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Fair Grade Allocation to Unfair Students: An Application of the Shapley Value to Solve the Free-Rider Problem

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Abstract

This study applies the Shapley value within the context of class-group based peer assessments by treating it as a cooperative game and giving fairness to each group member to cushion the effect of coalitions. Based on scenario analysis, the Shapley Value method is used to identify the fair marks which should be given/allocated to individuals with in groups where there is uneven participation. Scenarios are presented using two and three student member cases with situations under pinning the free rider problem. The Shapley value concept as applied in this study provides what may be viewed as a contribution consistent mark allocation for group assignments.

Key words*: Free-rider problem; Shapley value, peer-assessment; scenario analysis, cooperative games.*

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Introduction

Class based assessments fall into two major categories, (1) formative and (2) summative. Formative assessments can be considered as an assessment while learning whereas summative assessments can be considered as an assessment of learning. The latter are usually high stakes and take the form of examinations or major assignments. In line with student centred and peer learning, group based projects are encouraged by facilitators as they also help develop communication, teamwork and other tacit skills. Despite the many advantages of the group based assessment, a major pitfall is the free rider problem that persists in such activities. West (1994) explains that the free-rider problem exists because a constant score is given to the project outcome which counts as individual marks for each member for the course. This creates a situation where one or more members get the opportunity to reframe from "pulling their weight", limiting the work they invest, with the knowledge that they will nevertheless benefit from the efforts of other members.

The free rider problem is largely behavioural. People tend to behave differently and tend to have different goals in life. This is an inborn trait that is seen during school years. While a group could consist of friends of both low and high performance students, only low performers or only high performing students. We assume that a group consists of at least two types of students, (a) low achievers and (b) high achievers; they may or may not be friends. The free rider problem arises due to this combination of people in a group.

To combat this problem, this study seeks to allocate individual marks to each member of the group member based on each member's contribution towards the group assessment where peer contribution assessment is practised. Each member states the percentage of contribution they have made as well as those of other members. Each member is given this privilege. The major issue for this action is the formation of coalitions within the group. Due to the free rider problem, the free rider would (1) get marks he/she may not deserve, and (2) fail to understand the principle concepts the project hopes to convey. In this light, we propose to apply the Shapley value concept to help mitigate the mentioned problems.

The study's primary motivation stems from the many complaints received by students in group work from the teaching experiences of both the authors and the subsequent gap in the literature applying the Shapley value concept to mitigate this problem. Given this contribution, the study would be useful to instructors of any field in assigning grades consistent with student performance. The remainder of the study is outlined as follows, section 2 briefly reviews the literature surrounding the overview and applications of the Shapley value concept, section 3 describes the methodology, section 4 presents the analysis and discussions and lastly, section 5 concludes with some suggestions for classroom assessments.

Literature Review

In game theory, the Shapley value, Shapley (1953) is a solution concept in cooperative game theory. It is applied to isolate the effect of collusion in cooperative games. Cooperative games are also seen in educational spaces, particularly in group settings. This literature review highlights the numerous applications of the Shapley value in practice. The Shapley value is a decomposition-based technique to disentangle the combined effects of various intertwined causal factors.

Free-Riding Issue

The free-rider problem, also known as social loafing occurs when one or more members of a team do not do their fair share of work (Brooks & Ammons, 2003). This leads to other members having to do more than what was expected of them thus coining the

term "free-rider" for those who have contribute less than what was expected of them. Many businesses rely on teamwork and many recruiters also ask students of their experience with working in team settings (Ravenscroft, 1997). Potential evaluation of individual contributions to group work has a strong influence in ensuring that each member does their fair share of work (Karau & Williams, 1993) while Druskat and Wolff (1999) also found that peer appraisals can have a positive influence on a group's ability to work well together and on team members' satisfaction with the group. Brooks and Ammons (2003) found evidence that an evaluation system that provides feedback on specific criteria at both early and multiple points can reduce free-rider problems and lead students to view group experiences in a more positive light.

