Understanding of Function and Quadratic Function among Secondary School Students in Selangor

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ABSTRACT

This paper highlights the importance of developing a good understanding of the topic of function. Topic of function becomes a building block for students to understand many more complex topics of mathematics. Specifically, this study aimed to investigate the relationship of students' level of understanding between function and quadratic functions. A survey research was employed. A total of 103 samples have been involved in this study. The finding revealed that there was a strong, positive and very significant relationship of the two topics in additional mathematics among the Form Four students. It implies that the teaching and learning strategies of the two topics have to be considered simultaneously.

Keywords: *Understanding, Functions, Quadratic Functions, Students.*

BACKGROUND OF THE STUDY

At secondary school level, students should be helped to comprehend the basic concepts of mathematics including concepts of function (National Council of Teachers of Mathematics, 2000). As stated by Suzanne (2015), the topic of function is one of the main requirements as it provides the knowledge in being able to discover and seek solutions to common issues related to the real world. Similarly, Sajka (2003) stated that the concept of function is the fundamental knowledge in mathematics. Once a formal definition is introduced, then function becomes a challenge for students to grasp and learn. The basic concept of applying function in representation emerges from the aim to discover patterns among quantities (Muzaffer, 2013). However, students face many difficulties when they try to understand it (Chazan, Yerushalmy, Leikin, 2008; Ghazali, 2011). Formal definition and the concept of function generally is only taught to students once a student reaches middle and high school.

On the other hand, it is important to learn quadratics function because the concepts of quadratic function are used later in higher mathematics, especially when dealing with higher polynomial functions (Suzanne 2015). Applying quadratics functions enables the application of mathematical thinking and reasoning which involves making the paths of decision (Brown et al, 2007; Center, 2012). Hence, function and quadratic function are two important topics among Form Four students.

The contents of the Form Four Additional Mathematics syllabus have been arranged accordingly to its level of difficulty. Quadratic function is learned after the topic of function (Integrated Curriculum for Secondary School Curriculum Additional mathematics, 2006). Many studies reveal that secondary students have difficulties in understanding the concept of functions, and quadratic functions (Eraslan, 2008; Kotsopoulos, 2007; Metcalf, 2007; Zaslavsky, 1997). In fact, students have been struggling to understand this complex concept. Despite the differences in the abilities of students, the one common issue is the students' difficulties in generalizing a mathematical concept to real world problems. In addition, students are not explicitly instructed with regards to the specific tools they need to use. If the difficulties of function became an unsolved problem, it will cause many other learning difficulties in learning quadratic functions (Eraslan, 2008; Kotsopoulous, 2007; Metcalf, 2007; Sevim, 2011; Zaslavsky, 1997).

Getting the key concepts right is always emphasized in the teaching and learning of mathematics (Watson, Jones & Pratt, 2013). Hence, getting the concepts of functions right is always targeted in quadratic function class. It is also introduced in the algebra courses and the concepts and properties become building blocks to students' understanding of the concept of function (Metcalf, 2007; Zaslavsky, 1997). If students understand quadratic functions and its properties and applications, it becomes easier for them to build and develop a good understanding of more complex and different types of functions and concepts.

The belief that students' understanding of mathematics is built upon experience is also a direction for teachers to help students move forward for the understanding of quadratic function. Haylock (1982) highlighted that students need to make connections between the new experience and previous experiences. This indicates that understanding involves long term process of experiencing. Knowledge is not learned in isolation. When students are experiencing mathematical understanding of concepts they are making a lot of relational understanding in knowing both what to do and why (Skemp, 1978). They are guiding themselves to make connections. The way of understanding is flexible. Specifically, making understanding of any learning concepts means making connections of idea cognitively (Hiebert & Lefevre, 1986).

Based on the above mentioned arguments, this study aimed to examine the students' abilities in function and quadratic In addition the relationship between function and quadratic was investigated.

