

Growing mathematics teachers: Pre-service primary teachers' relationships with mathematics

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Received: 22 April 2018/ Accepted: 30 September 2018
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The 83 primary pre-service teachers beginning their three-year programme in 2015 at the University of Otago were participants in the Growing Mathematics Teachers' Project. In this longitudinal research, the pre-service teachers' mathematical content knowledge was examined, as well as other affective aspects of their relationships with mathematics, including beliefs and feelings about mathematics and teaching, and associated identities. At the beginning of their programme, 40% of the pre-service teachers did not meet the required standard for content knowledge, and other aspects of their relationships with mathematics were negative. Their programme was adjusted by supporting their content knowledge, teaching affective aspects explicitly, and positive role modelling. During their programme, the pre-service teachers' relationships with mathematics generally became more positive as their teaching identity grew, although their confidence remained somewhat conditional on their content knowledge and the age of the students they were placed with on professional experience.

Keywords pre-service teachers • affect • mathematical knowledge • affect • growth mindset • longitudinal

Introduction

In 2015, only 40% of pre-service teachers beginning their three-year undergraduate degree at the University of Otago, New Zealand, demonstrated they had the required content knowledge for teaching primary level mathematics. Furthermore, of those surveyed, only one quarter liked the subject of mathematics, and only 37% felt confident to teach mathematics.

These results were the catalyst for the *Growing Mathematics Teachers* project, where the lecturers in the programme (the authors) sought to support pre-service teachers to grow positive *relationships with mathematics* – the unique, dynamic connections between the pre-service teacher and mathematics. Various aspects of pre-service teachers' relationships with mathematics were tracked for three years from 2015 to 2017, including mathematical content knowledge, beliefs about the importance and nature of mathematics, feelings about mathematics and mathematics teaching, and identities related to becoming a mathematics teacher. A number of interventions were specifically designed to support the pre-service teachers' relationships with mathematics,

based on the on-going data collection. This project provided us with a structure to reflect on our own practices with the goal of adopting these interventions into our programme.

In this paper, we define and explain the meanings of the constructs within this project, and describe the project in some detail. Then we explore the pre-service teachers' relationships with mathematics, both at a cohort level and through the mathematical journeys of three pre-service teachers.

Pre-service teachers' relationships with mathematics

In our broad conception of an individual's relationship with mathematics, we include both mathematical knowledge and affective aspects. *Affect* is usually related to an individual's feelings and emotions (McLeod, 1992) but research in the affective domain can also include constructs such as anxiety, beliefs, efficacy, and identities. The experience of affect is an integral part of mathematical learning (Op 't Eynde, De Corte & Verschaffel, 2006). We include consideration of a range of affective factors, rather than singular notions of affect, because of the complex interactions between them (Grootenboer & Marshman, 2016). For example, there is evidence that an individual's perceptions of their mathematical ability (a view or belief about mathematics) has an impact on other affective factors related to mathematics and mathematics teaching, such as their confidence to teach.

We were informed by Ingram's (2011, 2015) longitudinal research, which merged affective and identity frameworks to explore secondary school students' relationships with mathematics. Using the students' perspectives gathered over two years and a grounded theory approach to analysis, Ingram (2011, 2015) developed a model of school students' relationships with mathematics with five elements: mathematical knowledge; views of mathematics (similar to beliefs); feelings about the subject; identities; and, mathematical engagement. These components interacted to provide a context for the ways students engaged in mathematics. They developed from students' experiences with mathematics in social environments as they consciously or unconsciously negotiated between new learning experiences and their relationship with the subject. Students with positive relationships with mathematics were more likely to thrive in mathematics and were therefore more likely to continue in the subject when it became non-compulsory.

In pre-service teacher research, there are similar broad conceptions of pre-service teachers' relationships with mathematics that include both mathematical knowledge and a range of affective factors. For example, Grootenboer and Jorgensen (2008) included knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions related to mathematics in their conception of pre-service teachers' mathematical identities. We were informed by this research, and notions of individuals having internal structures that relate to mathematics, variously described as 'self-systems' (Malmivuori, 2006) or 'mathematical identity' (Op 't Eynde, De Corte & Verschaffel, 2006). We view a pre-service teacher as developing a relationship with mathematics that includes five elements.

- **Mathematical knowledge for teaching.** Includes pedagogical content knowledge, curricular knowledge, and mathematical content knowledge – the facts, symbols, concepts and rules that constitute mathematics, including the strategies for accessing and using knowledge to solve problems (Op 't Eynde et al. 2006; Shulman, 1986).
- **Beliefs about mathematics.** Incorporates an individual's personal, internal and shared subjective conceptions about the nature of mathematics, mathematics teaching and learning, about themselves in relation to mathematics, and about the context (Ingram, 2011; Op 't Eynde et al. 2006).

- **Feelings.** An individual's overall feelings about the subject of mathematics, similar to global affects (De Bellis & Goldin, 2006) or macro-feelings (Ingram, 2011). These feelings contribute to the context within which an individual engages in a specific mathematical activity. When an individual has negative feelings for the subject of mathematics, they are more likely to experience negative affect during each mathematical situation.
- **Identities.** Pre-service teachers have unique and dynamic identities related to their becoming mathematics teachers, which develop through their interactions with others, and through their experiences of success and failure (Ingram, 2011; Grootenboer & Jorgensen, 2011).
- **Habits of engagement.** These habitual ways of doing mathematics include affective pathways and engagement skills, including perseverance, independence, cooperation and reflection (DeBellis & Goldin, 2006; Ingram, 2011).

We now focus on pre-service teacher research related to mathematical content knowledge, and mathematical affect, and then outline how other researchers and teacher educators sought to improve their pre-service teachers' relationships with mathematics. We include research that sought to understand singular aspects, as well broader conceptions of the relationship between pre-service teachers and mathematics.

Mathematics content knowledge

The knowledge required for teaching is classified by Shulman (1986) as subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. We acknowledge the important connections between these, but focus on mathematical content knowledge in this paper.

