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Keywords

equitable mathematics teaching and learning, urban education, reform-oriented teaching practices, inquiry-based learning

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In the National Council of Teachers of Mathematics' (NCTM) publication of the Principles and Standards for School Mathematics (2000), equity became a prominent focus. In light of these recommendations, the mathematics education research community has focused on addressing educational disparities such as academic achievement, graduation rates and the overall quality of education received by minorities and low income students. However, the enactment of this principle tends to be less visible in schools (Fisher, 2005; Ladson-Billings, 2000; Lubienski, 2002; Martin, 2009). Many African American (AA) students are under-performing in core subject areas such as Mathematics, Language Arts, Science and Social Studies (Fisher, 2005; Ladson-Billings, 2000; Mezuka, 2009; Moses-Snipes, 2005). Moreover, the consequences of these inequities are more salient for AA males because of how they are positioned in the larger societal context (Howard, 2008; Steele, 1997). African American males are perceived as unsuccessful and these stereotypes manifest in academic settings (Martin, 2003). Often times, AA males are presumed to lack the intellectual, social and behavioral skills necessary for accomplishing educational goals (Stinson, 2006). Moreover, teachers, administrators and school staff tend to have lowered expectations for them (Thompson & Lewis, 2005). In addition, the structure of schools reflect values, norms and cultural orientations that are more aligned to the dominant group (White, middle class culture) and favors learners that demonstrate mainstream characteristics more aligned to societal goals (i.e., abstraction and logic; Ladson-Billings, 2000). African American students often have a more relational, person-oriented style of learning (Martin, 2000; Moody, 2004). As a result, they are presumed to lack logic and abstraction skills and, therefore, are not provided with sufficient opportunities to apply their

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critical thinking and reasoning skills. In addition, curriculum materials and instructional practices emphasize strategies that do not fully take into account their learning style preferences and cultural experiences (Garibalidi, 2007). This suggests that the practices dominant in schools do not appropriately support their academic development. Moreover, AA students are more likely to receive less engaging modes of instruction (remediation, rote memorization, drill) not aligned to the recommendations of NCTM (Darling-Hammond, 1995; Lattimore, 2005; Lubienski, 2002; Stein, 2001). Subsequently, this leads to disengagement and a lack of interest in mathematics. Therefore, the quality of teaching and instructional materials is crucial because it impacts decisions to stay or drop out of school and dictates how individuals progress through the mathematics pipeline (Mau, 2003; Berry, 2008). This research study was designed to address two ways (low quality curricula and sub-standard teaching) in which AA males are disadvantaged mathematically. The curricular unit was designed to harness mathematics skills and place the participants in a more advantageous position. Geometry was the content strand of focus because it has specific features aligned to the research goal. First, males are inclined to perform well on spatial tasks, and spatial skills have been linked to achievement in geometry (Bishop, 2008; Clements & Battista, 1992; Geiser, Lehmann, & Eid, 2008). In addition, well-designed geometric activities promote interest and engagement (Outhred & Mitchelmore, 2001). Therefore, providing opportunities for AA males to work on a series of geometric activities has the capacity to engage them in the discipline and create learning opportunities to increase the likelihood of success, bolster mathematical self-efficacy and engender stronger academic identities. The research goal was to investigate how the implementation of a geometry curricular unit impacted understandings of sixth grade AA males. The specific research question posed was

How does the implementation of a curricular unit impact geometric understandings of elementary aged African American males?

Literature Review

The achievement gap that persists among AA students and their peers has been well-documented (Bottge, Rueda, Serlin, Hung, & Jung, 2007; Cummins, 2001; Darling-Hammond, & Sykes, 2003; Davis, 2003). However, the current literature tends to focus more on academic outcomes than schooling experiences. This overemphasis on achievement does not provide a holistic view of factors that contribute to educational disparities. Several researchers, however, challenge this perspective and shed light on cases of AA students that have succeeded academically and identified school as integral to success (Berry, 2008; Martin, 2000; O'Connor, 1997; Stinson, 2006).

Martin (2000) and O'Connor (1997) reported cases of AA students who achieved in school despite the challenges that they experienced. Prior researchers, such as Ogbu (1986) and Fordham (1988), argued that AA students were unsuccessful when they were aware of the negative societal structures that impacted their community. However, recent findings (Berry, 2008; Martin, 2000; O'Connor, 1997; Stinson, 2006) refute this perspective and highlight AA students that have been academically successful.

Regarding the learning of mathematics among AA males, Berry (2008) found that middle school AA students performed well when they recognized their academic abilities and were surrounded by a support system of individuals who advocated on their behalf. These findings are supported by Stinson (2006) and Thompson and Lewis (2005), who also found that AA males who were aware of their academic capacities succeeded mathematically in spite of the negative social and environmental factors that surrounded them.