METHODOLOGY

This study employed correlational research design to investigate the students' level of knowledge in function as well as quadratic function and their relationship. The population of this study was all Form Four with Additional Mathematics students in Selangor. A total of 103 students were randomly selected to participate in this study. The instruments are two different sets of test, namely function test and quadratic function test. Function test consists of 19 items with total marks of 48. Quadratic function consists of 13 items with total marks of 36. The tests were constructed using table of justification as in Appendix A. The table of justification illustrates the distribution of items according to the level of difficulties. The items have been validated by two experts in additional mathematics. The reliability of this study is supported by Cronbach's reliability statistic which gave a value of 0.936. Figure 1 presents sample items of the instrument for topic of function.

Given function $f: x \to -9 + 1$, find the

(a) image of -3

(b) object which has the image 19

Diagram 1 shows a function $f: x \to 5x + 9$ *and* $g: x \to x^2 - x\,x^2x$.

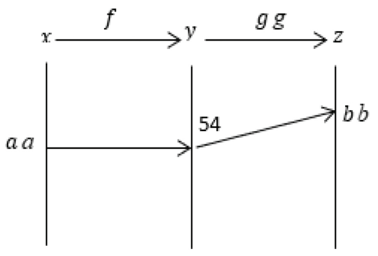


Diagram 1

Find

a) The values of a a and b b

b) $fg(x)$ $fg(x)$

c) The function which maps x x onto z z

Figure 1: Sample Items for the Topic of Functions

FINDINGS AND DISCUSSION

The results of achievement in functions and quadratic functions are presented in Table 1. For the topic of function, the mean and the standard deviation are 20.54 and 11.99 respectively. For the topic of quadratic function, the mean and standard deviation are 11.19 and 8.58 respectively.

Table 1: Results for Function and Quadratic Function

Measures	Achievement in topic Function (total mark =48 with 19 items)	Achievement in topic Quadratic Function (total mark=36 with 13 items)
Mean	20.54	11.19
Std. Deviation	11.99	8.58

This study reviewed that the students are relatively mostly able to comprehend concept of relation. Percentage of students (64.1%) in understanding relation was higher compared to other topics of functions, namely understanding of function (7.8 % are in the higher level), composite function (22.3% are in higher level), inverse function (24.3% are in the higher level), graph of function (23.3% are in the higher level) as stated in Table 2. The low percentage of performing high comprehend the concept of function has been related to the students' difficulties to translate verbal function or word problem function (Carlson, 1998). The verbal translation for the conceptual understanding of function needs to be explored since it is crucial to identify the relationship contained in algebraic expressions as claimed by Hohensee (2017). For example, the interpretation of a relationships between two variables focuses on direction and magnitude as well as rate of change within a range of values. Further discussion in terms of rate of change is also playing around on the change of a variable which will affect the change of the other variables. Hence, the early lessons of function require students to discuss around functional reasoning as emphasized by Kalchman & Koedinger (2005).

Table 2: Percentage of Students' Achievement in Function

	Percent					
Levels	Overall achievement	Understanding of Relation	Understanding of Function	Composite function	Inverse Function	Graph of Function
High	23.3	64.1	7.8	22.3	24.3	23.3
Average	39.8	22.3	59.2	29.1	37.9	28.2
Low	36.9	13.6	33.0	48.5	37.9	47.6
Total	100.0	100.0	100.0	100.0	100.0	100.0

The results also revealed that there are high percentages of students having low achievement in the topics of composite function (48.5% in the low achievement category) and inverse function (37.9% in the low achievement category). Both topics (composite function and inverse functions) involve operations. Justification of the further relationship in the operations need to be focused. For example the relation of function f(x)=5x and fg(x)=5x-10 which needs the explanation of composite function was not be able to be described in terms of function of a function. The importance of the variation reasoning for interpreting the relationship of variables as can be seen in a pattern which was emphasized by Angela, Kyle, Alyson & Matthew (2017).

Table 3 shows the achievement in quadratic functions. The percentages of students in low achievement were comparatively high in all the topics, namely understanding of quadratic equation (47.6% in lower level of achievement), find the maximum & minimum values (75.7% in lower level of achievement), graph of quadratic (96.1% in lower level of achievement) and quadratic inequalities (42.7% in lower level of achievement).

