The problem of Sunny's pennies: A multi-institutional study about the development of elementary preservice teachers' professional noticing

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In a multi-institutional study, the authors examine elementary preservice teachers' (PSTs') noticing skills before and after their engagement in a mathematics methods course. In particular, the authors examine their use of *Cognitively Guided Instruction* (CGI) as a means of supporting the development of PSTs' *professional noticing of children's mathematical thinking.* PSTs (n=123) enrolled in elementary teacher preparation programs across three institutions in the United States were recruited for the study. They were shown a video of four children solving a mathematics problem and asked questions designed to elicit their professional noticing. Analyses revealed that across all three sites the authors' use of CGI supported PSTs in developing two out of the three noticing skills that comprise the professional noticing framework (albeit to differing degrees). Comparing across institutions revealed important implications for (a) the ways in which teacher educators support PSTs' professional noticing and (b) teacher education programs more broadly.

Keywords · teacher noticing · elementary teacher preparation · mathematics teacher preparation · methods course

Introduction

Current visions of high-quality instruction both inside and outside the discipline of mathematics involve teachers' abilities to implement in-the-moment, pedagogical decision making that is responsive to and builds on student contributions. For instance, science teachers are encouraged to listen to and interpret student ideas and to subsequently use those ideas to support students' investigations of authentic questions (National Research Council, 2001, 2011). Within mathematics education, curricula from around the world foreground the importance of developing students' meaning-based understandings of mathematical topics (e.g., Australian Curriculum, n.d.; Ministry of Education, 2017; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010; Takahashi, Wantanabe, & Yoshida, 2008). Teachers are particularly encouraged to build on students' ways of thinking and to support students in making connections between those ways of thinking and conventional mathematics (e.g., National Council of Teachers of Mathematics, 2014). An important part of mathematics teacher preparation then becomes developing preservice teachers' (PSTs') abilities to notice and interpret students' mathematical thinking so that PSTs are prepared to act on it in the moment.

In this paper, we report the findings of a multi-institutional study wherein the authors aimed to support elementary PSTs' noticing of children's mathematical thinking. In particular, we used frameworks provided by the professional development and research project *Cognitively Guided Instruction* (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996) as a means of supporting PSTs' noticing and report on what it supported the noticing of at our respective institutions. We begin with a discussion of the various ways in which researchers have conceived of noticing and the potential of CGI to support PSTs' noticing of children's mathematical thinking. Our findings suggest that our respective use of the CGI frameworks supported PSTs' abilities to attend to the mathematical details contained in students' strategies as well as their abilities to interpret students' thinking (albeit to different degrees). We conclude by offering ideas regarding how to better support PSTs' noticing of children's mathematical thinking.

Relevant literature

Mathematics teachers' noticing

Those who research mathematics teachers' noticing have conceptualised *noticing* in different ways. Some have conceived of noticing as involving the process in which teachers *identify* important or noteworthy aspects of classroom activity. This conceptualization of noticing involves examining the aspects of instruction teachers do and do not attend to. For instance, Star and his colleagues examined what secondary PSTs noticed when watching a video lesson and whether PSTs' engagement in a mathematics methods course could improve that noticing (Star, Lynch, & Perova, 2011; Star & Strickland, 2008). They noted that, prior to PSTs' engagement with the course, PSTs "were not particularly observant of many classroom events" (Star & Strickland, 2008, p. 121). After PSTs' engagement with the course, however, PSTs were better able to notice classroom events in a subset of five observation categories (classroom environment, classroom management, tasks, mathematical content, and communication). Interestingly, the categories in which there were improvements in PSTs' noticing differed across the studies (even though the 2011 study was a replication of the 2008 study). This is consistent with Goodwin's (1994) argument that it is the interplay among the materials and the individuals that shapes what gets noticed.

Other researchers have conceived of noticing as involving the processes in which teachers *identify* important or noteworthy aspects of classroom activity and *interpret* that activity (e.g., Mason, 2008; Sherin & van Es, 2009). This conceptualization involves examining not only what teachers attend to but also how they make sense of what they attend to based on their knowledge of the particular students and content. This conceptualization is perhaps best exemplified by the work of Sherin and van Es (e.g., Sherin & van Es, 2005, 2009; van Es & Sherin, 2002, 2006, 2008). They have examined the development of both teachers' and PSTs' abilities to notice aspects of classroom activity and have found the use of video clips of classroom instruction to be particularly powerful in shaping what is noticed (e.g., shifting the focus of attention from teacher actions to student conceptions) and how it is interpreted (e.g., shifting the nature of the comments made from evaluative comments to evidence-based, interpretive comments).

Other researchers have conceived of noticing as involving a third process. For instance, Jacobs, Lamb, and Philipp (2010) conceived of noticing as involving the two processes discussed above (i.e., *attending* to and *interpreting* classroom activity) and *responding* to classroom activity. Moreover, Jacobs et al. (2010) selected a specific focus for noticing, namely children's mathematical thinking, and thus defined the *professional noticing of children's mathematical thinking* as involving three interrelated processes: (a) attending to the mathematically important details of children's strategies, (b) interpreting the nature of children's mathematical understandings based on the details of their strategies, and (c) making decisions regarding what to do next with a child based on inferences regarding his or her understanding. In doing so, these researchers concerned themselves less with the variety of things that teachers notice and more with the extent to which teachers notice children's mathematical thinking (Jacobs et al., 2010). Like Sherin and van Es, these researchers found the use of video clips to be effective in supporting teachers' professional noticing of children's mathematical thinking (Jacobs et al., 2010; Jacobs, Lamb, Philipp, & Schappelle, 2010; McDuffie et al., 2014; Santagata & Guarino, 2011). We found Jacobs et al.'s conception of noticing particularly useful since it is an in-the-moment expertise and thus aligns with the responsive nature of teaching highlighted in the previous section. In addition, this conception of noticing focuses on children's mathematical thinking as opposed to, for example, issues of equity (e.g., Hand, 2012) and therefore aligns with calls for instructional approaches that are responsive to children's thinking.

