IMPROVEMENT ALGEBRAIC THINKING ABILITY USING MULTIPLE REPRESENTATION STRATEGY ON REALISTIC MATHEMATICS EDUCATION

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Abstract

The study aimed at improving students' algebraic thinking ability in the eighth-grade junior high school student through multiple representation strategies using realistic approach. The multiple representation strategies consist of orientation, exploration, internalization, and evaluation. This is a quasi-experimental study with nonrandomized pretest-posttest control group design. The population of this study was the student the eighth grade in Kudus city. Two classes were selected and classified as a class experiment that subject are given multiple representation strategies using realistic approach, and one other class as a control class that subjects are given scientific approach. Data obtained was analyzed by the independent t-test and proportions test. The result showed that there was an interaction between the multiple representation strategies using the realistic approach on the ability of algebraic thinking. The students with multiple representation strategies had better algebraic thinking ability than those with current scientific learning. In addition, more than seventy-five percent of the students with multiple representation strategies using realistic approach fulfill the learning completeness.

Keywords: algebraic thinking, multiple representation strategy, Realistic Mathematics Education

Abstrak

Penelitian ini bertujuan untuk meningkatkan kemampuan berpikir aljabar pada siswa SMP kelas delapan melalui strategi multiple representasi menggunakan pendekatan realistis. Strategi multile representasi terdiri dari: orientasi, eksplorasi, internalisasi, dan evaluasi. Penelitian ini merupakan penelitian quasi eksperimen dengan desain pretest-postest non-acak. Populasi penelitian ini adalah siswa kelas delapan di kota Kudus. Dua kelas dipilih dan diklasifikasikan sebagai eksperimen kelas yang subjek diberikan strategi multiple representasi dengan pendekatan realistik, dan satu kelas lain sebagai kelas kontrol yang subjek diberikan pembelajaran dengan pendekatan scientifik. Data yang diperoleh dianalisis dengan uji t-test. Hasilnya menunjukkan bahwa ada pengaruh antara strategi multiple representasi dengan pendekatan realistik matematik pada kemampuan berpikir aljabar. Siswa dengan pembelajaran menggunakan strategi multiple representasi memiliki kemampuan berpikir aljabar yang lebih baik daripada siswa dengan pendekatan scientifik. Selain itu, lebih dari tujuh puluh lima persen siswa dengan strategi multiple representasi pada pembelajaran realistik memenuhi kriteria ketuntasan belajar.

Kata kunci: berpikir aljabar, strategi multipel representasi, pendidikan matematika realistik

Algebraic thinking can be interpreted as an approach to quantitative situations that emphasize aspects of public relations using tools that are not always symbols but can be used as a cognitive tool to introduce and retain the more traditional school algebra discourse (Kieran, 2004). Some scholars define algebraic thinking, one of which is Ameron (2002), defining that, algebraic thinking is a mental process such as reasoning with unknowns, generalizations, and formalizing the relationship between magnitude and developing the concept of variables. To highlight, it can be interpreted that algebra

thinking is a mental process with something unknown, generalize, and make the relationship formula between the scale and build the concept of variables.

Teachers are needed to know students' algebraic thinking skills, especially in junior high school students in math problems. Teachers must understand the way students think in algebra. The teacher's thoughts are important to consider when the teacher gives polyhedron material, numbers, functional relations, social arithmetic and others where the material requires the ability to use the algebraic form and its solution in the form of algebra. This is following the opinion of Kamol and Har (2002) show that to solve algebraic problems in mathematics learning students need to have mathematical reasoning.

According to Ntsohi (2013), algebraic thought is the use of symbols and mathematical tools to analyze different conditions by representing information mathematically regarding words, diagrams, tables, graphs and equations and using mathematical findings such as calculating unknown values, proving and determining relationships between functions. Affirming algebraic thinking, Kriegler (2002) points out that there are two components in algebraic thinking, namely the development of mathematical thinking tools and the study of the basic idea of algebra. The tools of mathematical thought, on the other hand, consist of three categories: tools for problem-solving skills, representational skills, and quantitative reasoning abilities. Meanwhile, the basic idea of algebra in question is algebra consist of the generalization form of arithmetic, algebra as the language of mathematics, and algebra as a tool for the functioning and modeling of mathematics (Kriegler, 2002).

