Epistemic Irony in Philosophical Narrative

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Abstract

This paper explores the use of irony in narratives that focus on the problem of knowing what we do not know. Sometimes issues arise on a grand scale, as in the literature of Socrates, Pascal, and Descartes, where the question really is whether we can know anything at all or whether all that we can know is that there is nothing worth knowing that we do know. The grand level does not apply only to philosophy, but also to mathematics and theoretical physics. They are disciplines that seek to state truths about the structure of the universe. Examples are Newton's laws; absolute space and time and Einstein's denial of absolute space and time; Maxwell's Equations and Heisenberg' Uncertainty Principle. There are problems about the nature of knowledge that arise on a smaller scale, at the petite level, for example where ignorance of a protagonist's own epistemic condition leads to tragedy, as in narratives from Sophocles, Shakespeare and Flaubert. These personal issues pertain to introspective knowledge, but not all petite issues deal with the intensely personal. Some deal with problems that cry out for statistical analysis, as in the medical sciences or in the social sciences more generally. Other more general issues do not attempt to generalize about the "deep structure" of nature and thought, but often take the form of "statistically significant" correlations. The paper argues that ironies about knowing test the limits of rational deliberation about what to do and what to believe, which in turn accounts for much that is perplexing in intellectual life, tragic in personal life and confused in public life. All this, it is concluded, encourages more humble assessments of the extent of human knowledge and therefore a greater commitment to intellectual and moral toleration.

Introduction

Philosophy is a strange subject in that its name suggests that it isn't a subject at all, but rather an affection: the love of wisdom; and yet, philosophy is also viewed as the parent of all theoretical knowledge. What then is the relation between wisdom and knowledge? Perhaps it is right to insist that the wise distinguish what they know from what they do not know. Broadly, the purpose of this project is to explore this suggestion.

Socrates was the first Western thinker to maintain something like the view that philosophers can distinguish what they know from what they do not know. In *Apology* Socrates reports that he consulted one who had a reputation for his wisdom, but that "in the process of talking with him and examining him," Socrates was driven to the conclusion that his companion wasn't wise at all. In fact, Socrates congratulates himself, because although he thinks that he doesn't know anything much worth knowing, at least he doesn't think that he knows something worth knowing.

> So, I left him, saying to myself as I went away: Well, although I do not suppose that either of us knows anything really worth knowing, I am at least wiser than this fellow – for he knows nothing, and thinks that he knows: I neither know nor think that I know. In this one little point, then, I seem to the advantage of over him.[1] (Plato/Jowett, *Apology*, 21df, p. 345)

[1]Plato, trans. Jowett, 1953/orig. 399 BCE, 21df, p. 345.

Ironically, Socrates seems to imply that there is at least one thing worth knowing, and that is knowing that one doesn't know anything (else?) worth knowing. Irony is achieved by using a fragment of language to assert or designate the exact opposite of its literal meaning. I suppose, if anything is bad news, it is that no one really knows anything worth knowing.

The ironic response to Socrates' narrative surely is the current cliché: "Good to know."

The Socratic Paradox: On a Grand Scale

The above passage from *Apology* has given rise to a series of puzzles collectively designated as the "Socratic Paradoxes." Indeed, if one really knows nothing worth knowing (or perhaps doesn't know anything at all) but thinks that one knows, one must be seriously mistaken about what knowledge is or what it takes to get it. Imbedded is a crucial distinction between being *aware* of a given proposition and not knowing whether it is true and *believing* that there might (or even must be) some factor that is *relevant* to knowing that proposition, but not knowing what that factor is, or else what to make of it. Issues of this sort arise in epistemology and metaphysics, in natural science, in analysis of personal introspection and in the statistical analysis of significant empirical correlations.

Newton and his successors knew that the "wobble" in the precession of the perihelion of Mercury's orbit around the sun posed a threat to Newton's own unified account of terrestrial and celestial motion. One might attribute the deviation to some sort of intervention by God or perhaps to the gravitational attraction of a hitherto unobserved object (which was tentatively named "Vulcan." during the 19th century). Yet for over 200 years no one suspected or could have even conceived that the explanation for Mercury's misbehavior lay hidden in the presupposition of Newtonian science that space is "absolute." Unfortunately for Newton, Einstein demonstrated that the geometry of space depends upon the presence of mass. Although space emptied of all mass would be Euclidean, actual space is Riemannian, meaning that Euclid's Fifth Postulate is false of actual space, where straight lines within a plane do not have parallels. It is therefore the "deformation" of space due to the mass of the sun that accounts for the "wobble" in the precession in the perihelion of Mercury.[2]

Sometimes the impossibility of knowledge of specific facts is a consequence of scientific theory itself. For example, a consequence of the Special Theory of Relativity, which treats of non-accelerating inertial frames, is illustrated by Makowski's famous space-time diagram, which illustrates the fact that space and time are not independent and as a consequence much of the universe is inaccessible to us.[3] Another obvious example comes from Heisenberg, who demonstrated that it is impossible to determine the momentum and position of certain small entities, like electrons.[4] More recently Stephen Hawking suggested that despite evidence to the contrary, "information" apparently lost as events are gobbled up by black holes is nevertheless stored (somehow) at the event horizon itself.[5]

These examples are extremely important from a philosophical point of view because they show that even the most certain theories can turn out to be false. The most important of these cases mentioned above is Euclidean geometry. For two thousand years philosophers thought that Euclidean geometry was the paradigm of a successful theory of the structure of physical space. Descartes, for example, thought that any theory as certain as Euclidean geometry was surely beyond doubt. The fact that that the finest minds were mistaken about the structure of physical space ought to give us pause. We all believe the deliverances of the best minds of our time, but mightn't they be mistaken? Perhaps we do not have a reason now to doubt

[2] Einstein and Infeld, 1938, pp. 209 – 239.
[3] SPGR, Stanford University, Lecture 3: Lecture 3 Spacetime Diagrams, Spacetime, Geometry, 2015, p.1.
[4] *Op. cit*., Einstein and Infeld, pp 287 – 91.
[5] Hawking, 2008, p.2.

the great theories of our time, but that might be only because we are *unaware* of pertinent data. All this is to say that we cannot assess the epistemic importance of information that we do not have, and, by hypothesis, we do not know just what information that might be.

Worries about mathematical physics are troubling enough, but there are still grander questions. The questions of theoretical science do not challenge our *capacity* to know. Irony arises at this level as well, most explicitly and famously in Descartes, Pascal and Hume. Descartes wonders whether all putative knowledge is undermined by an evil force so that even the most certain of beliefs systematically mislead us about the world and even about ourselves. Only God, Descartes insists, can extricate us from systemic doubt.[6] Yet as most commentators have insisted, we can hardly know that God rescues us from doubt without knowing that God exists, and we can hardly know that God exists unless we already know that God has given us reliable tools that will rescue us from doubt about our powers to know.