Shulman defines subject matter content knowledge as "the amount and organisation of knowledge per se in the mind of the teacher" (p. 9). Mason, Stephens and Watson (2009) further describe this content knowledge as a fusion of conceptual and procedural knowledge. Conceptual knowledge is a knowledge of mathematics in which representations are connected in multiple ways, and procedural knowledge is knowledge of a number of actions within a procedure with only minimal connections between the actions (Hiebert & Carpenter, 1992). Although mastering procedures is important in mathematics, it is also necessary to appreciate the mathematical structures that make the procedures effective and which provide criteria for appropriateness (Mason et al. 2009). Both conceptual and procedural knowledge are needed for solving problems effectively, particularly when the problems require interpretation of their solution (Homes, 2012).

In New Zealand, the mathematics content knowledge graduating pre-service teachers are required to demonstrate must be appropriate to learning areas and the students they will teach (NZTC, 2010). Knowledge of number concepts and operations is particularly important because number constitutes such a large part of the primary school mathematics curriculum. At Level 4 of the New Zealand Curriculum there is an emphasis on using and operating on fractions, decimals and percentages and extending understanding of place value to decimals. Solving non-routine problems involving rational number is therefore an indicator of the content knowledge required for becoming a primary school teacher and is consistent with Delaney, Ball, Hill, Schilling and Zopf's (2008) description of common content knowledge.

Primary teachers need to have sufficient knowledge of mathematical content to meet the challenging demands of teaching mathematics (Young-Loveridge, Bicknell & Mills, 2012), partly because effective teachers need to have a sound grasp of knowledge to assist students in developing mathematically grounded understandings (Anthony & Walshaw, 2009), and partly because of the relationship of knowledge to affective factors (Ingram, 2011, 2015).

Given the importance of mathematical knowledge to teaching, a continuing concern is the level of knowledge found in beginning pre-service primary teachers. Young-Loveridge et al. (2012), for example, described the mathematics knowledge of the 319 pre-service primary teachers in her research as “weak” (p. 44). Many participants did not use knowledge of number properties to find common factors, and many were unable to use calculation strategies based on number sense to add common fractions, or convert a fraction to a percentage. Over one third of participants added both numerators and denominators for addition of common fractions. This finding is not isolated to number knowledge or strategies. Shirvani (2015), for example, noted that, when preservice teachers sat a measurement assessment designed for 11-year old students, on average, the majority of preservice teachers achieved failing grades.

Affective factors

There is a body of research relating to pre-service primary teachers’ affect, especially in the areas of emotions (e.g., Coppola, Di Martino, Moolo, Pacelli & Sabena, 2013), anxiety (Gresham, 2008), beliefs (e.g., Beswick, 2006) and attitudes (e.g., Young-Loveridge et al. 2012). It can sometimes be difficult to distinguish between constructs in this body of research because of a lack of theorising and definitions (Ingram, 2017). In general, however, pre-service teachers often experience negative affect towards mathematics, which is linked to their past experiences and their future perspectives (Coppola et al. 2013).

In New Zealand, Young-Loveridge et al. (2012) found only around half of their pre-service teachers liked mathematics, and in South Africa, Moodley, Adendorff and Pather (2015) found just over half had positive attitudes in relation to their future roles as mathematics teachers. In Coppola et al.’s (2013) study of 90 pre-service primary teachers, more than three-quarters of them associated mathematics with negative emotions, such as fear, anxiety, anger, panic or unease.

Gresham (2008) studied the mathematics anxiety and mathematics teaching efficacy of 156 primary pre-service teachers, where *mathematics anxiety* was defined as an irrational dread of mathematics that interferes with solving mathematical problems in a variety of situations. They found that low mathematics anxiety is related to higher mathematics teacher efficacy, where *mathematics teaching efficacy* is the belief an individual has about his or her capacity to teach mathematics effectively (Newton, Leonard, Evans & Eastburn, 2012).

We know that how a teacher feels about mathematics, and their efficacy beliefs have an impact on their teaching. Using data from 167 Year 4 and 160 Year 8 mathematics teachers from 181 schools who participated in the National Monitoring Study of Student Achievement 2013 Mathematics and Statistics assessment, Ingram, Asil and Berg (2017) determined that teachers who had different levels of mathematics teaching efficacy, had unique pedagogical practices. The pedagogies of teachers with high efficacy were consistent with pedagogical practices known to be effective in mathematics teaching (Anthony & Walshaw, 2009). There was also some evidence that anxious teachers rely heavily on rules and algorithms as opposed to using richer contexts for teaching mathematics. Furthermore, mathematically anxious teachers may also perpetuate the growth of mathematics anxiety in their children through their classroom practices (Sloan, 2010), and affect the quality of students’ learning (Hannula, Liljedahl, Kaasila & Rosken, 2007)

Interventions supporting pre-service teachers

The challenge for pre-service teacher mathematics education programmes is to develop all aspects of pre-service teachers’ mathematical knowledge, as well as addressing affective factors (Beswick, 2006). A heartening finding of Coppola et al.’s (2015) study is that pre-service teachers want to reconstruct their relationship with mathematics and mathematics teaching, highlighting

the important role teacher educators have in supporting pre-service teachers to take on this challenge.

From our review of the relevant literature, there has surprisingly been only a small body of research on what interventions can improve pre-service teachers' mathematical content knowledge. Diagnostic assessment is an important starting point for pre-service teachers to appreciate their foundation knowledge and make decisions about their learning needs (Ryan & McCrae, 2005/2006). Following diagnostic assessment, growth in pre-service teachers' mathematics content knowledge is intended to result from the provision of mathematics methods courses. However, assisting pre-service teachers to develop their mathematics content knowledge within the constraints of university-based teacher education is a challenging task, particularly so because of time constraints within methods courses. Blömeke, Suhl, Kaiser, and Döhrmann (2012) reported results from 14,000 pre-service teachers in 500 teacher education programmes across 16 countries who participated in the Teacher Education and Development Study in Mathematics (TEDS-M). They found that opportunities to learn in mathematics courses and mathematics methods courses were both significant determinants for developing mathematics content knowledge. However, opportunities to learn taken in mathematics courses explained a substantial proportion of the variance between programmes whereas the proportion was relatively low in the case of opportunities to learn taken in mathematics methods courses. Research-based learning did not have a significant effect on the acquisition of mathematics content knowledge.

