



How Students with Mathematics Learning Disabilities Understands Fraction: A Case from the Indonesian Inclusive School

Trisno Ikhwanudin

MA., Indonesia University of Education, Indonesia, trisno.ikhwanudin@student.upi.edu

Didi Suryadi

Prof., Indonesia University of Education, Indonesia, didisuryadi1@gmail.com

This research aims to analyze ways of understanding of students with mathematics learning disabilities when learning fraction. The research was conducted in an Inclusive Junior High School in the West Java Province, Indonesia. This study is qualitative, with the single-case (holistic) designs. The case will focus on three students who suspected of having mathematics learning disabilities when they learn fraction. The data were collected through student tests, observations, document and media analysis, and teacher interview. The data analysis was conducted using interpretational analysis. The results showed that students with mathematics learning disabilities performed two mental acts with corresponding ways of understanding and ways of thinking; those are interpreting and problem-solving. The other interesting findings were: (1) students know the common denominator method in the addition of fractions; however, they incorrectly apply the method; (2) Surprisingly, students used the common denominator approach in the multiplication of fraction; (3) in the division of fraction, students mistakenly apply the invert multiply algorithm.

Keywords: fraction learning, students with mathematics learning disabilities, mental acts, ways of understanding, ways of thinking

INTRODUCTION

Inclusive education is an education service system that includes children with special needs to learn together with their peers in the regular school. The spirit of implementing inclusive education is to provide the widest possible opportunity or access to all children to obtain a good quality of education and to meet the individual needs of learners without discrimination.

In mathematics education, several communities have had an inclusive spirit in their principle. For example, the Equity Principle in the Principles and Standards for School Mathematics (NCTM, 2000, p.12): “*all students, regardless of their personal*

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characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn— mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.”

All of us agreed about the equity principle in the teaching and learning mathematics. However, studies that examine the learning of mathematics in the inclusive school are still not widely practiced. A study of the practice of inclusive education identified only five articles published between 2000 and 2013 relating to mathematics classes (McKenna et al., 2015).

Schools that implement inclusive education systems have the opportunity to receive all learners with their specific needs. Especially for learning mathematics, inclusive schools will likely receive two types of students, i.e., students who have difficulty in learning mathematics, and students who have no difficulty in learning mathematics.

In the inclusive school, many students struggle with mathematics because of different reasons; especially the difficulty of students with mathematics learning disabilities (MLD). Shalev (2007) estimated that 6% of students with MLD have difficulties that came from a cognitive origin. The other researchers showed that students with MLD have different error pattern if compared with their low achieving peers (Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013). Lewis (2014) suggested research on fraction learning with younger MLD students, such as elementary or junior-high-school students. Based on Lewis’s suggestion, this research was conducted in an inclusive junior-high-school student. The inclusive school was chosen because the school has the opportunity to receive all learners with their specific needs, including students with MLD.

Research on students' ways of understanding fractions in the inclusive school is needed as an essential first step toward the effective instructional methods. Thus, the problem in this research is how students with MLD understand fraction in the inclusive school.

LITERATURE REVIEW

The term difficulty in learning mathematics is called Mathematics Learning Disabilities (MLD). Some literature and researchers call MLD as dyscalculia. Research on dyscalculia continues until today. Therefore, understanding of dyscalculia will continue to grow. The following is some of the meanings of dyscalculia issued by both formal institutions and individual researchers. According to The National Center for Learning Disabilities, USA (2014), dyscalculia is a term related to the difficulty in learning mathematics. The characteristics of students with dyscalculia are as follows: (1) difficulty in counting, studying number facts, and performing mathematical calculations; (2) difficulty in measuring, timing, counting money, and estimating numbers; (3) struggle in mental mathematics and problem-solving methods.

Geary (2004) describes dyscalculia as a difficulty in numerical and arithmetic caused by brain injury. Geary uses this term to describe a population of 5 - 8% of school-age children who have a cognitive abnormality that affects their ability to study concepts or

procedures in one or more areas of mathematics. In general, dyscalculia is an umbrella term used for various difficulties in learning mathematics, such as developmental dyscalculia, mathematical difficulty, difficulty learning numerical concepts, and difficulty in learning the concept of numbers.