Algebra is not only important to be learned in school age, but also for adult life in which it is required for the period of work even during life. In the Piaget thinking stage, students at the school age of 7 - 15 years are at the formal operational stage. At this stage, an individual has begun to think about experiences outside of concrete experience, and think more abstractly, ideally, and logically. A specific operational understanding needs to look at the real elements A, B, and C to draw the logical conclusion that if A = B and B = C, then A = C. In contrast, formal operational experts can solve this problem even if the question is only presented in oral (NCTM, 2000).

Algebra is not only needed during education. However, in adult life algebra is also important not only in advanced education, but also in employment. In Piaget's thinking stage, students at the age of 15-16 are at the stage of thinking. Students at this stage should be able to use algebra in solving math problems, "In grades 9-12 all students should use algebra symbollic to represent and explain mathematical relationship" (Parton, 2012).

Some thoughts on algebra, the views of both classical and modern algebraic views (Usiskin, 1988; Kieran, 1996; Chevallard, 1999;), attempt to extract the basic concepts and methods that can be considered the essence of algebra. Kieran (2004) defines several components of algebra as follows: a) development of mathematical thinking tools, divided into three topics, namely: problem solving skills, representational skills, and quantitative reasoning skills; b) the idea of fundamental algebra, divided into three subjects, namely: algebra as general arithmetic, algebra as a language, and algebra as a tool

for mathematical functions and modeling.

Multiple representations consist of various formats of representation that can be used in the learning process. According to Kohl and Finkelstein (2006a) and Knight (2013), representations consist of (1) verbal representation, (2) diagrams, (3) graphical representations, and (4) mathematical representation, required when students solve quantitative problems using equations that match the information obtained. Note that mathematical representation is only one of the few and most in physics leads more to think and reasoning than solving equations (Knight, 2013). Multiple representations are widely used in mathematical research to improve students' mathematical concepts.

Ainsworth (2008) states that multiple representations are highly relevant and necessary in learning to build and develop an understanding of the concept of the situation in depth scientifically. Kohl's research, et al. (2008) shows that students who learned through multiple representations complete a set of mathematics tests better than thosewho learned through few representations.

Realistic mathematics education supporting learning through multiple representations

Realistic mathematics education (RME) is known as a learning approach developed based on Freudenthal's (1905-1990) idea arguing that mathematics is a human activity, in which it should be associated with real world. Freudenthal also stated that students could not be considered as passive recipients. Instead, they should be provided with opportunities to reinvent mathematics through teacher guidance through real-life experiences (Treffers, 1991; Gravemeijer, 1994; Lange, 1995; Panhuizen, 2010). The characteristics of RME, namely "real world" context, models, student production and construction, interactive and intertwinement, indicates that RME starts with real problems so that students can use the previous experience directly. Also, RME can also hel students develop a comprehensive particular concepts in mathematics as well as apply mathematical concepts to new fields and the real world problem.

In particular, Soedjadi (2001) describes the characteristics of RME as follows. The use of context means that students' daily environment or knowledge can be used as part of the contextual learning materials. The use of models means that problems or ideas in mathematics can be expressed in the form of models, both models of real situations and models leading to the abstract level. The student contribution means that problem-solving strategies or concepts are discovered based on students' idea contribution. The interactivity means that learning process activities are built by the interaction of students with students, students with teachers, students with the environment and so. Lastly, intertwinement means different topics can be integrated to generate an understanding of a particular concept simultaneously.

With regard to the relationship between applying RME and encourgaing students' multiple representation strategies, students are also given the opportunity to use the ideas acquired in solving problems related to daily life so that students are more likely to benefit from the material learned and to apply then the concepts they have in everyday life.

METHOD

Research Design

In this research, the quasi-experimental research was applied. Specifically, the nonrandomized Pretest-posttest control group design was selected for the quasi-experiment research. Sampling with simple random sampling technique. The subjects in this study were eighth graders on the topic of the polyhedron. The sample in this study with 72 students were divided into 36 students in the experimental class and 36 students in the control class. In the experiment class, the subjects were given multiple representation strategies with the realistic approach, and one other course as the subject control class is given learning with the scientific method.

The design used in this research was Nonrandomized Pretest-posttest Control Group design. At the beginning and end of the learning, the students of the experimental class and the contrast class were given pretest and posttest, i.e., algebraic thinking skills tests. At the end of the lesson, the students at the experimental and control class were given the initial and final test. Test instrument in this research was in the form of a description that consists of 5 items related to polyhedron problem. Problem description was used to measure students' algebraic thinking abilities that include aspects of identifying or constructing numerical patterns and geometry, explaining verbal patterns and representing with tables or symbols, determining and applying relationships among variables to make predictions, making and teaching generalizations relating facts and conditions, using graphs to illustrate patterns and make predictions, and using notations, symbols, and variables to express patterns, generalizations in various situations. The test instruments were given to the two classes in the beginning and in the end of the lesson.