Mathematics teachers' noticing and CGI

CGI aims not only to support teachers in focusing on mathematical content but also to support teachers in focusing on how children conceive of and engage with that content (Carpenter et al., 1999; Carpenter et al., 2003). At the core of CGI is the belief that instructional improvements can be attained by providing teachers access to research-based knowledge regarding children's mathematical thinking and helping teachers explore ways in which instruction can build on that thinking. Furthermore, CGI has documented gains in both teacher learning and student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996). This combined with research that shows teachers' noticing develops over time and that teacher educators can support the development of that noticing (Jacobs et al. 2010; Mason, 2008; McDuffie et al., 2014; Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder, 2013; Star et al., 2011) suggested to us that CGI was a viable candidate for use in supporting the development of our PSTs' noticing.

Although there is limited research examining the relationship between CGI and teachers' professional noticing of children's mathematical thinking, there are some studies that provide guidance for mathematics teacher educators when thinking about how to support and assess teachers' professional noticing.[1] In their study, Jacobs et al. (2010) examined the relationship between K-3 teachers' and PSTs' professional noticing and the amount of exposure they had to CGI tenets. They found that expertise in teachers' professional noticing is "neither something adults routinely know how to do nor is it expertise that teachers generally develop solely from many years teaching" (p. 184). Rather, expertise develops with participants' prolonged and sustained participation in professional development focused on children's mathematical

[1] From here on, *professional noticing* is used in reference to Jacob et al.'s (2010) *professional noticing of children's mathematical thinking*.

thinking. We were, thus, moved to examine the development that could take place in PSTs' professional noticing as a consequence of their engagement in a 15-week methods course focused on children's mathematical thinking.

To assess the participants' professional noticing, the participants in Jacobs et al.'s (2010) study were either asked to watch a video clip or to examine a set of student work and to respond, in writing, to various prompts. To assess *what* participants attended to, they were posed the following: "Please describe in detail what you think each child did in response to the problem." To assess participants' *interpretations* of the details they attended to in students' work, they were posed: "Please explain what you learned about these children's understandings." To assess participants' *decisions* about what to do next based on their inferences regarding students' understandings, participants were posed: "Pretend that you are the teacher of these children. What problem or problems might you pose next?" In our respective methods courses, we routinely watched video clips (from the publicly available collection of CGI DVDs[2]) of students engaged in problem solving activities and asked PSTs to make sense of students' engagement using prompts like those included here.

CGI

Since professional noticing is a domain specific expertise (Jacobs & Empson, 2016; Walkoe, 2015), we provide information here regarding the CGI frameworks that we leveraged in our courses. In particular, CGI provides teachers with frameworks for making sense of various problem types, the relative difficulty of those problem types for children, and for assessing the relative sophistication of the problem-solving strategies children use.

Consider, for instance, the Pennies Problem: *Sunny had 7 pennies. His dad gave him some more pennies. And now he has 11 pennies. How many pennies did his dad give him?* For many adults, this may be a simple problem requiring subtraction wherein subtracting 7 from 11 provides one with the unknown number of pennies given to Sunny by his father. However, the problem-type framework provided by CGI distinguishes among four basic classes of addition and subtraction problems (i.e., Join, Separate, Compare, and Part-Part-Whole) and in so doing helps teachers attend to important distinctions that exist for children. While *Join* and *Separate* problems involve a direct or implied action that takes place over time, *Compare* and *Part-Part-Whole* problems do not. In the Pennies Problem, Sunny's father *gives* Sunny pennies. The problem thus contains an action that unfolds over time.

To distinguish between Join and Separate problems, one looks to the nature of the action involved in the problem. Join problems involve action wherein a particular set is increased over time and Separate problems involve action wherein a set is decreased over time. In the Pennies Problem, Sunny is *given* pennies; thus, the set (i.e., the amount of pennies Sunny has) is increased rather than decreased (e.g., as would be the case if Sunny were to lose pennies or give some away). CGI's problem-type framework further classifies various types of Join and Separate Problems as Start, Change, or Result Unknown problems. As Join and Separate problems involve changes made to a set over time, there is naturally a starting amount of the set, an amount the set changes by, and a resulting amount. In the Pennies Problem, we know the starting amount of the set (Sunny starts out with 11 pennies), we do not know how much

[2] Carpenter, T. P., Fennema, E., & Franke, M. L. (Research Project Directors), & Empson, S. B. (Director). (1995). *Children's mathematics: Cognitively guided instruction* [DVD set]. United States: WPT Digital Videos.

the set is changed by (his dad gave him *some* more pennies), and we know the resulting amount of the set (now he has 11 pennies). Therefore, the Pennies Problem is a *Join Change Unknown* problem.

Furthermore, CGI provides a three-part framework for making sense of the relative sophistication and overall development of children's solution strategies. This framework includes *modelling*, *counting*, and *number fact* strategies. The most basic of these strategies are *direct modelling* strategies wherein children use physical objects to represent the action or relationships among the quantities described in problems. For instance, a child who solves the Pennies Problem via direct modelling might create a pile of 7 cubes to represent the number of pennies Sunny started with, add more cubes to the pile until there are 11 cubes, count the number of cubes that were added to the original pile, and conclude that Sunny's dad gave him 4 pennies. In a more sophisticated *modelling strategy* wherein all quantities are represented but the action is not followed (thus a modelling strategy but not a *direct* modelling strategy), a child might create a pile of 11 cubes to represent the number of pennies Sunny ended with, remove cubes from the pile until there are only 7 cubes left in the pile, count the number of cubes that were removed, and conclude that Sunny's dad gave him 4 pennies.