Some definitions of dyscalculia are relatively close to the medical area. In this study, the authors chose the definition of dyscalculia formulated from the view of education. So, in this study, the term chosen is the MLD. Here are some researchers' opinions about the criteria of students with MLD:

- a. students with an average IQ whose standardized test scores were below the 20th or 25th percentiles (Geary, 2004);
- b. slower and often error in processing numerical representation, such as the symbol number "3" and its counterpart "◆◆◆" (Piazza et al., 2010);
- c. making mistakes in comparing and estimating numbers (Mazzocco et al., 2011);
- d. wrong in performing arithmetic calculations (Geary et al., 2004);
- e. wrong in solving very easy numbers (e.g., $4 \times 5 = 20$; Mazzocco et al., 2008).

According to Clarke et al., (2010), a fraction is a symbol that represents the results of two numbers $\frac{a}{b}$ (with b not equal to zero). Thus, all rational numbers written in form $\frac{a}{b}$ are fractions, but rational numbers 1.45 are not fractions; however, if 1.45 is written $\frac{145}{100}$ then it is a fraction. The selection of fraction topic in this research is based on the result of teacher interview in the preliminary study. Many teachers in the inclusive school stated that fractions were one of the topics that difficult to teach, especially for students with MLD. In the Indonesia context, Wijaya (2017) mentioned three possible causes of students experiencing difficulties in fraction learning. The three causes are: (1) the lack of emphasis on the basic concept of fractions and the introduction of fraction operations that are too early; (2) In general, Indonesian textbook only presents the definition of fractions as part of a whole; and (3) in the class, various fraction representations are rarely used. The other causes stated by Lewis and Fisher's (2016) research finding that studying fractions require high-level or complex thinking skills.

Several researchers have done the study on fractions learning in students with MLD:

Lewis (2014) states that MLD students have different mindsets in understanding the fractions. She views students with MLD does not mean to have a lack of understanding of the concept of fractions, but there are different ways of thinking in understanding the fractions. She suggests other approaches to the interventions that are undertaken in students with MLD, not only as part of a whole approach but also using another approach such as the number line.

Lewis (2016a) states that students with MLD experience obstacles in fraction learning, especially on fraction comparison subjects, either fraction comparisons with the same denominator, or in fractions comparisons involving half fractions ($\frac{1}{2}$). In this study,

Lewis suggested examining students' understanding of the quantity of fractions. Lewis (2016b) states that partitioning activity may be at the root of understanding fraction quantities in regular students. Students with MLD may not follow this pattern of development.

Hunt et al. (2016) state that students with learning disabilities have difficulty in mastering the concept of fractions with part-whole model learning. Also, Mazzocco et al. (2013) stated that the difficulties in fraction learning are still felt by students with MLD until they are in grade 8. They also said the visual model could be used as an alternative when helping students with MLD in understanding the fraction. Newton et al. (2014) stated that the main pattern of error in fraction understanding on MLD students is the error in using traditional algorithms.

In this study, students and the teacher used Indonesia national curriculum textbook (Kemdikbud, 2016) for learning fraction; the textbook approach was called scientific approach. To analyze the students' thinking, we used Harel's theory (2008): Mental Act, Ways of Understanding and Ways of Thinking. This theory is chosen because the researchers want to know the student's way of thinking when they learn fractions in the inclusive class. By Harel theory, researchers could know the way of students' thinking through two ways: (1) knowing how students understand the fractions (ways of understanding); and (2) knowing how a student uses the strategies in solving the problem of fractions (ways of thinking).

Mental Acts, Ways of Understanding and Ways of Thinking

According to Harel (2008), mathematics is the set of ways of understanding (WoU) and ways of thinking (WoT). WoU is the product of mental acts; it consists of axioms, definitions, theorems, proofs, problems, and solutions. WoT is the characteristics of mental acts; it consists of all the ways of thinking that used to produce WoU.

