International Journal of Instruction
e-ISSN: 1308-1470 ● www.e-iji.net

April 2019 ● Vol.12, No.2
p-ISSN: 1694-609X
pp. 399-416

Received: 15/10/2018
Revision: 12/01/2019
Accepted: 23/01/2019
OnlineFirst:14/03/2019

Common Errors in Fractions and the Thinking Strategies That Accompany Them

Mohammad Ahmad Alkhateeb
Asst. Prof., Faculty of Educational Sciences, Hashemite University, Jordan, *mkm7879@hu.edu.jo*

This study aimed to reveal the common errors in fractions, the associated thinking strategies among 5th graders, and the persistence of these errors. A quantitative method was applied in this study through calculating the percentages of every type of error the students made in the diagnostic test, The qualitative part was performed through individual interviews and analyzing them to probe the thinking strategies used by the students that lie behind their common errors, The test was conducted for 240 students and 30 students were interviewed. The results showed a variety of common mistakes the students made, and that the rates of these mistakes were as follows: the highest was due to dealing with the fractions as integers, followed by errors about the basic concepts of the fractions. The results further showed a diversification in the students' thinking and the associated mistakes. One of the most noticeable mistakes was expressing the fraction without attention to the equal parts. The rates of other common errors were as follows: considering that the fractional number is always higher than the figure A/B, and that figure A/B is always less than one; treating the fractions as integers; misinterpretation of the relation between the numerator and the denominator with the actual value of the fraction, and ignoring the integer in the fractional number. The results also showed that more than 50% of the students made errors associated with solution strategies in the fractions issue.

Keywords: common errors, fractions, thinking strategies, thinking persistence, 5th grade

INTRODUCTION

The aim of learning mathematics is to build students' knowledge of mathematical concepts. Teachers need to assess how well students are accommodating to a topic. Understanding students' mistakes when solving mathematical problems gives teachers a sense of why and how these errors and solutions may affect them in the future.

Fractions are one of the richest and most complex subjects in mathematics teaching. The student begins to learn fractions from the first year of elementary school and continues

Citation: Alkhateeb, M. A. (2019). Common Errors in Fractions and the Thinking Strategies That Accompany Them. *International Journal of Instruction*, 12(2), 399-416. https://doi.org/10.29333/iji.2019.12226a

to the upper grade of high school (Jigyel & Afamasaga-Fuata'i, 2007). In addition, it is established that fractions have an important place in teaching advanced mathematical subjects such as algebra (Newton, 2008).

Fractions, as revealed by some studies, are considered as a challenging topic for students (e.g. Brown & Quinn, 2006; Bottge, Ma, Gassaway, Butler, Toland, 2014; Charalambous & Pitta-Pantazi, 2007; Maelasari & Jupri, 2017; Soylu, 2008). Several studies suggested some patterns of errors in fractions, such as interpretations of fractions, comparison of fractions, and adding and subtracting fractions. Students tend to use rule-based procedures to solve fraction problems, without understanding the problems (Pesen 2008).

Thus, the problem of the study is concentrated on trying to identify the common mistakes, and the recurrent patterns common among 5th graders, when dealing with ordinary decimal fraction concepts. It also focuses on revealing the thinking strategies associated with their mistakes, as well as the persistence of these strategies.

Since identifying the mistakes that basic stage students encounter while performing their arithmetic operations on fractions, revealing their reasons and finding the proper solutions will contribute to the students' development in maths learning (Devika , 2016; Kocaoglu, 2010) , the significance of this study is apparent in filling the gap of the need for a theory in maths teaching. It also deals with a vitally important issue, which has an important role in improving the students' learning. It is well established that the fractions topic is one of the most important within the maths curriculum. It is learnt in a hierarchical mode from the very beginning of the first grade to the end of the basic stage. Therefore, this study fulfils an urgent educational need, namely, exploring the mistakes that students make, especially throughout the basic education stages, when they start learning the principles and concepts of maths, their relations, and the arithmetical operations.

It is hoped that the findings highlighted in this article will help teachers to formulate teaching strategies and provide them with an idea of the types of errors and the reasons that lead to their occurrence. It is also expected that, with this information, teachers will be able to improve their instructional planning and pedagogical practices, so that students will acquire a deeper conceptual understanding of fractions. Furthermore, the instruments used would be beneficial to teachers in diagnosing problems faced by their students in fractions, and planning remedial work for them. Subsequently, the significance of this study stems from highlighting the difficulties that students face in learning the fractions topic, and providing a deep diagnosis of the weaknesses of these students. Accordingly, this study definitively answered the following questions:

- What are the common errors and types of their recurrence among fifth graders in fractions?
- What are the thinking strategies behind the common errors in fractions?

