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High School Algebra Students Busting the Myth about Mathematical Smartness: Counterstories to the Dominant Narrative "Get It Quick and Get It Right"

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Abstract: This article continues to challenge the robust myth that mathematical smartness is exemplified in individuals who consistently complete mathematics problems quickly and accurately. In so doing, I present a set of counterstories from three students in one ninth-grade Algebra 1 classroom. These students described transformative experiences in their perceptions of mathematical smartness. Analysis of interviews revealed four themes about their perceptions of mathematical smartness, including: (1) consistently and unapologetically affording time and space to value multiple solution strategies, (2) belief in mathematical justification and explanation as the goal for demonstrating mastery of mathematical content, (3) valuing mathematically valid ideas from all class members, and (4) valuing collaborative problem solving as a way to help group members, distribute mathematical knowledge and orient students toward learning with one another. I found that their interpretations of mathematical smartness are counter to the still-dominant myths around speed and accuracy. While the four themes that emerged have been previously studied in the frame of teacher practices, this research provides needed additional empirical evidence of students' voices describing what mathematical smartness can and should look like.

Keywords: mathematics; mathematics education; mathematics myth; equity; striving toward equity; student perspectives; counterstories; smartness

1. Introduction to the Myth: Mathematical Smartness Defined by "Get It Quick and Get It Right"

In February of 2018, I was sitting in a meeting with my peers and a mentor—a group of four assistant professors and a leader from the Center for Teaching at my University. The five of us are a part of a one-year teaching fellowship. While I am an assistant professor in our College of Education, the other teaching fellows come from the College of Engineering and the Law School. Part of the fellowship includes observing one another and debriefing our visits. We had just completed one such observation, observing a colleague who is teaching a course that contains a substantial amount of mathematics. During the debrief of that observation, the course instructor indicated an interest in exploring more opportunities for the undergraduate students to talk with one another about the mathematics during the course of a lesson. I offered a wonder to the group about whether there could be chances for the students to work on this material by exploring multiple solutions and multiple solution strategies. One of the other colleagues responded with a chuckle, saying, "But we teach math! All we care about is the answer!"

A couple of years ago, I worked with a sixth-grade teacher who was having trouble getting her students to draw a picture to represent fraction division in the expression $\frac{2}{3} \div \frac{1}{2}$. I asked her to show me how she presented the question to her students. On the students' handout, she had typed, "What is $\frac{2}{3} \div \frac{1}{2}$? Draw a picture." She told me that she was frustrated that students were following the memorized algorithm "flip and multiply" to find their answers to the "what is . . . " part of her question, without understanding the mathematical concept of division of fractions. She said that she was further discouraged that after her students found the "answer," they did not want to draw a representation of the problem. I suggested she make a small change, writing the problem statement instead as, "Draw a diagram to represent $\frac{2}{3} \div \frac{1}{2}$" The difference in what her students did was instantaneous. Students' complaints about drawing a picture to represent their understanding of fraction division disappeared. When the teacher changed the nature of her students' mathematical justification to move away from describing an algorithm and toward mathematical explanation rooted in representations, she linked the mathematics students worked on to opportunities for deep mathematical understanding. She said that after shifting the "problem" (the work of doing mathematics) away from an "answer," her students no longer worked on only solving problems for the sake of getting an answer.

The above examples are only a slice of the ever-dominant conception that the goal of mathematical thinking centers squarely around finding a correct solution to a given problem. Many mathematics educators and mathematics education researchers have previously offered empirical evidence of mathematical learning that works against the notion of speed and answer-oriented mathematics (see, e.g., [1–5]). Other studies have focused on mathematics practices that support historically marginalized students (e.g., [4–9]), including Black girls [3]. In this piece, I feature analysis of three high-school Algebra I girls' perspectives by sharing findings from interviews with each of them.

I contend that we gain a depth of understanding by letting students guide us in busting the "Get It Quick and Get It Right" myth. More explicitly, the research question posed is: How do high school mathematics students express counterstories against the dominant narratives about mathematical smartness? How do these students' experiences contribute to bust the myth around "Get It Right" and "Get it Quick"? As such, these students' counterstories contribute needed additional empirical evidence that high school mathematics is a subject whose main focus is to follow a quick procedure and obtain a correct answer. I will take the reader inside one classroom, in order to hear student perspectives about what mathematics is like when finding a correct answer is not the primary focus of being a mathematician in this classroom. Further, I aim to feature counterstories in order to develop a new dominant narrative as the field of mathematics education continues to push against traditional views of mathematics and mathematical smartness (see, e.g., [2,5,10–13]).

