies and gentlemen, what is our aver age? Then let's do it another way.

The student tried to equalize the number of unit cubes but the result was not the required arithmetic average. The student equalized the number of unit cubes by devising his/her own strategy and got 8 instead of 6. The student reduced the number to 3, but s/he was expected to find the average without reducing the number of cubes. Since Zuhal's level of knowledge was not sufficient to evaluate the student's answer, she approved it. Although she tried to improve the instruction in terms of the related component of MPCK by helping the students to understand the meaning of arithmetic average by using concrete models, as well as keeping them cognitively active by including them in the thinking process, she was unable to interpret and expand the student's response. She directed the students to use another approach when she could not master the situation.

Even though CK of some teacher-candidates were good, the instructional explanations were not comprehensible at all. For example Aydın's explanations were about the objective of *"Performing subtraction with numbers with 5 digits maximum."* The students could do subtraction operations with 3 and 4 digit numbers. It was quite obvious from Aydın's explanations that his knowledge of how to teach the algorithm for the subtraction operation was at the conceptual level. This situation was exhibited with a sample dialog in the classroom.

Aydın started to explain how to process the algorithm of the subtraction operation with the numbers 346 and 256 (Figure 6). He also tried to support his explanations with base ten blocks.

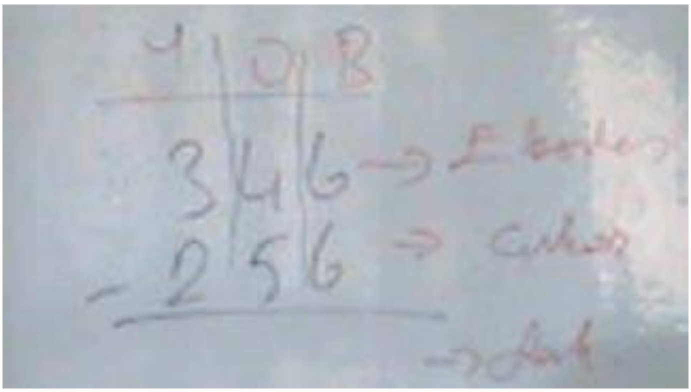

Figure 6. Aydın's explanation

Aydın: Suppose that we have 6 ones (he counted ones on the table)….
1-2-3-4-5-6, I have taken all these ones, what is left?
Student: Zero
Aydın: There is nothing left, right? So I am writing zero here.
Student: 5 can't be subtracted from 4.
Aydın: Not 4. Can we subtract 4 tens from 5 tens?
Student: We need to trade hundreds.
Aydın: How do we trade hundreds?
Hande: We take one from 3 hundreds.
Aydın: So, how many tens in one hundred?
Ayşe: 10
Aydın: There are 10. Can you count this? How many tens in one hudred?

(He gave one hundred to the student and asked her to count)
Student: 1-2-3-4-5-6-7-8-9-10
Aydın: OK, you count how many tens are here.
Student: 1-2-3-4-5-6-7-8-9-10

Aydın continued asking how many tens there are in one hundred, although the students had already given the correct answer. Repeating the process with too many students lessened the effect of the explanation.

Aydın: There are 14 tens. 1-2-3-4-5-6-7-8-9-10-11-12-13-14. (Counted out 14 of tens). Now, he wants me to remove 5 of the 14 tens, reduce 5, and put 5 of them somewhere else. 1-2-3-4-5. Batuhan, could you please count them and tell how many of them are here.
Batuhan: 1-2-3-4-5-6-7-8-9
Aydın: OK, you may sit down. We subtracted 5 tens from 14 tens. How many are left, Fatma? How many? I've taken 5 tens from 14.

Aydın kept up the explanation by asking the same question to different students. Even though he tried to get students to understand the role of bases in subtraction, it was not an efficient lesson for the students. His asking questions whose answers the students already knew during the instructional explanations and his making the same explanations repeatedly bored the students and made it hard for them to follow the lesson. Also, his carrying out some activities that were well below the knowledge level of the students deterred the students from gaining new objectives.