What is the extent of the fifth graders' adherence to the thinking strategies that lie behind the common errors in fractions?

METHOD

1- A quantitative method was applied in this study through calculating the percentages of every type of error the students made in the diagnostic test. (Mcmillan & Schumacher, 2010)

2- The qualitative part was performed through individual interviews and analyzing them to probe the thinking strategies used by the students that lie behind their common errors, the operations on them, and to explain their persistence with them. (Mcmillan & Schumacher, 2010; Yildirim & Simsek, 2006)

Population and Sample

The study sample was chosen by simple random selection from public schools that include a 5th grade in the City of Zarqa (Jordan) (n=20 schools), which has a total of 600 5th graders. Forty percent of these students were chosen as the study sample (n=240).

Pupils in the sample were taught the topic on fractions about three weeks before they sat the diagnostic test. In addition, they had been introduced to the initial concepts of fractions when they were in the 3rd primary grade, and were taught more operations on fractions when they were in the 4th grade. No revision was carried out prior to the diagnostic test, and the pupils were not informed in advance that they were to be given the test. The pupils worked individually while doing the test.

The Jordanian mathematics curriculum is currently in alignment with the National Council of Teachers of Mathematics (NTCM, 2000) standards, where students in grades 3–5 continue to refine their understanding of arithmetic operations on fractions and develop algorithms to compute the fractions.

Instruments:

The study relied on two instruments for data collection: the diagnostic test and individual interviews.

First: The Diagnostic Test

The test aims to reveal the students' degree of understanding of the basic concepts of fractions. It consists of 28 items of the word and short-answer type. It includes concepts and skills of reading regular and decimal fractions, comparing two fractions or two decimal numbers, comparing two regular numbers, writing a non-real fraction in the form of a decimal number, writing a decimal number in the form of a non-real fraction, writing a regular fraction in the form of a decimal fraction, and adding and subtracting regular and decimal fractions.

Test Construction

The researcher reviewed the previous studies conducted in this area, in terms of common errors in regular and decimal fractions and operations on them (Brown & Quinn, 2006; Bottge, Ma, Gassaway, Butler, Toland, 2014; Charalambous & Pitta-

Pantazi, 2007; Devika, 2016; Cramer, Wyberg & Leavitt, 2008; Maelasari & Jupri, 2017; Soylu, 2008). These errors were detected, classified and organized into basic ideas. The questions in every group were chosen to fit the 5th grade; other questions were added to certain errors through the experience of the researcher in this issue, and through suggestions provided by an experienced maths teacher. Namely, we carried out the following:

Monitoring the errors made by students concerning the concepts of fractions and the operations on them. We achieved this by consulting the previous studies that addressed this topic, and listing and arranging these errors within the related arithmetic concepts and operations.

Preparing a list of the common mistakes expected based on previous studies, the experience of the researcher and suggestions from teachers.

Paraphrasing the test items, taking into account that the items should represent the list of common errors, which was prepared in advance, and cover all the aspects of the topic of regular and decimal numbers.

Test Validity and Reliability

The test was introduced to a number of arbitrators to verify the content validity, and the amendments they suggested were made. The test was applied on an exploratory sample from outside the main sample (n=25 students) to identify vague and difficult phrases and paraphrase them if required. The time allocated for the test was (45) minutes, and the test reliability was assured through the test-retest and calculating the Pearson coefficient, which was (0.93). The test was applied in the middle of the academic year 2017/2018.

Second: The Individual Interviews

The interview aimed to identify the thinking strategies that 5th graders use that lead them to make mistakes when dealing with fractions, and how far they keep hold of these strategies. The interview assists in detecting the nature of the errors the students make through the students' own explanation of the way they reach the solution. It also helps to track incorrect answers and know how persistent they are by postering similar questions, and whether this mistake was random or based on incorrect persuasions and rules, or on the incorrect application of correct rules. This is because a failure in a certain part of the test does not give a clear image of the arithmetic operation the student used in seeking the solution. Furthermore, it is difficult for the student to think how to find the error without an individual interview.

Interview Procedure Steps

In the light of the results obtained following application of the test on the sample of participants, the students were categorized according to the number and type of mistakes each of them made, and then the researcher chose the students with more diverse mistakes, and interviewed (30) students. Arrangements were made to interview the students individually, where every student was asked about the questions where he had made mistakes -***which are the common errors***- and was given the opportunity to answer them again. The answers were recorded audibly. A form was prepared for this interview, which includes the questions the students were asked. For instance:

How did you solve this problem (the student is asked a question in which he answered wrongly in the diagnostic test)?

If you were attempting to explain this question to your colleague, how would you do that?