The above definition of mathematics implies a good pedagogical implementation. A mental act is a characteristic of thinking in line with the problems encountered. The flow of thinking formed from the mental act become WoT. When the flow of thinking proceeds and comes into contact with certain contexts to form an understanding, the path leads to WoU. The reasoning of human involves many mental acts such as interpreting, guessing, concluding, proving, explaining, composing, generalizing, applying, predicting, classifying, seeking and solving problems.

Based on Harel (2008), a way of understanding is a specific cognitive product of the mental act performed by an individual. For example, after looking at the $\frac{3}{4}$ symbol, one can interpret (one of the mental act) to generate meaning for $\frac{3}{4}$. The resulting interpretation is the way of understanding of a person to the $\frac{3}{4}$ symbol. Such means of understanding may differ depending on the context, and if judged by an observer, may

be considered true or false. For example, in a context one can interpret the $\frac{3}{4}$ symbol as

"3 objects of 4 objects," and others can be interpreted as "recurring sum: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$."

Furthermore, according to Harel (2008), a way of thinking is the cognitive characteristic of the mental act. The cognitive characteristics of the mental act are inferred from the observation of the way of understanding (the cognitive product of a mental act). For example, a teacher who follows a student's mathematical behavior may conclude that students' interpretations of mathematical symbols are inflexible, there is no quantitative view, or, for example, students' interpretations of symbols are flexible and connected with other concepts. Another example, teachers can conclude that the student's proof of mathematical statements is based on empirical evidence, or based on deductive reasoning.

When analyzing students' mathematical behavior regarding the way of understanding and way of thinking, a person starts with a mental act being observed, see the type of cognitive product (way of understanding associated with it), and tries to find the common cognitive nature between the way of this understanding. Every character that found is the way of thinking associated with the mental act (Harel, 2008).

METHOD

This research uses a qualitative approach, with the single-case (holistic) designs (Yin, 2009). The approach used in this research is more emphasis on the interpretative study of the data analysis. The qualitative research investigates events as they occur naturally (Bogdan and Biklen, 2007). The use of case study methods is applied to describe field findings related to the research problem, i.e., how students with MLD understands fractions in the inclusive school. As written by Bogdan and Biklen, the case study is "*a detailed examination of one setting or one single subject, or a single depository of documents, or one particular event*" (Bogdan and Biklen, 2007, p. 59). One single subject in this research is students with MLD, and particular event which studied are ways of understanding of students with MLD when they learn fractions.

Sample

This study involves purposeful sampling. The sample was chosen using researchers' judgment to select the instances that are information-rich by the phenomena studied (Gall, Gall, & Borg, 2010). This research was conducted in one of the Inclusion Junior High School in Cimahi City, West Java, Indonesia. The school location is in the urban area. The reason to gather data from this school is that the researchers want to know students with MLD's ways of understanding when they learn fraction in the inclusive school. Based on the preliminary interview, the teachers stated that their school received many students that have learning disability symptoms. The participants are students in the 7th grade; they learn fraction topic used Indonesia national curriculum textbook for the 7th grade. They have learned basic fraction concept from the 3rd grade, and in the 7th grade, they completed all the learning on fractions topic. The case will focus on

three students who suspected of having MLD when they learn fraction. The three students with MLD were selected from test results, teacher interviews, and observation.

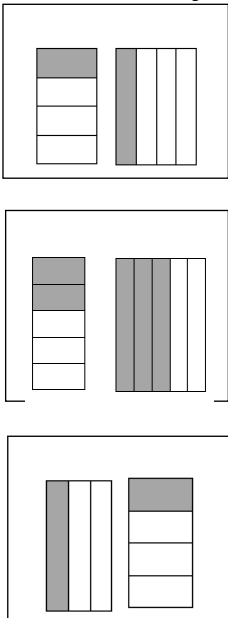
The selection stage of three students with MLD is as follows:

1. Based on the tests result, three students earned a score under the first quartile; they are students A, B, and C.
2. Based on the teacher's interview and observation; the following results are obtained:
 - Student A got the score 35.71. This score is below the first quartile, which is 66.9. Based on interviews with the teacher and class observation, student A experienced difficulties in learning mathematics. So, student A can be recorded to experience MLD.
 - Student B got the score 39.29. This score is below the first quartile, which is 66.9. Based on interviews with the teacher and class observation, student B experienced difficulties in learning mathematics. So, student B can be recorded to experienced MLD.
 - Student C got the score 64,29. This score is below the first quartile, which is 66.9. Based on interviews with the teacher and class observation, student C experienced difficulties in learning mathematics. So, student C can be recorded to experience MLD.