RESULTS AND DISCUSSION

The significance of the algebraic thinking ability was obtained by computing the N-Gain of the students that received multiple representation strategies and the students that received scientific learning. The description of the student's algebraic thinking ability can be shown in Table 1.

Table 1. N-Gain Table

Aspects	Learning Strategy	Data	N	$\bar{X}$	Sd
Algebraic Thinking Ability	Multiple Representation Strategy with RME Approach	Pre-scale	36	46.87	1.80
		Post-scale	36	76.72	1.32
		N-Gain	36	29.84	2.08
	Scientific Approach	Pre-scale	36	51.09	2.20
		Post-scale	36	62.18	1.82
		N-Gain	36	11.09	2.40

N-gain data were analyzed by applying an average deviation score test of two tests. The analysis used to find out the contribution of multiple representation strategies for algebraic thinking is an independent sample of the t-test. The results showed that the significant value of independent sample t-test N-Gain students' algebraic thinking score is 0.005 and then Ho is rejected. Thus, it can be concluded that the algebraic thinking ability of students who have received multiple representation strategies is better than the standard scientific strategy. Based on the data found, it can be concluded that multiple representation strategies can contribute significantly to the development of students' algebraic thinking abilities.

After the normality test and homogeneity test of variance and the result of normal and homogeneous distribution data, then the Independent Sample Test was examined. The results of statistical analysis the Independent Sample Test can be seen in Table 2.

Table 2 Independent Sample Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Postest	Equal variances assumed	.257	.614	-2.888	70	.005	-11.472	3.972	-19.395	-3.550
	Equal variances not assumed			-2.888	69.709	.005	-11.472	3.972	-19.395	-3.549

Table 2 shows the Levene Test for the postest resulting that with Equal variances assumed, F = 0.257 with the sig. = 0.614. Because 0.614> 0.05 the null hypothesis was accepted, whih means the second variant of the population is identical. Since the two variants of the population are identical, then to know t table on the independent sample test table using the base of the assumed Equal variant, and obtained *t* table = -2.888 with the sig value. (2-tailed) = 0.005. Since *t* table = 2.03011 and *t* count = -2.888, it means *t* table <- t table, so H_0 is rejected. Meanwhile, based on the value of significance, it was obtained that the sig value. (2-tailed) is 0.005. Since 0.005 <0.05 then H_0 is rejected. So both population averages are not identical. The difference is indicated in the next output.

Table 3. Posttest Average Value for Experiment Group and Control Group

	Group Statistics				
	Code	N	Mean	Std. Deviation	Std. Error Mean
Posttest value	Experiment	36	75.75	16.300	2.717
	Control	36	62.25	17.389	2.898

From the Table 3, it is obtained an average value posttest control class 62.25, and the average value of posttest in the experimental class is 75.75. This shows the average posttest score of the students in the experimental class is higher than the amount of students posttest in the control class.

The significance difference between the result posttest of algebraic thinking ability between control class and experimental class students is caused by different approaches used in the learning process. The learning process in the experimental class was directly related to real life using realistic approach through multiple representation strategies so that students' representations is more varied. The teacher presented realistic questions so that students could see, understand, the objects they learned in everyday life. In this study, the problems the students received were identical problems with everyday problems so that students could understand easily and represent the issue in the language, symbols, and notations that they make themselves using multiple representations.

The student mastery learning using multiple strategies of representation was examined with proportions test realistic approach whether it meets the criteria of mastery learning, which is more than 75% of the total students. Before performing the proportion test, the completeness of individual learning and mastery of classical learning were examined. The proportion test of the trial class can be seen in Table 4.

Table 4 Proportion Test

Individual completeness	Percentage Complete	Significance	$-Z\alpha$	Zobs	Criteria
0,78	0,75	0,05	-1,645	0,29	Complete

Table 4 shows that Z obs = 0.29 with a significance level of 0.05 with $-Z\alpha$ = -1,645. Since Zobs> $-Z\alpha$ (0.29> -1.645), then the null hypothesis is rejected and the alternative hypothesis is received. It shows that the percentage of students who have finished their study has reached more than 75%.