There are a variety of interventions described in research designed to improve affective aspects of pre-service teachers' relationships with mathematics. Sometimes it is assumed that affect will improve through participation in a mathematics methods courses (Sloan, 2010) or by increasing content knowledge. Some interventions, however, are designed to explicitly target an affective aspect, such as beliefs or anxiety.

For example, the key aim of an intervention by Gill, Ashton, and Algina (2004) was to move pre-service teachers' beliefs about mathematical learning from notions of passively learning rules and procedures, towards constructing understanding through active problem-solving. In this study, the intervention was designed to both help pre-service teachers develop an awareness of their underlying beliefs, and also work to challenge these beliefs through the use of "refutational texts" (Gill et al. 2004, p. 181).

Also using texts, an approach used for breaking down mathematics anxiety in pre-service teachers is bibliotherapy (e.g., Wilson, 2006). Bibliotherapy requires the pre-service teachers to reflect on an event or series of events that may have contributed to an emotional response to mathematics and uses guided readings and discussion as part of the reflection process to break down the emotional barriers that arise when participating in mathematics.

Mentoring pre-service teachers as a planned and intentional process can also positively impact on pre-service teachers' feelings about mathematics. "Exposure to best practice in the teaching of mathematics by supervising teachers as well as support and opportunities to lead mathematics lessons in a structured, planned and non-threatening environment, is essential for pre-service teachers" (Perkins, 2015, p. 502).

White, Way, Perry, and Southwell (2006) highlighted the importance of supporting both pre-service teachers' affective development and their mathematical knowledge. Pre-service teachers need to understand their own beliefs, feelings and practices, and these should be made explicit and examined with a pre-service teacher education programme. However, "positive attitudes are necessary, but not sufficient" (p. 47), and teachers need to know their mathematical content as well.

Methodology

As described earlier, the catalyst for this project was the achievement of our primary pre-service teachers' in a mathematics assessment at the beginning of their three-year programme in 2015, and evidence of their negative affective responses to mathematics. Informed by the literature, we knew this had the potential to affect their learning and teaching, both during their programme and when they began their professional lives as teachers.

The *Growing Mathematics Project* is a combination of getting to know our pre-service teachers and their relationships with mathematics, monitoring aspects of those relationships throughout their programme, and explicitly addressing some of these aspects with a variety of interventions in addition to a full programme of teaching and learning. We were guided by three research questions:

1. What was the nature of our pre-service teachers' relationships with mathematics?
2. How did these relationships change over their three-year primary pre-service programme?
3. What were the pre-service teachers' perceptions of our interventions designed to support their growth as teachers of mathematics?

Participants

The participants were the 83 pre-service teachers who began the Bachelor of Teaching programme at the University of Otago, New Zealand at the beginning of 2015. Thirteen pre-service teachers were male and 70 were female. Eighteen pre-service teachers were aged over 25 years. Seventy pre-service teachers were New Zealand European, and six identified as Maori. Data were collected from pre-service teachers until they graduated or left the programme. There were 60 pre-service teachers who graduated. Of the 23 pre-service teachers who did not, two pre-service teachers were scheduled to complete the programme in 2018, six pre-service teachers changed programmes, and fifteen withdrew from the university.

This cohort of pre-service teachers had unique relationships with mathematics with some commonalities, and these relationships were explored first across the cohort. They provided a context to examine the relationships with mathematics of three individuals in more detail. Maggie, George and Sonya were chosen as case studies as they represented the range of ages and a variety of relationships with mathematics.

Interventions

The pre-service primary teachers engaged in their normal programme, which grew their pedagogical content knowledge and curricula knowledge. Further to this, the pre-service teachers were supported in their growth as mathematics teachers by explicit interventions designed to grow their mathematical content knowledge, and affective aspects of their relationship with mathematics.

The main theme of the Growing Mathematics Teachers project was that the authors consciously adopted and enacted a growth perspective in their teaching and in the programme administration. We emphasised and were explicit about having a growth perspective throughout the mathematics programme. In conversations, in the terminology used in our course outlines, our teaching and our feedback on teaching, we consciously adopted growth language, rather than a deficit language. This growth perspective originates from the work of Dweck (1999). She presented two ways intelligence is perceived: as an entity, when intelligence is believed to be a fixed trait that cannot be changed; and incremental, when it is believed that intelligence can be

cultivated or grown through learning. People with an incremental view believe intelligence can be increased through effort and guidance, and even if they have low confidence in their intelligence, they thrive on challenge, throwing themselves wholeheartedly into difficult tasks – and sticking with them. A growth perspective challenges the belief that intelligence is fixed and rather posits the view anyone can learn mathematics. Ryan and William (2011) promoted the use of a growth model for pre-service teachers to reflect on their content knowledge.

Interventions related to mathematics content knowledge. Pre-service teachers' mathematical content knowledge is usually viewed from a deficit perspective (Ryan & William, 2011). Rather, our pre-service teachers were considered to have foundation content knowledge that could be built on (Linsell & Anakin, 2013; Anakin & Linsell, 2014). In this growth model all pre-service teachers can improve their content knowledge and have greater agency in determining this growth.