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Prior studies (Corey & Bower, 2005; Davis, 2003; Garibalidi, 2007; Moses-Snipes, 2005; Swanson, Cunningham, & Spencer, 2003) reported that AA males perform well in school until the middle grades (6^{th}, 7^{th} and 8^{th}), the stage documented as typically when many begin to avoid academic engagement. However, findings from these studies shed light on students who were engaged intellectually and performed well at this crucial stage of development and beyond.

Lattimore (2005) concluded from a series of interviews that he conducted with AA males that engagement, active participation and classroom discourse were elements that the students identified as important aspects of effective teaching of mathematics. Pedagogical strategies such as animation and collaboration were found to impact how they participated in classroom activities. This is validated by research which shows that AA students respond more effectively to interactive methods of teaching (Howard, 2013; Ladson-Billings, 2000; Moody, 2004). In addition, because many AA males tend to be kinesthetic learners, a didactic instructional approach is not necessarily the most effective (Corey & Bower, 2005; Rosseau & Tate, 2003; Townsend, 2000).

Geometry, the content strand of focus, has the potential to deeply engage AA males because students can use their intuitions of space to make sense of the mathematical ideas (Outhred & Mitchelmore, 2001). Therefore, a curricular unit that provides opportunities for active participation can engage young AA males in the discipline, support their understanding of mathematics and engender self-confidence in one's mathematics abilities. This assertion is supported by studies that demonstrated that AA males that were supported academically were able to maintain self-confidence in their mathematical abilities that enabled them to continue to succeed in school despite the widely held stereotypical perceptions about their abilities (Howard, Flennaugh, & Terry, 2012; Stinson, 2006).

The literature reviewed provides insight and highlights important aspects of the educational experiences of AA males. However, these studies are not sufficiently situated in the context of mathematics teaching and learning with explicit learning goals and how such learning goals were accomplished. As a result, our understanding of how to effectively support content knowledge development of AA male students is minimal. This paper describes how content knowledge in a specific area of mathematics (geometry) was impacted by the implementation of a curricular unit focused on active exploration and inquiry.

Theoretical Framework

Critical Race Theory (CRT) is the theoretical framework that guides this study. CRT has its roots in legal studies and is a philosophical theory that strives to advance social justice (Ladson-Billings & Tate, 1995; Solorzano & Yosso, 2001). Its basic tenets include: the centrality of race and the intersections of other forms of oppression as a component of analysis, racism as deeply embedded in society, and a strong commitment to social justice and equity (Solorzano & Yosso, 2001). Given the research goal, CRT is particularly appropriate because it recognizes that race impacts schooling practices and is endemic and crucial in determining the educational experiences of minority students (Ladson-Billings & Tate, 1995). CRT also challenges stereotypical assumptions regarding intelligence and academic abilities of people of color (Tate, 1997). As a result, it provides a medium to study phenomena with racial undertones with the aim of addressing these inequalities (Delgado & Stefancic, 2001). Regarding the context of this study, it shows the problem of AA male underachievement as resultant from social inequities in the form of lack of access and opportunities to quality educational experiences and provides a medium to understand how intersections of race, gender, and social class impact the lives of AA males. Explorations of the mathematical learning experiences of AA males are lacking in the literature. CRT provides a framework in which these mathematical learning experiences can be described with the goal of developing instructional materials and teaching practices appropriate for young AA males.

Curricular Unit

The curricular unit was designed to facilitate students' understanding of four quadrilaterals (parallelograms, rectangles, rhombi and squares) and their properties. It was further aimed to equip students to use the knowledge they gained to determine the relationships between quadrilaterals. The curriculum was implemented during regular school hours in a formal classroom setting for a duration of two weeks. Mathematical tasks were aligned to NCTM and the Common Core Standards for Mathematics, which states that students should be able to classify two-dimensional figures based on properties. Classroom activities were sequenced so that mathematical tasks that supported understanding of different characteristics of the shapes were completed before transitioning into activities where students explored relationships between shapes. The introductory lessons focused explicitly on angles (acute, right, and obtuse) and lines (segments, rays, parallel, and perpendicular). The subsequent part of the curricular unit focused on investigating relationships between pairs of quadrilaterals. Students were introduced to these concepts through concrete (protractors, rulers, geo-boards) and semi-concrete tools (technology). Given the sample of students, this was particularly important because it provided a medium for them to actively participate in mathematical tasks and draw valid conclusions from their interactions with the tools. For example, students were introduced to a broader conception of angles by creating protractors with construction paper and investigating how many wedges comprise different types of angles (See Figure 1). This concept was particularly important to integrate into the unit because elementary school students tend to understand angles only as the point of intersection on a ray (Mitchelmore, 1997; Mitchemore & White, 2000). This description is quite limiting and does not provide students with a complete understanding of what angles actually entail—the amount of space embedded within rays connected to a vertex point (Browning & Garza-Kling, 2008). The goal of the curriculum was to introduce students to this broader conception of angles and ensure that they had a robust understanding of different types of lines so that they can use this knowledge to identify relationships among the quadrilaterals explicitly explored in the curriculum. Specifically, segments, rays and lines were explored through Geometers Sketchpad (GSP), a dynamic technological tool that illustrates different geometric shapes dynamically. Concrete, hands-on tools (geo-boards, protractors) and semi-concrete tools (GSP) were used to investigate quadrilaterals.