In addition to the statistical test, the improvement of students' algebraic thinking ability is also seen based on the five Kriegler algebraic thinking indicators: 1) algebra as the language of mathematics (indicators: explaining the meaning and function of the variable, using variables to show the known or unknown information, communicating the result of the problem solving, carrying out algebraic manipulation in an algebraic equation, and determinig the value of the variabely asked), 2)

the ability of representation (indicators: representing the relationship of information from the question, generating various forms of representation from the subject, explaining information obtained from representation made), 3) problem solving ability (indicators: identifying the element that is known and asked, selecting problem-solving strategy, solving problem using students' own strategies, checking the accuracy of the selected plan and the correctness of problem-solving, explaining other approaches/solutions to open problem), 4) ability of quantitative reasoning (answering the question correctly with plausible reasons, using correct algebraic procedure, able to use inductive or deductive reasoning), and 5) algebra as a tool for mathematical functions and modeling (indicators: using patterns/rules in the form of words/equations, able to represent mathematical ideas on each question using equations, inequalities, tables, graphs, or words appropriately and consistently).

Here is one example of a polyhedron problem to measure students' algebraic thinking skills based on multiple representation strategies on realistic mathematical learning.

4. Mrs. Siti a traditional cake seller typical of Kudus City (Bugis Cake). The Bugis cake made by Mrs. Siti is a rectangular pyramid. The cake has measure of 6 cm x 6 cm x 5 cm.
   a. How much dough does it take to make a bugis cake?
   b. If a cup can accommodate a 2.4-liter batter, then determine how many cups Mrs. Siti needs to make 120 pieces of bugis! (Hint 1 liter = 1 dm^3)

Figure 1. Sample Question of Algebraic Thinking Ability

The students at Figure 1 solved the problem based on algebraic thinking indicator, with a different strategy, here is the sample of student's answer in the experiment class.

Figure 2. Answer Problem No. 4 Subject in experiment class

Figure 2 indicates that the student having responses on no 4 obtained score 4 meets since he meet all indicators of algebraic usage as a mathematical language, capable of using and explaining symbolic meanings to indicate known information, capable of performing algebraic manipulations and capable of determining variable values. Then the student also met the indicators of capable of performing a symbolic representation that can create equations. Also, the student used mathematical discovery and met all the problem-solving indicators. The problem-solving indicator is answering known and asked elements, and checking the selected problem-solving strategy.

Figure 3. Answer Problem No. 4 Subject in Control Class

Figure 3 shows that it is known that the student in the control class met some indicators of algebraic usage as a mathematical language, capable of using and explaining the symbols used to show information that is known, capable of manipulating algebra and capable of determining the value of a variable even though it is wrong to understand the problem. Students in control classes also meet symbolic representation indicators by creating equations. But the student has not been able to answer the known element correctly.

In general, the algebraic thinking ability scores in the experimental class is better than those in control class. The increased ability of algebraic thinking after learning through multiple representation strategies with realistic approach gives a positive impact for students on the material of polyhedron. With the increased student ability of algebraic thinking, student learning achievement was increasing (Widodo, Prahmana, & Purnami, 2018; Permatasari & Harta, 2018; Kurniati, *et al*. 2015; Fatah, *et al*. 2016; Widyatiningtyas, *et al*. 2015). It is the evident from the results of proportional tests that show that more than 70% of students meet learning completeness.

CONCLUSION

Learning by the multiple representation strategies with realistic approach affected the ability of students' algebraic thinking. The multiple representation strategies with realistic approach were able to improve the algebraic thinking. In general, the experimental class student obtained a higher score than the control class. The completeness learning of individual student using multiple representation strategies with realistic approach was over seventy percent. The ability of algebraic thinking of student on the course subject of polyhedron using multiple representation strategies with realistic

approach provided an algebraic thinking capability better than the scientific approach. This finding was supported by Koca (in Hwang, 2007) on Midwestern students, 96% of students agreed that mathematical problems could be solved using multi-representation. Although 66% of students liked using more than one representation to solve mathematical problems, it turned out that 72% agreed that it was easier to focus on just one representation. Furthermore, it was found that the learning by using multiple representation strategies with realistic approach took relatively more extended than the learning by a scientific method. Therefore, it is suggested for further research than when the multiple representation strategies are applied, the appropriate materials should be selected, and hence the learning mathematics will be more effective.

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