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Acquiring mathematics as a second language: A theoretical model to illustrate similarities in the acquisition of English as a second language and mathematics

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Mathematics has been defined by researchers as a 'second or third language' and, as a result, it should be taught as a second language. Results of the literature reviewed from the theories on the teaching of mathematics and English as a second language, as well as on mathematics learning and English as a second language acquisition, have resulted in the emergence of four themes, which are similar to the ones on the teaching and learning of both mathematics and English as a second language; these are: comprehensible input, language processing and interaction, output, and feedback. In this article, the themes are illustrated in a theoretical model and discussed to show how English as a second language and mathematics can be acquired simultaneously. (English as a second language in the South African context is referred to as English as a first additional language.)

Introduction

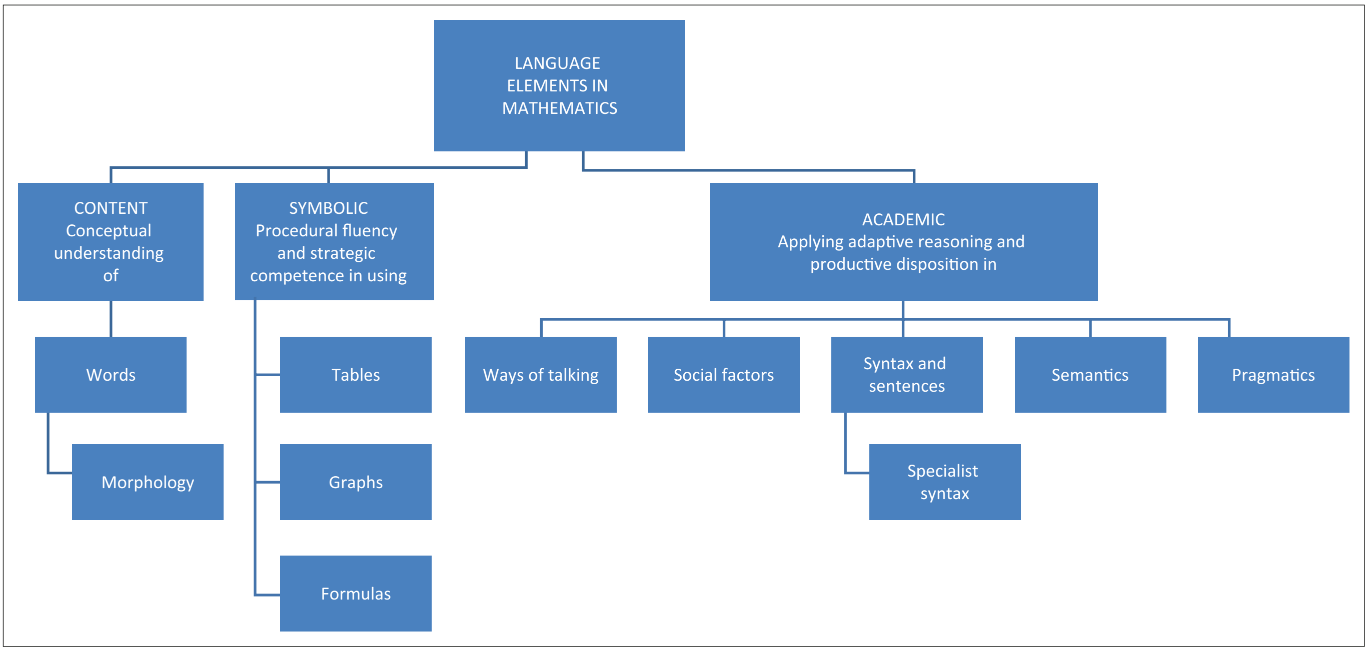
Many researchers have described mathematics as a language in itself (Esty, 1992; Setati, 2002). It, therefore, stands to reason that mathematics should be taught by teachers and acquired by learners in the same way as any other second or third language (Garrison & Mora, 1999). In South Africa, where the majority of learners have to learn through the medium of English as a second language, mathematics teachers have to teach and scaffold two languages, in effect, in their classrooms. This might be one of the reasons why language is often regarded as one of the challenges or barriers to learning in mathematics classrooms, especially in South Africa (Howie, 2003; Reddy et al., 2011). This article traces the parallel theories relating to the teaching and learning of mathematics and English as a second language (ESL) in order to integrate those theories into one coherent theoretical model for teaching and learning. The acquisition of mathematics and ESL, therefore, can be accommodated in one process and need not be two separate teaching and learning processes. Furthermore, due to the fact that questioning is an integral part of teaching and learning in mathematics classrooms (Brualdi, 1998; Rosenshine, Meister, & Chapman, 1996; Sutton & Krueger, 2002), the model also incorporates the functions of questions, questioning techniques and teacher strategies that can be used simultaneously for the acquisition of both ESL and mathematics. The model focuses on four crucial processes of language acquisition, namely comprehensible input, language processing and interaction, output, and feedback.

Mathematics as a language

Setati (2002) describes mathematics as a language as it uses notations, symbols, terminology, conventions, models and expressions to process and communicate information. Furthermore, Esty (1992) defines mathematics as a language, because, like other languages, it has its own grammar, syntax, vocabulary, word order, synonyms, conventions, idioms, abbreviations and sentence and paragraph structures.

The language that is specifically used in mathematics classrooms, classified as mathematical discourse, includes aspects summarised in Figure 1.

Mathematics educators are cautioned to pay more attention to language learning because, firstly, language learning is often an expected outcome of mathematics education and, secondly, there is evidence that language learning and mathematics learning are intimately related



Source: Adapted from Thompson, D.R., & Rubenstein, R.N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *Mathematics Teacher*, *93*(7), 568–574 (p. 569). Retrieved from http://www.jstor.org/stable/27971502

FIGURE 1: Types of language in mathematics.