Explain how you came to this answer.

(The student is given a question similar to the former). Solve this question and explain out loud how you get the solution.

The results of these interviews were the basis for identifying the strategies that students use when performing arithmetic operations with fractions, and how persistent they are with them, through the collection, analysis and assortment of the interview results.

The interview card content validity was verified by presenting it to a pool of arbitrators, and making the amendments they proposed. In addition, the reality of the interview card was verified by applying it to five students from among the exploratory sample of participants, who answered many of the items incorrectly. We aimed to determine how relevant it is to reveal the errors in the fractions and the operations on them, and to define the length of the individual interview and the student's ability to understand the questions included. One week later, the students were re-interviewed by the maths teacher in the school where the test on the explanatory sample was performed, using the same questions and the same method. The results of both the first and second interviews were posted, and were compared to reveal the degree of persistence of the answers of each student. The two analyses were to a large extent identical (92%). Finally, the tests were corrected and the students were sorted according to the nature and number of common errors they made.

The researcher obtained the agreement of the school principals to conduct the individual interviews with students, and the interviews each lasted half an hour.

Data Processing

The researcher corrected the diagnostic test for every participant in the sample, posted the answers to every question, and calculated the percentages and frequencies of the errors the students made in the test items. Errors that students made at a rate of more than 15% of the total answers were adopted as common errors. A list was prepared to describe the errors based on the students' results in the diagnostic test, and the percentages of students who committed these errors were listed.

The students who were designated with errors in the fractions were interviewed, and the interviewer listened to them to describe the thinking they followed to get the solution. For this purpose, questions were asked to uncover the strategies the students used to answer the test questions wrongly. The percentages of every wrong strategy the students used were obtained. The quality of the students' answers was revealed, whether it was random or as a result of a guess, and how far the students strongly held or persisted in using these strategies, by comparing the solution strategies accompanying the common errors made by the students in the diagnostic test and in the interview. The researcher further calculated separately the percentage of the persistence of the sample participants in using solution strategies that resulted in the common errors.

FINDINGS

Question 1: What are the common errors, and what are their frequency patterns with the fifth graders in fractions?

The results showed various common mistakes made by the students, and these mistakes fell into four groups. The researcher found that the highest percentage of mistakes resulted from dealing with the fractions as integers (52.17%). Errors about the basic concepts of fractions accounted for (46.65%), followed by errors from confusion between the concepts of fractions and the operations on them (36.5%), and finally, errors resulting from applying the algorithms (33.55%). To identify the common errors the students made, their errors were collected and sorted within categories of mistakes. These errors were then described under main headings, with an illustrative example of each error. Finally, the researcher obtained the percentages of all the types of errors the students made, as shown in Table 1.

Table 1
Percentages of all the types of error the students made

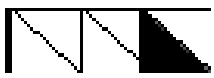
Error Description	Example	Error (%)
1- Errors resulting from dealing with the fractions as integers		52.17
*Writing regular and decimal fractions as two separate integers.	2.14 read as 14 divided by 2	5.35
* Comparing two decimal fractions based on the number of the decimal digits the fraction includes.	The greater of the two numbers 0.7 or 0.52 is 0.52	41.8
* Dealing with adding and subtracting decimal fractions as if adding and subtracting integers.	0.2 + 0.12= 0.14	72.31
* Dealing with adding two fractions or a fraction and fractional number, or fraction and integer, by adding the two numerators as a numerator of the fraction, and adding the two denominators as a denominator of the fractions.	$\frac{2}{3}+\frac{1}{3}=\frac{2}{3}$ $2+\frac{2}{3}=\frac{4}{3}$	44.2
2- Errors about the basic concepts of the fractions		46.65
* Writing the fraction that represents the shaded part as a part of another part, not a part of the whole.	The shaded part in Figure represents $\frac{1}{2}$ 	52.5
* Writing the fraction that denotes the shaded part of a given figure without attention to the equal parts inside the	The figure that represents is	40.8

