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Abstract

Writing in mathematics provides students with the opportunity to think critically about and reflect on their experiences while solving problems. While many studies have documented the benefit of writing in math, it is not clear which instructional methods should be used to help students learn how to use writing to support learning. In this study, we take a constructivist approach to building students understanding of effective writing by developing a series of active, student-centered lessons. The findings indicate that students wrote more effectively after the instructional sequence; in particular, they were better able to explain their reasoning and to make connections between abstract mathematics and the context of the problem. These results provide a pathway for improving both students' conceptual understanding, and their performance on open-ended state assessment items.

Keywords

math education, writing-to-learn, conceptual understanding, problem-solving

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Abstract: Writing in mathematics provides students with the opportunity to think critically about and reflect on their experiences while solving problems. While many studies have documented the benefit of writing in math, it is not clear which instructional methods should be used to help students learn how to use writing to support learning. In this study, we take a constructivist approach to building students understanding of effective writing by developing a series of active, student-centered lessons. The findings indicate that students wrote more effectively after the instructional sequence; in particular, they were better able to explain their reasoning and to make connections between abstract mathematics and the context of the problem. These results provide a pathway for improving both students' conceptual understanding, and their performance on open-ended state assessment items.

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Promoting Student Buy-in: Using Writing to Develop Mathematical Understanding

Beginning in 2014-2015, Georgia initiated the Georgia Milestones Assessment System. The new assessments include multiple-choice items similar to those found in the previous state assessment. However, as is the case with many states, the assessment also includes open-ended (constructed response and extended constructed response) items (Beaudette, 2014). The open-ended items require deeper levels of critical thinking as students are asked to explain how and why they obtained each answer (Few, 2014). These tests now encourage the same type of thinking promoted by the National Council of Teachers of Mathematics (2000) and the Common Core State Standards Initiative (2010). One method mathematics teachers can use to respond to the call for students to possess higher level thinking and reasoning skills is to make writing a regular part of the classroom experience.

Researchers have long documented the powerful role writing can play in learning (Bangert-Drowns, Hurley, & Wilkinson, 2004; Bean, 2011; Sanchez & Lewis, 2014). In the National Commission on Writing Report (2003, p.13), the authors argue, "At its best, writing is learning." In the mathematics classroom, writing helps students learn because it provides them with the opportunity to step back from their experiences while solving problems and enables them to interpret, clarify, and reflect on these experiences (Burns, 2004; Urquhart, 2009). It is this type of thinking and reflection that often leads to a deeper understanding of mathematical concepts (Hiebert et al., 1997; Urquhart, 2009). Writing can also be effective in helping students meet many of the goals outlined in the Standards for Mathematical Practice (Common Core State Standards Initiative, 2010), for example, reasoning abstractly and quantitatively (Standard

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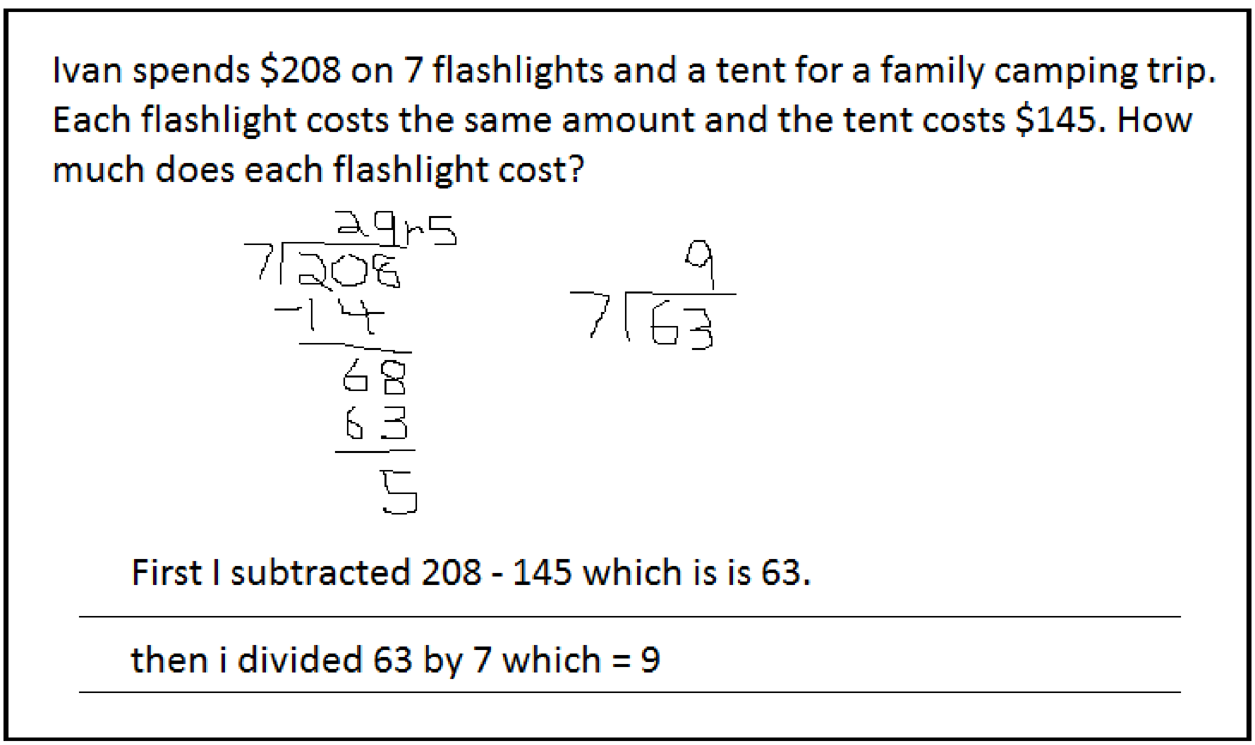
CCSS.MP2), constructing viable arguments and critiquing the reasoning of others (Standard CCSS.MP3), and attending to precision (Standard CCSS.MP6).