The design of the curriculum was informed by best practices advocated by the NCTM, Common Core and research (Battista, 2003; Boakes, 2009; Clements, 2000; Pittalis & Christou, 2010) that recommends a sequential approach to geometry instruction that begins with introducing students to the content and facilitating understanding by allowing them to investigate underlying concepts and using that knowledge to draw conclusions from their interactions with mathematical ideas.

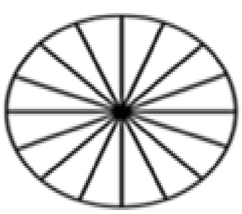

Figure 1. Paper Protractor

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Methodology

Setting

Claxton is a school situated in the metropolitan area of a large Midwestern city. Similar to many schools located in large urban areas, Claxton is surrounded by high levels of poverty and crime. The school demographic is comprised of a disproportionate number of students who identify as AA (99% and 1% multi-racial) and qualify for free or reduced-price lunch. Over the past decade, students from grades K-6 have scored below the state average according to the state standardized test in mathematics.

Participants

The participants were 16 sixth grade AA males from an “all boys” elementary school called Claxton Academy (a pseudonym, as are all proper names). Participants were identified through a professional development program designed to support the mathematical content knowledge of teachers in high needs schools in the district. The sample of students were selected because they are representative of males enrolled in high needs schools that comprise of a high proportion of AA students. As described in the setting, the students reside in neighborhoods characterized by high levels of poverty, crime, high incarceration rates, illiteracy and teenage pregnancies. These are often components of high needs schools located in other school districts in the U.S. Regarding the sample of students, the young males have been enrolled in the school since kindergarten and were either born or are long term residents of the community. They live in low income households led by females as many of the males from the community are incarcerated.

Due to the social and environmental factors that impact the lives of the students, teachers, administrators and school staff described many of the students as challenged both behaviorally and academically. Performances on state standardized tests demonstrated that students were under-performing in mathematics. School staff also reported that students struggle in core subject areas including mathematics.

Data Collection

Written assessments. Pre- and post-tests were administered prior to and after the implementation of the curricular unit. Assessment items were designed to focus explicitly on fundamental geometric concepts of angles and lines. Pre-tests were used to examine students' prior understandings of angles and lines. Post-tests were used to examine the same geometric concepts and also determine how students' geometric reasoning was impacted by the curricular intervention. Test items specifically consisted of questions that required students to describe their understanding of fundamental geometric principles of angles and lines.

Questions were open-ended to enable the researcher to have a broader understanding of how students reasoned geometrically. Test items comprised of a total score of 10 points, questions 2, 4 and 6 consisted of 2 points while questions 1, 3, 5 and 7 comprised of 1 point. To ensure reliability, test items were scored by three mathematics educators. Scorers met weekly for the duration of a month to evaluate student responses of each of the assessment items.

Mathematics interviews. All participants were interviewed. Initial interviews were conducted on the first day of the implementation of the curriculum and focused on students' views and knowledge of geometry regarding the relationships among quadrilaterals (rectangles, parallelograms, rhombi and squares). The second interview was conducted on the last day of the implementation of the unit and focused on students' learning experiences and their knowledge of geometry, specifically how they classified matched pairs of quadrilaterals. Interviews comprised of identical assessment items and were conducted with individual students for a duration of about

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30 minutes. Interview questions were used to examine students' level of geometric reasoning prior to the implementation of the curriculum and demonstrate how their reasoning developed throughout the course of the implementation of the unit.

Video-recordings. All mathematics activities included in the geometry curriculum were videotaped. Three cameras were used to record class sessions. Two cameras focused on groups of students working on classroom tasks for each activity. Students were recorded as they worked on activities focused on angles, lines and quadrilaterals. The third camera was used to capture teaching practices during whole-group conversations. Video-recordings of how the classroom teacher orchestrated mathematical discourse, led mathematical activities, established mathematical norms and interacted with the students were captured by the third camera.

Data Analysis

Quantitative Analysis

A dependent samples t-test was used to analyze the written assessments (pre- and post-tests). Two students were absent during the pre and post-test administration. As a result, these students were excluded from the analysis. Because the goal of the curricular unit was to examine how the implementation of an inquiry-based curricular unit impacted geometric reasoning, a dependent sample t-test was appropriate because it is used to test differences in means between two related groups.