Counting strategies are even more sophisticated because they tend to be more abstract and efficient than modelling strategies. Here, children no longer need to represent all of the quantities in a problem but can hold at least one quantity in their head. For instance, a child might solve the Pennies Problem by counting up from 7. *Number fact strategies* are the most sophisticated as they tend to be the most abstract and efficient of the strategy types.[3] When children are allowed to solve problems in ways that make sense to them, they often leverage known number facts to *derive facts*. For instance, a child may reason, "I know 7 and 3 is 10 so if Sunny's dad were to have given him 3 pennies, Sunny would have ended up with 10 pennies. But, Sunny ended up with 11 pennies so his dad must have given him one more than that." Here, the child leveraged a known fact (i.e., 7+3=10) to derive a fact (i.e., 7+4=11).

Methods

Our study involved two rounds of data collection: (a) an initial pre-assessment of PSTs' professional noticing, and (b) a post-assessment of PSTs' professional noticing. Below, we describe the study context, measures, and data analysis.

Study context

Participants. The 123 PSTs who participated in this study in the fall of 2014 were early childhood education majors enrolled in a semester-long mathematics methods course for elementary PSTs taught by one of the authors at his or her respective institution. The instructor at Site A taught two sections of the mathematics methods course. Site A is a large public 4-year university in the southeastern United States. The participants at this site were predominantly female, White, and native-English speakers (which is reflective of the larger teacher preparation program). Forty PSTs completed the pre-assessment at Site A and 42 PSTs completed the post-assessment.

[3] This assumes that a child understands the meaning of the facts they are using and their relation to the problem the child is solving.

The instructor at Site B also taught two sections of the mathematics methods course. Site B is a large public 4-year Hispanic Serving Institution in the south central United States. The majority of the participants at this site were female, Latina/o or White, and spoke Spanish and/or English as their first language(s). Fifty-one PSTs completed the pre-assessment at Site B and 47 completed the post-assessment.

The instructor at Site C taught one section of the mathematics methods course. Site C is a large public 4-year Minority Serving Institution in the western United States. The majority of the participants at this site were female, Latina/o or White, and spoke Spanish and/or English as their first language(s). Twenty-nine PSTs completed the pre-assessment at Site C and 30 completed the post-assessment.

Courses. Our courses were informed by CGI. In particular, we made use of Carpenter et al.'s (1999) book *Children's mathematics: Cognitively Guided Instruction* to support PSTs in learning:

1. how children interpret mathematical problems (as compared to how adults interpret mathematical problems);
2. whether and how a child's problem solving strategy is related to the type of mathematical problem he or she is solving; and
3. how children generally progress through different types of problem solving strategies.

Many class activities were aimed at supporting PSTs' abilities to notice details in mathematical story problems and children's solution strategies, and to subsequently categorise those problems and strategies. For instance, we each supported our respective groups of PSTs in attending to the mathematical details and structure of various story problems by asking our PSTs to act out the problems, thus making the presence and nature of the action contained in a problem more salient to the PSTs. Next, we supported our PSTs in attending to the details of children's strategies by asking PSTs to describe verbally and in writing everything they saw a child do and heard a child say while solving a problem.

To support PSTs in making inferences about the nature of children's mathematical understandings, we facilitated conversations regarding the relative importance of various aspects of children's problem solving strategies. We also supported our PSTs in focusing on the relationship between the details contained in the problem and those contained in the child's solution strategy by asking questions like "Did the child follow the action in the problem? How do you know?" PSTs' contributions to these discussions served as the bases for further discussion and corresponding rationale regarding what PSTs believed they should do next with the children.

There were some differences across the sites that should be noted here. First, the teacher preparation program at Site A includes two mathematics methods courses while the teacher preparation programs at Sites B and C include one such course. PSTs' professional noticing skills were thus assessed before and after either their *first* mathematics methods course (at Site A) or their *only* methods course (at Sites B and C). Second, the particular ways in which each instructor went about supporting and scaffolding his or her PSTs' professional noticing differed. While we all routinely made use of the problem-type and strategy-type frameworks offered by CGI as well as the professional noticing prompts (e.g., "Describe in detail what you think the child did in response to the problem"), when and how we coupled these frameworks and prompts with PSTs' viewing of video clips differed. Lastly, we did not show our respective groups of PSTs the same video clips during the semester. For example, while Instructor A tended to show video clips wherein one child worked to solve a problem, Instructor B tended to show video clips wherein multiple children worked to solve a problem.

Measures

The pre- and post-assessments were written instruments designed to examine PSTs' professional noticing while watching an 8-minute video wherein a teacher supported four students (Evelyn, Ashley, Sunny, and David) as they worked to solve the Pennies Problem. The video, referred to here as Sunny's Pennies, is from the publicly available collection of CGI DVDs. PSTs watched the video once at the beginning of the semester and once at the end of the semester. We did not show this particular video again nor discuss it during the semester. We did, however, show many other video clips from the CGI DVD collection. None of these clips included the teacher or the students from Sunny's Pennies.