figure.	$\frac{1}{4}$ the first and third 	
3- Errors resulting from confusion between the fractions' denominators and the operations on them.		36.5
* Obtaining a fraction equivalent to another by inverting the fraction.	$\frac{3}{4} = \frac{4}{3}$	28.7
* Converting an improper fraction into a fractional number by making the numerator an integer, and the denominator the place of decimals.	$\frac{6}{4} = 6.4$	44.3
4- Errors in conducting the algorithms		33.5
* Writing the fractional number in the form of an improper fraction, by adding the integer and the numerator of the fraction.	$4\frac{2}{5} = \frac{4+2}{5} = \frac{6}{5}$	23.5
* Equalizing the denominators by multiplying one of them by a number without changing the numerator.	$\frac{1}{3} + \frac{3}{12} = \frac{1}{4 \times 3} + \frac{3}{12} = \frac{4}{12}$	22.8
* Adding two fractions by equalizing the denominators and finding the common denominator instead of the common multiplier.	$5\frac{3}{8} + 2\frac{2}{4} = 7\frac{5}{4}$	19.7
*Converting a regular number into a decimal fraction by making the denominator an integer and the numerator the decimal part.	$\frac{5}{9} = 9.5$	42.8
*Misplacing the decimal point when carrying out the operations on it.	0.5 x 1.25 = 6.25	24.2
*Considering the regular fraction greater because its denominator is greater.	$\frac{1}{3} > \frac{1}{2}$ (Because 3 is greater than 2)	44.3
* Comparing an integer with the numerator of the fraction or its denominator	The greater of the numbers $\frac{4}{5}$ *or* 3 *is* $\frac{4}{5}$	49.5
* Comparing the two fractions while ignoring the integer.	$\frac{5}{3} > 2\frac{2}{3}$ because $\frac{5}{3} > \frac{2}{3}$	41.6

Question 2: What are the thinking strategies that accompany the errors in the fractions?

This question was answered through the individual interviews with 30 students who made about 21 wrong answers. The answers were collected, analyzed and sorted. The researcher identified the most important ways in which the students deal with fractions and the operations on them; then the percentage of every strategy was calculated based on the number of students who used the strategy.

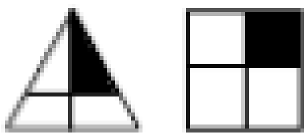
The results showed a variety of ways in which the students think, leading them to make mistakes. There were about nine ways that differed with the concept and the arithmetic operation the student performed. The most prominent strategies were as follows:

- Expressing the fraction without attention to the equal parts (92%);
- Considering the fractional number as always greater than the figure A/B, and that the figure A/B is less than integer one (76%);
- Treating the fractions as integers (73%);
- Misinterpretation of the relation of the numerator and denominator to the actual value of the fraction (60%).
- Ignoring the integer in the fractional number (60%).
- Use of incorrect algorithms (50%);
- Failure to relate the part to the whole and to other parts of it (40%);
- Reciprocation between the integer and the decimal fraction (30%);
- Finally, confusion between the concepts of fractions and the operations on them (30%).

Descriptions are given below of these strategies together with important conclusions about the ways of thinking that cause students' mistakes in the concepts of fractions and the operations on them.

1- Expressing the fraction without attention to the equal parts.

Students believed that any geometrical shape could denote the fraction regardless of the division among its parts. The researcher, by interviewing the students, found that a large proportion of them use this strategy (92%). For instance, to express a regular fraction, most students answered that they could use any geometrical shape regardless of whether the parts were equally drawn or not. For example, they considered that the two shaded areas below denote one quarter of the shape.



2- Considering the fractional number as always greater than the figure A/B and that the figure A/B is less than the integer one.

During the interviews, (76%) of the students believed that the existence of the integer in the fractional number makes it greater than the fraction, regardless of the type of fraction; and that the fractional figure A/B is always less than integer one. When the students were asked which number is the greater in: $\frac{6}{3}$ *or* $1\frac{1}{2}$

most of them answered that $1\frac{1}{2}$ is greater. When they were asked about the reason, some answered that in $1\frac{1}{2}$ we have one integer (1), while $\frac{6}{3}$ is a fraction, and a fraction is "parts, not a complete thing". And when they were asked which is greater: 1 *or* $\frac{6}{3}$?, (35%) of them answered that 1 is greater than $\frac{6}{3}$. They explained that "the fraction is always less than the integer one; it (the fraction) comes from parts, while the integer is a complete one, like a "loaf" that comes complete without any missing parts.

3- Treating the fractions as integers

The students dealt with regular and decimal fractions as if they were independent integers (73%). This strategy was clear in many aspects as follows:

A) Treating the integer part and the decimal part as integers with a certain separator

When the students were asked to write the decimal fractions in words and perform addition and subtraction operations on them, they believed that the decimal point is a "mere punctuation mark" (like a comma, full point, etc.). In addition, they could work on them independently by using the arithmetic rules of the integer. For instance, (50%) of them said that the decimal number 3.12 is read as "three and twelve". When they were asked to find the result of adding 1.3 + 0.13, about (78%) of them added the integers 1 + 0 to each other, and the decimal parts to each other 3 + 13 = 16, then combined the two results to get the answer 1.16. Many of them explained this answer saying that the number in front of them is a number right of the decimal point and a number left of it. Therefore, they add the two numbers on the left to each other and those on the right to each other, and then put them together.