A robust and adaptive assessment was given to the pre-service teachers at the beginning of their programme, with the explicit purpose they could use it to decide their own next steps for learning. This was the Tertiary Education Commission's (TEC) literacy and numeracy for adults' online assessment tool (Tertiary Education Commission, 2010), described further below. The pre-service teachers were later reassessed if they did not meet the standard. Pre-service teachers were provided with feedback immediately following their assessment and their possible next learning steps discussed with them. For example, we talked about not reaching a standard yet in the mathematics content assessment, rather than failing. We also asked all pre-service teachers to set goals related to mathematics, even for those pre-service teachers who had achieved the standard.

The nature and extent of the support they received was differentiated depending on the support required. The pre-service teachers who had met the target were encouraged to set goals for improving their mathematics content knowledge. Those close to meeting the standard were offered peer tutoring to help them before they had another attempt at the assessment. For those pre-service teachers, far away from the standard, we recommended that they enrol in a mathematics content course, Essential Mathematics for Teaching (EMAT), given by one of the lecturers. We further supported all pre-service teachers' mathematical content knowledge growth with content-based starters given within the pedagogy course in their first year, and we gave them access to content-based mathematics websites. For example, final year pre-service teachers completed online modules on e-ako, an online professional learning tool based on the New Zealand curriculum designed to support teachers in mathematical content and pedagogy. The pre-service teachers could choose modules, but were required to accumulate at least 1500 points of modules over the year.

Interventions related to affective aspects. Within their second year, we explicitly taught aspects of relationships with mathematics. Pre-service teachers were made aware of their own relationships with mathematics when we asked them to describe mathematics using metaphors, drawings of mathematicians and personal journey graphs. These served both as a data collection exercise and to support the pre-service teachers to understand their own relationships with mathematics, and are described later.

In mathematics education, metaphors have been used to gather pre-service teachers' affective views by Buerk (1996), Miller-Reilly (2006) and Ingram (2011). Metaphors are a useful alternative to interviews for collecting rich data. Pre-service teachers were asked to compare mathematics to everyday objects such as buildings or food. The verbal prompts used were adapted from Buerk's (1996) protocol for the collection of metaphors.

Pre-service teachers' views of mathematics and mathematicians were further captured through their drawings of mathematicians. Adapted from the work of Picker and Berry (2000)

and Ingram (2011), the pre-service teachers were given a blank sheet of paper and asked to draw a mathematician at work.

The pre-service teachers reflected on their journeys through mathematics by completing personal journey graphs. These were adapted from the work of Anderson (2005) and were previously used by Ingram (2011) to support secondary school students to describe how their relationships with mathematics had changed over time. The personal journey graph provided an opportunity for pre-service teachers to highlight significant experiences that had affected them. At the beginning of their second year, the pre-service teachers graphed their feelings about mathematics against the axis labels 'very good', 'neutral', and 'very bad'. They were asked to write as much information on the graph as they could to identify the reasons for any positioning or change in that positioning. These were given back to the pre-service teachers at the end of their third year to annotate.

Finally, currently practising teachers were invited as guest speakers to address the third-year pre-service teachers. These positive role models supported the pre-service teachers to build aspects of their identity on their journey towards becoming a teacher of mathematics. Furthermore, they made links between the research literature and practice. For example, one teacher presented a rationale of why she used heterogenous groupings, materials and rich tasks with her class by connecting with the relevant literature.

Research methodology and tools

To capture the pre-service teachers' journeys, qualitative research was undertaken. The inductive and interpretative practices (Denzin & Lincoln, 2005) of qualitative research allowed pre-service teachers' journeys to become more visible. Importantly, apart from their assessment results in content and pedagogy, the data is from the pre-service teachers' perspective. It was the meanings pre-service teachers took from a situation or experience that were important. The research was inductive in the sense that decisions were made about interventions in the programme because of pre-service teachers' responses and assessment results.

The affective domain has unique methodological issues and limitations because of its complexity, the ethical issues surrounding its personal nature, and the high level of inference required (Ingram, 2015). Often a research tool only gives glimpses of a moment in time and researchers rely on the participants' willingness or ability to describe their perceptions. Having length, breadth and depth in affective research is therefore important to capture the complexity of affective issues (Ingram, 2011) as "no single method can grasp all the subtle variations in ongoing human experience" (Denzin & Lincoln, 2005, p. 21). This research is longitudinal because we sought to find out how the pre-service teachers' relationships with mathematics changed over time. The data collection was broad because of the multiple data collection tools needed to capture the range of responses. This research has depth because it is descriptive case study research, where a *case study* examines individuals events, activities and processes in detail (Stake, 2005). Through the stories the pre-service teachers told, they describe their views of reality (Baxter & Jack, 2008) and we use these stories to facilitate our understanding of the research questions.

We first examined the cohort as a whole, and then three representative individuals have been analysed as separate case studies. Understanding of the cohort as a whole provided a context for understanding each individual. As they were operating within the same context, the cohort can be considered a single case, however the individual stories highlight that each of the pre-service teacher's relationships with mathematics were unique.

Two tools were employed to assess the pre-service teachers' mathematics content knowledge, with both tools being used to appraise whether pre-service teachers could demonstrate the required content knowledge for teaching in primary schools in New Zealand. Furthermore, a

range of tools were used to capture other affective aspects of pre-service teachers' relationships with mathematics.

In the TEC assessment, each pre-service teacher was asked 30 questions drawn from a large bank of questions on number knowledge, number strategies and measurement in meaningful contexts. We regard this content as being essential conceptual knowledge required for teaching children in the primary sector. Our threshold score requires pre-service teachers to achieve Step 6 (around 695 points), which included requirements for them to solve addition and subtraction problems involving fractions; solve multiplication or division problems with decimals, fractions and percentages; use multiplication and division strategies to solve problems that involve proportions, ratios and rates; and know the sequences of integers, fractions, decimals and percentages, forwards and backwards, from any given number. Full details of the progressions of requirements at each step are given on the TEC website <http://literacyandnumeracyforadults.com/The-Learning-Progressions>. The assessment tool adapts to whether a pre-service teacher is answering questions correctly or incorrectly. The software draws upon its large bank of questions and continuously adjusts the difficulty of questions presented to the pre-service teacher, until they respond correctly 70% of the time to questions at that level of numeracy. Therefore, it presents questions that are neither too hard nor too easy and is able to estimate the achievement level of each pre-service teacher and includes a measure of the uncertainty of the estimate. Linsell and Anakin (2013) have shown that the TEC tool's assessment of content knowledge is consistent with written exams previously used at the University of Otago College of Education, which also made use of meaningful contexts.