2. Literature Review

"Smartness" is a socially constructed concept, opposing those who "get it" (a small elite) against those who do not [14]. In spite of recent neurological research that you can grow your smartness or abilities in subject areas like mathematics [15,16], some people believe that smartness is about something you inherently possess—that you either have "it" or you do not. Why, then, use the term "smartness," a typically contentious, polarizing term? I do so in order to name and intentionally take back the assertion that not all students can be mathematically intelligent. As have others [12,13,17], I argue here that all students have mathematical smartness, whether they know it yet or not—but that our society, reified by structural inequities that offer certain students access to high-level mathematics thinking—does not currently believe that all students can be mathematically smart. That is to say that I believe that mathematical smartness has to do with accessing one's own mathematical intelligence, mathematical thinking, and as such, mathematical competence. In other words, too many people still believe that students' smartness in mathematics is predetermined by what they show aptitude for—not what they have had access to and what kinds of opportunities they either have or have not had.

I assert that classrooms in which students are supported to move their focus away from valuing speed and accuracy can be spaces in which teachers are "striving toward equity." I use the phrase striving toward equity based on three ideas. First, Cohen [18] asserts that, in equitable classrooms, teachers and students view everyone as competent and capable of learning high-level concepts. Cohen's conviction affirms that mathematical smartness is not narrowly defined by students who answer questions quickly and accurately; rather, Cohen asserts that, in equitable classrooms, all students are known to be capable of understanding high-level concepts. Second, equitable classrooms demonstrate a "fair distribution of opportunities to learn" ([19], p. 1010). Esmonde's construction of equity implies that students in equitable classrooms have access to opportunities to learn. Third, Gutiérrez [20] asserted that we will know we have made progress toward equitable learning opportunities when we diminish differences in access, opportunities, and outcomes based on different groupings of students. I use Cohen's, Esmonde's, and Gutiérrez's interpretations of equity, equitable classrooms, and smartness in order to outline how striving toward equity, particularly in mathematics education, implies ongoing work toward broadened access as well as higher expectations that all students can and will learn advanced mathematical concepts.

This study builds on previous work around the need for mathematics to move beyond valuing speed and accuracy (see, e.g., [11,21,22]). This study also works to contest dominant narratives around mathematical smartness (see, e.g., [3,23]). Previous work has reported on the need for equity-based mathematics teaching to move beyond specific curriculum or practices [24] and to move beyond dominant-culture narratives [3,9,25]. Whereas these scholars largely addressed the ways that teachers can enact these practices while striving toward equity, this piece addresses the nature of smartness from the student perspective by busting the myth around speed and accuracy while featuring the voices of students.

In previous analyses of the larger data set from which these data come, I found that the four student-centered teaching practices mentioned here had fostered mathematical smartness in this high-school Algebra I class [26]. Analyses also revealed that the teacher delegated mathematical authority to her students in particular ways that repositioned them as competent sense-makers [6]. In so doing, I corroborated other empirical evidence (e.g., [2]) that found that while the typically perceived "smart" mathematics student can quickly follow an algorithm to reach a correct solution [13,27], the teaching practices in this classroom pushed on addition ways of being smart, which allowed students to find success, often for the first time. I use this article to explain the students' perspectives. As such, this article is positioned to continue to bust the myth that mathematics, and in particular success in high school algebra, is centered around getting a right answer in the quickest way possible. In order to unpack this idea, I feature one high school Algebra 1 classroom in which I unpack themes that emerged around how students described four classroom practices that are already accepted as important in mathematics education: (1) valuing multiple solution strategies, (2) requiring student explanation and justification, (3) naming students' competence, and (4) valuing collaborative problem solving.

3. Materials and Methods

3.1. Site and Teacher Selection

Race and racism are crucial considerations in mathematics education research that strives toward equity [3,8,28,29]. As American classrooms continue to increase in linguistic, ethnic, and socioeconomic diversity, districts, schools, and teachers who strive for equitable learning opportunities are obliged to use pedagogies that are successful with diverse, heterogeneous populations of students [12,13,18,24]. As such, I chose to situate this study in an urban school in the Pacific Northwest with a racially, ethnically, linguistically, and socioeconomically diverse student population. I intentionally chose to work with a ninth-grade Algebra class because this school (and much of the district and region, at the time) also tracked ninth-graders into Geometry and Algebra II courses, and as such, students who took Algebra in ninth grade were "behind" their peers who took more advanced courses with respect to their mathematics learning.