One way in which Descartes was misled (indeed the most important way in my view), is that Descartes assumed that mathematics is the unshakeable foundation of physics. According to Descartes we clear and distinctly perceive the attribute of extension, which is the essence of matter, and therefore the mathematical truths about extension must describe the *physical* space in which matter is located. So, for Descartes, whatever is necessarily true of extension applies, *mutatis mutandis*, to matter and the space it occupies. Since we "know" that *Euclidean* geometry is necessarily true of extension, it must be necessarily true of physical space as well. As we have seen above, however, Einstein successfully argued that matter, (though extended) and the space that it occupies are not Euclidean, but rather are Riemannian – meaning that straight lines in the neighborhood of mass do not have parallels.

Pascal, who was reasonably threatened (intimidated?) by thoughts of the "infinite," accepts the idea that the grand questions of philosophy are imponderable.

> Pascal claims that it is incomprehensible that God should exist, and incomprehensible that he should not; that the soul should exist in the body, that we should have no soul; that the world should be created, that it should not; that original sin should exist, and that it should not; etc.[7]

Yet, Pascal concludes that one inescapably lives either on the basis of the existence of God or does not live on the basis of the existence of God. Arguing that the payoff for a correct bet on the existence of God is eternal bliss and that an incorrect bet against the existence of God is eternal damnation, Pascal concludes that the only wise course is to take a *chance* and go "all in" for God.[8]

There have been many objections to Pascal's line of reasoning about how to wager assuming that one "must" wager. Do we really know that the consequence of an incorrect choice is eternal damnation and of a correct choice eternal bliss? Isn't that very claim also open to open to Socratic doubt? Perhaps the right conclusion is that we just do not know how to choose. Perhaps Pascal concedes as much inasmuch as he insists that we "know" of God's existence only through faith. For many, however, that is just another way of saying that we really do not know at all; perhaps that is what Socrates would have said in response to Pascal.

Hume argues famously that our capacity to make simple predictions is flawed in that predictions presuppose that past and present are connected or related in a way that assures us that *the future will continue to*

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[6] Descartes, Rene, trans. Cottingham *et al*., 1984/orig. 1641, §23 6, ¶40f., pp. 27f.
[7] Pascal, Blaise, trans. Honor Levi, 1995/orig. 1670, §656, p. 148
[8] *Ibid*, §680, pp. 152 – 55.

resemble the past. But knowledge that the future will resemble the past can be inferred from *past* successes only if we already know precisely what we do not know, which is that the future *will continue* to resemble the past. There is an additional and even deeper problem, which is that we cannot ever be certain that we have *all* the evidence that is pertinent to a given knowledge claim or even that what has counted as pertinent evidence in the past will continue to be counted as pertinent evidence in the future. To summarize: We do not and cannot know that we have all the pertinent evidence to support a claim about the future, but even if we did know that in the past we had all the pertinent evidence, we could not infer that what was then pertinent evidence is still pertinent nor could be infer that facts that were not pertinent in the past would not become pertinent.[9]

The Retreat to Probable Knowledge and Statistics

Perhaps all this goes to show only that our assertions need to be qualified in order to count as certain knowledge. Perhaps we cannot know, for example, whether or not we shall die this year, but perhaps we can know the *probability* that we will die this year. However, as we shall see, claims to know the *probability* that an event will occur are just as problematic as knowledge claims. Indeed, probabilistic claims are uncertain for the very same reasons that unqualified, "unhedged" knowledge claims are problematic.

What we learn from statistics is not the probability of a given event but rather the *probability of a given event on certain evidence*. In order to explore this idea, I am going to refer extensively to an excellent example presented by James Joyce in the *Stanford Encyclopedia of Philosophy*. (The passage is fairly long, and it is reproduced for the reader's convenience at the end of this paper under the heading "Appendix ").

Joyce begins his example and analysis by referring to an "unconditioned probability."[10] Our goal in the example is to calculate the probability that "J. Doe" died during the year 2000. Our initial information comes from the U.S. Census Bureau, according to whom "*roughly*" (my emphasis) 2.4 million of the 275 million American citizens alive on January 1, 2000 died during the year 2000. According to Joyce, we might reasonably conjecture that the probability that J. Doe was one of those who died during 2000 is 2.4 m./275m., which is .00873.

Joyce calls this "probability" an "unconditional probability," and is absolutely right to do that. The reason is that the unqualified *form* of the resulting probability statement is: P(H) = .00873.[11] This is going to be the starting point for our analysis of the probability that J. Doe died during 2000. As Joyce implicitly acknowledges by qualifying his data report by the word "roughly", the calculation is probability not *absolutely* accurate.[12] So, if we hope to have absolutely certain knowledge of probabilities, we'll need to concede at the very outset that we are *not* dealing with input data that are absolutely certain. Therefore, the resulting conditional probability calculation will not be absolutely certain. Yet, perhaps this is all too quick. Perhaps someone will argue that lacking confidence that a calculation is precisely correct is pretty much

[9] Hume/Norton & Norton, 2007/orig. 1739-40, §3.2-3.4, pp. 52 – 58.

[10] What Joyce calls "unconditioned probabilities" are also called "prior probabilities" in explicitly Bayesian contexts.

[11] In this example, the unconditioned probability appears to be "objective,' meaning that it is more than a mere guess. Sometimes, as we shall see later, probability judgments are blatantly subjective. In that case, as intimated above in fn. 10, the starting point of a probability calculation is sometimes called a "prior probability,' which is also "unconditioned."

[12] Indeed, we could complicate the story by trying to measure the *probability* that the U.S., Census Bureau's estimation of deaths of American citizens during 2000 is *correct*. Problematic features might conclude that the difficulty of knowing just who actually are American citizens. (For example, recently and during the Bush and Obama administrations, difficulties arose in determining whether or not certain people born near the Texas/Mexico border really were born in the United States because well-meaning, charitable, sympathetic medical workers erroneously recorded the United States rather than Mexico as their patients' place or birth. (*Texas Tribune*.org, 2018/08/29.)

the same as being absolutely certain that it is roughly correct. Either way, however, belief in the present case that the unconditioned probability that J. Doe died during 2000 is .00873 is *fallible*.

That unconditioned initial probabilities are uncertain is also illustrated by the fact that many introductions to the subject, unlike the one by Joyce, begin with examples that are contrived. For example: There are three bags of marbles; two are filled with fifty red marbles each; the other is filled with fifty black marbles. The bags are put into an empty urn after each of the 150 marbles has been uniquely numbered from 1 to 150. What is the probability that #29 is red? We are inclined to say that it is certain that the unconditioned probability is 100/150 = .66667. It is tempting to think that here we have conquered the unknown, but that is not true. Someone might have miscounted the marbles in the bags or perhaps didn't get all the marbles back in their proper bags after numbering them. The point is that we often generate our unproblematic unconditioned probabilities by *ignoring* all the facts that might undermine the calculation., It would be better, I think, to qualify unconditioned probabilities by saying that they are unconditioned probabilities *for all we know or suppose.*

It must have occurred to the reader that in the previous case, it is natural, indeed essential to ask about how the marbles were numbered. The example really presupposes that the numbering was random. But suppose that one bag of red marbles was numbered from 1 to 50; the blacks were numbered from 51 to 100 and the third bag of reds was numbered from 101 – 150. In that case, the probability that #29 is red, *given the information about the numbering process and the other pertinent information about the case*, is 100%. So, if you don't know about the numbering process, you don't know that it is certain that #29 is red. What you know is that #29 is red for sure on *the condition* that the numbers were inserted *as described* and that the marbles were deposited from the three bags into an empty urn.