(Barwell, 2008). In fact, the academic language involved in mathematics has been referred to as a third language for English language learners since research has shown that native English-speaking learners learning academic language face many of the same challenges as learners learning ESL and, as a result, they should be paired during group work activities (Biro, Chatzis, Roper, & Sehr, 2005). The next section therefore discusses the relationship between mathematics and ESL in as far as their teaching and learning are concerned.

Learning and teaching both English as a second language and mathematics

Learning a second language is not a separate process that has no impact on mathematics learning (Barwell, 2008). In other words, the learning of mathematics in multilingual classrooms depends to a large extent on the acquisition of English as a second or third language. This interdependency of the learning of ESL and mathematics therefore allows for certain acquisition processes to take place simultaneously.

The important role of language in mathematics learning is succinctly captured by Harrison (2014, par. 12): 'language is the cement that allows us to build upon prior knowledge learning. If language is weak, so too is the ability to learn'.

Similarly, Thompson and Rubenstein (2000) argue that language plays at least three crucial roles in our classrooms:

- We teach through the medium of language. It is our major means of communication.
- Learners build understanding as they process ideas through language.
- We diagnose and assess learners' understanding by listening to their oral communication and by reading their mathematical writings.

The next section discusses the conditions for and the theories on the teaching and acquisition of ESL, and also on mathematics teaching and mathematics learning. These theories include a combination of second language acquisition (SLA) teaching and learning theories, and also their similarities to the principles of realistic mathematics education (RME).

Parallels in teaching mathematics and English as a second language

In the 1960s, mathematics education in most parts of the world and in the Netherlands was dominated by a mechanistic teaching approach (Van den Heuvel-Panhuizen & Drijvers, 2014). This means that learners sat passively in mathematics classrooms while teachers demonstrated how problems are solved. Also, teachers asked closed questions that were followed up by learners' answers and teachers' feedback, engaging learners in the Initiate-Response-Evaluate discourse in mathematics classrooms.

Similarly, the mechanistic teaching approach to ESL with regard to the audio-lingual method, emphasising the spoken language, became popular in the middle of the 20th century. It involved a systematic presentation of the structures of the second language, moving from simple to complex, in the form of drills that learners had to repeat. It was influenced by a belief that the fluent use of a language was essentially a 'set of "habits" that could be developed with much practice'. Much of this practice involved 'hours spent in the language laboratory repeating oral drills' (Yule, 2010, p. 190).

In reaction to the mechanistic approach to mathematics teaching, Freudenthal, a mathematician who became interested in mathematics education, propagated a method of teaching mathematics that is relevant for learners. His method included carrying out thought experiments to investigate how learners can be offered opportunities for guided reinvention of mathematics and, in this way, contributed to the development of the RME theory (Van den Heuvel-Panhuizen & Drijvers, 2014). The main characteristics of RME are problematisation, construction and reflection. The teacher is the activator in the process of problematisation and the tutor in the process of construction, 'taking learners' informal strategies as a starting point for the interactional development of mathematical concepts and insights' (Van Eerde, Hajer, & Prenger, 2008, p. 33).

Similarly, in recent years, contemporary language teaching has moved away from dogmatic practices of 'right' or 'wrong', becoming much more eclectic in its attitudes, and more willing to recognise the potential merits of a wide variety of methods and approaches. As a result, 'the interest in the contribution of the learners in the teaching/learning dichotomy was resurrected, accommodating the learning strategies that learners employ in the process of language learning' (Griffiths & Parr, 2001, p. 248), through methodologies such as task-based instruction (TBI). According to Powers (2008), in TBI, teachers prepare lessons that are constructed according to the language required to perform specific tasks. This means that learners learn language structures through induction as they focus on task completion and meaning. Their interaction during the tasks facilitates transfer of information they have previously learned and incorporates it with new information they receive as they perform the task.

The three characteristics of RME (problematisation, construction and reflection) correlate well with the three phases in a task-based language lesson as described by Ellis (2003), namely the 'pre-task phase, the during-task phase and the post-task phase'. During the pre-task phase, learners are provided with examples of similar problems and they are given time for strategic planning, so that they can plan how they will solve the specific problem or perform the task. In the during-task phase, learners are scaffolded so that they can discuss the problem or task while using the appropriate discourse and it allows them to take linguistic risks. The post-task phase allows learners to reflect on the task, so that they can develop the metacognitive strategies of planning, monitoring and evaluating in the process.

In a similar vein, Moschkovich (2002) proposes the following three perspectives for bilingual and ESL learners to 'communicate mathematically', both orally and in writing, and to participate in mathematical practices:

- *Acquiring vocabulary*: For ESL learners to communicate mathematically, they should acquire vocabulary, usually referred to as mathematical discourse. Acquiring vocabulary is emphasised in learning mathematics as it is the central issue that second language learners are grappling with when learning mathematics (Moschkovich, 2002). Learners can only communicate mathematically if they have acquired the vocabulary, which comprises the different types of languages shown in Figure 1.
- *Constructing meanings*: The second perspective describes mathematics learning as constructing multiple meanings for words rather than acquiring a list of words. Learning mathematics, therefore, involves a shift from everyday terms to more mathematical and precise meanings, referred to as 'mathematical register' (Moschkovich, 2002, p. 194). However, everyday meanings and learners' home language can also be used by the learners as resources to communicate mathematically.
- *Participating in discourse*: From this perspective, learning to communicate mathematically involves more than learning vocabulary or understanding meanings in different registers and, according to (Moschkovich, 2002), it is seen as using social, linguistic and material resources to participate in mathematical practices.

The integration of these theories will be discussed in terms of four crucial processes of language acquisition, namely comprehensible input, language processing and interaction, output, and feedback, in the teaching and learning of both English and mathematics.

Comprehensible input

Ellis (1986) defines input as the language that learners are exposed to. He further explains that it is possible for the input provided by the teachers and interlocutors to be 'comprehensible (i.e. input that learners can understand) or incomprehensible (i.e. input that they cannot understand)'; when it is incomprehensible, it becomes 'the impetus for learners to recognise the inadequacy of their own rule system' (Gass, Mackey, & Pica, 1998, p. 301).