B) Treating the numerator and denominator of the regular fraction as independent numbers

The interviews revealed that the students looked at the regular fraction as two numbers written one above the other. When dealing with regular fractions, about (70%) of the students regarded the fraction in that way. When they were asked how they added the two fractions

$\frac{1}{5}+\frac{2}{5}$, (84%) of them added the two numerators (1+2=3), then the two denominators (5+5=10), and the students saw the addition of the two numerators as the numerator of the result, and the addition of the two denominators as the denominator of the result (i.e. $\frac{3}{10}$). They explained that "in order to find the addition result, we see that the numbers 1 and 2 were written above (i.e. above the fraction sign), so we add them together. In the same way, the numbers 5 and 5 were written under the fraction sign, so we add them together, and then we write the "final" result". Furthermore, when they were asked about the fraction sign, about (54%) said that it "is nothing more than a hyphen (short line) (-) that comes between and separates two numbers".

C) Ignoring the decimal point

The results of the interview revealed that the students ignore the existence of the decimal point, and treat the decimal fractions as integers. When they compared two fractions, (65%) of the students believed that the fraction that contains more decimal parts is the greater in value. On the other hand, when they were presented with decimal fractions with the same number of decimal parts after the decimal point, most of them answered correctly. For instance, when the students were asked to compare 0.54 and 0.6, (50%) of the interviewed students answered that 0.54 is greater than 0.6, because the former has two digits and the latter has one.

D Ignoring the denominator of the integer number

When the students were asked to find the result of $2+\frac{3}{4}$, (50%) of them answered that "we first add 2+3=5, and then put the result in the form of $\frac{5}{4}$". When they explained this, they said that "if we look at the number 2, we find it without a denominator, so we add the two numbers (2 and 3) because they are on one "line", then we put the number 4 as it has no other number to add to".

4 Misinterpretation of the relation of the numerator and denominator with the actual value of the fraction

When the students were asked to convert the shape of the regular fraction $\frac{2}{10}$ to the shape of a decimal fraction, (60%) of them answered that the result is 10.2. Most of them explained that "the decimal fraction consists of the decimal point, a number right of the decimal point, and a number left of it. Therefore, we write the numerator beside the denominator and put the decimal point between them".

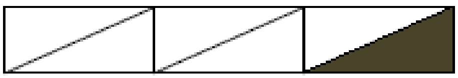
5 Ignoring the integer number in the fractional number

When comparing the regular fraction and the fractional number, (60%) of the students ignored the integer of the fractional number, and compared the two fractions with each other. When they were asked which is more, $\frac{5}{3}$ *or* $2\frac{2}{3}$, some answered that $\frac{5}{3}$ is greater, because the numerator of the first fraction is greater than that of the second (3 is more than 2). They explained that "the fraction is only comparable with a similar fraction, and the integer (1) is not a part of the number and is of no value; it is just a number written beside the fraction. We compare the two numbers that are like each other, i.e. numerator with numerator and integer with integer".

6 Use of incorrect algorithms

The researcher found that the students use incorrect steps to approach the answer without understanding these steps. When they were asked to convert the fractional number $1\frac{2}{5}$ into a fraction, (50%) added the numerator and the integer 1+2=3, and then put the result as a numerator in a fraction in which the denominator is 5, i.e. $\frac{3}{5}$. The students explained that "the 2 is related to the numerator. Therefore, to get a fraction, we have to add two numbers, because the fractional number results from dividing the numerator into the result of adding two numbers, so that one of them will be put as an integer and the other as a denominator of the fraction".

7 Failure to relate the part to the whole, but to other parts

When the students were asked about the fraction that denotes the shaded part within a given shape, they considered the shaded part as a part of the other part, not taking into account the fraction concept as a part of the whole. When they talked about the shaded part of the shape,  the researcher found that 12 of the 30 students expressed the fraction mistakenly and differently. Some said that the fraction that denotes the part is $\frac{1}{2}$, others said it is $\frac{1}{3}$, and four students said it is $\frac{1}{5}$. The students of the first group looked at the shaded part as a part of the first square, which is divided into two sections, the second group explained their answer as that the shaded part was a part of three triangles, not paying attention to the internal division. Finally, the third group said that the shaded part is one and the remaining five are not shaded, so the fraction is one fifth ($\frac{1}{5}$).

8 Reciprocation between the integer and the decimal fraction

The interviews revealed that (35%) of the students read the numbers in the same way that they read Arabic words (from right to left). They mistook the integer as a decimal fraction, and the fractional number as an integer. For instance, they read the number 3.17 as 17 and 0.3 (i.e. seventeen and zero point 3). Nine students explained this way of reading, saying "we read the number as we read the word in Arabic, from right to left".