After PSTs watched the video, they responded in writing to four prompts designed to assess their professional noticing skills. To assess *what* the PSTs attended to, we posed the prompt: "What do you notice or find noteworthy? Why?" We made our attending prompt more open than the one used by Jacobs et al. (2010) because we wanted a way to assess whether children's mathematical activity was something that our PSTs attended to prior to their engagement in our courses. Given how often we posed more specific versions of the prompt during the semester (e.g., "What did you notice about what the child[ren] did in response to the problem?"), we believed that our PSTs would interpret our prompt as being about children's mathematical activity at the end of the semester. We thus wanted a way to capture any changes in how PSTs responded to the prompt before and after their engagement in our courses. In addition, we believed that other prompts, specifically our interpreting prompt, would elicit responses about students' strategies particularly at the beginning of the semester. We were therefore careful to code across prompts for evidence of each professional noticing skill.

To assess how the PSTs *interpreted* the mathematical details that they noticed in the students' work, we posed the prompt: "Tell me about the students … What do they understand?" To assess the *decisions* the PSTs would make regarding what to do next with the students, we used two prompts: "Is there something you would have done differently than Ms. Keith? If so, what would you have done and why?" and "If Ms. Keith is to continue working with this group of students, what should she do next? Why?" We added the first of these deciding prompts to gather additional data on what our PSTs were attending to (e.g., whether a child was able to provide an answer, whether a child provided the correct answer).

With respect to Sunny's Pennies and in the context of professional noticing, we believed that our use of CGI should have supported our PSTs in recognizing the Pennies Problem as a Join Change Unknown problem. Our use of CGI should have also supported PSTs in making use of the identified problem type to judge the relative difficulty of a problem. In the video, Evelyn and Ashley struggled to solve the problem. According to CGI, their struggle could be due to structure of the problem, namely the location of the unknown, since change-unknown problems tend to be more difficult for children than result-unknown problems. Our use of CGI should have also supported PSTs in noticing the mathematical details in the children's respective solution strategies. In the video, David used a *direct modelling* strategy wherein he followed the action of the problem and used the cubes to represent the seven pennies Sunny started with, the additional pennies Sunny's dad gave him, and the 11 pennies Sunny ended with. David then found the solution to the problem by counting the number of cubes he added to his initial set. Sunny used a *modelling* strategy wherein he did not follow the action of the problem, but did use the cubes to represent the 11 pennies Sunny ended with, the amount of pennies Sunny's dad gave him, and the 7 pennies Sunny started with. Sunny then found the solution to the problem by using his fingers to keep track of how many cubes he removed from the set of 11 in order to get back to the initial set of 7 pennies. Sunny also made use of second strategy, a *derived fact* strategy, when he said that he knew "if three plus seven was ten then if you had four it would be eleven." Finally, we hoped that our use of the CGI problem-type and strategy-type frameworks would have supported PSTs in making specific inferences about the nature of the children's mathematical understandings. For example, the details in Sunny's modelling strategy could be used to infer that he is developing an understanding that addition and subtraction are inverse operations (since he used a separating action to solve a join problem) while his derived fact strategy could be used to infer that he is developing an understanding of the base-ten system (since he leveraged a group of 10 in his derived fact).

Data analysis

In this paper, we discuss the results of our analyses of pre- and post-assessments of our PSTs' professional noticing. Preliminary analyses of the third noticing skill, PSTs' abilities to decide how to respond, revealed only minor changes. This is consistent with Jacobs et al.'s (2010) finding that developing expertise in this skill takes significant time to develop. Thus, we focused our analytical efforts on the first two noticing skills: attending to the mathematical details of children's strategies and interpreting the details of children's strategies. That said, we coded PSTs' responses to all four prompts for evidence of these two noticing skills.

Analysis of the data began at the end of the semester after the PSTs' final grades were completed. First, the pre- and post-assessments were electronically transferred to a spreadsheet and the PSTs' names were removed from the sheet and replaced with numerical identifiers in order to protect the confidentiality of the participants. Based on the Sunny's Pennies video, we used content analysis (Krippendorf, 2012) to devise a series of codes regarding the mathematical details PSTs might notice and the understandings they might infer. Table 1 describes the codes we used and provides examples from the data for context. We separated Sunny and David from Evelyn and Ashley in our coding because, while Sunny and David solved the problem by making use of three different solution strategies (Sunny solved the problem in two ways and David solved the problem in one way, as described in the above section), Evelyn and Ashley struggled to make sense of the problem and did not solve it. Thus, responses that included descriptions of strategies were necessarily about Sunny's or David's mathematical activity. After we coded our PSTs' responses, we compiled and organised our analyses so that we could make comparisons within and across institutions.

Table 1
Codes and Subcodes Exemplified with Data Excerpts

	Code	Subcode	Example of Code from Data
Sunny and David	Does not focus on Sunny's or David's mathematical strategies or understandings	(N/A)	"Sunny and David are more confident and not as afraid to make a mistake."
	Strategy (S)	Unclear	"Both Sunny and David found a way to solve the problem though the two had different ways of solving it."
		Clear (description of strategy provided for one student)	"Sunny used derived & recalled facts to answer. He said that he knew 7+3=10. So, he knew 7+4 would be 1 more than 10."
		Clear and Complete (description provided for both students)	"David solved the problem using the cubes – He first put together 7 cubes, then counted each cube it took to get to 11. Sunny solved it by moving backwards – He started with 11 cubes, then counted each cube he took away to get to 7. It was interesting because he also did some mental math – He knew 7+3=10 so he could easily find out that 7+4=11."
	Understanding (U)	*Understanding* is used more generally	"Sunny and David both understand math better."
		Specific Claim is made, but is unsubstantiated by the video	"The students seem to understand that they need to find the difference between 11 and 7." "Sunny understands the inverse relationship between addition and subtraction."
		Specific Claim is appropriate	"Sunny – knows that subtraction can be used to solve addition problems. Instead of solving 7+__=11, he went backwards from there. He also uses base-10 knowledge – uses 7+3 to know 7+4."
Evelyn and Ashley	Confusion/ Struggle	Confusion/ Struggle is mentioned	"Ashley asks for help. She is confused." "Evelyn and Ashley were struggling a little bit."
		Source of Confusion/	"Evelyn thought the problem added 11 pennies not 4. Wording is key."