Data Collection Tools

In this study, the data were collected through student test (paper and pencil measure), observations, document and media analysis, and interview (Gall, Gall, & Borg, 2010). In the paper and pencil measure (test), the problem is as follows:

Table 1
The Test Instruments

No	Problems	Indicator	Reference
1	Draw the following fractions: $\frac{1}{2}$ and $\frac{2}{5}$ with two different ways.	Students can understand the representation of fractions in various forms	Lewis (2014)
2	Check which fraction images are larger: 	Students can understand the fraction comparisons in various forms of representation	Armstrong & Larson (1995)
3	a. Explain which one is larger: $\frac{1}{3}$ or $\frac{2}{3}$ b. Explain which one is larger: $\frac{3}{3}$ or $\frac{4}{4}$	Students can understand fraction comparisons	Lewis (2014)
4	Write down your way to find 2 fractions which are equal $\frac{1}{2}$?	Students can understand the equal fractions	Lewis (2014)
5	Complete the following problems with the steps. a. $\frac{1}{3} + \frac{1}{3} =$ b. $\frac{1}{3} + \frac{1}{2} =$	Students can perform the addition operation of fractions	Lewis (2014)
6	Complete the following problems with the steps. a. $\frac{4}{5} \times \frac{1}{3} =$ b. $\frac{9}{4} \div \frac{3}{5} =$	Students can perform the multiplication and division operation of fractions	Zembar (2015); Wyberg et al., (2011)
7	Susi ran $\frac{2}{5}$ km on Monday. On Tuesday, Susi ran $\frac{3}{7}$ km. Explain how many kilometers (km) Susi ran on both days?	Students can understand the fractions in a word problem	Brown and Quinn (2006)

We adapted the problem above from some previous research. Our research case is different from the previous one. The main difference is the focus of the research problem; in this research, we investigate students with MLD's ways of understanding when they learn fraction in the inclusive school. The case of Lewis (2014) was two students (19 and 18 years old) who were diagnosed having MLD; the other previous researchers (Armstrong & Larson, 1995; Zembat, 2015; Wyberg et al., 2011; Brown & Quinn, 2006) were researched the regular class setting.

After the instrument is composed, content and construct validation are done by experts. This test instrument was given at the end of the fraction learning session using the Indonesian national curriculum. The item indicator is the same as the fraction topic indicator that students have learned. Thus, the above problem evaluated students' understanding of the fraction topic that they have learned.

The Teacher Interview

After we obtained the result of students test, we interviewed the teacher. The purpose of the teacher interview was to confirm the result of the test that there were three students suspected of having MLD. The following table is the teacher interview guide:

Table 2

Teacher Interview Protocol

No	Question
1	Do the three students have difficulty in learning math in your classroom?
2	Are the students' difficulties caused by socioeconomic status factors?
3	Are the students' difficulties caused by language factors?
4	Are the students' difficulties caused by a lack of parental support factors?
5	Do the ability of the three students improve after being given fractional learning?
6	Do the three students have behavior and attention barriers?
7	Do the three students have math anxiety?

The result of the teacher interview revealed that the three students had difficulties in learning mathematics. The difficulties of the three students were not due to socioeconomic status, language, and a lack of parental support. The ability of the three students also did not increase after being given fractional learning. Furthermore, the three students had no behavior or attention barriers and did not experience math anxiety.

The observation

The observation was conducted when students did fractions learning and when they completed the test problems. The purpose of the observation is to make sure students learn and solve the fraction problem seriously. The following table is the observation guide:

Table 3

The Observation Protocol

No	Description	Yes	No
1	Students attend fractions learning activities seriously		
2	Students are active in group discussions during fractions learning		
3	Students undertake the tasks that teacher gives during fractions learning		
4	Students complete the test problems seriously		
5	Students complete the test within the specified time duration		

The results of the observations revealed that the three students that suspected of having MLD attended fractions learning activities seriously. They were also active in group discussions during fractions learning. Then, they did the tasks that the teacher gave during fractional learning. Furthermore, when the test was given, they also did it thoughtfully and within the specified time duration.