Unfortunately, in many mathematics classrooms, there is little written expression (Teuscher, Kulinna, & Crooker, 2015). Even when writing is encouraged, as was the case in the classroom where this action research project took place, students' writing often includes only a step-by-step description of the calculations used while solving the problem (see for example Figure 1). Often missing from students' writing is rich mathematical language and attempts to explain and reflect upon the mathematical concepts examined while solving the problem. In this article, we describe an instructional sequence designed to strengthen students' written justification skills as students created a writing rubric, explored the elements of the rubric, and used the rubric for self-evaluation.


Ivan spends $208 on 7 flashlights and a tent for a family camping trip. Each flashlight costs the same amount and the tent costs $145. How much does each flashlight cost?

First I subtracted 208 - 145 which is is 63.

then i divided 63 by 7 which = 9

Figure 1: A typical writing sample from the pre-assessment

Background

Writing in the mathematics classroom has been found to benefit the development of students' mathematical understanding (Countryman, 1992; National Council of Teachers of Mathematics, 2000; Silver, 1999). The reason for this might be because writing activates and supports students' cognition and metacognition (Bangert-Drowns, Hurley, & Wilkinson, 2004). As Smith (1994, p.36) explains, "We do not think and then write, . . . We find out what we think when we write and in the process put thinking to work—and increase its possibilities." For example, as students write mathematical justifications, they think deeply about the solution strategy they used to solve a given problem. Metcalfe and Shimamura (1994) explain that this type of thinking leads to understanding as students build knowledge about when and how to use particular strategies for solving problems. Pugalee (2004) also documented the benefits that can come from students' writing mathematical justifications. In a recent study comparing students who discussed their solution strategies either verbally or in writing, Pugalee (2004) found that those who wrote about their strategies were significantly more likely to exhibit metacognitive behaviors and were more likely to provide accurate solutions. As these results demonstrate, writing can play a critical role in helping students learn mathematics.

At the beginning of this project, we realized that the students in the current study often did not write in ways that promoted the development of mathematical understanding. Therefore, we set out to determine the best way to encourage students to engage deeply in the writing process. As we reviewed relevant literature, we realized many studies articulated the positive outcomes associated with using a rubric to help support students' writing (see for example O'Connell et al., 2005; Parker & Breyfogle, 2011). Although rubrics were originally created as assessment tools, they are now commonly used in instruction. For example, Parker and

Breyfogle (2011) found that presenting students with a rubric and using it to assess their own writing and that of others, led to improvements in students' writing on future assignments. In such studies, it was common for students to be given a rubric and then told how to use it by the teacher. In contrast, we wanted students to be actively involved in creating the rubric. Therefore, we used a problem-based approach in which students completed a genuine task asking them to determine the elements needed to produce an effective mathematical justification. By making the development of the rubrics' elements an active, student-centered process, we believed students would construct the knowledge necessary and be motivated to write more effectively.