I selected the teacher, Ms. Martin (all proper names are pseudonyms), because I had worked with her previously through mediated field experiences [7] and because, through my work with her, she told me that she wanted to challenge notions of who could be smart in mathematics and she cared about attending to classroom status [2,6,12,17,18,26,30]. In addition, Ms. Martin described her teaching as being founded on the principles of complex instruction (CI), a pedagogical approach to teaching in which teachers believe in each of their students as smart and capable of engaging in rigorous mathematics [2,4,6,12,13,17,18,26,30]. At the time of the study, Ms. Martin was in her fifth year of teaching; it was also her fifth year of teaching Algebra I at this school.

3.2. Participant Selection

Twenty-eight students took this Algebra I class during the year of the study. Twenty-two of the students identified as girls, and six students identified as boys. (Because demographic information was not available to me, I describe the classroom demographics on the basis of my own observation. Any error or misrepresentation of how students perceived their identities is my error.) Also, 17 of the 28 students identified as Black. (I use the term "Black" instead of African-American or African Immigrant intentionally. Black refers to individuals who are multigenerationally born and raised in the United States. Black is capitalized, whereas white is lowercase in order to disrupt the hegemonic interpretations of dominant U.S. culture.) Five students identified as Asian American, four students identified as white, and two students identified as Latina. When compared with the school's demographics, this class had an overrepresentation of Black students (61% vs. 30%), Latinas (7% vs. 4%), and girls (79% vs. 50%), and an underrepresentation of Asian-American students (18% vs. 41%), whites (14% vs. 40%), and boys (21% vs. 50%). I outline the class's demographics in detail because they were not representative of the make-up of the school's student body. Twenty-four of the 28 students formally agreed to participate in the study.

I invited all students in the class to participate in interviews about their mathematics learning, and 13 of the 28 students participated in at least one interview. Five students participated in interviews throughout the entire study. In this paper, I feature the voices of three students because they are representative of what the larger data says about their experiences in this class as a whole; further, these students completed all interviews and described significant transformations in their perceptions of competence over the course of the study.

Although in this paper I do not attempt to tell the individual storylines for each of the students who are featured, I attend to their voices as a way to identify trends in students' perceptions of mathematical smartness across this class. As such, Table 1 is offered as a reference for the voices of the three students featured: Neesha, Helen, and Jaelyn.

Table 1. Student demographics and self-perceptions of competence.

Student	Grade	Racial-Gender Identity	Initial Self-Perceptions	End-of-Study Perceptions
Neesha	9	Black female	I used to shut down.	Math is one of my favorite classes [31].
Helen	9	white female	I've never had much [mathematics] confidence.	Well now I can explain it to everyone I know [32]!
Jaelyn	10	Black female	Last year, I didn't like to go to class.	I was curious to get to class and see what everybody else's answer was [33].

3.3. Data Collection

I used a qualitative case study approach to offer a thick and rich description [34] of the mathematics perspectives of these Algebra I students. The data collection period took place during the first semester of Ms. Martin's Algebra I class. Ethnographic methods for data collection included (a) audio and video recordings of three semi-structured interviews with each of the three featured students; (i) one individual interview midway through the semester; (ii) one individual interview at the end of the semester; and (iii) one group interview (which other students participated in as well), (b) triangulation of data from (a) with the larger set of data in which I observed 50 class sessions in the first semester and video recorded 26 of them focused on the first and final units, (c) triangulation of data from (a) with student artifacts; including homework and quizzes from each of the 50 observation days, and (d) triangulation of data from (a) with the three semi-structured individual interviews and many informal conversations conducted with the teacher. (See Appendix A for individual and group interview questions.)

The goal of the interviews with students was to understand their experiences in the classroom, and in so doing, to capture their voices around these experiences. I intentionally did not lead with words like "speed" or "accuracy," but instead asked open-ended questions like, "What's it like to be a student in this class?" I did ask questions around smartness, including, "What do you think of when you think of someone being smart in math?" And, "Last week, Ms. Martin said that she is starting to know when students are going to groups and are acting as leaders and are going to make their group members smarter. What did you think about this idea?"

