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Place value without number sense: Exploring the need for mental mathematical skills assessment within the Annual National Assessments

Abstract

In this paper we examine the extent of the focus on number sense, enabled and accompanied by the development of efficient strategies for mental maths, in the foundation and intermediate phase. We do this through documentary analysis of the Curriculum and Assessment Policy Statements (CAPS) for these phases and the Annual National Assessments (ANAs). We argue that number sense and mental agility are critical for the development and understanding of algorithms and algebraic thinking introduced in the intermediate phase. However, we note from our work with learners, and broader evidence in the South African landscape, that counting-based strategies in the foundation phase are replaced in the intermediate phase with traditional algorithms. We share experiences in the form of vignettes to illuminate this problem. Whilst literature and the CAPS curriculum emphasise the important role of mental computation within number sense, we note that the ANAs do not include a "mental mathematics" component. This absence in assessment, where assessment often drives teaching, is problematic. We conclude with the suggestion that research be conducted into the viability/appropriateness of an orally administered mental mathematics assessment component in the ANAs as a way to establish a focus on number sense across the foundation and intermediate phases.

Keywords: assessment, mental strategies, number sense, CAPS, ANA, South Africa, primary mathematics

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Introduction

In our work as the SA Numeracy Chairs at Rhodes and Wits University respectively (first two authors) and as doctoral and master's research students in mathematics education across these institutions (last two authors) we collaborate with teachers and learners in over 25 primary schools in the broader Grahamstown and Johannesburg areas. The schools include both township and suburban, fee paying and non-fee paying schools. In all of our work (research and development) we pay particular attention to the development of number sense and mental strategies and fluency in young learners. In this paper we examine the extent of the focus on the development of number sense, enabled and accompanied by the development of efficient strategies for mental maths, in the foundation and intermediate phase. We do this through documentary analysis of the Curriculum and Assessment Policy Statements (CAPS) documents for these phases and the Annual National Assessments (ANAs). We supplement this analysis with vignettes drawn from the experiences of the four authors across our contexts.

Number sense is critical in the development of mathematical understanding throughout schooling and in everyday life. Greeno (1991) equates number sense with a set of capabilities for constructing and reasoning within mental models and includes flexible numerical computation, numerical estimation and quantitative judgement and inference. The CAPS document (South African Department of Basic Education [DBE], 2011a), as with previous curricula, has a similarly broad conception of number sense that would include both mental and written application. It suggests that number sense includes developing an understanding of the meaning of numbers, the relative size of numbers, the relationships between numbers, knowledge of different ways of representing numbers and the effect of operating with numbers. Number sense, especially in terms of relationships between numbers and operations in the form of mental models, is also the critical basis for the development of algebraic reasoning (Greeno, 1991). Without fluent and flexible knowledge of the commutative and distributive properties for example, one cannot efficiently calculate answers to sums like 3 + 49 (as 49 + 3) or 14 x 101 (as 14 x 100 + 14). McIntosh *et al.* (1992) concur with this view, viewing number sense in terms of: knowledge of and facility with numbers and operations, and applying this knowledge and facility to a range of computational settings.

Much South African research points to overdependence on concrete counting strategies (Hoadley, 2012; Schollar, 2008). This is consistent with our own research (Venkat, 2013; Graven & Stott, 2012; Stott & Graven, 2013; Stott & Graven, in press). For example we have seen learners well into the intermediate phase across our work draw two groups of tally lines with 2 and 98 in each of the groups to answer the question 2 + 98. Not knowing a basic bond to 10 like 2 + 8 = 10, or that 2 + 98 is the same as 98 + 2 so one can count on 2 from 98, renders this simple computation tedious and highly error prone. Wright, Martland, & Stafford (2006) argue that children who are low attainers in the early grades continue to be low attainers with the gap between learners with solid number sense and those without increasing as they move up the grades in school. This makes the development of foundational number sense and mental agility particularly important in the early years of schooling. From our work with teachers and learners it seems that whilst in the foundation phase, an emphasis on counting limits the development of number sense, in the intermediate phase, the introduction of algorithms for addition, subtraction, multiplication and division (including of decimals and fractions) further circumvents the need for number sense. Thus even those learners who had solved 2 + 98 previously by counting on 2 from 98 seem to simply move to less efficient column algorithm methods for these kinds of problems in the intermediate phase. Thus for example a learner might write 2 + 398 = 3 910 (since 8 + 2 = 10) when a quick counting on would give 400. The absence of estimation skills – of a sense of where the answer should roughly be which at the very least would indicate that an answer in the thousands would be clearly wrong – is telling here. In the next section we examine the extent to which mental agility with numbers is connected to number sense and its presence in the curriculum documents for intermediate and foundation phase.