Before designing the lessons, we felt it was important to examine sample writing rubrics to determine what elements we wanted to focus on during the lessons. We eventually agreed upon four elements (Denman, 2013; Dougherty, 2006; Parker & Breyfogle, 2011):

1. The use of precise mathematical language.
2. Explanation of the solution process.
3. Reasoning about why the solution process produced the correct answer.
4. Connections between abstract mathematical procedures and the context of the problem.

Although this list is not absolute, having an idea what was important to us was instrumental in creating the instructional tasks and guiding the class discussions towards the intended outcomes.

The Current Study

This action research study took place in a fifth-grade classroom in the Southeastern United States with 20 general education students. The students ranged in age from 10 to 11 years old. There were 13 boys and 9 girls in the class and thirty percent of the students were English Language Learners. Due to missing assignments, two male students were not included in the final analysis, resulting in a sample size of 18 for this study. The study involved three classroom sessions with additional time at home for students to solve problems and write justifications. The first session took approximately 90 minutes and sessions two and three took 45 to 50 minutes each. To help investigate the effectiveness of the instructional intervention a pre-assessment and post-assessment were administered to each student asking him or her to solve a problem and write a justification. The class-generated rubric was used to score both assessments.

The classroom teacher (*second author*) led all parts of the instructional sequence with support from a university supervisor (*first author*) and a colleague (*third author)*. Below we describe the tasks, provide a short description of the classroom where the tasks were used, and detail the reasons why the tasks were designed the way they were. At the end of the project, it was clear students had a better understanding of what is involved when writing mathematical justifications, and they were better able to demonstrate this understanding in their writing.

Instructional Sequence

Task 1: Creating a Writing Rubric

Our primary goal in the first task was to motivate students to discover the elements of effective mathematical writing. To do this, we created two justifications. One justification was designed to draw students' attention to the four elements we had identified and one was modeled after the type of justifications we typically saw from students. We created the lesson in this way to draw on and connect with students' prior knowledge about writing and to provide them with the opportunity to work with other students to build on this understanding. After individually solving the problem shown in Figure 1 and writing a justification, each student was given a paper with the following: Justification Sample A: I subtracted 145 from 28 and got 63. Then, I

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divided 63 by 7 and got the answer, 9. Justification Sample B: The total cost was $208, so I subtracted the price of the tent to find out how much money was spent on the flashlights, which was $145. The difference was $63. I know the 7 flashlights cost $63, so I divided 63 by 7 and got 9. The quotient, 9, tells me the price of each flashlight. Therefore, the price of each flashlight is $9.

After reading the justifications aloud in class, the teacher asked students to work in groups of four to create a poster that described the similarities and differences between the two justifications. As students worked on this assignment, the teacher encouraged their thinking by asking questions such as, “Which justification would you be more proud to put your name on and why?” and “What do you see in justification B that is not in justification A?” Many groups chose to use a Venn Diagram to record their thinking (see Figure 2) while others made t-charts or lists. Upon completion, each group hung their poster on the wall and shared what they found with the class.