Group Effort in Education

Collaborative learning and student centred learning in higher education are well recognised in academic literature. Collaborative learning provides students with a peer group outside of the formal classroom with whom they discuss new concepts and assimilate new ideas (Ravenscroft, 1997). These groups provide students to speak their mind and critique each other fearlessly as there is absence of a control figure that is the teacher. There are many reasons as to why students will not openly contribute in classes. Liu (2001) elaborated that there are four types of student behaviours in the classroom; full integration participation in the circumstances, marginal interaction and Silence observation. In full integration, students are actively engaged and say what they want to say whenever they want to say. Students here behave in a natural way and instincts. Participation in the circumstances is when students contribute when they are influenced by socio-cultural, cognitive, affective, linguistic and environment factors. Students here only speak at the most appropriate time. In marginal interaction, students are more of listeners than speakers. These students tend to listen and take notes rather than be part of the discussions. In silent observation, students tend to avoid oral participation in class. They usually prefer writing notes.

There are several factors that affect a student's participation in learning. One important factor is personality. Students with high self-efficacy showed better academic achievement and participate more in the classroom (Pajares, 1996 & Schunk, 1995). Self-efficacy trait raises curiosity and the exploring urge and motivates them to be more active (Mahyuddin, Elias, Cheong, Muhamad, Noordin, Abdullah, 2006) and this enhances their confidence level and become more active in class. On the contrary, students can become passive learners due to self-limitations such as focus during lecture time, fear of offence, low levels of self-confidence, not being prepared for class, fear of failing to show their intellectual ability, fear of criticism thus lead to becoming less engaged (Fassinger, 1995). The aforementioned factors explain the reason for free-riding as active participants mostly take control of groups or end up contributing more while passive participants either do their share of work or simply fail do to the expected. Additionally, group effort can lead to an understanding and appreciation of group dynamics fostering the development of inter-personal skills that are essential for the modern day work force.

Other Methods to Assign Group Grade

Brooks and Ammons (2003) used a multiple peer and self-evaluation method throughout the period of the group project. This method required each member of the group to evaluate each members' performance based on the criteria that included; Attendance, complete delivery on agreed upon parts of the project, meeting deadlines, volunteering for tasks during team meetings and contributing, accomplished fair share of the work in regards to the overall workload, showing enthusiasm and positive attitude towards team activities and members. Their findings noted a decline in variance of peer ratings between first and second assessments. Feedback allows students to identify areas of poor performance and ways to improve them.

Levin (2003) suggests three questions that every academic should consider when using group assessment; (1) to minimise incentive to free-ride, maximise incentive to work as a team, (2) prevent unintended free-riding, (3) to deal with free-ridding when it occurs. On average, students find group work to have a positive experience on them however they have difficulties with various aspects of the group work. These difficulties include, competing demands from other subjects, not having enough time for groups to speak with teachers about problems and lack of meeting space on campus. The word "frustrating" is mostly used by students for whom group work were a negative experience (Hall & Buzwell, 2012).

Cooperative and Non Cooperative Games

Nash (1953) stated that the word "cooperative" in cooperative games is used in situations involving two individuals with similar interests where they can discuss the situation and agree to a joint action. He also described non-cooperative game a one where it is impossible for players to communicate or collaborate in any way. Cooperative and Non-cooperative games can have two or more individuals in respective games.

Aumann and Dreze (1974), state that a coalition structure in an n–person game is a partition of the players and has been used in defining the various solution notions that constitute the bargaining set family that is the various bargaining sets (Aumann & Machler, 1964; Davis & Machler, 1967), the kernel (Davis & Machler, 1965) and the nucleolus (Schmeidler, 1969) in effect, these notions are defined separately for each coalition structure.

Applications of the Shapley Value

The Shapley value has widespread applications in poverty and inequality analysis. Accordingly, the Shapley value was applied by Shorrocks (2013) to disaggregate and study the determining factors of income inequality.

Moreover, the Shapley value is also applied to study the distributional impact of carbon emissions (Yu et al. 2014) in Chinese provinces. Other authors applying the Shapley value in the area of climate change and carbon emissions include Zhang et al. (2014) in the case of China and Liao et al. (2015) in the case of Shanghai.
In the areas of financial management, the Shapley value is applied in risk attribution and management by Tarashev et al. (2010); Land et al. (2001); Gauthier et al. (2012). Cost allocation applications include studies by Petrosjan and Zaccour (2003);
In a democratic system, each person has a vote and can influence the government's decision based on their own utility. Aumann & Kurz (1977) introduce a model whereby each agent's power is reflected in two spheres, namely policy and economics. Here, each agent has an initial endowment and a utility function, and, a tax and redistribution policy is decided by majority votes but every person can destroy part or all of his endowment (Aumann, 1994).