	Struggle named

Results

Table 2 shows the frequency of PSTs' responses at Sites A, B, and C before and after their engagement in their respective mathematics methods course.

Table 2
Frequency of Pre and Post PST Responses

	Site A (*n* = 40, 42)		**Site B (*n* = 51, 47)**		**Site C (*n* = 29, 30)**	
	Pre (%)	Post (%)	Pre (%)	Post (%)	Pre (%)	Post (%)
Sunny and David						
Neither	13	0	6	2	34	0
Strategy						
Unclear	33	2	31	6	52	10
Clear	15	21	22	30	7	60
Clear and Complete	5	60	6	57	3	27
Understanding						
General	38	17	24	2	21	7
Specific Claim, Unsubstantiated	15	5	29	17	0	3
Specific Claim, Appropriate	23	45	25	34	10	23
Evelyn and Ashley						
Confusion Mentioned	20	24	22	32	28	13
Confusion Named	48	36	24	32	24	40

Sunny and David (strategies)

Professional noticing involves the ability to *attend to* the mathematical details of children's strategies, details that could later be used to make inferences about the nature of those children's mathematical understandings. At the end of the semester, more PSTs provided evidence of attending to such details than at the beginning, and PSTs' responses tended to shift from unclear and incomplete descriptions of strategies to clear and complete descriptions of strategies.

Unclear. We considered a response about the strategies enacted in the video to be *unclear* when a PST appeared to attend to the process by which a child solved the problem but did not provide corresponding details that would allow someone else to replicate the process (see Table 1 for an example). There was a significant decrease in the percentage of PSTs who provided unclear descriptions of children's strategies at all sites (from 33% to 2% at Site A, 31% to 6% at Site B, and 52% to 10% at Site C). What follows suggests that our use of CGI supported the development of our PSTs' abilities to attend to the mathematical details contained in children's strategies.

Clear (description of strategy provided for one student). We noticed that some PSTs seemed to focus their noticing on a single child. We therefore considered a response about the strategies enacted in the video to be *clear* when a PST provided a description of how either Sunny or David (but not both) solved the problem that would allow someone else to replicate the strategy (see Table 1 for an example). The overall pattern for the percentage of PSTs who provided clear descriptions was similar across the three sites. At Site A, there was an increase from 15% to 21%; at Site B, there was an increase from 22% to 30%; at Site C, there was an even larger increase from 7% to 60%. In sum, there was an increase at all three sites in the percentage of PSTs who provided a clear description of one child's strategy.

Clear and complete (description of strategy provided for both students). We noticed that some PSTs provided evidence of attending to both Sunny's and David's problem-solving strategies and we thus considered such responses to be *clear and complete*. In other words, we considered a response about the strategies enacted in the video to be *clear and complete* when a PST provided a description of how both Sunny and David solved the problem that would allow someone else to replicate their strategies (see Table 1 for an example). At the end of the semester, PSTs at Site A provided the most evidence of attending to the mathematical details contained in both children's strategies, with 5% of the PSTs offering clear and complete responses at the beginning of the semester and 60% of the PSTs doing so at the end of the semester. Similarly, the PSTs at Site B provided significantly more evidence of attending to the details contained in both children's strategies at the end of the semester (from 6% to 57%). Although evidence of PSTs' noticing in this area also increased at Site C, from 3% to 27%, the spread was noticeably different (24 at Site C versus 55 and 51 at Sites A and B, respectively).

Overall. Across all three sites, the percentage of PSTs who provided evidence of attending to the mathematical details contained in at least one child's strategy increased (from 20% to 81% at Site A, 28% to 87% at Site B, and 10% to 87% at Site C). While the shift from unclear to clear descriptions of strategies occurred at all three sites, there were some differences across the sites. At Sites A and B, approximately one-third of the PSTs provided unclear descriptions of the mathematical details contained in children's strategies at the beginning of the semester. By the end of the semester, 81% of the PSTs at Site A provided clear descriptions of at least one child's strategy with the majority of these PSTs (60%) offering clear descriptions of *both* children's strategies. This was similar at Site B where 87% of the PSTs provided clear descriptions of at least one child's strategy at the end of the semester with the majority of these PSTs (57%) offering clear descriptions for both children's strategies.

In contrast, over one-half of the PSTs at Site C (i.e., 52%) provided unclear descriptions of the mathematical details contained in the children's strategies at the beginning of the semester. By the end of the semester, 87% of the PSTs provided clear descriptions of at least one child's strategy (the same percentage as Site B). However, the majority of these PSTs only provided evidence of attending to one child's strategy.

Together, these results suggest that our semester-long use of CGI supported our PSTs in developing the first of the three noticing skills that comprise teachers' professional noticing (i.e., the ability to notice the mathematical details contained in children's strategies). This is significant given prior research that highlights PSTs' inabilities to notice mathematical aspects of classroom events even after engaging in a semester-long mathematics methods course (e.g., Star & Strickland, 2008).

Sunny and David (understandings)

Professional noticing also involves the ability to *interpret* children's mathematical understandings based on the mathematical details contained in their strategies. At the end of the semester, more PSTs provided evidence of such interpretations than at the beginning and PSTs' responses shifted from those that included more general uses of the term *understanding* to those wherein specific understandings were identified (see Table 1 for examples). In contrast to the previous section, patterns of change varied from site to site.