Qualitative Analysis

Interviews were transcribed and coded through an inductive coding scheme that enabled the researcher to develop codes by examining the data. Codes were developed from excerpts of interview transcripts that demonstrated how students examined quadrilateral relationships. Specifically, two codes emerged from the data and were based on whether students classified quadrilaterals exclusively (saw pairs of quadrilaterals as completely separate figures) or inclusively (saw relationships between pairs of quadrilaterals). Codes were then grouped into hierarchical categories. So, codes were organized into categories based on the explanation that students provided to justify their perspective regarding exclusive or inclusive reasoning, the two major codes that emerged from the data. Regarding squares and rectangles, an example of categories that emerged from the data for inclusive classification include squares are rectangles because the sides of a square can be extended to become a rectangle (physical orientation justification), squares are rectangles because they both have right angles, parallel sides and opposite equal sides (property justification). For exclusive classification, squares are different from rectangles because squares have all equal sides and rectangles do not.

Video-recordings were watched to identify timed segments of conversations that demonstrated how students reasoned. Analyses were based on how students evaluated angles, lines and quadrilaterals and were used to complement findings from written assessments and mathematics student interviews.

Results

Written Assessments

Analysis of dependent sample t-test showed that learning gains from pre- to post-test were statistically significant, with $t(13) = -3.159$, $p = .008$. Means and standard deviations for pre-test were M = 4.57, SD =1.83; and for the post-test were M = 6.89, SD =2.25. Given the small sample size, findings from mathematics interviews and video-recordings of classroom episodes are used to complement findings from dependent samples t-test. In addition, individual assessment items on written tests were examined to compare student responses on the pre and post-test. Learning outcomes from video-recordings and mathematics interviews are sub-divided into three sections (angles, lines and quadrilaterals) and discussed in the subsequent sections.

Angles

Research findings from video-recordings demonstrated that students improved in their understanding of angles. During classroom activities, students created their own individual protractors and used them to measure angles. In addition, they engaged in interactive activities such as representing angles with objects in their initial classroom environment and identifying angles through hands on a clock. As discussed in previous section, AA males tend to be more kinesthetic and an interactive approach to learning is more appropriate for their learning style.

This is demonstrated in the classroom episode described below which shows that active participation enabled the students to draw valid conclusions of angles. The task was originally designed for students to use protractors to measure and identify different angles. However, students proceeded to estimate the angles without using the protractors. The classroom teacher encouraged this line of reasoning as she engaged them in mathematical discourse regarding the relative sizes of the different angles. The classroom excerpt shows how students examined the different angles depicted, see (Figure 2).

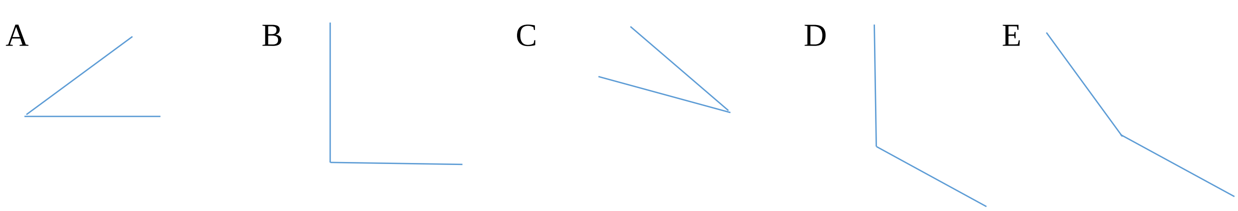

Fig 2. Evaluating Angles

Evaluating angles. The transcript below demonstrates how students compare relative sizes of different types of angles. All names provided are pseudonyms.

Teacher: So, we are starting [long pause] ok for angle A, what kind of angle do we have?

Cam: 45 or 50 degrees?

Teacher: Now, what type of angle is angle B?

Des: 90 degrees.

Teacher: So, do you agree that Cam is right with 45 or 50 degrees?

Phon: Yes, because A is an acute angle, an angle that measures between 1 and 89 degrees and B is a right angle, an angle that is 90 degrees.

Des: A is about half of B.

Teacher: What about C?

Tae: 32 degrees.

Teacher: First, when I ask you questions, I want you to explain why it's 32 degrees not because you are wrong or nothing.

Daniels: A and C look alike, acute they are less than 90 degrees, but C is smaller than A, 45 degrees is larger than the 32 degrees so I knew that the third angle had to be about 30 degrees and the first had to be around 45 or something like that.

Teacher: What about D?

Dric: 135

Teacher: Why?

Dric: Because the last is larger than the fourth, they are both obtuse angles. One is larger than the other so the last one is the biggest. Maybe 160 degrees and the fourth one is the second biggest, 135 degrees.

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Teacher: We've answered all our questions, I like the way you're thinking. This tells me you understand angles.

From the classroom excerpt, we learn that students not only estimated the different types of angles, but they also justified their estimated angles by providing a mathematical explanation in support of their responses. For example, Des justified that angle A is 45 degrees by explaining that angle A, (the acute angle) is half angle B, (the right angle). Daniels explained that angle C (acute) is smaller than angle A (the other acute angle), while Dric used the same line of reasoning to justify that angle E (obtuse) is larger than angle D (the other obtuse angle).