The only unconditioned probability calculations that are arguably absolutely certain are those that are *purely abstract*. Consider the integers from 1 to 100. What is the probability that a given integer from that group is also in the group from 1 to 40? The answer is clearly 40/100= .4. Here there isn't room for error because all the relevant information (the "givens") are themselves certain. But these are just the cases where statistical analysis is unnecessary. All probability calculations of this sort are "*a priori*," which is to say independent of contingent facts.[13,14]

Whatever we think of the unconditioned probability that J. Doe died during 2000, it is obvious that there might be factors that might affect the probability of J. Doe's death in 2000. Joyce selects an obvious one;

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[13] Even this claim needs to be more carefully circumscribed. In the case that we are considering infinite sets, the probability calculations are arguably problematic. Is the *probability* that an integer is greater than 40 *greater* than the *probability* that an integer is greater than 30? It might seem so, but the matter is complicated by the fact that the *cardinality* of the sets of integers greater than 30 is the same as the *cardinality* of the set of integers that are greater than 40. It is worth emphasizing that even in this case the certainty of the calculation presupposes that the calculation is done correctly. So, what doubts we have about our mathematical and logical abilities will of course affect the certainty of our calculation of probabilities. As Descartes suggests in *Meditations One*, perhaps the evil demon might deceive us in thinking that 2 + 3 = 5. Descartes, Cottingham, *et al*., *op. cit.*, §21, p. 14.

[14] There is a distinction often drawn between subjective and objective probability or interpretations of probability. Probabilities concerning purely abstract identities are calculated by purely deductive reasoning. They are "subjective' only to the degree that we doubt our own powers of reasoning. On the other hand, many beliefs about what is probable that are based upon introspection are purely subjective. Examples might be drawn from the testimony of mystics, who claim to have experiences of the supernatural that others do not have. In the middle are probable judgments of one degree or another. The notion of a *conditional* probability (X is probable to degree x on condition y), applies to those judgments that are more or less objective as well as those that are obviously subjective. Bayes Theorem instructs us to revise each probability assessment on new information. It applies equally to more or less objective as well as to more or less subjective probability claims. A detailed explanation of this understanding of probability claims and their relation to Bayes Theorem and statistical analysis is beyond the scope of this paper.

which is age. According to the U.S. Census Bureau, of the 16.6 million American citizens 75 or older, 1.36 million died during the year 2000. Now, suppose that we complicate our calculation in this way: Let E represent the information that J. Doe was a senior. What is the probability that J. Doe died during 2000 *on the condition that J. Dow was a senior*, that is, on the condition E? Now the probability that J. Doe was a senior in 2000 is 1.36m./275m. = .06036, and the probability that J. Doe was a senior who died during 2000 is P(H & E)=1.36 m./275m.= .00495 divided by P(E) = .06030. The probability of H in condition E is written as $P_E(H)$.

$P_E(H)$ = P(H & E)/P(E), which is:1.36 m./275m.= .00495, which in turn is divided by 06030 and = .082[15]

It may seem that the calculation of $P_E(H)$ is certain because it is a mathematical certainty. (Well, keep in mind that whether or not it is a mathematical certainty depends upon the skill of the people who perform the calculation.) Even if the $P_E(H)$ is a mathematical certainty, its certainty depends upon the prior calculations of P(H & E) and P(E), those calculations are themselves based upon data that may be imprecise or actually false.

The idea of conditional probability is made more precise by Bayes Theorem. In order to understand Bayes Theorem in terms of Joyce's example, we need to distinguish $P_E(H)$ from $P_H(E)$. $P_H(E)$ is the probability that J. Doe is a senior on the condition that he is an American citizen. The probability that J. Doe died in 2000 on the condition that he is an American citizen is

$P_H(E) = [P(H \,\&\, E)/P(E)]/P(H) = .57.$

As Joyce observes, $P_H(E)$ is the probability that J. Doe died in 2000 given that he is a senior who also is a part of the total population of American citizens. This number tells us that 57% of those who died among the total population of American citizens were seniors. The previous calculation of P_EH tells us that 8.2% of the senior population died in 2000, and hence the probability that J. Doe is among those who died on condition that he was a senior is 8.2%.

The calculations $P_E(H)$ and $P_H(E)$ make sense and are intuitively correct, but of course it is always possible that a conceptual error has crept into the calculation. More importantly, however, *the air of certainty encouraged by the mathematics is disguised by the fact that our data might be inaccurate*. Even more importantly, if we want to know the probability that J. Doe died in 2000, there will be many other variables that are relevant; for example, whether J. Doe is male or female; whether J. Doe smoked tobacco, and if so how frequently for how long; and if J. Doe was a consumer of alcoholic beverages or narcotics, and if J. Doe had genetic disorders, and so on. *The calculations of conditional probabilities will need to continue on and on in order to accurately assess J. Doe's chances*. And most importantly of all, we will never really know whether or not we have captured *all the salient facts* about J. Doe. So, while it looks as though we can be certain of what is probably true, assertions of probable truth seem to be open to the same worries that we have encountered previously concerning unqualified apodictic assertions. Error can creep in at any point, whether it is a mathematical error or a conceptual confusion or corrupt data.

[15] At first this may seem counter-intuitive, because it may not be obvious why it is that P(H & E) is divided by P(E). The answer is that P(H & E) is determined by the ratio *of* people who are American citizens and seniors who died *to* the entire population. But that portion of the total population includes seniors *only to the extent that seniors are a part of the total population* and the probability the Doe is among those who died is conditioned upon his membership in the smaller portion of the population who are seniors.

As Joyce continues, there is an important relation between a conditional probability and *its inverse*.[16] We have already seen this relation but perhaps have not noticed it. $P_E(H)$ is the inverse of $P_H(E)$. The reason this is important is because there is a relation between a conditional probability and its inverse that was captured by Thomas Bayes and is known as Bayes Theorem. It states:

$P_E(H) = [P(H)/P(E)]\ P_H(E)$

Recall that
$P_E(H) = .082$
$P_H(E) = .57$
$P(E) = .06030$
$P(H) = .00873$.

Thus, Bayes Theorem tells us that

$P_E(H) = (.00873/.06030)*.57$, where
$(.00873/.06030) = .144776$, and
$.144776 * .57 = .0082$,
Which is the value of $P_E(H)$.

Intuitively this equation tells us that the *ratio* of the deaths in the total population to the deaths in the population of seniors is the same as the *ratio* of the probability of the death of a given member of the population, J. Doe, given that he is a senior, to the probability of the death of a senior, J. Doe, given that he is a member of the general population. In other words:

$P_E(H)/P_H(E) = P(H)/P(E)$.