Garrison and Mora (1999) in their study on Latino mathematics learners recommend the use of Krashen's (1994) comprehensible input formula $i + 1$ in the teaching and learning of mathematics. Krashen's input hypothesis on second language acquisition claims that:

> an important condition for second language acquisition to occur is that the acquirer understands (via hearing and reading) input language that contains structure 'a bit beyond' his or her current level of competence. (Krashen, 1981, p. 100)

This formula is used in the theories on and principles for English SLA and mathematics learning with regard to the roles of input, together with teacher strategies applied. The formula $i + 1$ is recommended because it provides comprehensible input for English language learners to build mathematical concepts based on the principle of teaching the unknown from the known.

The role of input

From the definitions of input, one can outline the roles of input in language acquisition and mathematics learning as explained below.

Firstly, input plays a very important role in as far as language learning and acquisition is concerned. It provides the data that the learner must use to determine the rules of the target language. In the same way, the researchers of the universal grammar view input as a trigger that interacts with an innate system or the native language to promote learning (Gass & Mackey, 2006). Therefore, input forms the positive evidence that learners use as they construct their second language and mathematics grammars. This role of input has therefore resulted in many researchers describing the type of input learners receive in ESL classrooms as 'foreigner talk' (Gass & Mackey, 2006, p. 5).

Secondly, input, in the form of grammar rules (for both English and mathematics), information from mathematics textbooks, and knowledge from the language teachers and interlocutors, and also from the learners, provides the stepping stone for any form of learning to take place. It is up to the teachers and interlocutors to decide what they should do with all the input that they have to make it comprehensible for learners to learn and acquire the languages (i.e. ESL and mathematics).

Lastly, Seliger (1983) explains how the role of input gives credit to learners for successful acquisition to take place. The findings in the study showed that:

> learners referred to as High Input Generators maintained high levels of interaction in the second language, both in the classroom and outside, and progressed at a faster rate than learners who interacted little, referred to as Low Input Generators. (p. 262)

This is also supported by Cummins (1991, p. 85) when he states that 'appropriate input is clearly essential for development of all aspects of proficiency'. Teachers, at the beginning of a lesson, write down the vocabulary and symbols on the board. They discuss the definitions and representations (in mother tongue if necessary). The learners then have a reference to the meaning of the words and terminology as well as how to use it.

Even though Krashen in his input hypothesis does not credit the role of learners in as far as input is concerned when he states that 'comprehensible input is the only causative variable in SLA' (Brown, 2007, p. 297), many researchers with an increasing interest in social constructivist analyses of language acquisition focus on the characteristics of successful language learners. They have come up with learning strategies that successful learners apply with regard to input to acquire language, including mathematics language, by making it comprehensible, and thus crediting learners' role with regard to input.

Learning strategies

The learning strategies, according to Brown (2007), include:

- *Meta-cognitive strategies*: Metacognitive is a term used in information-processing theory to indicate an 'executive function', and it includes strategies that involve planning for learning, thinking about the learning process as it takes place, monitoring one's production, and evaluating learning after an activity has been completed; this evaluation includes self-monitoring, self-evaluation, advance organisers, and delayed production.
- *Cognitive strategies* are more limited to specific learning tasks and involve more direct manipulation of the learning material itself; these include repetition, resourcing, translation, grouping, note-taking, deduction, and others.
- *Socio-affective strategies* have to do with social-mediating activities and interacting with others, for example, cooperation ad asking questions for clarification. These also relate to output. Learning can be constrained by learners' or teachers' belief systems and attitudes towards mathematics and the nature of mathematics, and how it should be learned. These inform learners' decisions to avoid or embrace challenges; and these may influence the learners or teachers, attributing failure or success to cognitive (in)abilities rather than to effort. The content should therefore be meaningful to learners, and that links it to the reality principle of RME.

The reality principle in realistic mathematics education: The reality principle can be recognised in RME in two ways. Firstly, it expresses the importance that is attached to the goal of mathematics education, including learners' ability to apply mathematics in solving 'real-life' problems. Secondly, it stresses the point that mathematics education should start from 'problem situations that are meaningful to learners and that offer them opportunities to attach meaning to the mathematical constructs they develop when solving problems' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523).

Likewise, in TBI, as pointed out in Ellis (2006), a focus on form approach is valid as long as it includes an opportunity for learners to practise behaviour in communicative tasks, thus providing learners with opportunities also 'to apply mathematics in solving real-life problems'. The grammar taught emphasises not just form, but also the meanings and uses of different grammatical structures. As Krahnke (cited in Powers, 2008, p. 73) points out, 'connecting tasks to real-life situations contextualises language in a meaningful way and provides large amounts of input and feedback to assist learners in the learning process', especially in the second step of acquisition, namely language processing and interaction.

Language processing and interaction: Even if input is understood, according to Ellis (1986), it may not be processed by the learner's internal mechanisms. This is what Krashen means when he states that 'comprehensible input is not a sufficient condition for second language acquisition' (Ellis, 1986, p. 159). It is only when input becomes intake that SLA takes place. Input is the second language data that the learner hears; intake is that portion of the second language that is assimilated and fed into the inter-language system (Ellis, 1986) and, as a result, intake 'is the subset of all input that actually gets assigned to our long-term memory store' (Brown, 2007, p. 297). Interaction is, therefore crucial in the acquisition process of any language.

The role of interaction

The important role of interaction is revealed in the study conducted by Wong-Fillmore (1983, cited in Cummins, 1986) on Hispanic learners in ESL classrooms, which showed that learners learned more English in classrooms that provided opportunities for reciprocal interaction with teachers and peers. This reciprocal interaction can be achieved, according to Gass and Mackey (2006), when second language learners are presented with input that they do not understand, as that will force them to 'negotiate meaning by using confirmation checks, clarification requests, and comprehension checks, in order to change it into comprehensible input, thus making it the result of modified interaction' (Brown, 2007, p. 305). The combination of input and interaction, using forms of negotiation, makes input and interaction major players in the process of acquisition (Brown, 2007).