9 Confusion between the concept of fractions and the operations on them

The results of the interviews showed that the students confused the fractional concepts by applying an equivalence to obtain a fractional number, or to make the denominators similar when they multiply two fractions such as $\frac{1}{3}\times\frac{2}{6}=\frac{2\times1}{2\times3}\times\frac{2}{6}=\frac{4}{6}$, or do a reciprocal multiplication to obtain the result of multiplying two fractions, such as: $\frac{1}{5}\times\frac{2}{3}=\frac{1\times3}{2\times5}$. Table 2 shows the percentages of the thinking strategies that cause the errors with regular and decimal fractions, and the operations on them.

Table 2
Percentages of error-associated thinking strategies in the concepts of fractions

No.	Strategy	Percentage
1	Expressing the fraction without attention to the equal parts.	92%
2	Considering the fractional number as always greater than the figure A/B and that the figure A/B is less than integer one.	76%
3	Treating the fractions as integers.	73%
4	Misinterpretation of the relation of the numerator and denominator with the actual value of the fraction.	60%
5	Ignoring the integer number in the fractional number.	60%
6	Use of incorrect algorithms.	50%
7	Failure to relate the part to the whole, but to other parts.	40%
8	Reciprocating between the integer and the decimal fraction.	30%
9	Confusion between the concepts of fractions and the operations on them.	30%

Question 3: What is the extent of the fifth graders' adherence to the thinking strategies that lie behind the common errors in fractions?

The answer to this question was approached in the interviews by asking the student a similar question to one who answered incorrectly in the test, and finding the extent of persistence in using the same solution strategy. The researcher analyzed the interview form of every student. The answers were sorted and compared with those given in the test. Finally, the percentages of the students' persistence in the solution in the test and that given in the interview were calculated. The results showed the following:

- Generally, (90%) of the students persist in the strategies causing the common errors by expressing the fraction without attention to equal parts; they still consider that the fractional number is higher than the figure $\frac{A}{B}$, and that the figure $\frac{A}{B}$ is less than the integer one;

- (75%) of them persist in the strategy of not relating the part to the whole, but to other parts;
- (65%) insist on using the fractions as integers.
- (60%) insist on ignoring the integer in the fractional number;
- (40%) misinterpret the relation of the numerator and denominator with the actual value of the fraction;
- (30%) persist in confusing the concepts of fractions and the operations on them; and,
- (20%) insist on using incorrect algorithms.

Table 3 illustrates the percentages of the 5th graders persisting in using the thinking strategies that lead to the common errors with fractions.

Table 3
Percentages of 5th graders persisting in using thinking strategies that lead to the common errors with fractions

No.	Strategy	Percentage
1	Expressing the fraction without attention to the equal parts.	90%
2	Considering that the fractional number is higher than the figure $\frac{A}{B}$, and that the figure $\frac{A}{B}$ is less than integer one.	90%
3	Treating the fractions as integers.	65%
4	Misinterpretation of the relation of the numerator and denominator with the actual value of the fraction.	40%
5	Ignoring the integer in the fractional number.	60%
6	Using incorrect algorithms.	20%
7	Not relating the part to the whole, but to other parts.	75%
8	Reciprocation between the integer and the decimal fraction.	70%
9	Confusion between the concepts of fractions and the operations on them.	30%

DISCUSSION AND CONCLUSION

The results showed the students' poor ability in the four arithmetic operations on both regular and decimal fractions. The students add or subtract two fractions in the same way that they add and subtract the integers. For instance, the students added 0.2 +0.14 as if they were "integer" two plus "integer" fourteen, then they placed the decimal, so the answer became 0.16. This could be interpreted as failing to perceive the place value of the numbers contained in the fractions. They believe that the decimal point is a separator between two numbers. Therefore, the student adds (or subtracts) the decimal fractions as if they were integers, then places the decimal point to the left side of the resulting number.

This type of error indicates that the students' perception of the fractions is unclear due to overlapping of their concepts (Idris & Narayanan, 2011). This may be attributed to the use of unsuitable teaching patterns to teach fractions, such as introducing them without attention to their meaning. This, in turn, may impede the learning process and eliminate the opportunity to shift to similar situations (Fuchs, Schumacher, Long, Namkung, Malone, Wang & Changas, 2016).

Most of the errors the students made in this study were procedural and conceptual. Some students did not know how to compare and add fractions. Thus, they applied any procedures they were familiar with. Those mistakes included inaccurate use of models, crossed-multiplication, and crossed-addition strategy.