Showing that this equation is true is a simple calculation:

$P_E(H)/P_H(E) = .082/.57 = .144$.

and

$P(H)/P(E) = .00873/.06030 = .144$, allowing for rounding errors.

Joyce and many others think of Bayes Theorem as an obvious truth, and even a mathematical triviality. I suppose that this is so, but it is important to remember that for over two thousand years virtually everyone thought that Euclid's Fifth Postulate (that there is exactly one line parallel to any given straight line through a point outside the line but on its plane) is trivial and obvious -- until Gauss, that is, who introduced differential geometry and thereby enabled us *"to define the curvature of a surface at a point"*[17] In this way Gauss paved the way for the non-Euclidean geometries developed by Lobachevski and Riemann.[18]

I suppose that someone will say that the foregoing reinforces and elaborates Disraeli's famous quip that there are lies, damn lies and statistics. Yet, nothing could be further from the truth. Statistical analysis

[16] This distinction is illustrated by the following example. Let us suppose that of the people who come down with lung cancer, Y percent have been smokers. That obviously is different from the claim that of the people who have been smokers, Y percent come down with lung cancer.
[17] Boyer, 1968, p.568.
[18] *Ibid*., pp. 586 – 90.

has done more to sharpen our understanding of probability than any other tool. The point of this paper is not to criticize, much less to denigrate, a subtle, ingenious and essential tool in identifying the ways in which events are correlated with each other. The point is simply to emphasize the overlooked (or at least under emphasized fact) that merely qualifying a prediction by saying that it is "probable" or "very probable" even after the most rigorous statistical analysis, is *not sufficient to remove any possible doubt about its probability, whether unconditional or conditional.* The possibility of errors in reasoning, of falsely characterizing data, and miscategorizing true data are ever present.

When it comes to statistics and the analysis of probability, we just don't know what we do not know.

Still More Worries, on a "Smaller" Scale

It is wrong to assume that "grand" ironies are the ones that affect us most deeply. Knowing one's own position in life is not a grand issue, but that does not make it less important. Oedipus and Jocasta were in the dark about who they were. Had they known their identities, their lives would not have ended in tragedies that not only engulfed them but also were visited upon the next generation. Indeed, Oedipus was determined to avoid the fate that the oracle said was inevitable, and tragically it was his very effort to avoid the inevitable that resulted in the inevitable. It is indeed difficult to know just what the moral of the story is meant to be. Sometimes things don't work out no matter how careful we are, and sometimes it is the very care that we take that undermines us. Yet, sometimes tragedy multiplies itself as it does in the Theban tragedies, which end in the deaths of three of four of Jocasta's children. The only surviving child is Ismene who lives a cautious and conventional life, leaving fighting to *men*; as she says: "We are *women* and do not fight with men."[19] One of the lessons of the Theban tragedies is humility. The one who survives is not the one who tries too hard or takes herself to seriously.

In her own way, Ismene is humble, and I believe that one of the lessons of the Theban tragedies is humility. This is the conclusion that Sophocles draws as he ends *Antigone*, in the very the last lines of the tripartite tragedy:

> Wisdom is supreme for a blessed life,
> And reverence for the gods
> Must never cease.
> Great words, sprung from arrogance,
> Are punished by great blows.
> So it is one learns, in old age, to be wise[20]

Human knowledge and strength are limited, and it is only hubris that makes us *confident* that we shall succeed where all others have failed.

The case of Emma and Charles Bovary is comical as well as tragic. The problem is not that they were determined, like Oedipus, to avoid an inevitable, tragic outcome. Their failing was much simpler; they were simply unaware of the dreadful fate that they tempted. Charles was completely unaware of his wife's dissatisfaction with her and their life; indeed, it did not even *occur* to him that that he might not know his wife at all. Unfortunately for Charles, Emma felt imprisoned by her life of bourgeois complacency; as a nineteenth century Justice Ginsburg might have said: neither perched on a pedestal nor recumbent upon a soft couch, but rather imprisoned within in a cage. Emma longs for accomplishments of her own and a life of glamour and adventure, not the lazy comforts of a pretty little house on the French countryside. Charles's pathetic ignorance of his wife's undernourished identity persisted even beyond her death. It was simply never occurred to him that his wife could have wanted more than he could provide. After all, what could a

[19] Sophocles, trans. Paul Woodruff, 2001/orig. 442 BCE, p. 3 lines 58 - 62
[20] Ibid, p. 58, lines 1348 – 53.

woman want more than her own secure home, in a comfortable place, respected by her neighbors and loved by a "successful" husband?   Perhaps an identity of her own?

Emma too was in the dark insofar as she expected liberation from Charles to lead automatically to a life of excitement and fulfillment. Music is a hard master, and it is easy to think that one can play like the masters until one actually takes lessons and tries hard. More importantly, Emma's life of extravagance was not based upon her own modest achievements or a realistic assessment of her prospects away from Charles' secure home. As she took on more and more debt and became more and more desperate, she turns to a former lover, Rodolph, in an attempt to renew their love. At first, he responded ardently, but when it comes time for him to make a life for her, he thinks better of it. As he contemplates the inevitable burdens of a life with Emma, he concludes: "And besides, the worry, the expense! Ah no, no, no, no, a thousand times no."[21] It is as though Flaubert wryly observes that there is nothing that so quickly chills a renewed love affair more than a request for financial support.

It might be that there is a Iago ready to worm his (or her) way into every intimate relation, but there are similar worries about any friendship or professional association. The thought of not knowing what is really in the hearts and on the minds of others is the rot in which suspicion grows. From an emotional point of view, it affects us more deeply and tragically that the arcane doubts that undermine our confidence in theoretical science or even reason itself. Who, at one time or another, hasn't been an Othello? And then there are those who think that they can live again. Goethe's Faust longs for renewed love, but instead he only breaks his beloved's heart. His efforts to recover in acts of grotesque sensuality and vain searches for the extraordinary lead him instead to the perfectly ordinary: engineering land reclamation, which is a project that an old man of common sense might have undertaken in the first place. No fool like an old fool; they say, but apparently the saying comes only to the minds of those who already know.

These and innumerable other examples that could be offered show that in our relations with others and more importantly our relations with ourselves, true motives are often obscured, or misconstrued, or are obsessions determined to save us from the very truth that would save us. All these forms of ignorance (whether setting aside the important, or ignoring what others see, or mischaracterizing one's own intentions in compulsive self-deception) all of them are epistemological failures. We ought to know better, but we don't; and even when we do, it is often too late. Introspective knowledge is perhaps the least secure of all. There is no hope there from deductively valid reasoning or from subtle statistical analysis. The truth would be worth knowing, but a lesson of tragic literature is that truths worth knowing are often beyond us.