Students were randomly assigned to small groups about every other week, for a total of nine times across the semester. I asked interview questions about their small group experiences, including, for example, "Think of a small group you've worked with so far this year. What was it like to be a member of that group?" Then, I would follow that by asking students to talk about a different small group. This continued until they had described all of their small groups. In each case, I pointed to a diagram with students' names and locations in their group. After students described their experiences in a particular group, I asked the same question about their next group, until, across each of the individual interviews, they had talked about all groups.

3.4. Data Analysis

I started by transcribing and then open coding students' interviews using Studiocode as a way to understand how and when students described mathematical competence and smartness in this classroom [35,36]. (Studiocode is a software program used for video analysis/data coding. Studiocode was a part of Vosaic at the time of this paper submission but will be phased out of Vosaic by December 2019.) I used linguistic microanalysis [34] during a second phase of analysis with the data around perceptions of competence and smartness. I studied the ways students described themselves and others as mathematically competent and paid particular attention to whether and how they described speed or accuracy. I used the third phase of analysis to triangulate interview findings with the fieldnotes and classroom interaction data in search of commonalities between and across conversations about smartness. This third round of analysis is where the four themes these students used to describe mathematical competence emerged, including (1) consistently and unapologetically affording time and space to value multiple solution strategies, (2) belief in mathematical justification and explanation as the goal for demonstrating mastery of mathematical content, (3) valuing mathematically valid ideas from all class members, and (4) valuing collaborative problem solving as a way to help group members, distribute mathematical knowledge, and orient students toward learning with one another. I applied the constant comparison method across these rounds of analysis, in order to analyze segments of interviews alongside episodes of classroom interactions and the teacher's interview in order to find similarities and differences in the students' stories [36]. For example, in Table 1, I shared that at the end of the study Jaelyn said, when talking about a particular mathematics problem, "I was curious to get to class and see what everybody else's answer was." I analyzed the other student interviews to search for cases in which students invoked curiosity toward both the subject of mathematics and work with peers. From this, theme (3) emerged, in which I found that students consistently described valuing others' ideas.

3.5. Researcher Positioning

I share my positioning with respect to my work to be clear that about how who I am is reflected in this study and in the ways that I have interpreted these data. I am a white, middle-class, female secondary mathematics teacher educator and a former high school mathematics teacher. I believe passionately that all students and all people are competent in mathematics and are capable of achieving highly successful outcomes. I believe students who are not currently successful and who do not yet perceive themselves as competent in mathematics have lacked significant opportunities to show how they can be successful. I recognize that many of the students who have not had opportunities to be successful in mathematics have come from non-dominant backgrounds and that there are structural inequities preventing historically marginalized students' successes. As such, I have developed a strong commitment to supporting students to find the ways they are competent and successful in mathematics, and in particular, students who have been historically marginalized. This research is therefore motivated by the search for practices that support students to be successful. In order to examine my own biases, I kept daily fieldnotes of my classroom observations, took notes during interviews, and used the constant comparison method [34] to triangulate findings across the different kinds of data.

As a teacher educator and a researcher, I believe that all teachers and students should "view each student as capable of learning both basic skills and high-level concepts" ([18], p. 4). I support teachers to implement effective discourse and groupwork strategies that attend to ways students perceive their competence and work to increase equitable outcomes. I am interested in making a case for expanding what it means to "do mathematics" and to be competent in mathematics, so that society values more than speed and accuracy in mathematics learning. In other words, I endeavor to support change and to increase access to mathematics for all students. I was driven to conduct this study, in particular, by the desire to unpack teaching practices that strive toward equity and to uncover the counterstories of historically marginalized and underrepresented students and their perceptions of competence.

4. Results: Counterstories to "Get It Quick" and "Get It Right"

4.1. Consistently and Unapologetically Affording Time and Space to Value Multiple Solution Strategies

The students talked consistently about Ms. Martin's expectation to value multiple solution paths in her classroom. When describing what it was like to be a member of the class, Neesha described her belief that students in her class were meant to learn their own and their group members' methods for solving problems:

> So, you do need to be able to explain thoroughly and [in] several different ways ... So [that] everybody can get it. Or at least, if both of you have the right answer, you both have to explain how you got it, to each other. You might not have the same way, but you need to understand both ways [31].