Data Analysis

The data analysis of this study aims to analyze students with MLD's ways of understanding when they learn fractions. The data analysis was conducted using the qualitative analysis model of Milles & Hubberman (1994). The phases of the analysis were data reduction, data display, data verification and conclusion. Each data that has been collected from the field is written in the detailed form and forming daily reports. Considering the daily reports are so numerous and varied, then on the data collected, carried out data reduction, which is done by creating an abstraction. Data abstraction is a summary of research data, then selected and focused on the things that are important and related to understanding a fraction of MLD student. Further data are selected and categorized while coded. To assist the process of data analysis in this study, we used HyperRESEARCH version 7.3.7 software. HyperRESEARCH is one of several computer programs commonly used by qualitative researchers to assist data processing, coding, and development as well as theoretical testing resulting from a qualitative study.

Procedure

The procedure of this study is described in the form of three stages taken by researchers as follows.

1. Orientation Stage

This stage is the preliminary stage. At this stage, the researcher organizes the instrument and organizes the strategy for the next stage. This stage serves to understand the background situation of research.

2. Exploration Stage

This stage is a step follow up from the previous stage. The goal is to find in-depth information about the problem under study. That is, researchers plunge into the research arena and conduct research intensively. At this stage, tests on students are conducted, student observations and interviews of teachers are carried out.

3. Data Analysis and Interpretation Phase

At this stage, the data are analyzed with the qualitative analysis model of Milles & Hubberman (1994). The data are examining and grouping to describe, evaluate, and explain the phenomenon. The goals of this stage are to identify the ways of understanding of students with MLD when they learn fraction.

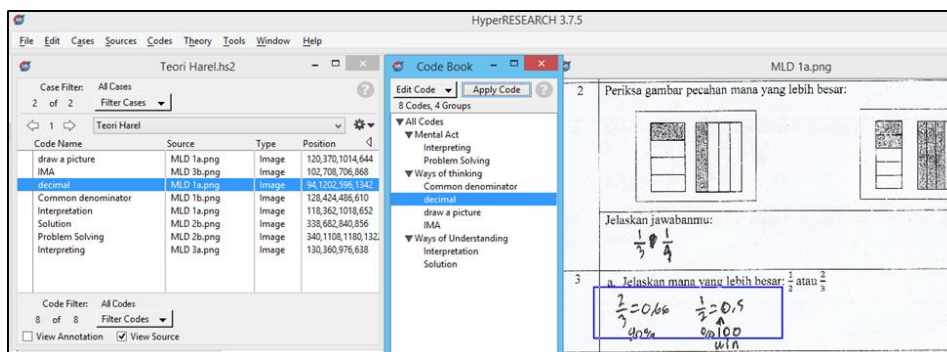
Validity and Reliability of the Study

To prove the validity and reliability of research data, the researcher performs the data validation process by the rules of validity and reliability in qualitative research. Validation of data is done through three strategies (Gall, Gall, & Borg, 2010), as follow:

1. Triangulation (triangulation), by analyzing different sources of data through examination of evidence derived from these sources. In this research, triangulation data is conducted by five different sources of data; there are paper and pencil measure, class observation, media/recording, teacher interview, and document analysis.
2. Using member checking to know the accuracy of research results. Checking members are made through final report confirmation or data descriptions of the teacher and participant.
3. Contextual completeness, i.e., the use of various reference materials to generate validity of information.