Some studies suggested that errors occur as teachers focus more on algorithms than on understanding the underlying reasoning behind the concept (Idris & Narayanan, 2011; Isik & Kar, 2012). Thus, in future, teachers should put more emphasis on students' understanding and reasoning to avoid such errors. This study clearly revealed a large number of mistakes. Most of these results were consistent with other studies that address this issue, e.g., Devika (2016), Cramer, Wyberg & Leavitt (2008) and Okur & Cakmak-Gurel (2016). These studies found mistakes about the basic concepts of fractions, such as defining the value of the shaded part in a given figure. The student considered the shaded part as a fraction of the un-shaded part, not as a part of the whole. Other errors were the result of inaccurate application of the algorithms.

The interview results showed a variety of thinking strategies that lead to students' errors in the concepts of fractions and the operations on them. The strategy of dealing with fractions as integers is the most noticeable; students treat the integer part and the decimal part as if they were all integers with a certain separator between them, and treat the numerator and denominator of the regular fraction as two independent integers. The students focus on the number of the decimal places without attention to the place value of the numbers when they compare decimal fractions. This could be attributed to incorrect knowledge of the systems and of how to build each system and interlink them. This result is in agreement with the studies by Aksoy & Yazlik (2017) and Unlu & Ertekin (2012).

This study is also in line with that of Trivena, Ningsih & Jupri (2017) in terms of the confusion between the concepts of the fractions and the operations on them. In this respect, the students applied the equivalence concept to obtain a fractional number from an improper fraction. This could be because they did not understand the basic concepts of the fraction, or the method of dealing with and perceiving the relation between them.

As for the strategy of relating the part not to the whole but to other parts, the results of the interviews indicated that the students see the shaded part in relation to another part, not as a part of a whole. This may be due to the lack of understanding of the relation between the part and the whole, as found in the study by Biber, Tuna & Aktas (2013).

The current study uniquely found strategies that were not dealt with in previous studies. Most important of these strategies is misinterpreting the relation between the numerator and denominator and the actual value of the fraction. When the students convert the regular fraction into a fractional number, they write the denominator of the improper fraction $\frac{7}{4}$ in the form of the addition of two components of the number 7 = (2+5), and put one of them as an integer and the other as a numerator of the fraction, $2\frac{5}{4}$.

The results further showed that more than half of the interviewed students persist in the strategies that lead to their making mistakes in the concepts of the fractions and the operations on them, between the first and second attempts in the test and interview. The researcher noticed that the students repeated the same errors they made in the test in the interview. This is an indication that the students' answers are systematic, that they have persistent, stable principles and beliefs; and that these strategies are not haphazard. Rather, there is a depth in the cognitive structure of the student. This result was further supported by Piaget, who emphasized that the confusion the students display in learning concepts could be due to the contradiction between prior and new knowledge. If the new knowledge contradicts the prior and is not related to it, the learner attempts to store the idea in any way. When he attempts to retrieve it, partial and confused remembering occurs, which leads to errors. This result is in line with the studies by Lestiana, Rejeki & Setyawan (2016), Young-Loveridge, Taylor, Hàwera & Sharma (2007), and Schumacher & Malone (2017).

In this research, it was found that most students apply random operations merely for the sake of finding a result. Similar to this finding is the study conducted by Kocaoglu and Yenilmez (2010) with 5th grade students, which found that students have difficulty in understanding the problems.

One of the causes that lead students to make errors is the previous errors they have made when operating with fractions. According to Hallet, Nunes and Bryant (2010), and Aliustaoglu, Tunab, and Biberc (2018), misunderstandings not only prevent students learning, but also negatively affect their subsequent learning. As seen, this conclusion supports the obtained findings.

For this reason, teachers need to be aware of the mistakes students can make, consider these mistakes in their lessons, and plan their lessons in a way that prevents students from making the same errors. Teachers need to be informed about the errors the students have, through both in-service training and seminars. Similarly, it is vitally important to present these errors to pre-service teachers in field education lessons.

The findings of this study should be a matter of serious concern and should lead school authorities to investigate the administrative and teaching techniques, which may be the causes of the low competency of the students in fractions. The current instruction and illustrations of fractions in the textbooks should be further examined, because the findings of this study are similar to those of Aksoy and Yazlik (2017) and Devika

(2016), which indicate a high occurrence of systematic errors made by students. It is suggested that a longitudinal study be done to examine whether or not the systematic errors are persistent. Future research should also include an error analysis of low and average achievers among primary and secondary students following the use of learning materials in learning the fractions.