The other tool used to assess content knowledge was a set of three tests employed during the course EMAT. These online tests required pre-service teachers to demonstrate mastery of the topics of measurement, whole number and rational number by scoring above 75% on each of three tests. We attempted to reduce test anxiety by permitting pre-service teachers multiple attempts at parallel versions of each test and emphasised that the tests were opportunities for pre-service teachers to demonstrate that they had acquired the knowledge, rather than being hurdles to jump over. Questions in the EMAT tests were comparable in level to the TEC tool.

The tools used in second year (the personal journey graph, the metaphors and the drawings of mathematicians), captured aspects of pre-service teachers' relationships with mathematics, such as pre-service teachers' views of mathematics and mathematics teaching, feelings about the subject, and ideas about their growing identities as teachers of mathematics.

A survey tool from the Google suite of software was used each year to capture further glimpses of these aspects and their perceptions on the usefulness of the interventions in the Growing Mathematics Teacher project. The survey each year included four questions: *How do you feel about mathematics?* *How confident do you feel in your ability to teach mathematics?*, *How much do you enjoy teaching mathematics?*, and, *How confident are you in your ability to teach mathematics*, and *Did you have the content knowledge you needed while teaching on professional experience?*, as well as questions designed to gather data about specific interventions. Pre-service teachers responded according to a four-point Likert scale for each of the questions, and were asked to comment on their response.

Analysis

The analysis of the data was designed to aid our understanding of pre-service teachers' relationships with mathematics, how these changed over the programme, and the pre-service teachers' perceptions of the interventions. The pre-service teachers' responses to the range of research tools were loosely coded and related to aspects of their relationship with mathematics. Systematic ordering and re-grouping refined and consolidated these. Each aspect remained

directly linked to the original data allowing the accumulation of evidence towards it. For example, to answer the research question: what was the nature of the pre-service teachers' relationships with mathematics, a range of codes developed. These codes were categorised into their knowledge (e.g., knowing), beliefs (e.g., own school, importance) and feelings about mathematics and mathematics teaching (e.g., anxiety, conditional), and their growth towards becoming a teacher of mathematics (e.g., confidence, satisfaction). This process was similar to open coding "because to uncover, name and develop concepts, we must open up the text and expose the thoughts, ideas and meaning contained therein" (Strauss & Corbin, 1998, p. 102). However, the elements of pre-service teachers' relationships with mathematics, identified by Ingram (2015) and the research questions were kept in mind.

The cohort's relationship with mathematics provided a context for individual relationships with mathematics to be understood. Individual mathematical journeys were then explored by capturing their unique relationship with mathematics over time through case studies. The data set relating each of the three pre-service teachers was separated from the main body of data, and read through in its entirety. Then, each pre-service teacher's mathematical journey was chronologically mapped in terms of each element of their relationship with mathematics and mathematics teaching. To understand how typical each pre-service teacher was, they were compared to the class as a whole. In other words, the internal themes unique to each pre-service teacher were considered in relation to the relationship with mathematics across the cohort. Through constant comparison, the analysis moved to and from between the general and the specific (Strauss & Corbin, 1998).

Reporting the Results

The elements of pre-service teachers' relationships with mathematics were complex and interacted with each other. It is therefore difficult to write in a linear way about these topics. Pre-service teachers' mathematical knowledge has been dealt with first, and then other affective aspects of their relationship with mathematics are discussed. Pseudonyms have been used, proportions of representation have been used where possible, and, if a quote is given, it is representative of quotes within the body of evidence towards a theme.

Results and Discussion

Pre-service teachers' relationships with mathematics and mathematics teaching

Through analysing the data across the cohort, there was strong evidence that the pre-service teachers had dynamic relationships with mathematics and mathematics teaching that changed over their three-year programme. There were differences in the amount of foundation content knowledge they had. The pre-service teachers therefore had varying perceptions of their ability in mathematics, associated feelings, and varying confidence levels in their teaching of mathematics. These complex relationships, and the changes during the programme, are described across the cohort.

Mathematical Content Knowledge.

The number of pre-service teachers demonstrating the required content knowledge for teaching rose from 33 pre-service teachers (40% of cohort) at the start of their first academic year to 73 pre-service teachers (88% of cohort) at the end of that year. One of the pre-service teachers who did not demonstrate the required content knowledge by the end of the first year worked independently over the summer and demonstrated the required knowledge before the start of

second year. She completed the programme, with the remaining nine pre-service teachers either withdrawing from the programme or making repeated attempts and not achieving the required level.

Other Aspects of Pre-service Teachers' Relationship with Mathematics.

In 2015, near the end of their first year of the programme, 23 (43%) of the 53 pre-service teachers who completed the survey were positive about mathematics, 21 (40%) were neutral, and nine (17%) were negative. Through their survey responses and their personal journey graphs, all of the pre-service teachers who responded attributed their feelings about mathematics to their experiences of learning mathematics at school or their perception of their ability. Gina and Donna are representative examples of these pre-service teachers.

[At school] if I didn't get the problem or if I was confused, the other students got it. I never spoke up for fear of looking like I was dumb therefore I don't like math. (Gina, 2015)

I didn't achieve well at school in maths so I have a sense that I am not very good. I don't like it. (Donna, 2015)

Although 77% of those pre-service teachers surveyed in 2015 felt they had the content knowledge they needed for teaching mathematics (they were surveyed at the end of the year after many of them had met the standard), only 37% felt confident in their ability to teach the subject and 40% had some concerns. Twenty-two percent were lacking in confidence or had no confidence. However, after their professional experience out in schools in first year, 45% enjoyed teaching mathematics, and 45% were neutral and 9% were negative in terms of their enjoyment of teaching the subject. Eleven of the pre-service teachers directly related their enjoyment of teaching mathematics with their confidence and/or the perception of their ability, examples of which are below.