Neesha's explanation that she was required to explain her own method and also to listen to her partner's method illustrates her belief that she was regularly encouraged to share her own method and to learn her group members' ways of working on a problem.

In one interview question, I asked Helen to describe what it was like to be a student in this class. She described her experience learning mathematics this year as significantly different than previous years:

> I know [Ms. Martin] teaches us differently. I know it's more like you're thinking for yourself. And you're thinking outside the box or something. And instead of learning a formula or something, you're learning it in your way, instead of like the structured way, or whatever [32].

Helen's reflection on how math was "different" indicated her belief that learning in this class was "outside the box" because students were learning in their "own way," and they were not learning formulas from memory or by procedure. Helen's comment also illustrates her belief that students should leverage their own strategies when doing mathematics.

The examples from Neesha and Helen are representative of the ways that learning multiple solution paths became a classroom norm. This finding was triangulated with my own observations of how Ms. Martin engaged students in learning Algebra I. For example, the students' first assignment, "The Garden Border" problem, was a group-worthy task [37] in which students had to find at least four different methods for tiling the perimeter of a square garden of an unknown size. (This problem may have been adapted from the Border Problem, [38]. See Appendix B for Ms. Martin's version of "The Garden Border Problem.") Later in the semester, the consistent value Ms. Martin placed on learning one's own and other group members' multiple methods was realized by emphasizing, for example, that when students used a table to determine rate of change and a linear equation for a given set of data, they could orient the table to any set of numbers they wanted. For example, Ms. Martin invited students to adjust x–y tables for rates of change to any set of numbers they wanted, rather than suggesting they use the much more typical $x = -1, 0, 1, 2 \ldots$, I observed some group members choosing to make their independent-variable table (x-table) entries change "by 1 s," others chose to make their table entries change "by 2s," and still others, "by 3s." No matter which spacing students chose for their table's x-values, students were expected to explain their reasoning to their group members, asking them to explain the different ways they chose to look at and utilize writing their tables.

These data revealed that students' belief in their own abilities to see problems in their own ways also fostered the concept of mathematical smartness as valuing multiple methods for solving mathematics problems. Movement away from a focus on mathematics in order to find an answer, and toward mathematics as a means to understand a solution, advanced the mathematical learning of the classroom community. And implicit in valuing multiple solutions for a particular problem is the movement away from quickly getting to one right answer and moving on. In so doing, this finding supports the notion that slowing down to understand not only your own method, but also a classmate's method for solving a problem, is a way to be mathematically smart. Further, because the students talked about thinking for themselves and not memorizing a formula, they pushed the dominant narrative that the goal of mathematics is to quickly solve a problem using an easily-accessible procedure.

4.2. Students' Beliefs in Mathematical Justification and Explanation as the Goal for Demonstrating Mastery of Mathematics Content

Students from this class spent the majority of class time working in their small groups. In a previous study [26], I found that students were randomly assigned to groups every two weeks and that 72% of class time was spent in small groups across the 26 video-recorded days in the first and last units of the semester. I also found that a significant extent of the mathematical justification that took place in this classroom happened as students explained mathematics to one another in small groups [6,26]. In this study, I analyzed what students said about working with their peers. Across all of the small groups the students were assigned to during the semester (nine small groups) the students described each group as developing ways to justify the mathematics to one another. Helen's way of describing her work in one group was typical of the evidence of mathematical justification that I found to be normative in this class. She described,

> We all made our statements, and it just worked out. And it helped. If they said something I didn't understand, I could ask questions. And then they would say, "Oh no, it's this . . . " [32].

Across the interviews, the students described these group interactions as consistently going beyond quickly confirming a correct answer. Because Helen shared at her first interview that she had previously "never had much confidence" in her mathematics abilities, the fact that she said she could ask questions to her group and "it just worked out," pointed to the ways that students described groupwork as focusing beyond valuing speed and accuracy.