Public Policy

It is difficult to think of an area of human endeavor in which it is more important than public policy to take care not to act in or from ignorance.   It was Donald Rumsfeld, former Secretary of Defense of the United States, who emphasized the problem of acting on unknowns. The issue arose at a NATO conference in Prague during 2002. The purpose of the meeting was to determine NATO's response to global terrorism. Secretary Rumsfeld was worried especially about difficulties in planning that seem to require intelligence that NATO did not have. The principal issues that arose concerned terrorism and weapons of mass destruction. The following is a part of the news conference:

[21] Flaubert, Gustave, 2008/orig. 1857, p. 250.

> **Q:** Regarding terrorism and weapons of mass destruction, you said something to the effect that the real situation is worse than the facts show. I wonder if you could tell us what is worse than is generally understood.
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> **Rumsfeld:** Sure. All of us in this business read intelligence information. And we read it daily and we think about it and it becomes, in our minds, essentially what exists. And that's wrong. It is not what exists.
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> I say that because I have had experiences where I have gone back and done a great deal of work and analysis on intelligence information and looked at important countries, target countries, looked at important subject matters with respect to those target countries and asked, probed deeper and deeper and kept probing until I found out what it is we knew, and when we learned it, and when it actually had existed. And I found that, not to my surprise, but I think anytime you look at it that way what you find is that there are very important pieces of intelligence information that countries, that spend a lot of money, and a lot of time with a lot of wonderful people trying to learn more about what's going in the world, did not know some significant event for two years after it happened, for four years after it happened, for six years after it happened, in some cases 11 and 12 and 13 years after it happened.
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> Now what is the message there? The message is that there are no "knowns." There are things we know that we know. There are known unknowns. That is to say there are things that we now know we don't know. But there are also unknown unknowns. There are things we don't know we don't know. So, when we do the best we can and we pull all this information together, and we then say well that's basically what we see as the situation that is really only the known knowns and the known unknowns. And each year, we discover a few more of those unknown unknowns.
>
> It sounds like a riddle. It isn't a riddle. It is a very serious, important matter.
>
> There's another way to phrase that and that is that the absence of evidence is not evidence of absence. It is basically saying the same thing in a different way. Simply because you do not have evidence that something exists does not mean that you have evidence that it doesn't exist. And yet almost always, when we make our threat assessments, when we look at the world, we end up basing it on the first two pieces of that puzzle, rather than all three.[22]

I have reproduced Secretary Rumsfeld's remarks because I think that they are of interest right now in light of worries about the spread of nuclear weapons. Rumsfeld's reply is also interesting in that it illustrates the complexity of Socrates's worries and the awkwardness of even describing them. There are, as Rumsfeld says, "the known knowns, but there are also unknown unknowns." They are tricky because we don't know them, but "each year we discover a few more of those unknowns." The secretary closes with an interesting epistemological insight, where he asserts "because you do not have evidence that something exists does not mean that you do have evidence that it does not exist." That obviously applies only to the happy case in which you are aware of possible evidence, but then there are the unhappy cases in which you do not have

[22] Rumsfeld, D., 2002.

any idea of what you are looking for, and yet the unhappier case in which you are simply unaware that there is anything to look for at all.

It is simply impossible to exaggerate the consequences of the inadequate intelligence concerning weapons of mass destruction at this juncture. The decision that was taken to go to war in Iraq over the issue of weapons of mass destruction crucially depended upon the existence of those weapons. And that, unfortunately, was uncertain. Many believed that President George W. Bush deliberately overstated the case for war. Many even attributed bad motives to the president. Perhaps they were right. But it is also possible that the president thought that the consequences of a wrong decision would be so horrific that he really had no choice but to go to war on the evidence (or perhaps suspicion) that he had.

This case is especially difficult because we do not have a group of cases that are very similar, and so we cannot mimic the earlier calculation about the probability of the death of J. Doe in 2000. In that case we could count the number of individuals with a certain property and distinguish them by that property from the whole. This is what happened in the case concerning the deaths of seniors.[23] Cases like those concerning the development of nuclear weapons as well as the personal stories developed from great literature tend to respond to subjectivist accounts of probability. What makes a theory more subjective is the subjectivity of the initial "unconditioned" probability. In the examples of the senior citizens, we can simply find the ratio of seniors to the total population by counting. Sure, we might miscount; we might leave some out, we might not really know whether or not an actual person is an American citizen, but even so, there appears to be a more or less objective standard to be applied in calculating the initial unconditioned probability. In the one-off cases, our initial probability is more speculative. In fact, if we cannot compare a part of a set to the whole set, just how are we to assign a *value* to a probability function when we consider the more subjective cases?

This leads us to consider a more subjectivist account of probability. On this account, probability assignments are not meant to *represent* the world (e.g., how many senior citizens will die next year), but rather are mean to *represent probability as a willingness to bet*. Here the magnitude as well as the "direction" of the bet needs to be taken into consideration. Obviously, President Bush and Secretary Rumsfeld attached great importance to the matter of weapons of mass destruction. The negative value of the presence of those weapons led both officials to assign a huge expected negative value to the possession of nuclear weapons by Iraq, which justified a very large bet (the "ante") or cost of conflict with a very low probability of success. That meant, as President Kennedy might have said, that it would be rational to "bear any burden and to pay any price" to disarm the adversary.

To be sure, a decision to place a bet is not always an indicator of uncertainty. Suppose, for example, that we are placing bets on which card will be randomly drawn next from an ordinary deck of 52 cards. The probability that a Heart will be drawn is ¼ because there are 13 hearts in every deck with 52 cards. Thus, we actually know that a fair bet is 1:4. If we put up one dollar, and a heart comes up, we should expect to

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[23]On the other hand, it should not be thought that all public policy is like the calculation to go to war over possible weapons of mass destruction. For example, policies that are developed on the basis of epidemiological models are like the cases that we previously analyzed concerning the probable death of a senior. In general, epidemiologists try to develop drugs that result in the great increase of healthy years among the population at the lowest cost. So, medications to reduce blood pressure are cheap to produce and distribute and have an enormously positive effect on a great mass of the population. Successive applications of the drugs will be tailored to avoid objectionable side effects. Obviously, calculations of this sort involve conditional probabilities that are continually revised.

receive four dollars (the one we put up plus three more).[24] Someone taking the other side, who puts up three dollars should expect only one dollar (plus the original three put up) if a non-Heart comes up, since the probability that a non-heart will be drawn is ¾ (because there are 39 non-hearts in a deck of 52 cards). On the other hand, if a non-Heart is drawn, we should expect to lose our dollar.

This reasoning depends upon the assumption that the deck of cards is good and that the draw is fair, that is, that each card has an equal chance of being drawn. That, of course, is quite an assumption, and we are loath to take it for granted. However, *given* that the assumptions are correct, the probability calculation is certain, and we can actually know whether a bet is fair or whether it is to our advantage or disadvantage. In this context, although we are thinking of probability on a subjective basis, as indicated by a willingness to bet, the *probability calculation* is merely a matter of counting, and not a matter of chance, even though the card actually drawn is a matter of chance.