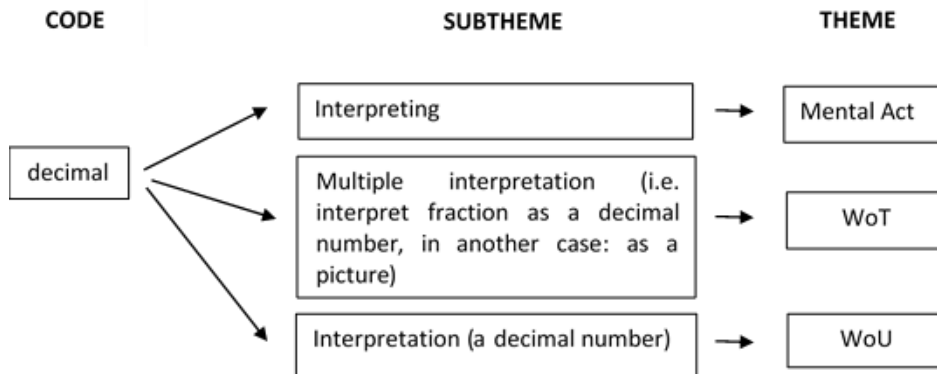
FINDINGS

In this study, we arrange the data as follows: the themes are mental acts, ways of understanding, and ways of thinking. The subthemes are problem-solving and interpreting (for mental acts theme), solution and interpretation (for ways of understanding theme), problem-solving approach and multiple interpretation strategies (for ways of thinking theme). We created a code for each students' answer; then the code is matched with the subtheme. To organize data, we use HyperResearch software. HyperResearch is used to facilitate display data. Here is one illustration of HyperResearch usage.



In the picture above, the code (decimal) corresponds to ways of thinking (multiple interpretations as a decimal number), mental acts (interpreting), and ways of understanding (interpretation). The blue box on the right shows the student's work indicating the decimal code.

From the student's answer above, we created a diagram that represents the interrelation between the code, subtheme, and theme as follows:



Here are the results of the data analysis from three students with MLD; we found mental acts, ways of understanding, and ways of thinking as follows:

Problem Solving

Here is one example of student work that used mental act problem-solving:

5	Selesaikan soal berikut dengan langkah-langkahnya.
a.	$\frac{1}{3} + \frac{1}{3} = \frac{8}{9}$
b.	$\frac{1}{3} + \frac{1}{2} = \frac{5}{12}$

In the above problem, the student is asked to complete two questions about the sum of the fractions. In the first question, the student is asked to solve the problem $\frac{1}{3} + \frac{1}{3} = \dots$. This problem aims to identify students' understanding of the addition of fractions with the same denominator. For this problem, students answer: $\frac{1}{3} + \frac{1}{3} = \frac{8}{9}$. Student thinking

can be analyzed as follows: $\frac{1}{3} + \frac{1}{3} = \frac{(1+3)}{(3 \times 3)} + \frac{(1+3)}{(3 \times 3)} = \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$.

The student is familiar with the common denominator procedure in the fraction addition operation. So, when students saw the $\frac{1}{3} + \frac{1}{3}$ problem, he applied the common denominator procedure by changing 3 to 9. He did not recognize that the denominator is the same, i.e., 3. There are other interesting things done by the students. He adds to the numerator by 3, not multiplying it, as follows: $\frac{(1+3)}{(3 \times 3)} + \frac{(1+3)}{(3 \times 3)}$. The student already knows

that if the denominator is multiplied by 3, then the numerator is also multiplied by 3. However, in fact, the numerator is added by 3 instead of multiplying by 3. In other words, the student mistakenly applies the procedures that already known.

In the second question, the student is asked to solve the problem $\frac{1}{3} + \frac{1}{2} = \dots$. This problem aims to know students' understanding of the addition of fractions with different

denominators. For this problem, students answer: $\frac{1}{3} + \frac{1}{2} = \frac{5}{12}$. Student thinking can be

analyzed as follows: $\frac{1}{3} + \frac{1}{2} = \frac{(1+4)}{(3 \times 4)} + \frac{\dots}{\dots} = \frac{5}{12}$. Students are familiar with the common denominator procedure in the fraction addition operation. So, when students saw the

$\frac{1}{3} + \frac{1}{2}$ problem, he did the common denominator procedure by changing 3 to 12. Unfortunately, he did only in the first part. Just like before, there are other interesting things done by students. He added the numerator by 4 instead of multiplying it, as

follows: $\frac{1}{3} + \frac{1}{2} = \frac{(1+4)}{(3 \times 4)} + \frac{\dots}{\dots}$. Thus, the student already knows that if the denominator multiplied by 4, then the numerator is also multiplied by 4. However, in fact, the numerator is added by 4, not multiplied by 4. In other words, the students mistakenly apply the procedures that already known. From the observation and the teacher interview, the student already knows the common denominator procedure.