Table 3: Percentage of Students' Achievement in Quadratic Function

			Percent		
Levels	Overall achievement	Understanding of quadratic equation	find the maximum and minimum values of quadratic functions	Graph of quadratic	Quadratic inequalities
High	10.7	23.3	6.8	1.9	22.3
Average	33.0	28.2	17.5	1.9	35.0
Low	54.4	47.6	75.7	96.1	42.7
Total	100.0	100.0	100.0	100.0	100.0

The findings below described the relationship between students' achievement in function and quadratic function. Table 4 shows that there is a significant correlation between students' achievement in the topic of function and quadratic function with p-value < 0.05. The correlation coefficient is 0.770. This value represents a moderately strong positive correlation between the achievement of students in the topics of function and quadratic function.

Table 4: Correlation between Achievement in Function and Quadratic Function

Variable		Quadratic Function
Function	coefficient correlation	0.770
	sig-value	0.00

Further analyses provide more inputs about the students' understanding of function and quadratic function. A few samples of the students' work were analysed. Figure 3 shows that there is a careless mistake in part (a) of the question. The mistake also indicates misunderstanding of getting a minimum value of a quadratic function. In getting the minimum value, an extra careful of ensuring values of the two parts in the function should be taken. The two parts in the function, namely $(x+p)^2$ and the subtraction or addition of a constant should be clearly separated in a completing square process. The two parts have conceptually reminded the determination of minimum or maximum value. In this example, concept of (x+p) =0 for ensuring the value -4 is the minimum value needs to be presented to find the value of p. On the other hand, the student has confusion in presenting axis of symmetry. A symmetry is a function, namely x = 5 (for this question), hence the answer can be presented in a graph as presented in Figure 2. The result reflects that students need to develop meaningful interpretation and use of function in various representational and settings. The misunderstanding of changing variables and applying the roles of variables in terms of object (x) and image (y) can be a major a major challenge for most students (Carlson & Oehrtman, 2017).

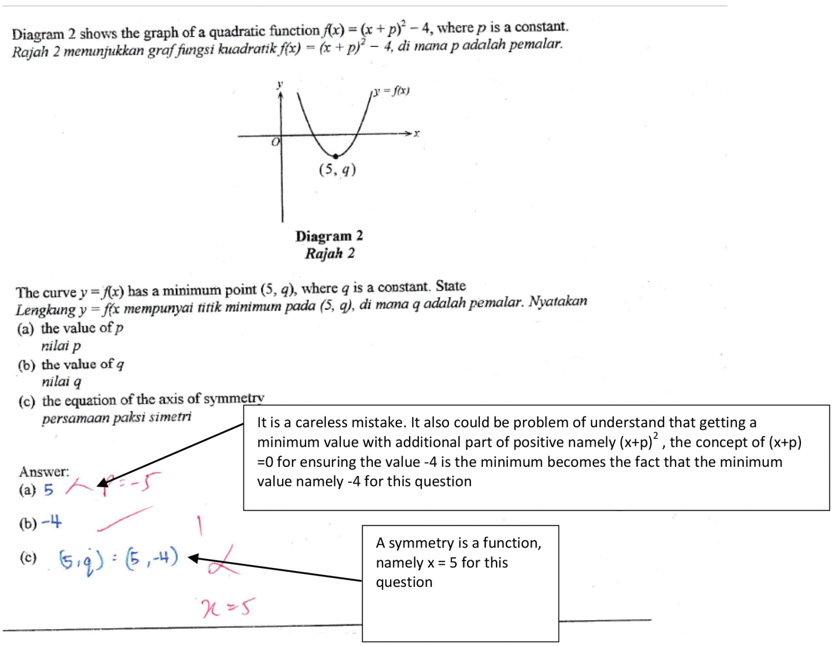


Diagram 2 shows the graph of a quadratic function $f(x) = (x + p)^2 - 4$, where p is a constant.
Rajah 2 menunjukkan graf fungsi kuadratik $f(x) = (x + p)^2 - 4$, di mana p adalah pemalar.