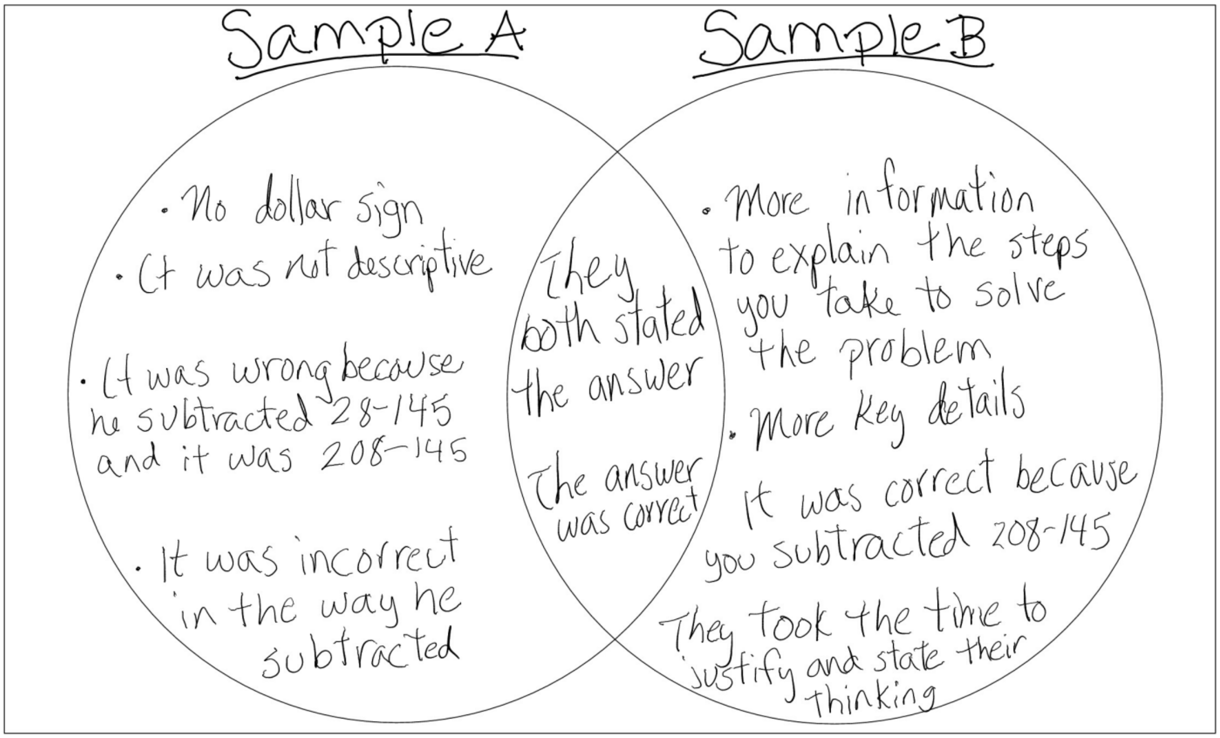

Figure 2: One group's representation of the similarities and differences between the two sample justifications

We wanted students to synthesize this information with the goal of eventually making a list of elements students should use in effective writing. Therefore, the teacher asked each group to look for similarities among the posters, while primarily focusing on the writing in justification B. As students discussed their ideas with the class, the teacher asked them to refer specifically to the comments on the posters to support their findings.

As an example, a student noticed a similarity between these three statements written on different posters, "it has better language (quotient)," "has math language," and "uses juicy words." The teacher asked several students to summarize what the three comments had in common and eventually wrote *mathematical language* on the board. Many students noticed that every poster had something written referencing justification B having more details. As a result, the teacher added *details* to the list on the board.

One student noticed a similarity between the following remarks, “he explains how he gets the answer” and “they took time to justify and state their thinking.” As the class discussed the meaning of the comments, it was apparent the students were having trouble differentiating between what the teacher deemed as mathematical procedures (steps) and mathematical reasoning. The teacher emphasized the distinction between the two by explaining that procedures are what you do, and reasoning is why you do it. With the class’s agreement, she erased “details” and wrote “mathematical steps and mathematical reasoning” on the board.

One element we wanted students to identify that went unnoticed was the fact that the answer in justification B was written using the context of the problem. We purposely left out any connection to the context from justification A, believing that students would notice this right away. In reality, the only comments about the solution written on the posters concluded that both students answered the problem correctly. To draw students’ attention to the different ways the solution was written, the teacher read the last sentence from each justification and asked the students, “If you could choose one way to write your answer on your paper, which would you choose and why?” The class quickly agreed—the sentence written in justification B. The teacher asked students to think about why the writing in justification B was more effective. Student responses ranged from “it’s longer” to “writing it that way helps you think about what the answer means.”

At the conclusion of the lesson, four elements were written on the board. These became the foundation for the class writing rubric (shown in Table 1). After school, the authors created the scoring portion of the rubric and introduced it to the students the next day. By giving students the opportunity to take an active role in creating the elements of the rubric, we believed students would understand the rubric better and be more likely to use it to support their writing.

Task 2: Unwrapping the Rubric

In the second task of the instructional sequence, we wanted to develop further students' understanding of the four elements in the rubric. We asked students to code the sample justifications and score them using the writing rubric developed in class on the previous day. The instructions for coding the justifications were as follows: a) *Circle* **math terminology** and vocabulary used to communicate ideas; b) *Underline* all the **mathematical steps** used in the solution process; c) *Place two underlines* where **mathematical reasoning** was described; and d) Place a *square* around the **answer** if it is **explained in** the **context** of the problem.