Students did not show this level of reasoning on the pre-test. However, students were more effective in describing angle size on the post-test. They used the "wedges" in the paper protractor that they created in class to explain the amount of space embedded in an angle. This shows that the wedge activity integrated in the curricular unit seemed to have supported students' understanding of the concept of angles.

In this classroom episode, the teacher also played an integral role in facilitating students' understanding. First, she realized that, because they did not need to use protractors to measure the angles, students were reasoning at a higher level; as a result, she adapted the mathematical task accordingly. Second, she provided students with sufficient opportunities to explain their reasoning regarding the relative sizes of the angles explored. This is particularly important because AA males are often not recognized for their academic potential (Howard, 2013). Instructional strategies optimized are typically focused on remediation, rote memorization and drills; approaches not necessarily capable of supporting deep understandings of mathematics (Berry, 2008).

Parallel lines. On the pre-test, students were able to draw parallel lines, but they were unable to explain why parallel lines do not meet. Geometer's Sketchpad, the interactive technological tool integrated into the curriculum, was used to support students' constructions of the equidistant property of parallel lines, the underlying reasoning why parallel lines do not meet. The transcript below demonstrates how Geometer's Sketchpad was used to support how students reasoned about the equidistant property of parallel lines.

Parallel lines and equidistance. The transcript below documents how students examined the equidistant property of parallel lines.

Teacher: What do you see on the screen? [Displays parallel lines on the screen].

Jeff: Parallel lines.

Teacher: Do you think that the distance between these two points are the same?

[Puts two pairs of perpendicular points on the parallel line to demonstrate the equidistant property of lines].

Cam: Yes.

Teacher: I want you to explain why you think that?

Cam: Looks like the two points are on the same line and these two are on the same line.

Teacher: Looks like these two points are on the same line. Ok, so, do you think that the distance between here and here [points to perpendicular points on the parallel line] is going to be the same as the distance between here and here [refers to the second pair of perpendicular points on the parallel lines]. Ok, Mario agrees, who else? Terry, Cam, Tae, Veon, Velt, Daniels and Phon. Ok, you don't think so? [Referring to Kenji]. Alright let's do something. B to A is 3 cm [measures the distance with the measure function of GSP] and C to D is 3 cm [also measures the distance with the measure function of GSP]. What do you see on the screen now? Kenji.

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Kenji: A to B and C and D is the same. Equidis….

Teacher: Equidistant [Teacher helps Kenji with pronouncing the rest of the word]

Perpendicular and intersecting lines. On the pre-test, students were able to define intersecting and perpendicular lines, but were unable to differentiate between both lines. A classroom activity was specifically designed for students to explore these two types of lines. Analysis of classroom data showed that students actively participated in this activity as they used transparent pieces of construction paper to investigate perpendicular and intersecting lines. This activity supported students' understanding because students were better able to differentiate between both lines on the post-test. An excerpt from the paper activity is documented and described.

Teacher: So [pause] now everyone has patty paper, right? I want you to draw a line segment and label that line segment line m. Make one end of the segment fit on top of the other, ok. Then trace over the crease line and label it line l. Make one part sit on top of the other. Can, you tell me what kind of lines these are?

Jeff: Intersecting.

Shawn: Why? They're perpendicular.

Jeff: Intersecting lines, right angles?

Tuan: X's [referring to intersecting lines sketched on the board] don't have right angles in them.

Teacher: Let's use our protractors to investigate if these are really 90-degree angles. Use your paper protractor or plastic protractor to measure the angles that you make with those intersecting lines. What kind of angles did you make?

Jeff: Right angles.

Teacher: How can you tell?

Jeff: I can tell it is 90 degrees it takes 4 wedges.

Tuan: 4, I got 4.

Shawn: Perpendicular lines are intersecting lines.

Teacher: Yes.

Shawn: That has 90 degrees?

Teacher: What are perpendicular lines again?

Shawn: Lines that intersect but have right angles in them.

Teacher: What about these intersecting lines I drew earlier? Tuan said they are X's, Tuan?

Tuan: Those are lines that cross but not at 90 degrees, obtuse and acute angles.

For this activity, students were required to make intersecting lines on construction paper and use protractors to measure the angles created from the intersection of the lines. Students either used paper protractors or the standard protractor to measure the four angles created from the intersection of the perpendicular lines. They also compared their perpendicular lines (on construction paper) to intersecting lines (sketched on the board by the teacher and referred to by Tuan as an X).

As the classroom conversations progressed, it became apparent that students were making sense of perpendicular lines, particularly the idea that perpendicular lines intersect at four right angles, but intersecting lines can also be formed with acute and obtuse angles. During the classroom activity, effective communication particularly seemed to support the construction of mathematical ideas. From the excerpt, we learn that Jeff recognized perpendicular lines have right angles even before the teacher instructed them to use their protractor to measure the four angles created from the intersection of the perpendicular lines. Shawn was also able to identify perpendicular lines as intersecting lines that form right angles. Towards the end of the transcript, Tuan used angles to effectively differentiate between both lines (perpendicular lines make right angles and intersecting lines, X's make obtuse and acute angles). Prior to this classroom activity, students did not necessarily demonstrate a thorough understanding of perpendicular and intersecting lines. On the post-test, they used right angles to effectively differentiate between both lines.