Sometimes I feel like the children know more than I do with maths, which is stressful and makes it less enjoyable to teach. (Robyn, 2015)

In their first year, we only get glimpses of the pre-service teachers' views on the mathematics. Only seven pre-service teachers described mathematics as an important subject in their comments about their feelings.

I was not very good at school but since being out of school I realise the importance of maths in our everyday lives and I am more confident and keen to explore with maths. (Daisy, 2015)

However, the following year, we began to understand more about their views of the subject and identities. The pre-service teachers had varying views of the subject that were often related to their feelings, as evidenced by their metaphors (similes) for mathematics.

Maths is like a metal rock band. You know it is supposed to sound like music but it just sounds like a lot of mumble jumble all squished together to make sense. (Gina, 2016)

Maths is like a building that has definite and sharp features. It is a solid colour. This is because there is only one definite answer in maths. It is either right or wrong with no room for disagreement. (Leanne, 2016).

Maths is like Wall Street. Big and complicated with everyone being hasty. (Henry, 2016)

Maths helps us make sense of the world around us. Maths is like a very tall building because it can seem overwhelming when you are standing on the ground and looking up you cannot even see the top of the building there. But wait – inside you have choices – choices that can scaffold your understanding step by step like stairs. At the top of the stairs you feel satisfaction (Donna, 2016).

For Gina, mathematics does not make sense, whereas for Leanne and Donna there is a lot of sense, with varying degrees of openness, choice and connections with the real world. For Henry, mathematics is a subject to be done quickly, and perhaps competitively. These views of mathematics helped us to plan for the problem-solving unit, to counter pre-service teachers' views that might impede their growth as mathematics teachers, such as mathematics is a series of topics with little connections between them.

Of the eight males and 52 female pre-service teachers who drew a mathematician at work, 19 drew a gender-neutral representation, six of the pre-service teachers drew females and 35 of the pre-service teachers drew males. Their comments ranged from physical descriptions to descriptions of teaching style.

I instantly thought of a male with the word 'mathematician'. Glasses come to mind as someone with poor eyesight but a brilliant mind. Nerdily dressed and fully engrossed in sums with not much else in their life but maths. (Moir, 2016).

Barry favours students who understand things. They are in the top group. He doesn't like them talking about the equation. He isn't a fun teacher. He's scary. (Gwen, 2016).

These pre-service teachers rarely identified with the idea of being mathematicians or teachers of mathematics. Indeed, it was clear they disassociated themselves from the idea. The pre-service teachers who drew neutral or female characters had quite different responses. These pre-service teachers were more accepting of their growing identity as a mathematics teacher.

Maths isn't just for 'brainy' looking people with glasses and books. It's for any person with no stereotype – someone that likes maths will be happy at work and convincing others of [the] enjoyment (Rita, 2016).

This is ... the teacher in my last placement. She loved maths and loved teaching it. Generally, she would sit on the floor with the kids in a circle around her and use resources to explain maths problems to the kids. I believe she was an extremely effective maths teacher and mathematician. (Eve, 2016)

A theme emerged that some pre-service teachers' confidence was higher when it came to teaching younger students. Ten of the pre-service teachers did not want to teach older primary students. Levi, for example, was concerned when he was placed in a Year 8 class.

I feel slightly unsettled to teach upper primary mathematics. (Levi, 2015)

Although 74 of the pre-service teachers who provided evidence had the content knowledge required for teaching mathematics, many did not have positive relationships with the subject, and there is evidence their views about mathematics would perhaps prohibit them from practising effective pedagogy.

Changes in pre-service teachers' relationship with mathematics and mathematics teaching

Over the three-year programme, the pre-service teachers' relationships with mathematics changed. In 2017, their final year, of the 40 who responded to the survey, none of them disliked mathematics, and 83% of them liked the subject (17% were neutral). Similarly, 83% enjoyed teaching mathematics and 58% were confident to teach it. Forty percent still had some concerns, and only one pre-service teacher claimed to be lacking in confidence. 95% of the pre-service teachers believed they had the content knowledge required to teach the subject.

Out of the 40 pre-service teachers who responded to the survey, 53% directly commented on having a change in their feelings about mathematics during the programme even though we did

not ask this question explicitly. All but one of those pre-service teachers described how their feelings, and their subsequent enjoyment when teaching mathematics, had grown more positive.

My feelings towards mathematics have changed over my time at college. I feel more confident in teaching maths and assessing students in maths. It is not my favourite subject to teach however I do not dread it anymore!!! Sometimes I even look forward to it, depending on the topic. (Sarah, 2017)

It's funny how I used to highly dislike maths and now I kinda like it! (Eve, 2017)

For one of the pre-service teachers, however, her feelings had grown more negative.

I am nervous and unconfident when teaching and scared that I don't know what they need to know or if I'm teaching it right. I've always liked maths throughout my school learning. Since learning it here my feelings have changed drastically. I am now scared that I cannot teach my students the correct maths. (Fiona, 2017)

Also, there was still a theme of worry about teaching older children.

Help us not to be afraid to teach intermediate maths. (Fiona, 2017)

Generally, however, the cohort of final year pre-service teachers seemed to move from thinking about their own relationships with mathematics to fostering the children's relationships with mathematics and realising the potential of the subject. These pre-service teachers were getting satisfaction from their teaching.

I'm starting to realise I need to be positive so I can be that teacher who inspires. (Karen, 2017).