Throughout this task, we were pleased with the amount of conversation created while students coded the justifications. For example, we again noticed that differentiating between mathematical steps and mathematical reasoning was often difficult. The debates over how to code the justifications helped the class develop two guidelines to assist students in identifying mathematical reasoning. If the writing connected the procedure with the problem context or explained why a given procedure was completed, then we would call this *reasoning*. Such discussions helped students better understand the elements and prepared students to write effectively.

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Table 1

Class-created writing rubric

Mathematical Justification Writing Rubric				
Element	***2***	***1***	***0***	***Score***
Mathematical Language	***Clearly*** uses accurate math vocabulary to communicate ideas.	***Sometimes*** uses accurate math vocabulary to communicate ideas.	***Does not*** use math vocabulary to communicate ideas or uses vocabulary inaccurately.	
Mathematical Steps	***Clearly*** provides the mathematical steps used to solve the problem.	***Sometimes*** provides the mathematical steps used to solve the problem.	***Does not*** provide the mathematical steps used to solve the problem.	
Mathematical Reasoning	***Clearly*** describes the reasoning used to solve the problem. Includes connections between the numbers used in the strategy and the problem context.	***Sometimes*** describes the reasoning used to solve the problem. Sometimes includes connections between the numbers used in the strategy and problem context.	***Does not*** describe the reasoning used to solve the problem. Does not include connections between the numbers used in the strategy and the problem context.	
Solution in Context	The final answer is ***clearly*** explained in the context of the problem.	The final answer is ***somewhat*** explained in the context of the problem.	The final answer is ***not*** explained in the context of the problem.	
			Self-Evaluation Score →	

Task 3: Using the Rubric for Self-evaluation

In the third task, we wanted students to put into action their new understanding about the rubric to help them write an effective mathematical justification, and later, to use the rubric to evaluate their writing. We used self-evaluation because we wanted students to reflect on their writing, and we believed that this reflection would lead to improvement. At the start of Task 3, students were asked to solve a problem, write a justification, and then code and score the justification. Upon completion of this work, a few students had the opportunity to read their justifications to the class and explain their self-evaluations. The class then discussed the evaluation and connected the scoring with the writing. This process was completed three times before giving students the post-assessment (a sample copy of the post-assessment with self-evaluation is shown in Figure 3).