Quadrilaterals

Throughout the course of the implementation of the unit, students were introduced to four different types of quadrilaterals (parallelograms, rectangles, rhombi and squares) and their properties. Both concrete and technological tools (GSP) were used to support students' understanding of the shapes. For the mathematics interview, students were tasked with explaining the relationship between three pairs of quadrilaterals (rectangles and parallelograms, squares and rectangles, squares and rhombi). On the initial interviews, four students explained that some of the quadrilaterals were related. Andre, Des and Daniels used transformations to justify the relationship between all three pairs of quadrilaterals. Transformations, in this context, refer to adapting or modifying a shape by adapting the side length to change it to another shape. For example, students explained that squares and rectangles are related because the side lengths of a square can be extended to transform the square into a rectangle. The same line of reasoning was applied to rectangles and parallelograms (tilt the sides of rectangles and squares to change them to parallelograms and rhombi).

During the initial mathematics interview, Jeff was the only student that correctly used properties to evaluate the relationship between a pair of quadrilaterals (squares and rectangles). He explained that both figures have sides that don't ever touch or meet and used the parallel line reasoning to justify that a rectangle is indeed a parallelogram with right angles.

Research findings from the post interview transcript however demonstrated that students improved in their understanding of rectangles and parallelograms. With one exception, students successfully used the parallel line reasoning to justify that rectangles were parallelograms—a rectangle is a parallelogram with right angles because it has two pairs of parallel sides. Students also classified squares as rectangles, although their analysis was not as robust as that of rectangles and parallelograms. About half of the students used transformations to justify that squares were rectangles.

Regarding squares and rhombi, students typically used transformations to explain that squares and rhombi were related. Only two students (Phon and Velt) used geometric properties to classify squares as rhombi and explained that squares are special rhombi with right angles. However, most of the class based their reasoning on adapting physical orientation, tilting the side length of a rhombus to transform it to a square.

An overwhelming number of students used geometric properties to justify the relationship between parallelograms and squares. This was quite interesting as the square can be classified as a rhombus in a similar fashion to how a rectangle can be classified as a parallelogram [squares are rhombi with right angles; rectangles are parallelograms with right angles]. Students were more effective at identifying properties associated with parallelograms and this enabled them to appropriately classify rectangles as parallelograms.

Rectangles and parallelograms. Analysis of video-recordings showed that GSP seemed to support students' construction of this concept. Documented below is an excerpt from that particular classroom episode.

Teacher: I'm getting ready to do something. So, I need everyone focused in on my screen ok. You saw I just clicked on the line and the point and used that to create parallel lines. Remember,

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the quadrilateral has two pair of parallel lines. So [pause] now I'm gonna construct another pair of parallel lines. [Constructing lines on GSP]. Are these lines gonna meet?

Students: No [Altogether].

Tuan: We did this in the lab, construct parallel lines with….

Teacher: GSP, I'm gonna get rid of the lines and we are gonna focus on our four-sided shape.

Velt: It looks like a rectangle.

Teacher: It does, doesn't it, but isn't a rectangle a parallelogram?

Velt: Yes, hide all the lines and you forgot to change it to units.

Teacher: Thanks Velt, you are so observant. Let's see…is rectangle ABCD a parallelogram?

Students: Yes [Altogether].

Teacher: Why?

Jeff: Because we used parallel lines for both.

Shawn: And they could be rectangle or parallelogram inside the shape.

Teacher: Ok, let's get some measurements. Here, let's go to measure function (pulls up angle and side measures).

Teacher: Ok, so now is a rectangle a parallelogram?

Phon: Yes.

Jeff: It's also a quadrilateral.

Teacher: Good, ok why is a rectangle a parallelogram?

Jeff: It has opposite sides that are parallel and equal like parallelograms.

Teacher: Looking at that shape does the rectangle have parallel lines, do parallelograms have parallel lines?

Phon: Yes, top and bottom are parallel and two sides are parallel it has all the properties of the parallelograms. Opposite sides equal and angles too.

The classroom excerpt documented shows that the use of GSP to construct the parallelogram illuminated for students an understanding of parallelogram properties. Students' responses demonstrated that they used properties to determine the relationships between rectangles and parallelograms. After the teacher constructed the parallelogram, Jeff and Shawn asserted that the figure embedded within the parallel lines could either be a rectangle or a parallelogram because parallel lines were used to construct both shapes. Throughout the classroom episode, students used properties (parallel sides, opposite equal sides and angles) to justify that rectangles were parallelograms. This was quite different for squares and rectangles. Students were not necessarily in agreement in their evaluation of squares and rectangles. Although they engaged in worthwhile mathematical discourse, their responses were quite varied.