I have enjoyed the shift from anxiety about maths to enjoying it and finding that children find it an exciting classroom subject as well. My feelings about teaching maths have moved from one of feeling ill-equipped to feeling that it is an area that is engaging to teach. (Donna, 2017)

It's really exciting to witness the 'lightbulb' moments when they finally understand something and can confidently explain how they worked out a problem. (Nadine, 2017)

Pre-service teachers' perceptions of interventions

As described earlier, various interventions over the course of the programme were designed to directly focus on building the pre-service teachers' knowledge, both because of them needing to provide evidence they had sufficient knowledge for teaching, but also because their knowledge interacted with affective elements of their relationship with mathematics. All of the pre-service teachers who responded to the survey at the end of their first year who had taken the EMAT course and all of the pre-service teachers who had peer tutoring found the extra support useful.

Doing the extra mathematics classes in first year, since I did not meet the requirement from the test, helped me gain confidence in just basic maths skills and knowledge. I really do feel having taken that paper that I am one step ahead. Those who did not take that paper were far less confident and unable to use the strategies in which I was able to. (Hannah, 2015)

I was in the mathematics content class ... and have learnt so much and feel 100 times more confident in teaching maths! (Robyn, 2015)

Indeed, for those pre-service teachers who met the requirements in the initial TEC assessment and for whom the mathematics content course was not suggested, several of them commented they were at a disadvantage. One pre-service teacher said that she wished she had "failed" the assessment.

All of the pre-service teachers who responded to the survey at the end of their first year also thought the number knowledge tasks in the first-year pedagogy class and the access to online

support were useful. The pre-service teachers' perceptions of the online-support (e-ako) provided in third year was less clear. Out of the 40 pre-service teachers who responded to the survey, 19 found it helpful, and 13 thought it was neither helpful or unhelpful. Nine did not find this support helpful. The comments indicated that, although it was a useful tool for learning content, clarifying terminology and progressions, and supporting them with planning, they would have preferred it earlier in the programme. Despite this, 60% of the 60 pre-service teachers in the third-year programme chose to engage in the support beyond the 1500 points stipulated.

There was some evidence that pre-service teachers were influenced by the elements of their programme specifically designed to support them to grow affectively. The pre-service teachers who responded to the survey also commented on the usefulness of the direct teaching about relationships with mathematics, which was set in a context of solving problems.

The different methods got us thinking like mathematicians. (Vivien, 2016).

This helped me to see how I felt about maths to start with and I got to see my opinion change over the course of the unit. (Nina, 2016)

It's important for teachers to understand themselves before they teach children. (Barbara, 2016)

The 40 pre-service teachers who responded to the survey at the end of the programme appreciated the chance to hear the experience of practising teachers, particularly those who had recently graduated. These teachers, along with their mentor teachers during their professional experience, provided role models for the pre-service teachers.

It was helpful to hear about real life experiences and to know that it is ok to feel freaked out sometimes because it is not nice to feel like you are in it alone over your head sometimes! (Ruth, 2017)

I really enjoyed these [visiting teachers]!!! It made me feel confident about going into the real world. Before they came I felt as if I had to know everything. Some of them had just recently graduated and gave a realistic picture of our first year out. (Sandra, 2017)

[My mentor's] lessons were fun, related to real life concepts e.g. money and her students (with complex behaviours) were engaged and on task. I aspire to teach maths like that. (Sonya, 2015)

Importantly, by their third year, there was no mention of ability nor other evidence of a fixed mindset in their comments. Rather, their comments centred around needing to gain more knowledge to improve their teaching.

When I don't feel confident in my teaching of maths I refresh my knowledge so that I am confident. Mathematics intrigues me, I love looking for various resources to enable me to challenge my knowledge and teaching abilities, to improve my skills for the future. (Tara, 2017)

You learn ... ways on which you can improve next time. It was good for me as I have a love/hate relationship with fractions, so I had to make sure I was up to scratch to teach my students. (Anna, 2017)

This cohort of pre-service teachers, at the beginning of their programme, described a variety of feelings about mathematics and stated a range of levels of confidence. They became more confident in time and, at the end of 2017, ready to apply for teaching jobs, they generally enjoyed the subject of mathematics, loved teaching it, and were mostly confident, although this was still somewhat conditional on the age of students.

Reporting on the case studies - Pre-service teachers' growing relationships with mathematics: Maggie, George and Sonya.

Once we understood more about the pre-service teachers' relationship with mathematics across the cohort, we can examine the relationships of three individuals in more detail. Maggie, George and Sonya represent the range of pre-service teachers in terms of their relationship with mathematics.

Maggie. In the first year of the programme, Maggie was 18 years old from a rural background, who had just moved to live and study in Dunedin at the University of Otago. In her mathematics assessment at the beginning of the year, she achieved 600 points, 95 points short of what she needed to meet the standard set, and the fifth lowest mark of the graduating cohort. Maggie enrolled in the EMAT course, which supported her to meet the standard by the end of her first year. For the rest of her time as a pre-service teacher she was able to meet the standard in her mathematics pedagogy and other courses and she graduated with her class at the end of 2017.

Maggie's view of mathematics in her first year was that it is "either right or wrong" and "not something you can waffle your way through." She felt she was "never any good" at the subject. As Maggie said, "I like maths when I understand it and when I can do it. I find it very frustrating when I can't do it, which makes me think I don't like it." Despite this and recognising the need to grow her knowledge more, Maggie enjoyed teaching mathematics at the end of first year,

I loved teaching maths. It was a new entrant class, so I was able to understand the math content quite easily. I feel my everyday math skills can get me by, but for me to teach maths confidently and be helpful to children, I need to know and understand more myself. I loved teaching it but would need to further develop my own skills and understanding to teach the older primary year levels confidently.

Moving through the two subsequent years, Maggie described how breaks in mathematics courses during the programme made it difficult for her to maintain a positive relationship. The pedagogy courses only went for one semester of each year.

I forget ... very quickly, so need to constantly re-learn things. The mathematics programme should run for the whole three years. If there are so many teachers out there that are not confident in teaching math, why is there nothing more done about it? Throughout the semester the relationship changed, but the instant we stop learning math, the relationship goes back to what it was.