A purpose of taxation is to raise funds to finance development of public goods. A continuum of agents, endowed with resources and voting rights, take place in the production of non-exclusive public goods. Therefore, when a coalition forms, they choose one strategy from the available, which together with the complementary coalition's choice, determines which bundle of public goods to be produced (Aumann, 1994).

Ma, Liu, Chiu, Mishra & Rubenstein (2008) used Shapley Value to address Internet Service Provider's (hereon referred to as ISP) issue of the correct financial compensation to be paid to other ISP's. ISP's used bilateral settlements to decide these payoffs' which resulted in disputes and sometimes led to disgraceful consequences.

Findings and Synthesis
It should be note that the Shapley value concept has been applied in fields as diverse as Government decision making, internet service providers, and exercising voting rights. However, scant attention is paid in applying the Shapley concept to solving the free rider issue prevalent in group based assessments. It was suggested that every member of the group assess the contribution of their group members and their own to a maximum of a 100%. This assessment should have done thrice, once in the first quarter of the duration of the assessment, once in the mid-point of the assessment and once at the end of the group project as found effective by Brooks and Ammons (2003).All the three assessments are averaged for every member to indicate their average contribution towards the project. To offset the problem of coalitions, these scores are used in the Shapley Value method to calculate the fair contribution of each member. The project gets a particular score (x) and every member gets this score if their fair contribution is greater than equal to 100/n , n being the number of group members. If a group member's fair contribution is less than100/n, their mark will be the proportion of their contribution to their required contribution against the group score (x).

Gap
This article addresses the issue of coalitions which may be formed among groups that practice peer assessments. This study addresses the coalition issue in group assessments so that each member of a group or team gets a fair mark or reward. While many educators use contributions of each student to allocate marks using peer assessments, there are no studies that consider the issue of coalitions. Thus, in this article the Shapley Value is used to allocate fairer marks to students based on their contributions from peer assessments.

Methodology

Shapley Value
In 1953, Lloyd Shapley proposed that it might be possible to numerically evaluate the value of playing a cooperative game. The designed function for this purpose came to be called Shapley Value (Roth, 1988). This paper applies the Shapley Value method to allocate students' grade in a group assessment according to their contributions. Each member of a group is expected to equally contribute to the final output of the group assessment. Peer Assessment is done whereby each member of the group states the percentage of contribution he/she has made as well as percentage contribution of their peers. This Peer assessment may be done three times during the duration of the group assessment as found effective by Brooks and Ammons (2003). The mean of all contributions should be used as the final contribution by each member.

Each permutation of the group is then used to assign the contribution each group member has made using the Shapley Value method. This is done to capture the marginal contribution of each group member averaging over all the different sequences according to which the grand coalition could be built up. The mean value of each member gives us the Shapley value of each member.

The Shapley Value of each member is then used to determine whether a member equally contributed to the group assessment. If the Shapley Value is greater than or equal to the equal contribution required by each member, the respective member is given the full marks of the assessment score. However, if a group member's Shapley value is lower than the equal contribution that was required, he or she will only get the marks according to their contribution to the group assessment. The method is clearly spelt out in the following section.

Ethical Approval
The usage of this method to allocate student grades should have a prior approval of the respective University.

Analysis and Discussion

This research has applied the Shapley value in the field of education. The study aims to ascertain an appropriate guideline that teachers can follow to allocate marks within group assignments based on dissimilar contributions by group members. This paper treats a group assessment similar to cooperative game because there could be members who may form coalitions in order to disadvantage other group members. The study aims to propose a method for tackling the free-rider problem within the context of group based activity while giving fairness to each member's contribution who may be a victim to a coalition by other members. While there are a lot of methods to allocate based on contributions in teams and groups, there are no studies that see a team or group as a cooperative game and therefore no one considers the possibility of coalitions in them. The likely hood of coalitions could mean the only member who has done the most work would get the least mark if all other members formed a coalition to say that they all did most of the work. Thus, the Shapley Value method was used to give fairness to contributions of each member and weeding out any possible disadvantage a member may have because of any possible coalition/collusion.