Discussion/Conclusion

In the related literature, studies tried to quantitatively identify the levels of pre-service teachers' MCK and MPCK, and the relationships between MCK and MPCK (Blömeke & Delaney, 2012; Krauss et al, 2008). On the other hand, this paper tries to qualitatively illuminate how MCK predicts the learning and teaching process while carrying out the requirements of MPCK. This study also examined the interrelationships of MCK and MPCK under the three themes with their sub components.

The inadequacy of teacher candidate's MCK might negatively affect her/his MPCK application in the classroom. For example, some teacher candidates' applications and explanations by taking into account factors such as choosing a good number of sufficient activities, actively participating students to the class as the center of the lesson, and paying attention to the prior knowledge of students were at a suitable level. However, their lack of mathematical content knowledge put them in a difficult situation in their interaction with students. The weaknesses in their MCK were apparent in their evaluation and interpretation of the student's responses and explanations, as well as in giving feedback to these comments. In some cases, the teacher candidates evaluated correct answers as incorrect; in others, they assumed the answers given by the students were correct because they did not understand their answers clearly enough. In still other cases, the pre-service teachers could not interpret the students' responses from a mathematical point of view. Similarly, several studies showed that the level of the content knowledge of teacher candidates put them in a difficult situation while they interpret students' opinions (Capraro et al, 2005; Halim & Meraah, 2002). The pre-service primary teachers found themselves in risky situations more frequently, because they complied with the requirements of the student-centered approach and tried to keep the students active; however, their MCK was not deep enough to expand on the discussions with the students. The lack of the teacher candidates' MCK affected the finalizing process of the activities. The mathematical result attained at the end of activities was too simple for the students. The activity was finalized without attaining the required mathematical knowledge.

On the other hand, in cases where pre-service teachers' MCK was good, but their MPCK was insufficient, the lessons were also ineffective. In these instances, the pre-service teachers carried out activities that were not appropriate for the level of students, had difficulties in reducing the level of instruction to the level of the students, and used some knowledge which had not been mastered by the students. Although pre-service primary teachers' MCK was good, the lack of their MPCK knowledge caused the lessons to be inefficient for the students. Choi, Ahn, and Kennedy (2008) likewise stated that MCK alone is not sufficient for teachers to improve student understanding, although teachers' MCK is necessary for the high level cognitive development of students.

Pre-service teachers' MCK affected their instructional explanations. Even though their MCK was on conceptual level, their instructional explanations for the students were not suitable. While pre-service primary teachers made their instructional explanations, they repeated similar explanations several times, they were not able to connect with different kinds of representations, or they were not able to use mathematical terminology effectively many times. Such situations reduced the efficiency of the lesson for the students and distracted their attention. Hence, pre-service teachers had difficulty in class management. All of the pre-service teachers had difficulties in using mathematical language and terminology while they were giving instructional explanations. Researchers indicated that teachers and especially pre-service teachers have difficulties in their instructional explanation. (Kinach, 2002, Thanheiser, 2009).This situation is quite understandable, since the pre-service teacher had not effectively used mathematical terminology before s/he began courses in teacher education should be revised and improved so as to support pre-service teachers in terms of giving instructional explanations.

As some studies on experienced teachers show (Van Driel & Berry, 2010), although CK forms the basis of MPCK, it is not sufficient for its development. Therefore in the process of teacher education, problems concerning CK should already be resolved before pre-service teachers are exposed to teaching practice. In fact, during instruction teachers should focus on making knowledge meaningful for students and MPCK should become more active and significant. It can be said that the failures observed in pre-service primary teachers reflect the situations that emerge in teacher training. Those situations can get better through practice. According to Van Driel and Berry (2010), giving pre-service teachers an opportunity for self-reflection on CK and in-class applications can facilitate the development of their PCK. Hence, pre-service teachers need more feedback during the courses of teaching practices where they use MPCK effectively, and where theoretical knowledge is combined with practice.

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