In the process of solving the problem, students have done the mental act problem solving, although the answer is not right. Ways of understanding students have also been identified, i.e. in the form of a given solution. While students' way of thinking has also been identified, students use the common denominator approach to solve the problem.

There is again an example of an interesting student answer, as follows:

6	Selesaikan soal berikut dengan langkah-langkahnya. a. $\frac{4}{5} \times \frac{1}{3} = \frac{12}{15} \times \frac{5}{15} = \frac{60}{15} = 4$
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In the above problem, the student is asked to complete problem of fraction

multiplication. In this problem, the student is asked to solve the problem $\frac{4}{5} \times \frac{1}{3} = \dots$.

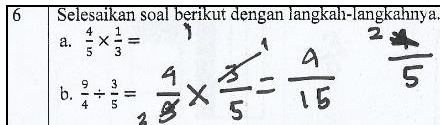
This problem aims to reveal students' understanding of fraction multiplication. In this

case, the students answer: $\frac{4}{5} \times \frac{1}{3} = \frac{12}{15} \times \frac{5}{15} = \frac{60}{15} = 4$. There is an interesting point; student applied the common denominator (fraction addition procedure) for the fraction

multiplication operation. So, when students saw the problem $\frac{4}{5} \times \frac{1}{3} = \dots$, he did the common denominator procedure on the first part by converting 5 to 15 and in the second part by changing 3 to 15. There are other interesting things done by the students. He

only did multiplication of the numerator, that is: $\frac{12}{15} \times \frac{5}{15} = \frac{60}{15}$. Surprisingly, students apply the fraction addition procedure to the fraction multiplication operations.

Another example of a student with MLD work is as follows:



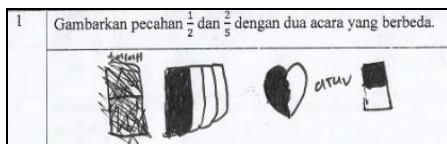
In the second problem (b), the student is asked to solve the problem $\frac{9}{4} \div \frac{3}{5} = \dots$. This problem aims to uncover students' understanding of fraction division operations. In this case, the students appear to have known the operating procedure of the fraction division. However, there is an interesting, students use the method of multiplying the opposite or invert multiply algorithm (Zembar, 2015), but the invert is not the second term, but the first term. Consider the following student answer illustrations:

$\frac{9}{4} \div \frac{3}{5} = \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$. So, the answer is backward, it should be $\frac{15}{4}$, but the answer is

$\frac{4}{15}$. Again, although the answers of MLD students are mistaken, the mental act problem solving, the way of understanding, and the way of thinking can still be identified.

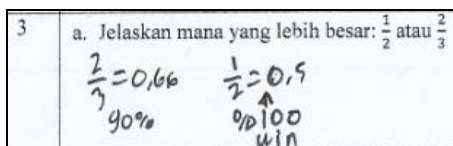
Interpreting

The second identifiable mental act of MLD students is interpreting. The example of student work is as follows:



In the above problem, the student is asked to describe fractions $\frac{1}{2}$ and $\frac{2}{5}$ in two different ways. Students have been able to interpret $\frac{1}{2}$ with two different interpretation, which is the heart and rectangle picture. While in the $\frac{2}{5}$ fraction, the student is only able to draw a representation that is a kind of rectangle, with two shaded segments.

There is also a student who has been able to interpret the fractions as a decimal number as follows:



In the picture above, the student is asked to explain which fractions are greater between $\frac{1}{2}$ and $\frac{2}{3}$. The student interprets fractions as decimals to explain which fractions are larger.

The fraction $\frac{2}{3}$ is changed to 0.66 and the fraction $\frac{1}{2}$ to 0.5. However, the student also listed the writing: 90% and %100 win (with the percent symbol in front of the number), without giving any other details.