Shuffle Quizzes happened about once a unit during the semester of the study. Shuffle Quizzes were developed by the teachers of Railside High School (See [2,12,13,17].) In this class, the teacher would come over, shuffle the papers, and hold the student whose paper landed on top accountable for sharing what the group understood. If the student gave a description that the teacher found satisfactory, the group could move on. If not, the teacher would ask the group to keep discussing, and she would come back and ask that same student to describe the group's understanding again. Shuffle Quizzes were opportunities for one student to tell the teacher what the group had learned before being allowed to move on to the next task or problem. In the first interview, Jaelyn mentioned that she found Shuffle Quizzes stressful. I followed up in her last interview, asking, "So, earlier when we were talking, both in the group interview and then you mentioned a little bit—There are some stresses that go along with the Shuffle Quiz or a Group Quiz—Do you think that the stresses make it not worth it? Or do you think that there are some benefits?" Jaelyn replied,

> Yeah, there's some benefits. Because if I'm in a group with somebody, and I know they don't get it, and then like, if the Shuffle Quiz comes to them, and they'll be like . . . Oh, you know, a little like . . . Because . . . I don't know, I feel like I can relate to it, because I was that person last year. So, if I see somebody struggling, I can be like, "Hey . . . " You know, when she [Ms. Martin] walks away and she's giving us another chance she'll be like, "Okay, what exactly didn't you get?" "So, you know, when she comes back around, and she picks you again, will you be able to explain it?" [33].

Jaelyn's description of Shuffle Quizzes indicated the ways that she experienced the mathematical accountability an explanation that Shuffle Quizzes required [6,12,13,17]. In particular, Jaelyn described helping her group members to engage in mathematical justification by checking in with them about whether they are ready to explain on behalf of their group. Jaelyn also said that she should ask her group members, "Okay, what exactly didn't you get?" if Ms. Martin suggested they needed to talk more about the mathematics so that they would each be ready to justify their mathematical thinking when Ms. Martin asked them to.

The students' descriptions of group members' talk supported advancing one another's understanding of the mathematics. Helen explained how students would help and correct one another's explanations. Jaelyn described how group members engaged in justification while preparing for a Shuffle Quiz. What separates these examples from the typical expectations for students to use mathematical justification is the ways in which each of these students described unequivocally valuing explanations over answers as a way of demonstrating they understood mathematics in this classroom. Further, a value on explanation over answers implies these students worked against speed, by slowing down to understand a mathematical answer through classroom activities such as small group work and Shuffle Quizzes.

4.3. Mathematically Valid Ideas Come from Everyone

I found that the students from this study also described how they knew their mathematical thinking was valid. Neesha described how Ms. Martin told her that she knew that she was capable of finding a mathematically correct solution, and then validated her finding that solution, even while being physically located away from the group.

> Ms. Martin, she'll come toward you and talk with you. And she won't give you the answer; she'll walk away. But even if she walks away, you can tell she's still listening, just by her posture. Like, she might be away, but she's kinda slightly towards you, so she can definitely still hear your group's conversation. And then, when you figure it out, she's like, "I knew you would," walking by . . . She just does things like that. So, you can tell that she's paying attention to almost every group at the same time [31].

Neesha's reflection on Ms. Martin indicated her belief that Ms. Martin not only paid attention to how students solved mathematics problems, but she subsequently identified students' smartness by allowing them to explain the mathematics to one another, then to follow up with statements like "I knew you would." Helen said she noticed Ms. Martin tell her ways she was smart in mathematics by "not just complimenting her" but by drawing attention to what she understood:

> Ms. Martin points out what I know, and it's not just like she's complimenting me all the time or something. But it's like, she's more encouraging, in that she knows that I know how to do the problem. [She knows] I know enough information to put it all together [32].

Helen perception of Ms. Martin's support explains how Ms. Martin assigned her competence [13,17] for solving mathematics problems in her own way.

Ms. Martin assigned competence to Jaelyn by suggesting her group members use her as a mathematics resource. Ms. Martin said, "Hey [student], do you ever *bug* Jaelyn? Because I know *she* knows how to do it!" Because Jaelyn was a sophomore student repeating Algebra I, asking other students to "bug" her was a particularly powerful move to assign her mathematical competence. In this case, Ms. Martin asked the other group member to "bug Jaelyn" so they could learn her method for simplifying an algebraic equation using Lab Gear. As a part of assigning Jaelyn competence for her mathematical explanations, Ms. Martin oriented her group to learn Jaelyn's method. During her second individual interview, Jaelyn and I looked together at the video excerpt in which Ms. Martin asked her classmates to "bug" her, and I asked her how watching it made her feel. Jaelyn responded, "Like the leader in the group!" This example is a representative example of how Jaelyn's new perception of mathematical competence positioned her to have increased access to creating and leading mathematical ideas, which fostered her smartness and gave her classmates increased access to her mathematical ideas.