Long, cited in Brown (2007, p. 305), in his interaction hypothesis, posits that comprehensible input is the result of modified interaction. Similarly, Ellis (2006, p. 100) states that 'input-based feedback models the correct form for the learner (e.g. by means of a recast)' and 'output-based feedback elicits production of the correct form from the learner (e.g. by means of a clarification request)'. These different types of interactions, referred to as modifications or negotiations, are applied by teachers and interlocutors to make input comprehensible to the learners in ESL and mathematics classrooms.

- *Confirmation checks*: A confirmation check is defined by Long as 'any expression … following an utterance by the interlocutor which are designed to elicit confirmation that the utterance has been correctly heard or understood by the speaker' (cited in Gass & Mackey, 2006, p. 7). It can be used for learners to receive comprehensible input.
- *Clarification requests or paraphrases*: A clarification request is any expression designed to elicit clarification of the interlocutor's preceding utterances (Gass & Mackey, 2006). It can be applied by saying an incorrect utterance in a rising intonation for the learner to reflect on the answer provided and come up with the correct utterance. For example, if the learner says *denominator*, instead of *numerator*, the teacher could say, *denominator* with a rising intonation, and that could result in the learner reflecting on the wrong answer and saying the correct answer, *numerator.*
- *Comprehension checks*: A comprehension check is an attempt 'to anticipate and prevent a breakdown in communication' (Gass & Mackey, 2006, p. 8). It can be used also in the form of a Yes or No question by the teacher to check if the learner understands the meaning of one of the utterances spoken for communication to continue, for example:

  Given the right-angled triangle ABC, with $\angle A = 90°$, and $AB = 3$ cm, and $BC = 4$ cm, use Pythagoras' theorem to find the length of the hypotenuse. Do you know the Pythagoras theorem?

  In response to the learner's negative answer, the teacher will draw a right-angled triangle and show learners what they mean by the word 'hypotenuse', thus assisting them in how to calculate the values of the hypotenuse of any right-angled triangle. This exercise would enable the learners to find the length of the hypotenuse in the problem initially given.
- *Recasts*: Another form of negotiation for the learners' feedback is recasts, defined as 'utterances that rephrase a child's utterance by changing one or more sentence components (subject, verb, or object) while still referring to its central meaning' (Gass & Mackey, 2006, p. 8). Recasts involve the teacher's reformulation of all or part of a learner's utterance minus the error (Gass & Mackey, 2006). In response to the learners' incorrect answer, the teacher may repeat the correct answer for the learner to identify the error that they have committed so as not to make the same error in the future.

From the examples of negotiations and modifications given, one could say that these negotiations or modifications alert the learners to the mistakes they have made in their utterances, thus providing them with opportunities to 'focus their attention on language and the correct mathematical concepts; 'to search for more input in their future utterances; and to be more aware of their hypotheses about language and mathematics' (Gass & Mackey, 2006, p. 12).

Also, modifications in interactions, according to Long (cited in Menezes, 2013, p. 405), are consistently found in successful SLA; therefore, they should be applied in ESL as well as in mathematics classrooms. When these are applied by the teachers and interlocutors, they provide learners with opportunities to process their utterances and responses mentally before they can produce them, and also help them to reflect on their learning process, thus enhancing their acquisition and learning. This is confirmed by Cummins (2000, p. 74), when he states that 'BICS [*basic interpersonal communication skills*] and CALP [*cognitive academic language proficiency*] both develop within a matrix of social interaction'. As learners respond to the modifications and interactions discussed, they are actively involved in their own learning, and ultimately produce output.

The activity principle in realistic mathematics education

The activity principle emphasises that learners should be treated as 'active participants in the learning process since mathematics is best learned by doing mathematics' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523). This is strongly reflected in Freudenthal's interpretation of mathematics as a human activity. Learners should not be passive listeners but active participants in mathematics classrooms, and this can be achieved if learners are taught learning strategies such as *metacognition* to think about their learning process so as to make input comprehensible.

Similarly, TBI is based on Krashen's language acquisition hypothesis. Krahnke, cited in Powers (2008, p. 73), explains that the theory asserts that the ability to use language is gained through exposure to and use of it, thus discouraging learners from being passive and to rather be active participants in the learning situation. Krahnke, as cited in Powers (2008, p. 73) goes on to explain that 'TBI develops communicative competence including linguistic, sociolinguistic, discourse and strategic competence', thus processing the information used during specific tasks through understandable input to provide students with linguistic and sociolinguistic competence in a systematic, step-by-step process (cited in Powers, 2008, p. 73), relevant to the level principle of RME.

The level principle in realistic mathematics education

The level principle underlines that learning mathematics means that learners pass various levels of understanding: from informal context-related solutions, through creating various levels of shortcuts and schematisations, to acquiring insight into how concepts and strategies are related (Van den Heuvel-Panhuizen & Drijvers, 2014). Particularly for teaching operating with numbers, this level principle is reflected in the didactical method of 'progressive schematisation' where transparent whole number methods of calculation gradually evolve into digit-based algorithms (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523).

Similarly, in his interaction hypothesis, Long (1985, 1996) explains in detail how input is made comprehensible, thereby picking up where Krashen left off. He posits that comprehensible input is the result of modified interaction (Brown, 2007), and it includes various types of interactions, such as *clarification requests*, *paraphrases* and *comprehension checks* for learners to interact and process the language and integrate knowledge from different domains in order to make sense of their own learning.