Thus, the student has been able to perform mental act interpreting. Their interpretation of fractions had identified the student's way of understanding. Similarly, the student's way of thinking had been identified by the characteristics of various interpretations of fractions, which is fractions as decimal numbers and fractions in the form of pictures.

We summarize these findings in the following table:

Table 4

The Mental Act, Way of Understanding, and Way of Thinking

Mental Act	Way of Understanding	Way of Thinking
Problem-solving	Solution	Problem-solving approach: common denominator approach, invert multiply algorithm
Interpreting	Interpretation	Multiple interpretations (as pictures and decimal number)

DISCUSSION

We found only two mental acts, which are problem-solving and interpreting. The students with MLD could not develop other mental acts like explaining or inferring. We think students with MLD could only solve the problem procedurally. They used the common denominator approach, drawing a picture, change to a decimal strategy, direct multiplied strategy, and invert multiply algorithm; they cannot use the other strategies like a benchmark or residual which demands the ability to infer and explain.

In mental problem-solving, we found interesting things as follows: students have known the procedure of common denominator approach to the operation of fraction addition, but in the application, students mistakenly apply the procedure. Similarly, in multiplication and divisions, students are familiar with the procedure, but in practice, they mistakenly apply the procedure. This finding is consistent with Newton et al. (2014) research, they stated that the main pattern of error in fraction understanding on MLD students is the use of traditional false algorithms.

These findings also in line with the research of Mazzocco et al. (2013) which stated that the difficulties in fraction learning are still felt by MLD students until they are in grade 8 (grade two junior high in Indonesia). Other researchers also disclosed the same thing that MLD students make a mistake in performing arithmetic calculations (Geary et al., 2004).

From previous research, Lewis (2014) stated that students with MLD have a different way of thinking in understanding fractions. She thought students with MLD did not mean to have a lack of understanding of fractions, but there are different ways of thinking in understanding fractions. In our finding, students have different ways of thinking in understanding the operation of fractions addition. In the addition operation, students differently understand the common denominator approach, students add, not multiply the numerator by the same number with the denominator. In multiplication operations, students only perform multiplication operations on the numerator; they do not multiply the denominator.

Another finding from Lewis (2016a) concluded that adolescent students with MLD are experiencing difficulties in fractions learning, especially on fraction comparison subjects, either fractional comparisons with the same denominator or in fractional comparisons involving a half fraction. She suggested to researching with younger MLD students as the subject. Although this study involved younger students with MLD, a similar result is found: students with MLD have difficulties in resolving the problem of fractional comparisons.

Furthermore, Lewis (2016b) explained that partitioning activity was probably the root of understanding the quantity of fractions in regular students. Students with MLD may not follow this pattern of development. In our finding, Indonesia's national curriculum teaches fractions with partitioning activities, which are beneficial for regular students, but not necessarily helpful to students with MLD; this can happen because MLD students do not follow a developmental pattern like their regular peers.

Finally, we think that students with MLD understand fraction procedurally. One possible cause is due to the lack of multiple representations in fractions learning in the context of Indonesian curriculum; because of that, we suggest increasing the use of multiple representations to fractions learning in our curriculum. We believe the use of multiple representation provides an opportunity for students with MLD to gain a better understanding of fractions.

CONCLUSION

The inclusive school system is our future, the study of students' way of thinking when understanding mathematics in inclusive class is very important to do, both for students with and without MLD. The results of this study can be made as one of the didactic anticipations when teachers teach the concept of fractions. We found only two mental acts with corresponding WoU and WoT, namely problem-solving and interpreting. On the analysis of MLD students, it was found an interesting thing in the mental act problem solving, i.e., the student knew the common denominator approach in the operation of fraction addition, but the practice is still wrong. The same thing is also

found in multiplication and division operation. Surprisingly, students use the common denominator approach in the fraction multiplication. In the division of fraction, students mistakenly apply the invert multiply algorithm.

Subsequent research is suggested to examine more specific student in the inclusive school, for example, the way of mathematical thinking in students with disabilities, such as the student with visual impairment, hearing impairment, mental retardation, and students with autism syndrome.

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