I found that Ms. Martin did not just call on students who had correct answers. Instead, she regularly found opportunities to tell students who had not previously had success in mathematics how they were mathematically smart. In spite of previous negative experiences in mathematics, this meant that Neesha, Helen, and Jaelyn each described the different ways Ms. Martin featured their algebraic competence throughout the semester. In turn, each of the three students described improving her perceptions of mathematical competence over the course of only a few months in this class.

4.4. Valuing Collaborative Problem Solving as a Way to Help Group Members, Distribute Mathematical Knowledge, and Orient Students toward Learning with One Another

Neesha's first individual interview took place about eight weeks into the school year, by which point Ms. Martin had randomly assigned her to four different small groups for a total of about 30 h of instruction. During that interview, I asked Neesha to describe her class. She responded that she (already) perceived her classmates to be hard workers, willing to learn, positive, and capable of doing the mathematics:

> I really like our class ... I feel like ... they're really hard workers, or, if they're not hard workers, then they'll ask a lot [of questions], so they can get the answers, and they're willing to learn. Ms. Martin knows that we're all talking. And I feel like that all talking thing really helps, like she wants to make sure everybody gets it. ... I feel like our class is extremely positive. [The students] seem to get the problems, and they can go up on the board and do the problems and get the right answer [31].

Neesha's perspective of collaboration this early in the year is particularly striking because her comments emerged from the context of a class full of students who had not had prior success with mathematics and whose peers were often taking Geometry or Algebra 2. Because this was a typical response of students in this class, Neesha's perspective provided evidence for the ways that Ms. Martin developed a classroom in which that being a mathematician was about collaboration, willingness to learn, and believing in each group member.

When talking about how she worked with others, Neesha described her mathematics class as a community who was helping one another, "It's a lot more community, in this class at least. And just really, like, helpful, with everybody. Like, they wanna make sure everybody gets it." When I asked Neesha to expand on how working with her group members was helpful, she replied,

> Talking out loud, or thinking out loud really, it like helps you ... so then you know. Cuz' that's something I struggle for, because I could do it on paper, but then, when I have to present to somebody else, the table group or in front of the class, it gets a lot harder. Cuz' you're like, "Wait, how did I do that, in the first place?" [31].

Neesha reflected that "thinking out loud" with her group was helpful. Her comments reveal how she was oriented to work with her classmates and how groupwork allowed everyone to make sense of the mathematics in a way that working it out alone ("on paper") would not, thus distributing the mathematical knowledge across group members.

When Helen talked about her experience working in a group in this classroom, she compared her experience to her previous mathematics class:

> This year, I think I'm a better math student. Because the other math environment I was in was more independent, and you had to get things on your own. This year, I think it's the thing they do in the math department: We're sitting in a group, and I think that really helps me [32].

Helen concluded that groupwork in Ms. Martin's class helped make her a better student because she did not have to "get things" on her own. Jaelyn also talked about how Ms. Martin oriented students to help group members, saying, "Ms. Martin will make everyone in your group help you out and stuff." Jaelyn also talked excitedly about how receiving her first "A" on a mathematics test was related to her work in groups.

> Once I got the hang of it down, for the first time in years, when we took our linear equations test, I got an A on it. And then I was like, running home, and I was like telling my mom, because I never get A's on math tests [33].

In response to my question, "What do you think has to happen for you to continue getting A's on math tests?" Jaelyn again talked about her group: "As a learner, you have to like ... if you don't understand it, this is where a group comes in. You should feel comfortable asking your group." I later read that quote back to her during another interview and I asked her to comment further. Jaelyn replied with another example that showed how working together was about both distributing mathematics knowledge and learning it better herself:

> Even like, yesterday, I think it was yesterday, I was like, "Ms. Martin, I give up!" and then Ms. Martin was like, "No, no, no! You don't." And then to Qianna, Ms. Martin was like, "Qianna, help her. Explain!" You know. "Explain it to her!" And she was like explaining. And then I was like, "Oh, okay, I get it now!" [33].

The more I asked Jaelyn to reflect on the connection between "getting A's" and groupwork, the more examples she offered, linking group members to mathematics understanding.