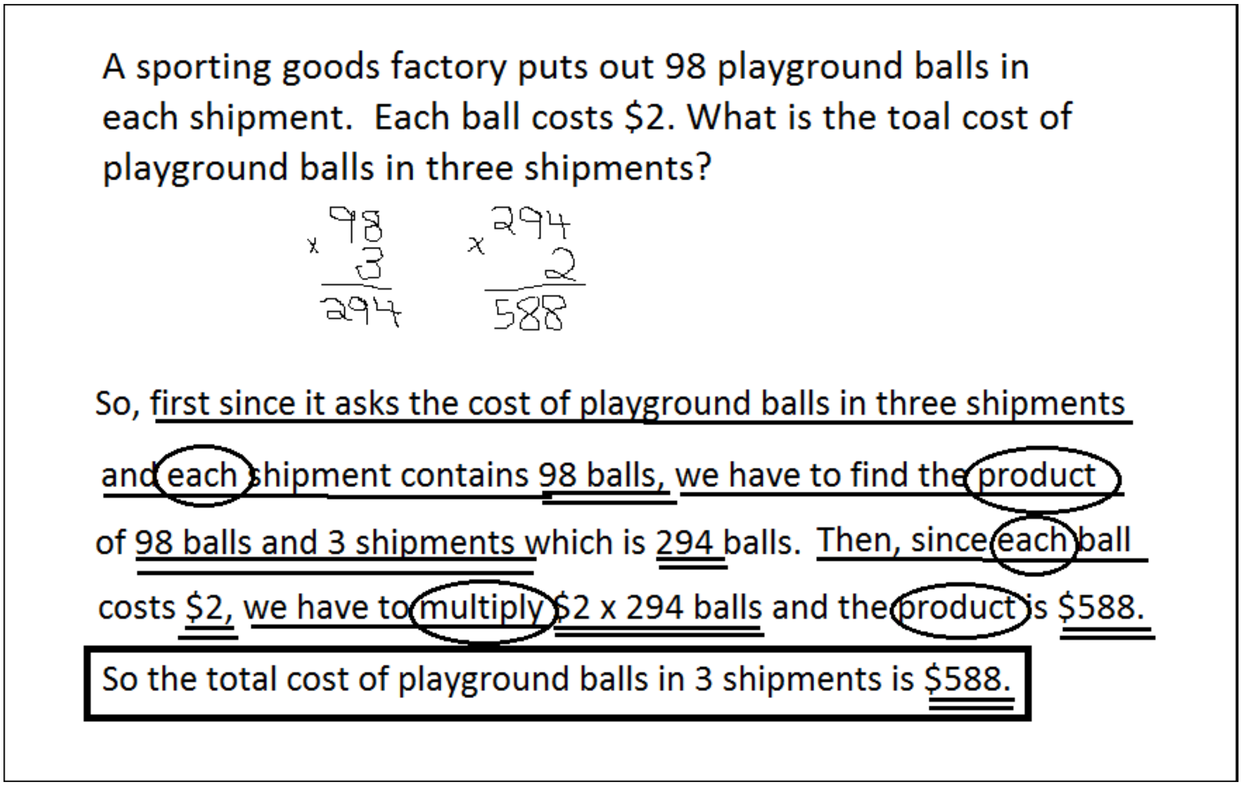

Figure 3. Sample post-assessment response

Results

Overall Learning Gains

We began the analysis by examining the learning gains from the pre-assessment to the post-assessment. To score each assessment we used the class-generated rubric. The class average on the pre-assessment was 3.1 on the eight-point scale. On the post-assessment, student scores more than doubled with a class average of 6.9. As shown in Figure 4, students' scores for both the pre- and post-assessments were highest in the area of mathematical steps. This was not surprising since going into the study the teacher identified this as one of her students' strengths. The next highest scores were in the area of mathematical language where the average score improved from 0.88 to 1.81. While examining the assessments, one change we noticed in mathematical language was a shift from using symbols to describe mathematical operations to using words. The largest area of improvement was in providing mathematical reasoning where the average score increased from 0.38 to 1.44. As an example, one student wrote, "Since it asks the cost of playground balls in three shipments and each shipment contains 98 balls, we have to find the product of 98 balls and 3 shipments, which is 294 balls." Before the instructional sequence, a more likely response would have been "I multiplied 3 and 98. The answer is 294 balls." Another area of improvement was in relating the answer back to the context of the problem. On the pre-assessment, only 30% of the students wrote the solution in context, while 80% of the students did so on the post-assessment.