Documentary analysis of the role of mental mathematics in the foundation and intermediate phase CAPS documents

The development of number sense is connected with developing mental models and mental strategies for computation. For example across the foundation and intermediate phase CAPS documents the definition of mathematics ends with:

> It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision making (DBE, 2011a:8-9; DBE, 2011b:8-9).

Additionally there is an on-going reference to mental mathematics throughout the curriculum documents and in relation to the outlined concepts and skills. So, for example the foundation phase CAPS document includes Section 2.7.3 Mental Mathematics and states, "Mental mathematics plays a very important role in the curriculum … Mental mathematics therefore features strongly in both the counting and the number concept development sections relating to the topics Number and Patterns and may also occur during Measurement and data handling activities" (DBE, 2011a:13). In the intermediate phase curriculum in the tables headed "Time Allocation Per Topic" for Grades 4, 5 and 6 (34; 122; 212) a total of 30 hours out of 240 hours of available teaching time is recommended to be spent on mental mathematics spread across all four terms and at the start of each lesson. That is 12.5% of time in each of these Grades. Additionally the "clarification notes" column for number sentences under patterns, functions and algebra for Grade 4 says: "All concepts and techniques developed here can be practiced throughout the year in the Mental Maths programme" (DBE, 2011b:42). Similar comments are made in Grade 5 and 6. Thus we see that an emphasis on mental programmes and the use of mental strategies to support the development of concepts continues right through the intermediate phase.

This pairing of the development of number sense and mental strategies is consistent with international literature (Burns, 2007a, 2007b; Greeno, 1991; NCTM, 1989). Across our experience of working with learners and teachers in the foundation and intermediate phase we have noted episodes which illuminate for us the critical problems relating to the absence of the development of number sense and mental agility. We share these as vignettes below that illustrate the problem in a way which national data in the form of extremely low average marks cannot. Indeed national data across our provinces indicate that foundational number sense is not developed in the vast majority of our learners by the end of Grade 3. The 2012 ANA results indicate that only 36.3% of Grade 3 learners nationally achieve more than 50% for the ANAs. The Grade 3 national average for Mathematics was 28% in 2011 and 41% in 2012 (DBE, 2012) . We can thus conclude that the majority of South African Grade 3 learners have not developed foundational number sense before entering the intermediate phase – which is premised on the notion of building on this foundational knowledge.

Vignettes from the Eastern Cape and Gauteng

The authors work across projects based at Rhodes and Wits University in the Eastern Cape and Gauteng. We shared the following anecdotes when we began considering this paper. We have edited them for readability:

Lise's anecdote: Based on my current experience in Grade 3 classrooms, it seems that the development of children's mental agility and number sense are not prioritised. Mental mathematics is reduced to skip counting, decomposition of numbers and simple addition, subtraction, multiplication and division number facts. With regards to the effect of operating with numbers some of the strategies suggested in CAPs are being taught as *formal* methods. The use of a single formal method impedes the ability of children to think flexibly when operating with numbers.

In a class where children were asked to solve 110 – 96, *all* the children used the same method involving expanded notation (I was surprised that no children "counted on" from 96 or "added ten and four"). While the expanded notation method may be useful when solving subtraction problems that do not require regrouping, the method led to a number of errors when regrouping was required. Three common errors when solving 110 – 96 involves which number to regroup:

(~~100~~^{0}) + (^{1}10 – 90) + (0 – 6)

20 + 6

= 26

(100) + (10 – 90) + (0 – 6)

100 + 80 + 6

= 186

(~~100~~^{0}) + (^{1}10 – ~~90~~^{80}) + (^{1}0 – 6)

30 + 4

= 34

It appears that: (1) counting, writing number names and decomposing numbers alone does not develop number sense; (2) children are being taught new strategies for solving problems which are not always suited to the problem; and (3) children are not being taught to think and work flexibly with numbers and number operations.

Mel's anecdote: Yesterday I was visiting one of our Grade 3 clubs and a learner kept doing the following when adding numbers in a pyramid:

58 + 23 = 711 (i.e. 5+2 = 7 and 8+3 = 11) and similarly 62 + 73 = 135.

However the same learner had managed questions such as 35 + 24 = 59 easily and quickly.

A visitor to the club noted that she saw this repeatedly in classes that she visited and I concurred that I too had seen such errors repeatedly across our assessments.

When I requested that the learner do the problem again he was quickly able to see what was happening and why his answers were incorrect. Thus, orally he said, "50 + 20 is 70 and the units 8 + 3 is 11 so its 81."