The examples about collaborative problem solving from Neesha, Helen, and Jaelyn illustrate that working together in their class went beyond quickly moving toward getting a correct solution. Neesha said she believed students used groupwork time to "make sure everyone gets it" and to prepare for sharing their thinking in front of the class, while Helen reported working in a group was helpful to her understanding, and Jaelyn said she believed helping was expected: "Ms. Martin will make everyone in your group help you." These examples illustrate the ways in which collaborative problem solving went beyond an expectation to help group members who did not understand. Each student described the ways groupwork can strive toward equity, by coming to value being oriented to work with others in this class in order to distribute mathematics knowledge across the class.

5. Discussion

In this paper, I argued that while we, as a mathematics education community, have made strides in pushing mathematics classrooms to consistently and unapologetically value student-centered learning, we continue to have room to develop our practices in ways that support all students to be successful. As such, the importance of placing these findings in a racially, ethnically and linguistically diverse classroom cannot be overstated. The findings here demonstrate that students who face ongoing marginalization and minoritization in mathematics, in this case, girls and Black girls, can take on the robust myth that mathematical smartness is fundamentally housed inside speed and accuracy. This study demonstrates that these students experienced mathematical smartness through their interactions with Ms. Martin, their classmates, and with mathematics content.

First, I found that Neesha, Helen, and Jaelyn experienced valuing multiple solution paths [39]. As such, Neesha, Helen, and Jaelyn consistently and unapologetically explained opportunities for affording time and space to value multiple solution strategies, which pushes against the dominant narrative around finding the quickest answer. Second, students began to value mathematical explanation and justification [40]. By describing mathematical justification and explanation as the expectation for demonstrating mastery, moving beyond the dominant expectation that the goal of mathematical thinking is to find an answer. Third, these students experienced Ms. Martin assigning them competence [6,12,13,17,26] to the students who had not previously found successes. Assigning competence allowed students to validate when mathematical contributions were correct [2,10,41]. Fourth, the students believed it was their job to help one another. This belief also empowered groups to take on the responsibility to use mathematical sense-making to solve problems and evaluate the reasonableness of their solutions. Justifying mathematics further oriented students to being accountable to their group members [42].

I shared an analysis of students' experiences with mathematical smartness in this classroom in order to reveal the ways that these four themes mediated their perceptions of and performance in high school Algebra I. Through their work with Ms. Martin—Neesha, Helen, and Jaelyn, students who had not previously found themselves mathematically competent—each came to believe and then found themselves doing well in mathematics. Neesha and Helen observed Ms. Martin surveying groups and pointing out their accomplishments when they solved a particular problem in their own ways. After failing Algebra I the year before, Jaelyn was positioned to be a leader. Each student, in her own way, noticed that she was told she was capable of working on authentic mathematics, and then found herself doing just that.

Each of the ways that these students described their experiences with mathematical smartness is also fundamental to the kinds of pedagogical practices aligned with using complex instruction (CI). (CI is a cooperative learning pedagogy developed by Elizabeth Cohen and Rachel Lotan and has been linked to equitable teaching and learning opportunities.) While many mathematics curricula and pedagogical practice still conflate procedural fluency with speed, Ms. Martin's implementation of CI made space for students to engage in mathematical smartness in ways that played to their strengths. Honoring students' multiple solution paths allowed students to decide which solution method to use [39]. Assigning competence allowed them to validate when mathematical contributions were correct [2]. Requiring students to help one another also empowered groups to take on the responsibility to use mathematical sense-making to solve problems and evaluate the reasonableness of their solutions. Justifying the mathematics oriented the students to being accountable to their group [42]

Neesha, Helen, and Jaelyn reveal how these pedagogical practices were experienced in particular ways in their classrooms over the course of the semester. The themes contribute to an understanding of the ways that students can play a role in cultivating a classroom environment in which being mathematically smart is about more than being quick and being correct.

The construction of mathematical smartness in this classroom has important implications for students' access to smartness. Fostering students' mathematical smartness is about increasing the presence of and access to students' mathematically competent contributions. Pushing against and busting the myth that speed and accuracy are the only way to value mathematical smartness is also about moving classrooms to be spaces in which students are treated as, and act as, mathematicians. The student experiences of Neeesha, Helen, and Jaelyn around increased access to mathematical smartness moved the classroom community to strive toward equity.

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