By the end of the programme, Maggie's feelings about mathematics and teaching were still somewhat conditional on her knowledge. She continued to prefer teaching younger students and did not enjoy dealing with mathematics problems that required proportional reasoning.

There's certain parts I enjoy and parts I would rather run from, like fractions. I love teaching it, but only the topics I feel comfortable in. [When] I was teaching number, I felt like I had used my knowledge but the children still didn't understand [and] I didn't know where else to look.

George. George, 19 years old at the beginning of his primary pre-service programme, met the standard required by the TEC mathematics assessment with over 100 points to spare. He then achieved well in his pedagogy papers, graduating at the end of three years. George had strong, positive feelings about both the subject of mathematics and teaching it. He described mathematics as "relaxed," like a "dolphin swimming through a calm blue sea" and he had positive associations with a mathematician as being a "normal semi-hipster person and reasonably outgoing with a bit of swagger." Throughout the programme, mathematics remained his favourite subject to teach and he described himself as very confident, with a strong knowledge of content. Despite this confidence, George acknowledged the need to revise content before teaching.

I enjoy teaching mathematics, which makes it easier as I think my enthusiasm rubs off on kids. I make sure I'm confident in content I'm teaching by preparing before I teach it

George had some frustrations with the programme, because he felt there needed to be less time spent on content within the pedagogy courses. He also expressed concern that mathematics did not go across all semesters during the three-year programme because of its importance and the knowledge of other pre-service teachers.

Maths is such a vital part of school and I feel many of the other pre-service teachers do not have the math ability to confidently teach it.

Sonya. Sonya was a mature pre-service teacher who got 681 points in the TEC assessment at the beginning of her year, just below the standard. She enrolled in the EMAT course and like Maggie, met the standard by the end of the year. She received very high marks for her pedagogy papers and graduated at the end of her third year.

Sonya had “disliked maths for as long as” she could remember. In primary school, she described “massive anxiety,” and a feeling of being “slow,” which continued through high school and her working life. However, beginning the programme, Sonya found a positive and friendly environment, where dialogue and a range of strategies were valued. She experienced “aha” moments and found professional experience inspiring.

I started College in a very bad place mathematically. I was actually terrified. Coming into maths class would make me feel nauseous, and I was in a state of high anxiety. Now, I am in a much better place.

At the beginning of her second year, Sonya, drew a split picture when thinking about a mathematician; one side of the page represented a traditional teacher-centred classroom with a “dictatorial, inflexible and cold” teacher, who believed it was his “way or the highway” and another side was a representation of a group of children sitting around a table with a teacher engaging in “dialogue as equals, interactive sharing ideas” and asking questions such as “why,” “I think,” and “what would happen if.” Sonya had grown to enjoy mathematics and felt she had sufficient content knowledge, yet she continued to experience a range of affective responses to mathematics, describing mathematics in the writing below

Tall and structured with two on each side. Or is it four? Nowhere to hide. Intimidation, revelation, freedom. In the symmetry and balance of what man has communicated.

She also sometimes lacked in confidence in her teaching and was vulnerable to feeling anxious.

When teaching I felt like a rabbit caught in the head lights. I thought the content knowledge would be enough, but [maths] still fills me with dread when I'm not able to grasp the concepts and the anxiety begins to feel uncomfortable. ... I feel unconfident when students who are strong at maths ask difficult questions that I need time to process. I am confident at a basic level.

In her final year, she was placed with a class of six-year-old children.

I'm really enjoying teaching mathematics because I have such a clear understanding of maths at this level and I'm able to create natural connections for the children between everyday life and their maths. Even the reluctant mathematicians in the class are enjoying their maths. This gives me confidence. I've developed a growth mathematical mindset. I've realised that with new learning and hard work I can understand and teach maths.

Maggie, George and Sonya began the programme with different relationships with mathematics, yet by the end of the programme each of them understood the importance of developing their own knowledge in preparing to teach children. All of them had a growth mindset when it came to aspects of their relationship with mathematics.

Conclusion

The pre-service teachers' relationships with mathematics were unique and dynamic and changed over time as they experienced mathematics within their teaching programme and interacted with interventions designed to grow them into teachers of mathematics. The pre-service teachers found all interventions useful to some extent. Although some pre-service teachers' confidence in teaching remained subject to the age of student they taught during professional experience, all of them understood the importance of growing their confidence by building their mathematical content knowledge for teaching.

Our findings reinforce other research (e.g., Young-Loveridge et al. 2012), that the mathematical content knowledge of a large proportion of our primary pre-service teachers at the beginning of their programme does not meet the required standard, and other affective aspects are not consistent with a positive relationship with mathematics. Without further growth, these pre-service teachers are at some risk of moving into teaching without understanding the impact of their own relationship with the subject and the potential impact they could have on children they are teaching.

There are three ways this research has added to research in this field. Firstly, this research is holistic because it considers the pre-service teachers' relationships with mathematics, thus acknowledging the interactions between mathematical content knowledge and affective elements. Addressing pre-service teachers' content knowledge, along with being explicit about the importance of affect, resulted in noticeable change in their growth towards becoming mathematics teachers. The interaction between pre-service teachers' content knowledge and their mathematical identities was apparent in the data for the whole cohort and in the narratives of the three individuals.

Secondly it tracks these relationships over time and therefore provides further understanding of the dynamism of these relationships. Ingram (2011) has previously documented the way in which high school students' mathematical identities relating to their perceptions of mathematical ability developed from their experiences with mathematics in social environments. Our study has shown how pre-service teachers' mathematical identities grew within the context of a college of education.

Thirdly, this research highlights the potential of adopting a growth perspective throughout initial teacher education programmes. Our longitudinal study has provided data on how a cohort of pre-service teachers have grown. Our third-year pre-service teachers have, generally, graduated from the programme knowing they can work towards learning more mathematics and can ready themselves for teaching children in engaging and effective ways.

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