Squares and rectangles. The following classroom excerpt demonstrates how students examined squares and rectangles.

Teacher: Is a square a rectangle?

Daniels: No, it's not.

Phon: Yes it is, it's a longer square.

Daniels: Uhmuhm [disagrees].

Tae: Square has equal sides and rectangle don't.

Phon: Rectangle is longer and stretched out but it's the same. If you close in a rectangle, it'll become a square.

Veon: Phon, but its way longer than a square.

Teacher: Can a square be a rectangle or no?

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Daniels: Polystrips are gonna break if you change a square to a rectangle [referring to the hands-on tool that was used to make squares and rectangles during the lesson].

Teacher: Can I not make a square into a rectangle? Can it not be a rectangle?

Velt: No, it can't there is no way it could be. This is a rectangle cut it in half and we still gonna have a rectangle. It's still a rectangle so a rectangle can't turn into a square.

Des: Both have right angles in them, the only difference are one is longer so they can be one and the same.

Phon: Yes, opposite sides are equal, they both have right angles, squares are rectangles only short. They have the same things.

Daniels: Opposite sides are not equal, all the sides on squares are equal.

Phon: Yes, they are [addressing Daniels that the opposite sides of a square are indeed equal].

Teacher: Wait, wait………… hold up disagreement is good, Phon, come and demonstrate what you mean by opposite sides are equal.

Phon: Top and bottom and the sides are equal for both [using the board to demonstrate to the class that opposite sides are equal on the square and the rectangle].

Velt: Ok, I kinda see what Phon's saying.

Daniels: Wo o wo o when you cut it or fold it, it's still gonna be a rectangle [still disagrees with Phon].

Velt: It is still a rectangle but a square becomes a rectangle when stretched out.

Veon: Rectangles are squares sometimes when it's made short to a square.

Daniels: I see, I got you.

In this classroom excerpt, students evaluated the shapes quite differently. Some (Daniels, Tae, Velt) agreed that squares and rectangles were completely different shapes, while others saw connections and either used geometric properties (Phon, Des) or transformations (Veon) to justify why they classified squares as rectangles. As the classroom conversations progressed, students that initially did not see a connection between squares and rectangles seemed to reason more inclusively. However, this level of understanding was not reflected on the mathematics interviews. Perhaps, students needed to individually investigate this concept on their own and draw valid conclusions from their interactions with the ideas. Nonetheless, students' understandings of angles, lines and quadrilaterals generally improved as demonstrated by performances on the written assessments and mathematics interviews.

Discussion

Research findings from this study indicate that students drew valid conclusions from their engagement with quality tasks. Students reasoned and articulated their understanding of angles, lines and quadrilaterals. In addition, they actively participated in classroom activities. Contrary to the dominant perspective, results from this study suggests that AA males want to learn when they are appropriately challenged and engaged (Berry, 2008; Martin, 2000). As demonstrated in the classroom episodes documented in this study, students can be challenged and motivated to learn throughout the implementation of the curricular unit that provides opportunities for active engagement and exploration.

Analyses of data provide evidence to suggest that students developed understandings of angles and lines. One of the goals of the curriculum was to introduce students to a broader concept of angles. The post-test showed that this goal was achieved because students applied an "amount of space" conception to determine which angle was larger.

Allowing students to explore angles and lines in dynamic and versatile ways, that include integration of hands-on tools, technology and paper folding activities, facilitated learning. Given

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the CRT framework that guided this study, such interactive activities were integrated into the unit to address educational inequities that many of these students typically experience. Moreover, because AA males tend to be more relational and personal, these mathematical experiences enabled them to optimize their learning style preferences and placed them in an advantageous position that enabled them to take ownership of their learning. Therefore, activities such as these should be integrated into lessons when introducing AA males to foundational geometric ideas. However, to do so effectively requires teachers to be cognizant of learning style preferences and the difficulties that students tend to experience. Moreover as CRT tenet posits, neglecting to do so only perpetuates social injustices regarding academic achievement. Therefore, teachers have to understand the mathematical content and be familiar with concepts that students typically struggle with in geometry. For example, they need to be aware that students typically struggle with measuring angles with the standard protractor and are generally unfamiliar with the notion of angles as space embedded within rays. In this regard, appropriate professional training focused on mathematics content, race, class and gender need to be designed for educators teaching in school districts with a high proportion of AA students so that they are better able to teach these concepts effectively.