REFERENCES

Aksoy, N. & Yazlik, D. (2017). Student errors in fractions and possible causes of these errors. *Journal of Education and Training Studies, 5*,(11), 219-233. doi:10.11114/jets.v5i11.2679

Aliustaoğlua, F., Tunab, A. & Biberc, A. (2018). Misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education, 1*0(5), 591-599. doi:10.26822/iejee.2018541308

Biber, A., Tuna, A. & Aktas, O. (2013). Students' misconceptions of fractions and its effect on solving fractions problems. *Trakya University Journal of Education, 3*(2), 152-162.

Bottge, B., Ma, X., Gassaway, L., Butler, M. & Toland, M. (2014). Detecting and correcting fractions computation error patterns. *Exceptional Children, 80*(2), 237–255. doi.org/10.1177/001440291408000207

Brown, G. & Quinn, R. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher, 62*(4), 28-40.

Charalambous, C. & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*(3), 293-316. doi.org/10.1007/s10649-006-9036-2

Cramer, K., Wyberg, T. & Leavitt, S. (2008). The role of representations in fraction addition and subtraction. *Mathematics Teaching in the Middle School*, *13*(8), 490-496. Retrieved from http://www.jstor.org/stable/41182601

Devika, R. (2016). Handling fractions and errors in fractions. *I-Manager's Journal of Mathematics, 5*(3), 1-14. doi.org/10.26634/jmat.5.3.8223

Fuchs, L., Schumacher, R., Long, J., Namkung, J., Malone, A., Wang, A., Changas, P. (2016). Effects of intervention to improve at-risk fourth graders' performance on fraction understanding, calculations, and word problems. *The Elementary School Journal, 16*(4), 625–651. doi.org/10.1086/686303

Hallet, D., Nunes, T., Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology*, *102*(2), 395–406. doi.org/10.1037/a0017486

Idris, N. & Narayanan, L. (2011). Error patterns in addition and subtraction of fractions among form two students. *Journal of Mathematics Education*, *4(*2), 35-54.

Isik, C. & Kar, T. (2012). Analyzing problems posed by 7th grade students for addition operation with fractions. *Elementary Education Online, 11*(4), 1021-1035. doi: 10.17051/io.2014.13224.

Jigyel, K. & Afamasaga-Fuata'i, K. (2007). Students' conceptions of models of fractions and equivalence. *Australian Mathematics Teacher, 63*(4), 17-25.

Kocaoglu, Y. (2010). 5th grade students' mistakes and misconceptions in solving problems about fractions. *Dicle University Journal of Ziya Gökalp Education Faculty*, 14, 71-85.

Lestiana, H., Rejeki, S. & Setyawan, F. (2016). Identifying students' errors on fractions. *Journal of Research and Advances in Mathematics Education*, 1/2, 131-139.

Maelasari, E. & Jupri, A. (2017). Analysis of student errors on division of fractions. *Journal of Physics: Conf. Series* 812 (2017) 012033 doi:10.1088/1742-6596/812/1/012033

Mcmillan, H. & Schumacher, S. (2010). *Research in education*. Boston, USA: Pearson Education

Newton, K. (2008). An extensive analysis of pre-service elementary teachers' knowledge of fractions. *American Educational Research Journal*, *45*(4), 1080-1110. /doi.org/10.3102/0002831208320851

Okur, M. & Cakmak-Gurel, Z. (2016). 6th and 7th grade secondary school students' misconceptions about fractions. *Erzincan University Journal of Education Faculty*, *18*(2), 922-952.

Pesen, C. (2008). Students' learning difficulties and misconceptions in pointing the fractions on the number line. *Journal of Inonu University Faculty of Education*, *9*(15), 157-168.

Schumacher, R. & Malone, A. (2017). Error patterns with fraction calculations at fourth grade as a function of students' mathematics achievement status. *The Elementary School Journal, 118*(1), 105-127. doi: 10.1086/692914

Soylu, Y. (2008). Students' errors and misunderstanding at fraction and primary school teachers candidates' prediction skills of error and misunderstanding. *Contemporary Education Journal*, 33(356), 31-39.

Trivena, V., Ningsih, A. & Jupri, A. (2017). Misconception on addition and subtraction of fraction at primary school students in fifth-grade. *Journal of Physics: Conf. Series* 895/2017 012139. doi:10.1088/1742-6596/895/1/012139

Unlu, M. & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions? *Procedia-Social and Behavioral Sciences*. 46, 490-494. doi.org/10.1016/j.sbspro.2012.05.148.

Yildirim, A., & Simsek, H. (2006). *Qualitative research methods in the social sciences*. Seckin Publishing House, Ankara.

Young-Loveridge, J., Taylor, M., Hàwera, N., & Sharma, S. (2007). Year 7–8 students' solution strategies for a task involving addition of unlike fractions. *Findings from the New Zealand Numeracy Development Projects 2006*, 67-86.