My suspicion was that the written version of column addition, or the stated rule of "add these two numbers and then those two numbers and then write your answers", which works well if answers to each of these additions are less than 10, got in the way of this learner's number sense as he was quite happy from his written work to say the answer was seven hundred and eleven.

I took a look at the departmentally issued workbooks for Grade 4 and noted that they don't push column addition but instead emphasise the horizontal expansion method, i.e. (DBE, 2013:48).

134 + 123

= [100] [30] [4] + [100] [20] [3]

= 100 + 100 + 30 + 20 + 4 + 3

= 200 + 50 + 7

= 257

This place value decomposition method certainly links better with what learners tend to do when solving such problems mentally and so I'm left wondering where the push to the formal vertical algorithm for Grade 3 learners is coming from? Perhaps it is historical learning, or it could be a result of where teachers are most comfortable having learnt maths this way themselves?

The problem with the method above however, as Lise's story shows, is that it does not translate directly to subtraction. So as the Realistic Mathematics Education (RME) literature suggests perhaps a better way to begin might be by modelling a number-line-based method of counting-on that keeps the first number "whole" as this method does transfer to subtraction. In RME this method is known as the "N10" procedure, and it is contrasted from the "1010" procedure which splits both numbers to be operated upon (Klein, Beishuizen & Treffers, 1998). We discuss this further below when reviewing related literature.

Hamsa's anecdote: We were in a G7 class of one of the teachers participating in our 20 day primary maths teacher development course. He had set this sum: 397 + 65 + 3.

I watch a girl who answers this sum correctly by setting up the sum in column addition form. I say well done and ask her if there might be a quicker way. She shrugs and shakes her head. I say, so what might happen if we add the 397 and 3 first and then come to the last number. She says answer for 397 + 3 is "three hundred and ninety ten". When I ask her to check this by counting on in 1s from 397, she gets 400 and then says: "Oh it's 465".

There is quite a complex interplay here I think between column arithmetic's early introduction negating the need for number sense in the early intermediate phase, and then later becoming a way of circumventing the lack of number sense, with calculators then becoming the prop of choice in high schools.

Herman's anecdote: In an intervention designed to develop number sense in Grade 4 learners, the Wits Maths Connect – Primary project, of which I am a part, administered a pre-test with 10 addition and subtraction problems. The last question was a "naked sum" written as a number sentence (92 – 87). Preliminary results revealed that only 9 out of 42 learners were getting the last question (92 – 87) right. That is, only 21% of the learners in the class were able to answer the question correctly in the pre-test. Of the 33 that got it wrong, 20 said the answer is 15. Looking across learners' responses, the column and the decomposition methods – both of which rely heavily on the 1010 procedure – were the preferred solution methods. 61% of the learners subtracted the smaller from the larger digit in each column:

$$\begin{array}{r} 92 \\ \underline{-87} \\ 15 \end{array}$$

This offers us a prima facie case that corroborates two very important research findings. Firstly, that two-digit subtraction problems that require "borrowing" pose a serious challenge to learners. Secondly, that "set-type solution methods" that utilise "the strategy of splitting numbers into tens and ones often leads to erroneous solution methods with subtraction" (Beishuizen, 1993, cited in Gravemeijer & Stephan, 2002:150). In other words, when learners rely on procedures without an appreciation for the quantities that underpin them they are bound to experience difficulties judging the rationality of computational results (Yang, 2003). By contrast, Kilpatrick et al. (2001:122-123) have noted that when "students learn to subtract with understanding, they rarely make this error".

What was an even more striking finding from the pre-test was that the structure of the question seemed to prompt in learners very specific solution methods. The word problems seemed to prompt learners to use tally counting, whilst the number sentence questions seemed to prompt them towards column addition/ subtraction methods. This raises the question as to which solution methods would learners have used had the test been administered orally. Could it be that the lack of development of number sense is a result of a lopsided proficiency in early written computation where carrying and borrowing are not required?

Discussion based on the vignettes – Number sense lost?

It is both useful and important to note that these vignettes are not that different from what teachers note across the world. So, for example a teacher cited in the USA in Carpenter, Franke & Levi's (2003) book states:

> I recently interviewed ten fourth and fifth graders whose mathematics instruction had not focused on big ideas. These children all had been taught procedures that didn't make sense to them, so they didn't remember them. They often pieced together different algorithms in senseless ways. It was painful

> to watch them solve problems; I could have cried. They had no number sense (Carpenter, Franke & Levi, 2003:xiii).