The intertwinement principle in realistic mathematics education

The intertwinement principle means mathematical content domains such as number, geometry, measurement and data handling are not considered as isolated curriculum chapters, but as heavily integrated (Van den Heuvel-Panhuizen & Drijvers, 2014). Learners are offered rich problems in which they can use various mathematical tools and knowledge. This principle also applies within domains. For example, within the domain of angles, triangles, sines and cosines, quadrilaterals are taught in close connection with each other. In other words, different sections of mathematics should not be taught in isolation, but as a unit showing relationships between one another so as to make sense to the learners (Van den Heuvel-Panhuizen & Drijvers, 2014).

Similarly, content-based instruction, a modified form of TBI is defined by Brinton, Snow and Wesche (1989) as the concurrent study of language and subject matter, with the form and sequence of language presentation dictated by content material. In other words, content and language are not taught in isolation or separately, but always within a meaningful context. Content-based instruction can take place at all educational levels, and it refers to total immersion (approximately 90% of school time in the second language), or it can refer to content-based themes in language classes (Cenoz, 2015).

Output

Reading and listening are not enough for learners to learn the language; therefore, teachers should provide learners with vast opportunities to try out and produce language using pair and group work activities. The three major functions of output in SLA, according to Swain (2005), emphasise the role of output in language production. These are similar to the conditions of mathematics learning as listed by Van Eerde et al. (2008):

- Learners become self-informed through their input.
- There should be ample opportunities for language production.
- Language learners need feedback on their utterances.

Firstly, learners become self-informed through their output. This condition claims that learners while attempting to produce the target language may notice their erroneous attempts to convey meaning. This prompts them to recognise their linguistic shortcomings, thus becoming self-informed about their output. Output helps the learners to 'try out' one's language: to test various hypotheses that are forming. Speech and writing can offer a means for learners to productively reflect on language and mathematical language in interaction with peers.

Furthermore, Swain (cited in Gass & Mackey, 2006) also suggests that output provides an opportunity for learners to test hypotheses about the target language, and modify them where necessary. Also, for modified output to be useful, most interaction researchers suggest that it is necessary for learners to notice the relationships between their initially erroneous forms, the feedback they receive and their output, since it is possible for learners' perceptions to differ according to the type of feedback they receive and the focus (Swain, cited in Gass & Mackey, 2006).

Similarly, the second condition for mathematical language development emphasises language production. In other words, the condition for output in mathematics learning is the promotion of active participation of pupils, giving them the opportunity to construct and verbalise their mathematical solutions, promoting classroom discussions and asking for clarifications and justifications (Van Eerde et al., 2008).

Output is defined as 'the process of producing language in the form of speaking and/or writing' (Brown, 2007, p. 293). Similarly, Gass and Mackey (2006, p. 13) define it as 'the language that learners produce'. Krashen has been criticised by other researchers for disregarding the function of learners' output in SLA when he says that 'output is too scarce to make any important impact on language development' (Brown, 2007, p. 298). De Bot (1996, p. 529), argues that 'output serves an important role in second language acquisition … because it generates highly specific input the cognitive system needs to build up a coherent set of knowledge'. As a result, interaction research, according to Gass and Mackey, focuses on output that has been modified, and therefore modified output promotes learning since it stimulates learners to reflect on their original language. This is done by utilising a number of communication strategies.

Communication strategies

Learners use a variety of communication strategies to request assistance, to modify the output produced with the feedback they receive from their interlocutors, and thereby produce modified output. These are strategies that learners use as 'potentially conscious plans for solving what, to an individual, presents itself as a problem in reaching a particular communicative goal' (Brown, 2007, p. 137). These include:

- *Avoidance strategies* include message abandonment, leaving a message unfinished because of language difficulties, and topic avoidance, avoiding topic areas or concepts that pose language difficulties (Brown, 2007). Learners use these strategies by changing the topic or pretending not to understand it because it is too difficult for them to express.
- *Compensatory strategies* are used for compensation for missing knowledge, and these include code-switching, circumlocution, appeal for help and non-linguistic signals like miming, among others.
- *Memory strategies* include creating mental linkages, applying images and sounds, reviewing well and employing action. For example, in order for learners to remember how they learn sections in geometry, they can simply look at the shape of their desks and use that mentally to remember the formulae for calculating the area of rectangles, squares, parallelograms, etc.
- *Cognitive and metacognitive strategies* were discussed under the Learning strategies section.
- *Affective strategies* include lowering your anxiety, encouraging yourself, and taking your emotional temperature.
- *Social strategies* include asking questions, cooperating with others and empathising with others.

Learning strategies 'relate to input', whereas communication strategies 'relate to output' (Brown, 2007, p. 132). These strategies help learners to interact meaningfully in the course of their learning.

The interactivity principle in realistic mathematics education

The interactivity principle signifies that learning mathematics is not only an individual activity, but also a social activity (Van den Heuvel-Panhuizen & Drijvers, 2014). It encourages teachers to make full use of group work and whole-class discussions to provide learners with opportunities to share ideas and strategies on how they solve mathematical problems and, in this way, produce output. As learners share their ideas, 'they evoke reflection, which enables them to reach a higher level of understanding' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523).

Similarly, proponents of TBI (Ellis, 2003) agree that communicative activities used during pair and group work are appropriate vehicles and that language learning activities should directly reflect what learners 'potentially or actually need to do with the target language' (Swan, 2005, p. 377). Also, the role of the teacher in the TBI classroom is to supply task-related vocabulary where necessary, offering recasts or acting as interlocutors, 'casting the teacher's role as a manager and facilitator of communicative activity rather than an important source of new language' (Swan, 2005, p. 391).

For ESL and mathematics learning to take place successfully, learners need support in the form of feedback from their teachers, interlocutors, adults and peers, so as to perfect acquisition and the learning process. This brings us back to the third condition of mathematics learning, namely that language learners need feedback on their utterances (Van Eerde et al., 2008).