Diagram 2
Rajah 2

The curve $y = f(x)$ has a minimum point $(5, q)$, where q is a constant. State
Lengkung $y = f(x$ mempunyai titik minimum pada $(5, q)$, di mana q adalah pemalar. Nyatakan
(a) the value of p
nilai p
(b) the value of q
nilai q
(c) the equation of the axis of symmetry
persamaan paksi simetri

Answer:
(a) 5 $p = -5$
(b) -4
(c) (5, q) = (5, -4)
$x = 5$

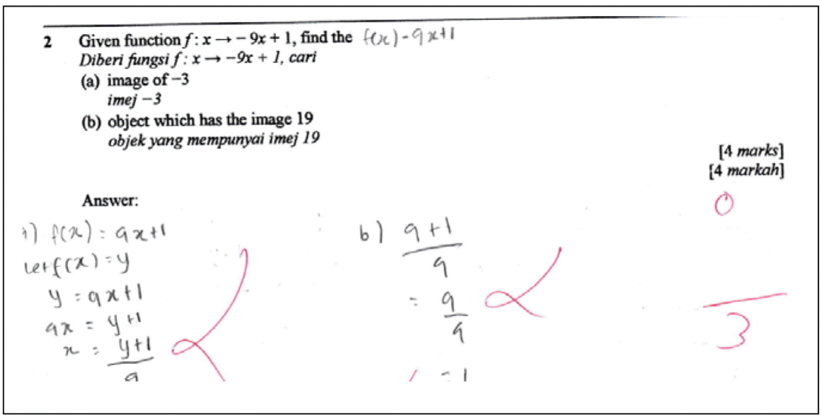
It is a careless mistake. It also could be problem of understand that getting a minimum value with additional part of positive namely $(x+p)^2$, the concept of (x+p) =0 for ensuring the value -4 is the minimum becomes the fact that the minimum value namely -4 for this question

A symmetry is a function, namely x = 5 for this question

Figure 2: Sample Work 1

The above results echo Ozaltun's & Bukova Guzel's (2017) findings which emphasized the importance of understanding and relating the concept of symmetry in determining the maximum and minimum points as well as drawing the graph. Hence, the low achievement in determining maximum and minimum points is related to drawing a graph function. The abilities of representation of quadratic functions from a given quadratic functions and vice versa (namely the writing of equation from a graph) need to be observed among students for their cognitive development in quadratic function (Ali Eraslan, 2005). Many students' low achievement in quadratic function was caused by the lack understanding of the graphing concepts in quadratic function. The symmetry concept was a critical point for the quadratic functions and the student could find the vertex and different points by using the axis of symmetry.

The relationship between concepts of function and concepts of quadratic functions can be observed in Figure 3. Figure 3 shows that the concept of object and image in the topic of function are not in the student's concern. The student was not aware of the scope in discussing the elements (object and image) of a function.


2 Given function $f: x \rightarrow -9x + 1$, find the $f(x)-9x+1$

Diberi fungsi $f: x \rightarrow -9x + 1$, cari

(a) image of -3

imej -3

(b) object which has the image 19

objek yang mempunyai imej 19

[4 *marks*]

[4 *markah*]

Answer:

a) $f(x) = 9x+1$

let $f(x) = y$

$y = 9x+1$

$9x = y+1$

$x = \frac{y+1}{9}$

b) $\frac{9+1}{9}$

$= \frac{9}{9}$

$= 1$

0

3

Figure 3: Sample 2

Usually, different views of concepts of function will cause confusion among the students. The serious condition has been highlighted by Dubinsky & Harel (1992) who perceived that action view of functions may contribute to the understanding of function when move along the learning process from functions and all related topics of functions. The action of view of function is also hoped to direct students to work flexibly in dealing with functions. Specifically, flexible thinking guides students to apply functions and further to equations and hence enable them to distinguish functions and equations separately (Breidenbach et al., 1992).

CONCLUSION

In general, the students' overall achievements (in terms of percentage of high achievement) for both function and quadratic function were low. The achievements were presented in two important concepts of quadratic function namely (1) find the maximum and minimum values of quadratic function (namely 6.8% students in high level of achievement) and (2) construct and apply graph of quadratic function are lower than other concepts in quadratic function (1.9% in high level of achievement). Hence, more effort among educators is needed to assist and promote students' understanding in function focusing on graphing with related concepts of both functions and quadratic functions.

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