Across these vignettes we note a similar lack of number sense underlying children's attempts to use taught methods. All of our vignettes indicate that the move from counting methods to decomposition into place value based methods is achieved with a stark leaving behind of a sense of "quantity" (Thompson, 1999). Thus, children solving 33 + 65 are writing 30 + 60; 3 + 5, but when calculating the 30 + 60 are saying 3 + 6. The relationships between 3 + 6 and 30 + 60 are left behind – there is no "echo" of the quantity underlying the digit in the enactment of the algorithm. Some might argue that the "echo" that we are calling for simply muddies the water. We argue that these glances back into quantity are important within a number sense oriented frame where estimation skills depend on a sense of quantity. This helps to avoid the errors we have illustrated above, but also emphasises the importance of the relationship between numbers that is fundamental to successfully working with addition and subtraction problems.

Additionally, pressure to keep up with the intermediate phase curriculum often means it is difficult for teachers to address the backlog of foundational understanding of learners. Thus while number sense may not be critically important for applying the rules of an algorithm correctly it is important for understanding how and why the algorithm works and is central to the development of abstract algebraic reasoning. However, many teachers feel they do not have the time to go back to developing this number sense if it is not already there.

One might have looked to the curriculum documents for reasons for poor learner number sense and mental agility. However, our documentary analysis of the CAPS documents, as discussed above, reveals that these are repeatedly foregrounded in both documents and thus even while algorithms are introduced for solving larger addition, multiplication, division and subtraction problems there is continued emphasis on checking these solutions with mental estimation and daily emphasis on ten minutes of mental mathematics at the start of each lesson. One issue could be that the curriculum tends to present models as alternatives, rather than noting which are useful in what problem situations for flexibility or efficiency. For instance, Yang (2003), in an intervention study of Grade 5 learners in Taiwan, observed that subtraction problems tended to lend themselves to being solved using the N10 (as opposed to the 1010) procedure.

Some evidence from our prior work in foundation phase points to random, rather than systematic sequencing of examples combined with a lack of attention to connecting between examples (Venkat & Naidoo, 2012). Perhaps interpretation of mental mathematics involves random mental questions rather than systematic varied questions aimed at getting learners to see and consolidate patterns and relationships. Learning rules for mental computation will not support number sense unless learners are able to make sense of the properties and relations (Greeno, 1991).

The problem can, of course, be addressed in various ways. Teacher development programmes, and indeed our work across the Numeracy Chair projects, are focused on supporting teachers to develop mental strategies and fluency with learners in ways that support connected number sense. However, it seems fair to ask that if such mental mathematics and number sense is foregrounded in the curriculum we might ask whether it is also foregrounded in the Annual National Assessments. There is much research that indicates that assessment can drive teaching and thus we now turn to analysing the role of mental agility and number sense in the ANAs that were introduced nationally in 2011. In the next section we thus turn to examining the extent to which mental agility with numbers is present within the ANAs.

Examining the assessment of mental agility within the ANAs

Broad nationwide implementation of the Annual National Assessments began in 2011. The ANA was explicitly focused on providing system wide information on learner performance for both formative (providing class teachers with information on what learners were able to do) and summative (providing progress information to parents and allowing for comparisons between schools, districts and provinces) purposes (DBE, 2012a). The ANAs were written by all public school Grades 1-6 and Grade 9 learners in September 2012 in literacy and numeracy in the foundation phase and language and mathematics in the intermediate phase.

Weitz and Venkat (forthcoming) argue that the Grade 1 ANA, in awarding marks for answers rather than strategies, tends to downplay the need for moves to more efficient strategies. The flexibility and efficiency that underlie number sense are therefore sidelined here. Examining the ANAs for Grade 3-6 where there is no oral component (unlike for Grades 1 and 2 where questions are read to learners) we note that whilst the competences that are described as comprising number sense are likely to support improved performance in the ANA, the mental agility that has been described as critical within number sense, is assessed to a very limited extent. Instead, the reliance on algorithms that we have indicated in our vignettes is accepted within the ANAs.

We noted in relation to the curriculum that number sense involves selecting the most appropriate strategy to use in specific problem situations. Van den Heuvel-Panhuizen & Treffers (2009) highlight the two faces of subtraction as *take away* and as determining the *difference.* They have found that it is crucial for learners to be made aware of the inverse relation between the operations of addition and subtraction so as to be placed in a position where they can harness the power of an addition strategy for a subtraction problem because "in the subtraction problems the context open[s] up the indirect addition strategy" (Van den Heuvel-Panhuizen & Treffers, 2009:109). This move elicits the use of a count-on strategy to support the calculation – a counting strategy with which learners should already be familiar by the time they are expected to successfully solve a "difference" question. In the example presented in Herman's vignette, a difference based strategy is more efficient than a "take away" based strategy. This emphasises that the examples that "surface" or encourage the need for flexible strategy selection should be incorporated into assessments.