Understanding is used more generally. We considered a response about understanding to be more *general* when a PST appeared to make inferences about a child's cognition but did not specify the nature of that cognition. For example, we coded the response "Sunny and David both understand math better" as general because it constitutes a statement about cognition but does not provide information regarding the particular understanding that the PST believed Sunny and David had developed that Evelyn and Ashley had not. The percentage of PSTs who provided general responses about the children's understandings decreased over the semester at all three sites (38% to 17% at Site A, 24% to 2% at Site B, and 21% to 7% at Site C). What follows suggests that our use of CGI supported the development of our PSTs' abilities to make specific inferences about the nature of children's mathematical understandings.

Specific claim is made, but is unsubstantiated by the video. We considered a response about understanding to be *specific but unsubstantiated* when a PST's claim could not be supported by the events in the video. We do not necessarily consider such responses "better" than the aforementioned general responses. Rather, our goal here was to capture PSTs' movement toward specificity. For example, we coded the response "The students seem to understand that they need to find the difference between 11 and 7" as specific but unsubstantiated for two reasons. First, this statement cannot be applied to all "students" in the video because not all of the students understood or solved the problem. Second, it is unclear that the students who did understand and subsequently solve the problem interpreted the problem in terms of a "difference." In fact, David counted up when solving the problem and never used the term "difference" nor related his strategy to a counting down or subtraction strategy. Thus, there is no evidence to suggest that he understood he needed "to find the difference between 11 and 7." At Site A, the percentage of PSTs who made a specific but unsubstantiated claim about students' understanding decreased from the pre- to the post-assessment, from 15% to 5%. At Site B, there was also a decrease, from 29% to 17%. In contrast, there was a slight increase in the percentage of PSTs who made a specific but unsubstantiated claim about students' understanding from the pre- to the post-assessment at Site C, from 0% to 3%.

Specific claim is appropriate. We considered a response about understanding to be *specific and appropriate* when a PST's response could be supported by the events in the video (see Table 1 for an example). The percentage of PSTs who provided specific and appropriate responses about children's understandings increased over the semester at all three sites (23% to 45% at Site A, 25% to 34% at Site B, and 10% to 23% at Site C).

Overall. From the pre- to the post-assessments, the percentage of PSTs who provided general responses about the nature of children's understanding decreased while the percentage of PSTs who provided specific and appropriate responses about said understanding increased. This suggests that our use of CGI supported the refinement of PSTs' abilities to interpret children's mathematical thinking. This is significant given prior work that highlights the amount of sustained and prolonged support needed to develop teachers' abilities to notice the thinking of children (e.g., Jacobs et al., 2010). The relative sizes of the changes in our percentages support previous findings that this second component of professional noticing takes more time to develop than the first.

We were surprised when we considered the percentage of responses that *did not contain reference* to students' understandings. At Site B, the percentage of responses that contained no mention of understanding more than doubled from the pre- to the post-assessment (from 22% to 47%). At Site A, this percentage also increased (albeit to a lesser degree from 24% to 33%) whereas at Site C this percentage remained relatively stable and even saw a slight decrease from 69% to 67%. Thus, while there was an increase in the percentage of PSTs who were able to make specific and appropriate inferences about the nature of children's mathematical thinking at all of the three sites, there was a decrease in the overall percentage of PSTs who made any reference to said thinking. Rather, these PSTs' responses tended to remain at the strategy level.

This suggests there is more to learn about how we can make use of video to support more of our respective PSTs in interpreting children's thinking. Similarly to previous research (Vacc & Bright, 1999), our results highlight the challenge it is for PSTs to make specific, clear, and complete claims about children's mathematical strategies *and* understandings when watching a video clip of multiple children solving a CGI task.

Evelyn and Ashley

We were interested in examining PSTs' noticing of Evelyn's and Ashley's mathematical activity even though they did not solve the problem. Such examination marks a departure from the professional noticing framework as this framework necessitates the existence of a problem-solving strategy. That said, there is much to be learned about students' understandings even when they are unable to devise a solution strategy (e.g., from a student's statements, questions, and gestures).

Confusion mentioned. The percentage of PSTs who noted *that* Evelyn and Ashley struggled but did not provide corresponding *rationale* for the source of said struggle decreased at Site C (from 28% to 13%) and increased at Sites A and B (from 20% to 24% at Site A and 22% to 32% at Site B).

Confusion named. The percentage of PSTs who noted *and* provided possible rationale for the source of Evelyn and Ashley's confusion increased at Site B, from 24% to 32%, and at Site C, from 24% to 40%. This percentage decreased at Site A, from 48% to 36%.

Overall. While the overall percentage of PSTs who attended to Evelyn's and Ashley's mathematical activity increased at Sites B and C (from 46% to 64% at Site B and 52% to 53% at

Site C), it decreased at Site A (68% to 60%). We find it interesting that the site in which there was a decrease corresponded with the site that had the highest percentages of PSTs who offered clear and complete descriptions of strategies and who made specific and appropriate claims about students' mathematical understandings. One possible reason for this finding is that, in the context of group work, PSTs make (unconscious) decisions about which aspects of children's mathematical activity to commit their noticing efforts to (e.g., the activity of the children who solved the problem over the activity of the children who did not solve the problem). That said, more work needs to be done to test this conjecture.

Discussion

When reflecting on and making sense of these results, we began to have more detailed conversations about our teacher preparation courses and about how we were choosing and using video clips in our respective courses. We conclude this paper by discussing three factors that may be significant when considering how to support the development of PSTs' professional noticing: the number of mathematics courses that comprise the teacher preparation program, how to scaffold PSTs' professional noticing skills, and how to choose videos from the CGI DVD collection.