Feedback

The important role of output has resulted in many researchers claiming that output provides the forum for receiving feedback (Gass & Mackey, 2006). In other words, when learners produce first language utterances, they rely on the interlocutors' feedback to see if they are on the right track in terms of language acquisition and mathematics learning. This is also emphasised in RME's guidance principle

The guidance principle in realistic mathematics education

The guidance principle refers to Freudenthal's idea of 'guided re-invention' of mathematics. It implies that 'RME teachers should have a proactive role in learners' learning and that educational programmes should contain scenarios that have the potential to work as a lever to reach shifts in learners' understanding' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523). To realise this, the teaching and the programmes should be based on coherent long-term teaching-learning trajectories. According to Gravemeijer (2009, p. 114), the principle means that 'learners should be provided with the opportunity to experience a process similar to the process by which a given piece of mathematics was invented'. The role of the teacher in this case is to revise the sections taught in previous classes that will enable them to perform the tasks before giving them the actual problem to solve, thus providing them with scaffolding.

For example, a question like *Find the sum of 49 + 58*, should not require learners to crack their heads with adding the two numbers, but they should simply think of the internalised method of factorising common factors by applying the Distributive Law taught in class, and thus group like and unlike terms by expanding *49* into *40 + 9* and *58* into *50 + 8,* and find the sum of like terms *40 + 50 = 90*, and that of units *9 + 8 = 17* to get the answer *107*. In doing so, the learners would be outsourcing guidance or scaffolding by remembering what their teachers taught them before and also what they learnt from mathematics textbooks. The role of the teacher in this case is to revise with the learners the previous sections taught that are required for them to perform the tasks in the example given before giving them this exercise, thus providing them with scaffolding.

Similarly, in ESL classrooms, Vygotsky's theory on the zone of proximal development (ZPD) stresses the fact that learners acquire language in the social world and, as a result, individuals learn best when working together with others during joint collaboration. It is through such 'collaborative endeavours with more skilled persons that learners learn and internalise new concepts, psychological tools, and skills' (Shabani, Khatib, & Ebadi, 2010, p. 237). The fact that learners need input from a more knowledgeable other in Vygotsky's ZPD also links to the qualification of the type of input specified in Krashen's input hypothesis $i + 1$ (Krashen, 1981, 1994), namely that learners should be exposed to input at a level higher than their current level of language proficiency.

Likewise, Ellis (2006, p. 102) believes that 'corrective feedback is important for learning grammar in ESL and that it is best conducted using a mixture of implicit and explicit feedback types that are both input-based and output-based'. To provide scaffolding, teachers can provide learners, for example, with the vocabulary that is required to perform a particular task, or design fill-in-the-gap conversations to practise as they simulate the task and to give them the means by which they can produce appropriate output.

For learners' utterances and mathematics language to be perfected, learners need support or what is termed 'scaffolding', in the form of feedback from the teachers, interlocutors, peers and adults, for them to be in a position to reflect on and correct the mistakes made and to perfect the acquisition and learning process. Scaffolding is understood as the assistance learners get from others (teachers, relatives, classmates) and it enables them to perform learning tasks (Menezes, 2013). Feedback from the teacher on pupils' contributions should not be immediate, but 'delayed to promote contributions from different pupils and horizontal interaction between pupils' (Van Eerde et al., 2008, p. 36).

In this instance, Vygotsky's theory of ZPD is also applicable in mathematics classrooms. The theory, as a result, puts emphasis on the role of feedback. Teachers and parents (as the more knowledgeable others) are therefore advised to offer learners this assistance and support for successful learning and language development to take place in both ESL and mathematics classrooms, using the following strategies:

Strategies for scaffolding

In a study in which a teacher was encouraged to employ seven strategies in a multilingual classroom, the results showed that the strategies used in Table 1 promoted pupils' language development (Smit & Van Eerde, 2013).

Similarly, Biro et al. (2005) encourages mathematics teachers to help English language learners to develop and practise academic language for learning mathematics using scaffolding strategies, such as having learners restate other learners' comments, using graphic organisers or gestures, correcting errors and providing positive feedback, providing handouts to help learners structure and guide their work, among others.

A constructivist approach to teaching and learning (summarised below) should also be applied for assessment in English SLA and mathematics language learning classrooms to take place in such a way that learners are able to reach the intended outcomes discussed.

Using a constructivist/open-ended approach to teaching and learning

A constructivist approach to teaching and learning, according to Mahlobo (2009), is characterised by the use of open-ended tasks or questions, and it encompasses the following:

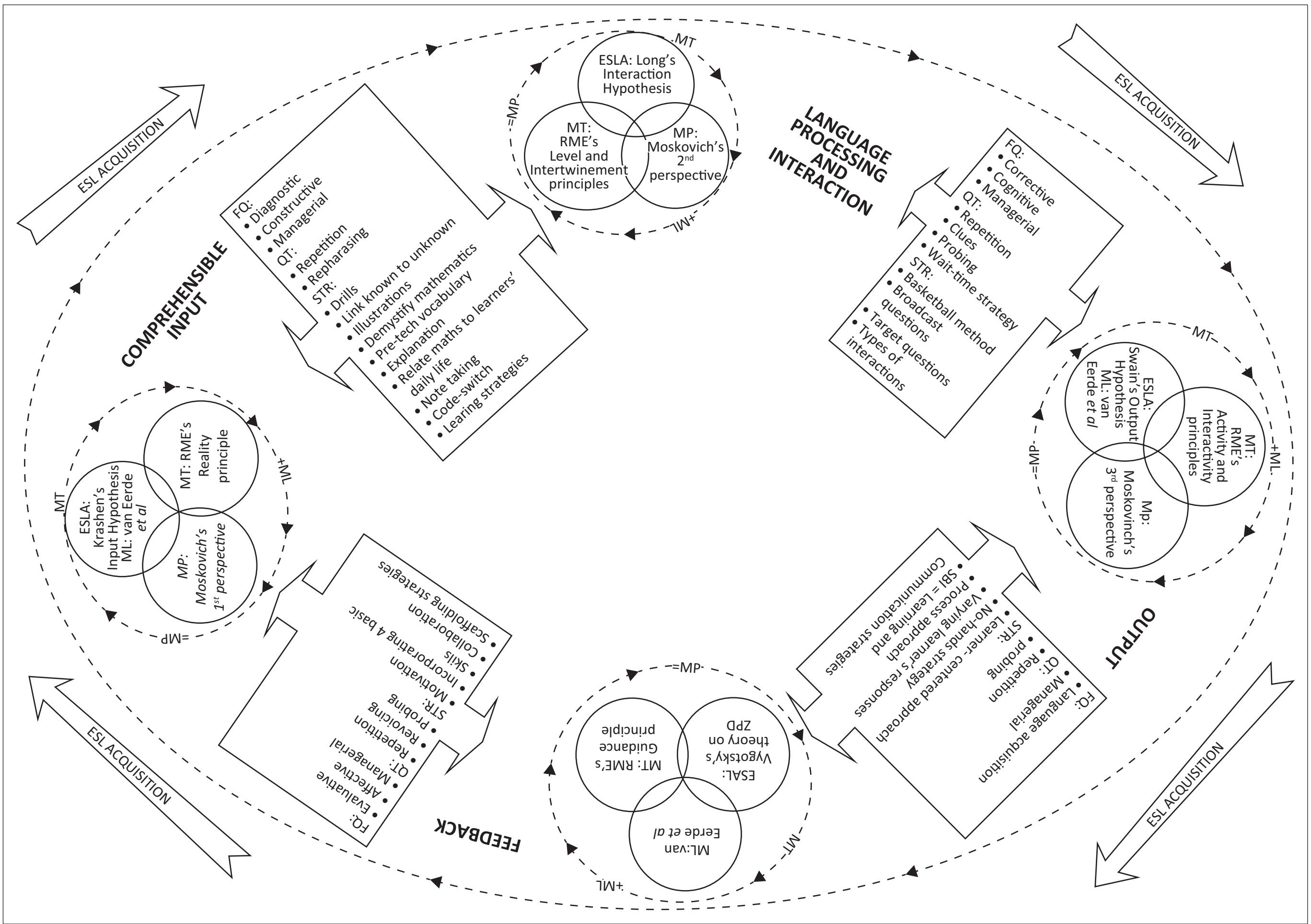
The learners:

(a) Take the initiative in solving mathematical problems and do not depend on the teacher.
(b) Determine their own approach when solving problems.
(c) Express their own ideas more frequently when solving mathematical problems.

TABLE 1: Strategies for scaffolding language and examples for each strategy.

Strategy	Example for strategy
Reformulate pupils' utterances (spoken or written) into more academic language	[In response to the *graph goes higher and higher up*:] *Yes, the graph does rise steeply*.
Ask pupils to be more precise in spoken language or to improve their spoken language	*What do you mean by 'it'*?
Repeat correct pupil utterances	*Yes, the graph does descend slowly.*
Refer to features of the text type (interpretative description of a line graph)	*Into how many segments can we split the graph?*
Use gestures or drawings to support verbal reasoning	Gesturing a horizontal axis when discussing this concept
Remind pupils (by gesturing or verbally) to use a designed scaffold (i.e. word list or writing plan) as a supporting material	*Look, the word you are looking for is written down here.*
Ask pupils how written text can be produced or improved	*How can we rewrite this in more mathematical language?*

Source: Smit, J., & Van Eerde, D. (2013). What counts as evidence for the long-term realization of whole-class scaffolding. *Learning, Culture and Social Interaction*, *2*(1), 22–31 (p. 24). https://doi.org/10.1016/j.lcsi.2012.12.006



Source: Ledibane, M.M. (2016). *A model for mathematics teachers to promote ESL acquisition through questioning strategies*. Unpublished doctoral dissertation, North-West University, Potchefstroom, South Africa (p. 103). Retrieved from http://hdl.handle.net/10394/18904

ESL, English as a second language; ESLA, ESL acquisition; FQ, functions of questions; ML, Mathematics learning; MP, mathematical proficiency; MT, Mathematics teaching; QT, questioning techniques; RME, realistic mathematics education; STR, teacher strategies; ZPD, zone of proximal development.

FIGURE 2: A theoretical model to illustrate the simultaneous acquisition of a second language and mathematics.

(d) Modify other learners' ideas.
(e) Can stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes. (p. 38)

This article has elaborated on the teaching of both mathematics and ESL, and also on the learning of mathematics and ESL acquisition. Therefore, the theoretical model in Figure 2 shows the similarities between the theories on the teaching of both ESL and mathematics, and also on the learning of mathematics and ESL acquisition. From the literature reviewed, the themes that emerged, namely comprehensible input, language processing and interaction, output, and feedback, are in response to the research question, namely:

> *What are the theories that underpin the effective questioning techniques and strategies to promote ESL acquisition?*

The theoretical model

This article has elaborated on the teaching and learning of both mathematics and ESL, and also on the learning of mathematics and ESL acquisition. As a result, the theoretical model in Figure 2 shows the similarities between the theories on the teaching of mathematics and ESL (TESL) and English second language acquisition (ESLA) and mathematical proficiency. These are indicated through different Venn diagrams. From the literature reviewed, the themes that emerged, namely comprehensible input, language processing and interaction, output, and feedback, are indicated. These processes are listed next to the relevant Venn diagrams and in a circle in order to indicate the cyclical nature of language acquisition and the interactivity and dynamics of the different processes. The movement from one process to another is stimulated by scaffolding of the teacher and output by the learners in the form of different strategies. In addition, the inner circle shows that ESLA is taking place throughout the processes, while the outer concentric circle shows that mathematics teaching, together with mathematics learning, produces mathematical proficiency in all four of the processes discussed. Also, the cyclical arrows show movement created by questions, questioning techniques and teacher strategies, thus moving learners from input to language processing and interaction, from language processing and interaction to output, from output to feedback, and vice versa.