Elementary school students often reason exclusively, so they typically identify quadrilaterals as two distinct and separate shapes (Clements, Wilson, & Sarama, 2004; Jones, 2000; Monaghan, 2000). Additionally, instructional materials tend to present geometric figures in prototypical orientations (Zaslavsky & Shir, 2005). So, students are not introduced to different representations of figures. As a result, students tend to recognize squares, for example, as figures that always have four “equal” sides and rectangles as figures that always have two equal “longer” sides and two equal “shorter” sides. The geometry curriculum was designed to address these misconceptions by engaging students in tasks capable of supporting understandings of inclusive ideas; identifying possible relationships between shapes. In general, students' understanding of quadrilaterals improved, as demonstrated by performances on post-assessments. However, understandings were not necessarily robust. Although students identified possible relationships between quadrilaterals, they did not necessarily use properties or attributes of the shapes to justify these connections.

Elementary and middle school students do struggle with inclusive classifications because these ideas are quite dense and abstract (Hoyles & Kuchemann, 2000; Johnson-Laird, 2001). Although, they tend to be better at evaluating connections between quadrilaterals when they can appropriately identify the attributes of different shapes (Fujita, 2012; Leung, 2008). This finding is validated by the learning outcomes presented in this study because students did not necessarily have a complete understanding of properties. As a result, they seemed to struggle with distinguishing between quadrilaterals and determining features necessary to classify one as a special case of the other. In evaluating squares and rectangles, for example, students used transformations to classify squares as rectangles even after investigating properties with concrete tools (rulers and protractors). It is important to note that the curricular intervention was relatively short (two weeks) and students seemed to need more time to learn these concepts. Nonetheless, a few students questioned this line of reasoning. They explained that squares were rectangles because both have opposite sides that are equal irrespective of the four equal sides of the square. This was an attempt to clarify for their peers the notion that squares are indeed special rectangles without adapting or modifying the physical orientation of the shape. Although this line of reasoning was atypical, it provides evidence to suggest that some students were able to inclusively classify one quadrilateral as the other. Furthermore, it demonstrates that AA males

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can engage in worthwhile mathematical discourse and interact appropriately when provided with adequate educational opportunities.

To appropriately facilitate students' understanding of inclusive classifications, concrete and technological tools were integrated into the curriculum. Regarding parallelograms, the technological tool (Geometer's Sketchpad) played an integral role in enabling students to understand the properties of parallelograms. This deepened their understanding of parallelograms and its connection to other quadrilaterals because it enabled them to see how parallelograms are related to these shapes. Geometer's Sketchpad also supported students' reasoning about abstract concepts that would otherwise have been very difficult with paper and pencil because it enables shapes to be constructed dynamically.

Limitations

Research findings provide evidence to suggest that AA males will learn when challenged intellectually and provided with appropriate reinforcement and educational support. In spite of these learning outcomes, the context of this study has limitations for potential findings. First, the study was relatively small scale. School and student demographics also affected the results because the students were not diverse. Research findings may be applicable to schools with similar student composition. However, results are likely to be different for schools that serve a different demographic of students such as AA males in more affluent settings.

The measures used for pre- and post-test evaluation of student achievement also created a limitation because the measures did not undergo analyses to determine reliability and validity. Items were constructed to be of equal difficulty. However, a complete analysis of assessment items would have strengthened research findings.

Conclusion

Despite the master narrative that laments the under-achievement of AA males, this study shows that students can learn when provided with equitable educational opportunities. Educational activities comprised of rigorous tasks that optimized hands-on activities and the integration of technology facilitated geometric reasoning. Students learned geometry because they had access to opportunities that enabled them to engage in active exploration and draw valid conclusions from their observations of these concepts.

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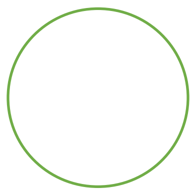
Appendix A

Pre/Post-test Items

1. In the space listed below, please describe your understanding of an angle.

2. Use the hour and minute hands of the clock below to show an obtuse angle.



3. Parallel lines are lines that never meet. Is this a true or a false statement?

   ____

4. What does a perpendicular line have that an intersecting line does not necessarily have?

5. Circle the image that has the larger angle.

6. Describe your understanding of a right angle in the space listed below.

7. Line symmetry is when a figure can be folded on a line so that the two halves match. Is this a true or false statement? _________________________________

Appendix B

Mathematics Interview Questions

1. Draw a quadrilateral that has the following

    a. Opposite equal sides

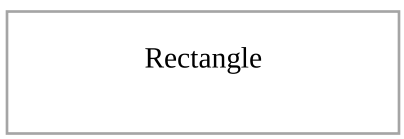
    b. Right angles

    Is your quadrilateral also a parallelogram? Why or Why not?

2. 

    a. What is the name of this quadrilateral?

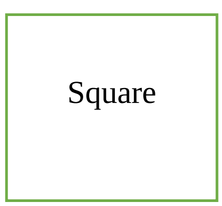
    b. Is this figure a rhombus? Why or Why not?

3. 

    a. Are all the angles 90 degree right angles?

    b. Are the opposite sides equal?

    c. Are opposite angles equal?

4. 

    a. Does a square have these three properties?

    b. What do a square and a rectangle have in common?

    c. How is a square different from a rectangle?

    d. Is a square a rectangle? Why or Why not? Explain your reasoning

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