The number of mathematics courses that comprise the teacher preparation program

The teacher preparation program at Site A includes two mathematics *methods* courses whereas the preparation programs at Sites B and C include one such course. At all sites, PSTs' professional noticing skills were assessed before and after their engagement in their *first* mathematics methods course. At Sites B and C, PSTs' first mathematics methods course was also their *only* such course. Thus, the instructors at Sites B and C were compelled to incorporate more (e.g., mathematical topics outside the domain of number and operations like Geometry, and pedagogy like supporting English Language Learners) into their respective 15-week semesters. This difference thus helped us understand why it was reasonable for more of the PSTs at Site A to have provided evidence of both attending to children's problem-solving strategies and appropriately interpreting their mathematical thinking than at Sites B or C. When teacher preparation programs provide the time and opportunity, teacher educators are better able to support the development of higher levels of noticing skills (Jacobs et al., 2010; McDuffie et al., 2014).

Interestingly, the number of mathematics-specific courses comprising the teacher preparation programs across the three sites was the same. While the teacher preparation programs at Sites B and C contain only one mathematics *methods* course, they contain two mathematics *content* courses. This is different at Site A where the teacher education program contains two methods courses, but only one content course. Might we be missing opportunities to further support the development of PSTs' professional noticing skills in content courses? How might the results of our study have been different had our PSTs had more time across multiple courses to develop and refine their professional noticing lens? Research shows that supporting teachers (especially PSTs) to refine this lens takes time and multiple iterations of opportunities to practice (McDuffie et al., 2014; Santagata & Guarino, 2011; Schack et al., 2013; Sherin & Van Es, 2009). It is thus important to reflect on how we can provide PSTs with the amount of support needed to develop higher-level professional noticing skills so that they are better positioned to enact the kind of high-quality instruction that leverages student thinking. One way to do this is to look across mathematics-specific courses and incorporate noticing activities into both methods and content courses.

How to scaffold PSTs' professional noticing skills

After analysing our data, we discussed (in greater detail) the ways in which we scaffolded the development of our respective PSTs' professional noticing skills. Implications of previous research suggested that providing prompts or frameworks for viewing video acts to scaffold higher levels of noticing skills (Star & Strickland, 2008; McDuffie et al., 2014). We therefore all made use of the CGI problem-type and strategy-type frameworks during instruction and used professional noticing prompts during video activities like "What did you notice regarding how the child solved the problem?" To support our PSTs in interpreting the details of what they noticed, we used prompts like, "Given the relationship between the problem the child was solving and what the child did, what does the child seem to understand?" These scaffolds yielded positive results as more PSTs at all three sites provided evidence of being able to provide clear and complete descriptions of strategies and specific and appropriate claims about children's understandings from the pre- to the post-assessment.

Given research that shows gains in the second noticing skill continue with a long-term focus on students' mathematical thinking (whereas gains in the first noticing skill seem to level off at a certain point; Jacobs et al., 2010), it may be the case that additional scaffolds in this area are necessary. Consider, for example, two different methods used to scaffold our PSTs' professional noticing. To support the development of PSTs' abilities to interpret the nature of children's mathematical understandings, Instructor B provided her PSTs with opportunities to practice creating, analysing, and adapting CGI tasks and to discuss how children might solve those tasks using a variety of different strategy types. She then showed her PSTs corresponding video clips of children solving those same problems and asked questions like the ones above regarding the mathematical details of children's strategies and nature of their mathematical understandings.

The instructor at Site A created and used an additional tool during noticing activities involving video clips (see Figure 1). For example, she would:

- press play;
- press pause after the interviewer in the clip finished reading the problem to the child;
- ask PSTs to act out the problem exactly as written, write down a number sentence that could be used to model the problem, and categorise the problem using CGI's problem-type framework;
- press play;
- press pause after the child in the video finished solving the problem;
- ask PSTs to write down everything they remembered about what the child did and said, discuss what they noticed with the members of their group to make sure they all saw and heard the same things, and categorise the child's solution strategy using CGI's strategy-type framework; and then
- ask PSTs to look *across* the details of the problem type and solution strategy to infer a child's understanding of mathematical ideas (e.g., *inverse operations, commutativity* with respect to the topics of addition and multiplication, *distributivity* with respect to the topics of multiplication and division).

This additional tool likely acted to scaffold PSTs' professional noticing skills by making the first two components of PSTs' professional noticing more explicit to PSTs. Such scaffolds may explain why Sites A and B saw the greatest percentage of PSTs who provided clear and complete descriptions of strategies (60% and 57% at Sites A and B, respectively, as compared to 27% at Site C). The explicit nature of the scaffold used at Site A may explain why Site A saw the greatest percentage of PSTs who provided specific and appropriate claims regarding students' mathematical understandings (45% as compared to 34% at Site B and 23% at Site C), in the post-assessment. Given that Instructor A was so explicit in her scaffolding of PSTs' professional noticing, it may be surprising that *only* 45% of her PSTs were able to identify specific and appropriate mathematical ideas in the Sunny's Pennies video. This again speaks to previous observations that developing expertise in this noticing skill benefits from extensive and prolonged support.

<table>
<tr><th colspan="2">Video x</th></tr>
<tr><td>Problem Type</td><td></td></tr>
<tr><td>Number Sentence</td><td></td></tr>
<tr><td>Student's Strategy</td><td>Description:

Strategy Type:</td></tr>
<tr><td>Big Idea</td><td></td></tr>
<tr><td>Justification of Big Idea</td><td></td></tr>
</table>

Figure 1. Instructor A's scaffolding tool.