Comprehensible input

According to Krashen's input hypothesis in ESLA (Krashen, 1981, 1994), as well as Van Eerde et al. (2008)'s first condition for mathematics learning, learners in ESL and mathematics classrooms should be provided with comprehensible input at level $i + 1$, that is, input that is challenging to the learners and not input that is very easy at level $i + 0$, or difficult at level $i + 2$. Furthermore, for learners to be able to understand what is taught in mathematics classrooms and achieve mathematical proficiency, according to Moschkovich (2002)'s first perspective, provisions should be made for learners to acquire vocabulary referred to as 'mathematical discourse'. Furthermore, learners should be given real-life problems that are meaningful to them, according to RME's reality principle (Van den Heuvel-Panhuizen & Drijvers, 2014), for them to understand what is taught in mathematics classrooms (Ledibane, 2016).

The functions of questions (FQ), the questioning techniques (QT) and teacher strategies (STR) indicated, together with learning strategies, when used in these classrooms, will assist learners to comprehend the input provided. These should also be used in mathematics classrooms.

Language processing and interaction

For learners to be able to process and interact using language, according to Long's interaction hypothesis (Long, 1985, 1996), comprehensible input has to be modified using the different types of interactions. Similarly, Moschkovich (2002)'s second principle emphasises *constructing meaning*, implying that everyday meanings and learners' home language can be used for mathematical formulations and concepts for learners to acquire mathematical proficiency. Furthermore, RME's level and the intertwinement principles (Van den Heuvel-Panhuizen & Drijvers, 2014) underline that learning mathematics means that learners should be taught in such a way that they see the concepts taught as inter-related, and not isolated from each other (Ledibane, 2016).

The functions of questions (FQ), the questioning techniques (QT), and teacher strategies (STR) indicated, together with types of interactions or modifications, when used in these classrooms, will provide learners with opportunities to process and interact using the language. Even though the interactions are used in ESL classrooms, they can also be used in mathematics classrooms for learners to achieve the intended outcomes.

Output

For learners to produce output, according to Swain's output hypothesis in ESLA, and Van Eerde et al (2008)'s second condition for mathematics learning, learners should be provided with opportunities to produce output. Also, to achieve mathematical proficiency, according to Moschkovich (2002)'s third perspective, learners should be given opportunities to *participate in discourse*. Similarly, RME's activity and interactivity principles (Van den Heuvel-Panhuizen & Drijvers, 2014) emphasise that learners should be treated as active participants in mathematics classrooms, and therefore group work activities should be used in these classrooms for learners to be provided with opportunities to produce output (Ledibane, 2016).

The functions of questions (FQ), the questioning techniques (QT) and the teacher strategies (STR) indicated, together with a combination of learning and communication strategies referred to as strategies-based instruction that assist learners to try out and produce language in ESL classrooms (Brown, 2007) are also captured. All these should be used in both ESL and mathematics classrooms.

Feedback

For learners to do well in acquiring both ESL and mathematical discourse, they should be provided with feedback on their utterances. This is emphasised in Vygotsky's ZPD theory on feedback and also in Van Eerde et al. (2008)'s third condition for mathematics learning. This is also stated in RME's guidance principle (Van den Heuvel-Panhuizen & Drijvers, 2014), where teachers are encouraged to provide scaffolding or support in the form of feedback on learners' utterances (Ledibane, 2016). Once again, the quality of the feedback can be linked to Krashen's input hypothesis $i + 1$, as the purpose of the input is to improve proficiency, and therefore needs to be at a level beyond the learners' current proficiency.

The functions of questions (FQ), the questioning techniques (QT) and teacher strategies (STR) indicated, together with scaffolding strategies, will provide learners with feedback on their utterances in ESL and mathematics classrooms.

In addition, the reflection processes described in the visual representation are cyclical in nature as the teachers provide input throughout the learning situation when they also reflect on the input provided to learners and on the output produced. Similarly, the learners reflect on what is taught and also on what they bring into the learning environment by applying meta-cognitive processes to speed up the process of producing language and acquiring it in the long run. Hence, the arrows on both sides show reflections throughout the learning process as teachers and learners reflect on the input provided and output received by using meta-cognitive knowledge (e.g. declarative, procedural and conditional; person, task and strategy variables) as well as the self-regulated processes (e.g. planning, monitoring, evaluation).

Conclusion

Current research on second language acquisition and mathematics learning shows that learners go through similar processes when acquiring both subjects, namely they need to actively use comprehensible input, to process language through interactions, to produce new linguistic elements in meaningful contexts and to receive feedback to integrate new knowledge into their existing knowledge systems (Krashen, 1981, 1994; Long, 1985, 1996; Moschkovich, 2002; Swain, 2005; Van den Heuvel-Panhuizen & Drijvers, 2014; Van Eerde et al., 2008; Vygotsky, 1978).

The researchers on second language acquisition and mathematics learning have brought good news for mathematics teachers, specifically in Grade 10–12 classes, who are struggling to bear the burden of teaching both language and mathematics in their classrooms. When basing their planning and presentation of lessons on the proposed model, mathematics teachers will not necessarily have to carry that burden. If they carefully plan the strategy instruction (from both the fields of ESL and mathematics), using the functions of questions, questioning techniques and teacher strategies associated with moving learners from one acquisition process to the next, they can teach both mathematics and English in such a way that learners acquire these simultaneously. However, very few mathematics teachers have been trained as language teachers as well, so they will have to be trained in order to create awareness of the similarities and useful functions of questions, questioning techniques and teacher strategies associated with the acquisition of a second language. The hope is, therefore, that this model could assist teacher educators in the pre-service and in-service training of Grade 10 mathematics teachers who have to teach through the medium of a second language, and prepare learners to perform well in the final Grade 12 mathematics examination papers.

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Authors' contributions

M.M.L. and K.K. conceptualised the article, with major contributions from M.v.d.W. The data was collected and analysed by M.L. All three authors were involved in the interpretation of the theories and the framework was designed with major contributions from K.K. and M.v.d.W.

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