This kind of highly purposive incorporation of examples appears rather patchy in the ANAs. The practice exercise for the Grade 3 ANA in 2012 seems to demonstrate possible strategies (mental or written) in written form by showing 125 + 64 as solved in three different ways: Using expanded notation and reordered; grouping the units, tens and hundreds and 125 + 64 →185 + 4 →189. This supports the point we made in relation to the CAPS curriculum – of strategies simply as possible alternatives, rather than as purposive selections. There are some questions in the Grade 3 2012 ANA that lend themselves to quick mental strategies such as Q1: Complete the table where the table has "count in 25s" and the first number 75 is given; "count backwards in 20s" where 678 is given as the starting number, and "count in 50s" where 250 is given as the 3rd number in the sequence. Similarly some of the four operations sums lend themselves to mental strategies – or at least mental strategies would support a written strategy (e.g. Q4 b) 31 x 3 and c) 84 ÷ 4). On the other hand in the 2012 ANA the Grade 4 ANAs tend to provide numbers which are not specifically geared towards mental strategies and could be just as efficiently done using algorithms. For example the 2010 ANA written in 2011, page 5 Q4 assessment of the four operations included the following:

6 832 + 2 594 =   3 648 – 555 =   156 x 24 =   9 296 ÷ 8 =

The Grade 4 ANA in 2012 had similar calculation questions and similar questions in the Grade 5 and 6 ANAs except in a higher number range. While some number sense and mental agility is assessed across the intermediate phase ANAs in the form of: multiple choice questions such as "which comes next in the pattern" "which number is 40 000 more than 54 562" (see 2012 Gr 5 Q 1) and doubling and halving questions (see 2012 Grade 4 Q 7 & 8) and flow chart questions (e.g. x 3 and ÷ 6 see 2012 Gr 5 Q 6), the ANA does not specifically push for mental calculation within work with the four operations.

Without assessments that include mental mathematics or the development of items that promote flexible mental strategies, learners do not need to develop, maintain or continually sharpen their flexible mental strategies that enable them, for example, to efficiently add 297 and 303 or to multiply 11 by 50 (without pen and paper). Our sense is that inclusion of a mental arithmetic component within the ANA structure may support the development of number sense in the foundation and intermediate phases. Research has shown that students' number sense can be effectively developed "through establishing a classroom environment that encourages communication, exploration, discussion, thinking and reasoning" (Yang, 2003:132). And since these practices of communication, exploration, discussion, thinking and reasoning unfold orally in the mathematics classroom, it can be argued that the room should be created in assessment for the mathematical reasoning behind a solution to be provided orally. In this way we can better ensure that curriculum aims, and the practices within instruction and assessment coalesce, with the added benefit that learners are not placed at a disadvantage for their lack of skill in providing written explanations. In the foundation phase, non-pencil and paper methods may provide the impetus to move beyond the tally counting that has been so widely documented; in the intermediate phase, an emphasis on mental methods can shift emphasis to the rounding and estimation skills that require a strong sense of both the number and number relations that have been described as critical to number sense.

Concluding remarks

Our documentary analysis of the CAPS documents and the ANAs for the foundation and intermediate phase in mathematics would seem to suggest a mismatch between what is promoted in the curriculum and what is assessed. Our experiences which we shared through vignettes would indicate that a greater emphasis on number sense and mental strategies of efficiency and estimation are required when algorithms are introduced for the four operations if we are to avoid learners providing non sensible answers such as 2 + 398 = 3 910. We thus ask – would it not make progressive sense therefore to include assessment tools that "capture important learning goals and processes [that] more directly connect assessment to ongoing instruction"? (Shepard, 2000:8). In particular, we suggest that the ANAs incorporate an oral mental math component going right through from Grade 1 to Grade 9. In doing so, what types of questions should be asked in order to assess whether learners have developed efficient mental strategies which imply knowledge of key relationships between numbers and operations (such as commutativity over addition and multiplication)? Would this inclusion support the continuation of the development of strong flexible and efficient number sense? Would this exacerbate some of the stress that young learners experience when writing the ANAs (Graven & Venkat, 2013)? On the other hand might it reduce learner stress by providing a familiar oral component? We thus conclude by suggesting that research be conducted into the viability and appropriateness of the inclusion of an orally administered mental mathematics assessment component in the Grade 4 – Grade 6 ANA's as a way of maintaining a focus on number sense and mental agility throughout the intermediate phase.

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