It is useful to consider probability as a measure of willingness to place a bet where the probability of an outcome cannot be determined as a matter of ordinary mathematical calculation. Cases of this sort might arise in card games as well. Suppose that we are playing Blackjack (also called "21") In this case we bet on whether or not the sum of series of cards drawn from a fair deck (or usually four fair decks) at random will come closer to 21 than the sum of the series of cards drawn by our opponent. Now, as the game progresses, some cards are drawn and *discarded*. So, if we could only keep them in mind, we would know how the probability of drawing a given card, say the 3 of Hearts, changes after each draw. However, the unfortunate truth is that most people cannot keep all the drawn cards in mind, and even if they could, they could not calculate the changes in probability *fast enough* to place a favorable bet.

How shall we understand a bet in these circumstances? Perhaps a bet is just a matter or irrational commitment, a thought that might well provoke the conclusion that honest labor really is better than gambling. But never mind! Someone who places a bet in those circumstances is acting upon *incomplete information* and a flawed understanding of the circumstances in which the bet is made. Unfortunately, the decisions made in the ordinary course of life, viewed as bets, are just like this. So, the question that we all face is *just how much to risk on the basis of imperfect understanding*. The subjectivist says that the probability that is actually assigned by a person should be understood, for better or worse, *as a bet* placed on the basis of incomplete information. The broader philosophical point is that due to the fact that there are always "unknown unknowns," every probability assignment, except those that are purely abstract, should be viewed as a bet.

As Ian Hacking observes, during the nineteenth century, statistical generalizations were thought to be "reducible" to deterministic calculations of the probability of events. This is essentially the "classical" model of statistical analysis, and perhaps it is still the prevailing theory, which is illustrated by the example concerning the probability of the death of J. Doe. However, by the end of the nineteenth century, the reducibility of statistical generalizations to deterministic models began to be questioned and statistical

[23] Stark, P.B., 2016, "A fair bet is one for which the **EXPECTED VALUE** of the payoff is zero, after accounting for the cost of the bet. For example, suppose I offer to pay you $2 if a fair coin lands heads, but you must **ANTE** up $1 to play. Your expected payoff is −$1+ $0×P(tails) + $2×P(heads) = −$1 + $2×50% = $0. This is a fair bet—in the long run, if you made this bet over and over again, you would expect to break even. The expected value of a **RANDOM VARIABLE** is the long-term limiting average of its values in independent repeated experiments. The expected value of the random variable X is denoted EX or E(X). For a discrete random variable (one that has a **COUNTABLE** number of possible values) the expected value is the weighted average of its possible values, where the weight assigned to each possible value is the chance that the random variable takes that value. One can think of the expected value of a random variable as the point at which its **PROBABILITY HISTOGRAM** would balance, if it were cut out of a uniform material. Taking the expected value is a **LINEAR** operation: if X and Y are two random variables, the expected value of their sum is the sum of their expected values (E(X+Y) = E(X) + E(Y)), and the expected value of a constant *a* times a random variable X is the constant times the expected value of X (E(*a*×X) = *a*× E(X))."

“laws” were thought to capture regularities among natural phenomena.[25] Thus the subjectivist model has gained favor throughout the contemporary era. However, the subjectivist model is not new. Colin Howson traces this understanding of statistical analysis back to David Hume and seventeenth century philosophy of science., Many philosophers now have taken up the subjective approach implied or at least anticipated by Hume.[26] In fact, this more subjective view of probability can be traced even farther back, all the way to philosophers like Pascal.[27] As we have seen, on this theory, rationality is thought to be exhibited as a kind of betting.

The *interpretation* of even very simple subjectivist probabilistic calculations can be misleading and must be undertaken cautiously. For example, suppose that there are two research proposals, R and R*, that promise to develop a new pesticide that will increase food production. Suppose that R and R* each require $1 m. in research funding. (This is the analogue of the “ante” in the previous example concerning betting on cards.) Suppose that the research committee judges each proposal to be equally promising. But the committee is convinced that if R works out, the estimated payoff is 4m; whereas the R* works out the estimated payoff is 5m. (In each case the original 1m is included in the payout, just as “the ante” was included in the previous example concerning cards.). Now, from these expectations it follows that a “fair bet” on R requires that the probability that R will be successful is .25. On the other hand, a bet on R* is a fair if the probability of a successful outcome is .2. Now the rational bet is R*, which may initially seem to be counter-intuitive because it appears that the rational bet is on the less probable outcome. Nevertheless, by hypothesis R and R* are equally likely to succeed. And the previous calculation does *not* undermine the hypothesis. The above calculation shows that the committee can afford *to take a greater risk on R* than on R because the payout on R* is greater.* The “probability” deduced in the example from the ante and payoff is not a function of the likelihood of the success of the research proposal *but merely the probability that is required to make the research grant a “fair bet” on the funded research*. That indeed is exactly what common sense would expect: the higher the payout on the same investment, the greater the risk that may be rationally accepted.

In this connection is instructive to return to the example of the spread of nuclear weapons. In this case of the decision to go to war against Iraq, it is clear that Secretary Rumsfeld and President Bush thought that the expected value of the decision to go to prevent the spread of nuclear weapons was very high and that it justified an effort even if the effort had a low probability of success. In the event, President Bush did not emphasize the possible cost, and perhaps because he thought that the effort was so important that it would be worth pursuing even if the probability of success were very low. Yet, many Americans were worried that the cost, both direct and indirect, might be very high, and in the ensuing event they came to think that the costs were very high. What might well have misled people about the decision to go to war could very well have been a different assessment about what its costs would be and *whether or not a successful outcome could justify those costs.* That is why it pays to consider probable costs, benefits and risks in explicit detail. In that way, it is more likely that communication failures can be avoided. Nevertheless, critics may claim that the Bush administration *exaggerated* the probability of a positive outcome in order to justify the decision to war. Perhaps that would explain why it is that so many people were outraged when in the end weapons of mass destruction were not discovered and the cost of the war was very high. It was not that he was mistaken, it was rather he *represented the probability of success that he believed would justify the war as the probability that the war would actually be successful.*

There are many “everyday” examples of the ways in which incomplete knowledge can undermine rational deliberation when it comes to betting. We all know that driving while intoxicated increases the risk of fatal accidents. Exactly how much the rate is increased for a given person, however, appears to be imponderable.

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[25] Hacking, 1991, pp. vii; 1 – 10.
[26] Howson, 2000, pp. pp. 116 – 11.
[27] Van Fraassen, 1989, pp. 293 – 300.