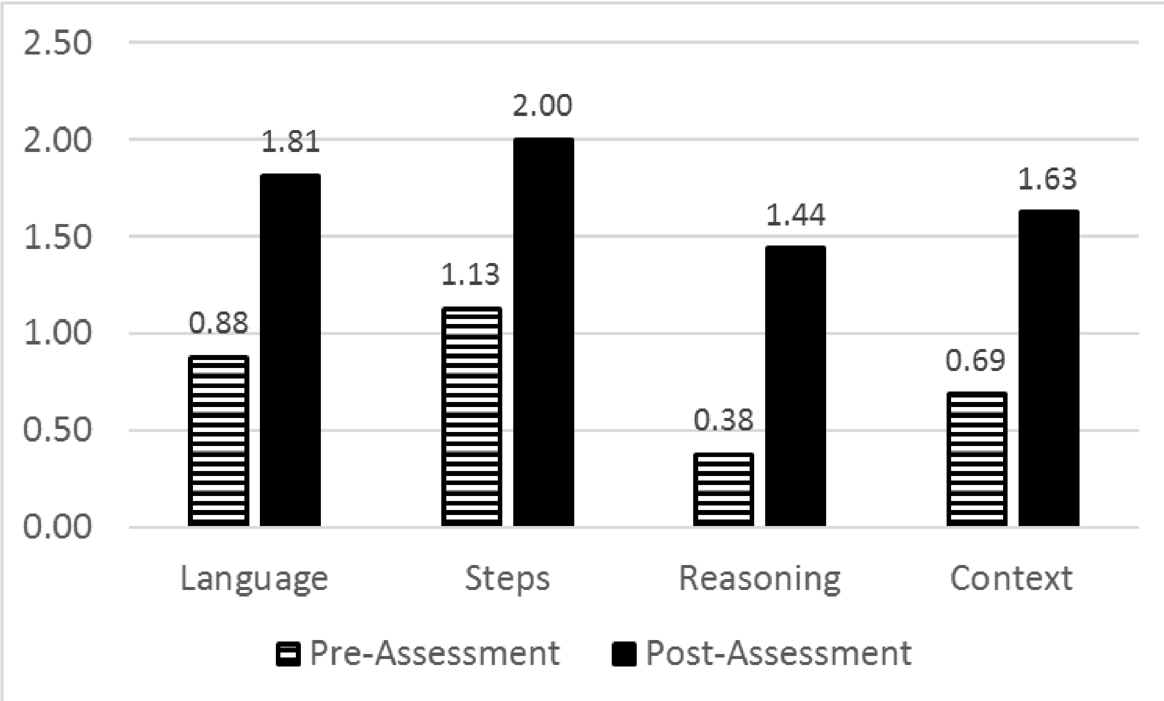

Figure 4. Pre- and post-assessment results

Self-evaluation Scores

In order to examine how students were assessing their writing, we compared the post-assessment scores with the self-evaluation scores completed by the students. The students' self-evaluations were somewhat higher, with an average score of 7.6, compared to 6.9. We determined the majority of this score differential related to a difference in the expectations needed to describe one's reasoning. It was common for us to score a student with a one on this element while students would score themselves with a two. This discrepancy reinforced our notion that the most challenging aspect of justification writing is successfully communicating one's reasoning.

Conclusion

At a time when state standards and assessments are requiring that students think deeply about mathematical concepts and effectively communicate this thinking, writing in the mathematics classroom has the potential to help bridge the gap between completing procedures and building conceptual understanding. In the current study, an instructional sequence was designed to empower students to write in a manner that would promote the type of thinking and reflection needed to build mathematical understanding. Pre- and post-assessment results confirmed that students in this sample wrote more effective justifications after participating in the instructional sequence. While it is true that this result cannot be generalized to students outside the sample, we believe the research-based principles used to develop the instructional sequence suggest it would be successful beyond a single classroom.

When developing the sequence, we focused on incorporating three instructional strategies that have been shown to build understanding, problem-based learning, opportunities for reflection, and opportunities for communication (Hiebert et al., 1997; Jonassen, 2000; Marshall & Horton, 2011). We incorporated problem-based learning in the opening task by asking students to explore the elements needed to write an effective justification. Instead of telling students the elements, they struggled to discover these elements for themselves. Communication was an essential part of this process as students worked together to compare and contrast the two sample justifications and to develop the elements for the class rubric. Students were also provided with ample opportunities for reflection as they were asked to evaluate their own written justifications using the class rubric. This process allowed them to step back and think deeply about whether their writing was effective or not. By designing the instructional sequence in a way that enabled students to problem solve, reflect, and communicate, we are optimistic that this sequence will be effective in other settings.

Recommendations

While the results in this study are promising, we learned several lessons that will guide how we use the instructional sequence moving forward. Most notably, we realized it is a challenge for students to write about the mathematical reasoning they use while solving problems. We attribute some of this challenge to the type of problems predominately used in this study. Most of the problems required students to review material they had previously learned. We decided to use review problems because we believed this would enable students to focus more intently on their writing. In retrospect, however, this approach may have led some students to omit their reasoning because they assumed the reasons were obvious. We now believe using problems designed to develop new understandings would encourage students to write more deeply about their reasoning. In the future, we plan to use a greater variety of problems in hopes that this will inspire students to explain their thinking more thoroughly.

In addition, we have three suggestions for strategies teachers can use in the classroom that we believe would greatly benefit students' capacity to write about their reasoning. First, ask students as frequently as possible questions such as, "Why did you do that? Explain to me why that makes sense. Tell me what you were thinking." Second, provide examples. Let students see samples of writing that clearly describes mathematical reasoning. Also, provide contrasting cases showing effective and ineffective writing and ask students to compare and contrast the two samples (similar to the two justifications presented in Task 1). Lastly, draw on students' prior experiences by asking them to construct an argument about why it is important to explain your reasoning in addition to showing the steps taken while solving a problem.

Throughout the remainder of the school-year, we noticed several improvements in students' math work, beyond those seen in their writing. We noticed increased effectiveness in oral communication as students began to incorporate the elements from the rubric into their classroom conversations. In general, we observed that students were more careful about using precise mathematical vocabulary and worked harder to describe their reasoning. Also, we noticed an increase in students' ability to solve problems. We believe that providing students opportunities to reflect on their problem solving through writing, helped them think more deeply about the process and enabled them to be more effective at monitoring their progress during the problem-solving process. Although our informal observations are promising, future research should consider how this instructional sequence is related to improvements in students' oral communication and problem-solving skills.

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