The choice of which video clips to show

In this section, we focus on how much data are contained within the perceptual field during noticing activities and the nature of the data contained within the perceptual field. With respect to the former, the instructor at Site A, where the greatest percentage of PSTs provided specific and appropriate claims about the nature of children's mathematical understandings in the post-assessment, tended to choose video clips that showed one child engaged in problem-solving activities. In contrast, Instructor B typically selected video clips like Sunny's Pennies, which showed multiple students engaged in such activities. It may be that reducing the amount of perceptual input in the initial stages of PSTs' development acts as an additional scaffold of their noticing skills.

With respect to the nature of the data contained within the perceptual field, the instructor at Site A tended to choose video clips that showed a child solving a problem in a way that did not follow the action or structure of the problem. While children often solve problems by following the action in the problem (like David did in Sunny's Pennies), their strategies transition as they develop more sophisticated ways of thinking. For instance, Sunny was able to start with the ending amount of pennies (i.e., 11) and subtract until he arrived at the beginning amount of pennies (i.e., 7). He then counted how many blocks were removed to find the answer to the problem. The CGI DVDs contain many such examples of children providing explanations for why these actions, which are typically associated with subtraction (here, removing pennies from a set), can be used in situations that call for joining actions (e.g., when someone is being given more of something), which are typically associated with addition. Videos like this, wherein a child is making sense of a problem and solving it without following the action or structure of a problem, are rich with opportunities to make inferences regarding the big mathematical ideas that children are developing, like the notion that addition and subtraction are inverses of one another. This is not to say that these are the only videos wherein such inferences can be made, as Sunny showed with his second problem-solving strategy wherein he followed the action in the problem while leveraging a bundle of 10 to find the answer to the problem.

In contrast, the instructor at Site C focused almost entirely on CGI's problem-type and strategy-type frameworks, helping PSTs to refine their lens for noticing distinctions between various problem types and strategy types, and helping PSTs match particular problems with *typical* solution strategies. For example, if a PST solved a Join Change Unknown problem by using subtraction instead of addition (as Sunny did in his first problem solving strategy in Sunny's Pennies), Instructor C would remind the PST that young children might not solve the problem in that way since they may have to follow the action contained in the problem. Thus, Instructor C encouraged the PSTs to make use of strategies that are more reflective of the actions suggested in problems (e.g., counting up from 7 to 11). Because Instructor C's primary goal was to support PSTs in learning about what children typically do early in their development, he tended to use video clips that showed children solving problems in ways that follow the action contained in the problems. Consequently, Site C teachers rarely saw strategies that exhibited understandings of, for instance, inverse operations. The decision not to push forward in terms of mathematically more sophisticated strategies is one that Instructor C wrestled with. In the end, his decision was based on his belief that PSTs, particularly those struggling with children's mathematical thinking, really need to understand the CGI frameworks and that bringing in inverse operations may potentially confuse PSTs' understandings of those frameworks.

References

Australian Curriculum. (n.d.). Retrieved from: https://www.australiancurriculum.edu.au/

Carpenter, T. P., Fennema, E., & Franke, M. L. (Research Project Directors), & Empson, S. B. (Director). (1995). *Children's mathematics: Cognitively guided instruction* [DVD set]. United States: WPT Digital Videos.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499–531.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*, 403–434.

Goodwin, C. (1994). Professional vision. *American Anthropologist, 96*, 606–633.

Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational Studies in Mathematics, 80*(1–2), 233–247.

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM, 48*(1–2), 185–197.

Jacobs, V. R., Lamb, L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*, 169–202.

Jacobs, V. R., Lamb, L. L., Philipp, R. A., & Schappelle, B. P. (2010). Deciding how to respond on the basis of children's understandings. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 97–116). New York: Routledge.

Krippendorff, K. (2012). *Content analysis: An introduction to its methodology*. Los Angeles: Sage.

Mason, J. (2008). Being mathematical with and in front of learners: Attention, awareness, and attitude as sources of difference between teacher educators, teachers and learners. In B. Jaworski & T. Wood (Eds.), *Handbook of mathematics teacher education: Vol. 4. The Mathematics teacher educator as a developing professional* (pp. 31–56). Rotterdam, The Netherlands: Sense.

McDuffie, A. R., Foote, M. Q., Bolson, C., Tuner, E. E., Aguirre, J. M., Bartell, T. G., …, Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education, 17*, 245–270.

Ministry of Education, Singapore. (2017). *Mathematics syllabus: Primary one to five*. Retrieved from: https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/primary_mathematics_syllabus_pri1_to_pri5.pdf

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education.* Washington, DC: National Academy Press.

National Research Council. (2011). *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas.* Washington, DC: National Academies Press.

Santagata, R., & Guarino, J. (2011). Using video to teach future teachers to learn from teaching. *ZDM Mathematics Education*, *43*, 133–145.

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education, 16*, 379–397.

Sherin, M. G., & van Es, E. A. (2005). Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education, 13*, 475–491.

Sherin, M. G., & Van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education*, *60*, 20–37.

Star, J., Lynch, K., & Perova, N. (2011). Using video to improve preservice teachers' abilities to attend to classroom features: A replication study. In M. G. Sherin, V. R., Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117–133). New York: Routledge.

Star, J., & Strickland, S. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*, 107–125.

Takahashi, A., Wantanabe, T., & Yoshida, M. (2008). *English translation of the Japanese mathematics curricula in the course of study*. Retrieved from: http://ncm.gu.se/media/kursplaner/andralander/Japanese_COS2008Math.pdf

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education*, *30,* 89–110.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*, 571–596.

van Es, E. A., & Sherin, M. G. (2006). How different video club designs support teachers in "learning to notice." *Journal of Computing in Teacher Education, 22*, 125–135.

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*, 244–276.

Walkoe, J. (2015). Exploring teacher noticing of student algebraic thinking in a video club. *Journal of Mathematics Teacher Education, 18*(6), 523–550.

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