It is true that there might well be statistics showing how many people on average who have given level of blood alcohol cause fatal accidents while driving. Yet, there are some who believe that they can "handle it." They might attach a very small probability that they will cause a fatal accident with the standard permitted limit of alcohol in them. When they take the wheel, they are in effect betting their own lives (and the lives of others) on their assessment that the risk they are assuming is not significant. The probability of a negative outcome *in that case* is significantly lower than the probability that would be calculated assuming that *each person is equally affected by blood alcohol levels*. Absent this reasonable assumption, there is cognitive space for a delusory, reckless driver to hide from the obvious conclusion that the risk being assumed is too high given the usual aversion to death. But some evidently believe, as reflected by the bet they take in getting behind the wheel, that their risk is significantly lower than average.[28]

Putting It Altogether

The conclusion that appears to be warranted from all this is that it is best to remain extremely cautious in making knowledge claims. And here it, as everywhere, it is best to practice what we preach. Socrates said that he neither knows nor thinks he knows anything worth much; but how about that piece of wisdom itself? Wisdom is certainly worth something (as Socrates implies) and so it would be worth knowing whether or not one is wise.

We have observed that even many "time-honored" (or perhaps merely persistent) "facts" of pure mathematics must be qualified. Great theories of mathematical physics haven fallen. Not only are there non-Euclidean, purely abstract geometries, but physical space itself is non-Euclidean. We have seen our conception of nature at the atomic level has changed to the point that we now no longer think that every "object" has a definite position and momentum at any given time. Perhaps that means that our very conception of reality at the atomic level has been separated from the common-sense reality of medium size physical objects.

Someone might have thought that we could have real knowledge if only we are more cautious in characterizing that knowledge. Perhaps knowledge is essentially statistical. The field of statistics is of course immensely complicated. Some statistical analyses appear to be more or less objective, like the analysis of the probability that J. Doe died in 2000 based upon statistical information about American citizens and the age distribution of deaths among them. Although in those cases the mathematics of conditional probability and Bayes Theorem make sense, we saw that the possibility of error always creeps in as we identify the unconditioned probabilities that are refined, over and over, by adding new conditions to our probability judgments.

Moreover, it appears that many probability calculations are very subjective. When we decide to go to war, there doesn't seem to be a place for statistics at all. Information about possible enemies is unavailable, and momentous decisions are ultimately based upon "unknowns," sometimes on nothing more than mere suspicion. It just is not true that these cases are rare or are consequential only when the decision affects millions, or perhaps everyone on Earth. Individual tragedy often hinges upon misinformation or perhaps on total ignorance of salient facts. How do we know what we don't know when we are completely "clueless"? In all these cases, probable calculations appear to be essentially *bets*. What makes a bet rational depends upon the probability of a favorable outcome and the value of the outcome. But when the value is immense (whether positive or negative), very small probabilities are overwhelmed, and the resulting

[28] Examples like the foregoing are instructive in analyzing the logic of self-deception. Self-deception is not a matter of deliberately ignoring pertinent evidence (How could it be?) but is rather a matter of mischaracterizing the strength of the evidence one has. Who, for example, wants to be bothered by the anxiety and inconvenience of a trip to the doctor over a mere mole on the hand?

expected values approach “infinity” or zero. Outcomes of bad bets are often tragic, not because whoever made the bet is foolish (or evil) for having made it, but rather because tragic outcomes occur despite our best, good faith efforts,

Wisdom

Surely wisdom consists in at least recognizing and accepting what we might call epistemological contingency. We can be fully confident in our belief systems and schemes of rational choice only if we know that their underlying assumptions are true, and that is exactly what Socrates claims that the wise do not think that they know. Yet, that raises another, even more difficult question. Just what does “fully confident” actually mean? In particular, how are we to understand the contrast between “fully confident” and “partially confident,” “somewhat confident,” or “more or less confident? More precisely, this is a request for an understanding of the meaning of “probable” itself. As we have seen, one of the most important ways of construing *probability* is to consider it in contexts where we face choices and therefore a choice among various bets.[29]

This conception of probability assignments (that is, as bets) is to be contrasted with classical models of probability that were illustrated previously in calculating the probability of the death of a certain person in a certain year. As previously noted, this is the type of calculation that arises in fields like epidemiology. In addition to these conceptions of probability, there are innumerable others. This essay has only touched the surface of the subject. As we have also observed, there are issues about knowledge that do not seem to involve probabilistic thinking in an obvious way, like grand issues of philosophy and mathematics and physics. Moreover, there are issues about self-knowledge, which appear to involve “knowledge” gained only by introspection, which is often augmented by psychoanalysis.

These disparate conceptions of *deductive knowledge*, *introspective knowledge* and *knowledge of probable outcomes* tempt us to think that knowledge is really *contextual*. What counts as knowledge depends upon the context of the question and the discipline that has been designed to answer them. Yet, even if “knowledge” is contextual, the philosophical mind is drawn to the possibility that there is a “broader,” more encompassing conception of knowledge that can unify all the others by providing a comprehensive account of the *methods* that can be applied in disparate cases. In a way it would be a “super-context,” broad enough, with methods powerful enough, to account for all the others.

I believe, however, that this vision is seriously misguided. It is true that we yearn here and elsewhere for super-contexts, but a super-context looks suspiciously like the *context of all contexts*. The context of all contexts would define a method by which we could assess the relative merits of all the various methods of coming to know. Unfortunately, in order to make a case for the context of all contexts we shall encounter the Cartesian dilemma. How shall the deliverances of the context of all contexts be evaluated? If by its own standards, it will surely be deemed to be question-begging; if not by its own standards, then by which, since all the others are to be evaluated by it? The idea that knowledge is fragmentary is offensive to the philosophical mind. From Plato on we have tried to convince ourselves that there is a special state of mind in which all judgments will be validated. That state of mind has often been defined by its objects: in Plato’s case the forms; in Descartes’ innate ideas; in Kant’s the pure categories of the understanding and the pure forms of intuition. Yet, all these attempts are at least inconclusive because they cannot reasonably take account of themselves and according to them, there is nowhere else to turn. All this is unavoidably vague, but perhaps it can be best understood as a return to pragmatism. On the contrary, each discipline focuses on methods that have proved successful, and the ensuing division of labor strongly suggests the conclusion that each discipline, usually by trial and error, finds its own methodology and assesses its validity by its own methods. That is perhaps the best that can be done, and although the best is not the realization of all

[29]Admittedly, this interpretation of probability is only one among several. See: Romeijan, 2014, §4.3.1

our hopes for objectivity, it has unquestionably served us well enough -- for more ideas about just how and how well, see Romeijan.[30]

Toleration

Perhaps a final irony is that Socratic frustration might well turn out to be one of humankind's greatest blessings. Justifying the imposition of our will on others depends upon knowing what is good for them in the circumstances in which they find themselves. We require parents to provide medical care for their children, even if those same parents are sure that God will look after their children better than medical doctors. We require students to master physics even though we know that physical theories come and go; even though we know that mathematics describes abstract structures and not necessarily physical space and time. Finally, when it comes to matters of the heart, the unfortunate truth appears to be that we do not really know much of anything at all, including our own hearts.

All this suggests that the wise course is not to proselytize. This doctrine of toleration applies not only within families, to parents and spouses with "big ideas" for those they love, but also to large enterprises that rigidly enforce polices that discourage and even undermine creativity. Even more importantly, following Montesquieu, toleration must apply to cultures and nations with differing ideas about what is good and about how best to govern themselves.

Aristotle distinguished intellectual virtue from moral virtue. Intellectual virtue properly guides belief formation; moral virtue properly guides our actions. Toleration, I suggest, is an example of a virtue that is both intellectual and moral. Intellectual toleration recognizes that beliefs that have been inviolate for centuries can nonetheless be overturned. More importantly, toleration recognizes that *it cannot be its own judge*; it cannot determine whether what it thinks it knows, it really does know; and it cannot determine whether or not it has taken all "unknowns" into account. This intellectual modesty leads to moral tolerance. We can disagree with others about how best to live and what to value, but that disagreement must be tempered by the confession that nothing is so certain that it can be justifiably forced upon others.

Even so, our conclusions about toleration must be cautious and couched. One can go too far with toleration: To the point of tolerating intolerance. Some things just *cannot* be tolerated, like genocide, which perhaps is the ultimate act of intolerance. To be sure, there aren't many absolutely clear cases of the intolerable, and the best remedy for it is hardly ever obvious. Surely doing the least violence is best, even if in the last resort it is sometimes unavoidable. After all, it is one thing to say that we really do not know how best to live; it is quite another thing to say that we might as well live up to someone else's standards and allow them to dominate our lives. The more important the issues, the more difficult it is to sort them out.

Good to know!

Summary and Conclusion

We began by reflecting upon the wisdom Socrates imparts to those who had voted to put Socrates to death for corrupting their youth, for teaching them to question values that their elders cherished. Socrates' only possible defense, the one he made, is that he didn't really teach the young anything positive at all; he only taught them not to accept without question what others taught. Perhaps we shall rediscover Socratic wisdom in our time. This analysis, as brief and inadequate as it is, suggests that assessing our own epistemic state, even when it is carefully couched as probabilistic knowledge, must be cautious, if only because we cannot consider completely unknown factors or factors that our methods cannot treat adequately. Our conclusion is that all knowledge is tentative, including this knowledge. Moreover, knowledge depends

[30]Romeijan, *op. cit*. §2 – 5.

upon method, which in turn depends upon context. The various branches of knowledge are fragmented, and there isn't likely to be a single standard that will emerge by which to adjudicate all its claims. However, all this leads to an intellectual toleration that embraces moral toleration. Although moral toleration cannot be coherently extended to the point of tolerating intolerance, it constrains dogmatic judgments about the ways of other people and their cultures. Some may feel liberated in this way; others perhaps sorely disappointed to forego the unity of thought sought by great forebears like Plato, Descartes and Kant.

Either way, we are left with Socratic wisdom: It is easy to think we know when we do not know; as one might enigmatically say: We just do not know what we do not know, but perhaps it is enough, therefore, to know that it is best to tread lightly; to refrain from judging others; to do no harm.

Good to know; really – I think!

Appendix

> "The probability of a hypothesis *H* conditional on a given body of data *E* is the ratio of the unconditional probability of the conjunction of the hypothesis with the data to the unconditional probability of the data alone.
>
> **Definition**.
> The probability of *H* conditional on *E* is defined as $\mathbf{P}_E(H) = \mathbf{P}(H \,\&\, E)/\mathbf{P}(E)$, provided that both terms of this ratio exist and $\mathbf{P}(E) > 0$.[1]
>
> To illustrate, suppose J. Doe is a randomly chosen American who was alive on January 1, 2000. According to the United States Center for Disease Control, `the 2000 calendar year. Among the approximately 16.6 million senior citizens (age 75 or greater) about 1.36 million died. The unconditional probability of the hypothesis that our J. Doe died during 2000, *H*, is just the population-wide mortality rate $\mathbf{P}(H) = 2.4\text{M}/275\text{M} = 0.00873$. To find the probability of J. Doe's death conditional on the information, *E*, that he or she was a senior citizen, we divide the probability that he or she was a senior who died, $\mathbf{P}(H \,\&\, E) = 1.36\text{M}/275\text{M} = 0.00495$, by the probability that he or she was a senior citizen, $\mathbf{P}(E) = 16.6\text{M}/275\text{M} = 0.06036$. Thus, the probability of J. Doe's death given that he or she was a senior is $\mathbf{P}_E(H) = \mathbf{P}(H \,\&\, E)/\mathbf{P}(E) = 0.00495/0.06036 = 0.082$. Notice how the size of the *total* population factors out of this equation, so that $\mathbf{P}_E(H)$ is just the proportion of seniors who died. One should contrast this quantity, which gives the mortality rate among senior citizens, with the "inverse" probability of *E* conditional on *H*, $\mathbf{P}_H(E) = \mathbf{P}(H \,\&\, E)/\mathbf{P}(H) = 0.00495/0.00873 = 0.57$, which is the proportion of deaths *in the total population* that occurred among seniors.
>
> Here are some straightforward consequences of (1.1):

- *Probability*. $\mathbf{P}_E$ is a probability function.[2]
- *Logical Consequence*. If *E* entails *H*, then $\mathbf{P}_E(H) = 1$.
- *Preservation of Certainties*. If $\mathbf{P}(H) = 1$, then $\mathbf{P}_E(H) = 1$.
- *Mixing*. $\mathbf{P}(H) = \mathbf{P}(E)\mathbf{P}_E(H) + \mathbf{P}(\sim E)\mathbf{P}_{\sim E}(H)$.[3]

The most important fact about conditional probabilities is undoubtedly *Bayes' Theorem*, whose significance was first appreciated by the British cleric Thomas Bayes in his posthumously published masterwork, "An Essay Toward Solving a Problem in the Doctrine of Chances" (Bayes 1764). Bayes' Theorem relates the "direct" probability of a hypothesis conditional on a given body of data, $\mathbf{P}_E(H)$, to the "inverse" probability of the data conditional on the hypothesis, $\mathbf{P}_H(E)$.

Bayes' Theorem.

$$\mathbf{P}_E(H) = [\mathbf{P}(H)/\mathbf{P}(E)]\ \mathbf{P}_H(E)$$

In an unfortunate, but now unavoidable, choice of terminology, statisticians refer to the inverse probability $\mathbf{P}_H(E)$ as the "likelihood" of H on E. It expresses the degree to which the hypothesis *predicts* the data given the background information codified in the probability $\mathbf{P}$.

In the example discussed above, the condition that J. Doe died during 2000 is a fairly strong predictor of senior citizenship. Indeed, the equation $\mathbf{P}_H(E) = 0.57$ tells us that 57% of the total deaths occurred among seniors that year. Bayes' theorem lets us use this information to compute the "direct" probability of J. Doe dying given that he or she was a senior citizen. We do this by multiplying the "prediction term" $\mathbf{P}_H(E)$ by the ratio of the total number of deaths in the population to the number of senior citizens in the population, $\mathbf{P}(H)/\mathbf{P}(E) = 2.4\text{M}/16.6\text{M} = 0.144$. The result is $\mathbf{P}_E(H) = 0.57 \times 0.144 = 0.082$, just as expected."[31]

[31] Joyce, James, 2003, §1.1; 11.2.

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