Assumptions

The first condition of this proposed methodology is that each member should equally contribute towards the group assessment that is (100/n). The second condition is that each member states the percent of their contribution and that of other members. The final condition is that, if Shapley Value is greater than or equal to 100%, the member is allocated the full mark of the group and; if, Shapley Value is less than 100%, final mark is calculated by ((Contribution by Player X/100) * Final Mark).

The maximum a player can allocate is 100% as it depicts the contribution of each player towards the common goal which would be 100%. The mean score of contributions allocated by all students are used as values for respective students in the calculation of the Shapley value. If the Shapley value for a respective student is greater than equal to (100/n), the student gets the full score of the graded assessment. However, if the student's Shapley Value score is less than equal to (100/n), their grade is calculated as (Shapley Value of Student X/(100/n)) X Assessment Grade.

Scenario 1: The Case of Two Students with group mark of 80/100

Table 1 shows the responses by each member as to the amount of their contribution and the other members of the group. Two students, A and B give unequal contribution towards the group assessment and indicate that they contributed more to the assessment than the other. In the case of two students, each has to contribute 50% (100/n) towards the group assessment. The mean of all responses give the percentage contribution of each member. These means are then used as the contribution by each member of the group towards the calculation of the Shapley Value.

Table 1:

Two Students with unequal contribution towards Group Assessment

	A	B
Student A	60%	40%
Student B	50%	50%
Total	55% (110/2)	45% (90/2)[1]

[1] Averaging the contribution was considered here but with a large number of players, this would produce too many decimal places. Therefore, for simplicity, the total contributions were used.

Shapley Value Calculation – Scenario 1

Table 2:
Shapley Value with two students

	A	B
AB	55%	45%
BA	55%	45%
Shapley Value	55% (110%/2)	45% (90%/2)
Mark Allocation	80 ((50/50)*80	72 ((45%/50%)*80

According to the Shapley Value (Table 2), Student A will get 80 Marks since their contribution is 55% (Greater than or equal to 100/n). Student B on the other hand, will get 72 [(45 %/(100/2))*80].

Scenario 2: The Case of Three Students (with group mark of 80/100)

Table 3:
Three students with unequal contribution towards Group Assessment

	A	B	C
Student A	40%	30%	30%
Student B	50%	50%	0
Student C	35%	30%	35%
Total	125/3%	110/3%	65/3%

Table 3 shows the responses by each member as to the amount of their contribution and the other members of the group. Three students, A, B and C give unequal contribution towards the group assessment and indicate that they contributed more to the assessment than the other. In the case of three students, each has to contribute 33.33% (100/3) towards the group assessment. All responses are averaged to give the percentage contribution of each member. These means are then used as the contribution by each member of the group towards the calculation of the Shapley Value.

Shapley Value Calculation – Scenario 2

Table 4:
Shapley Value with three students

	A	B	C
ABC	125/3%	25%	100/3%
ACB	125/3%	100/3%	25%
BAC	30%	110/3%	100/3%
BCA	100/3%	110/3%	30%
CAB	45%	100/3%	65/3%
CBA	100/3%	45%	65/3%
Shapley Value	225/6 = 37.5%	210/6 = 35%	165/6 = 27.5%
Mark Allocation	80	80	[27.5/(100/3)]*80 = 66

According to the Shapley Value, Student A and B will get 80 Marks since their contribution is 37.5% and 35% respectively (Greater than or equal to 100/n). Student C on the other hand, will get a score of 66 [(27.5 %/ (100/3))*80].

Implied Grading Procedure for Instructors

Instructors may have peer assessments in terms of contribution towards a group assessment thrice during the duration of the group assessment. Instructors may have the first peer evaluations in the first quarter of the group assessment, second peer evaluation in the middle of the group assessment and the third peer evaluation at the end of the group assessment as found very effective by Brooks and Ammons (2003). The mean of the three peer evaluations can be used as contribution scores for each student to calculate the Shapley Value. If the Shapley value for a respective student is greater than equal to (100/n), the student gets the full score of the graded assessment. However, if the student's Shapley Value score is less than (100/n), their score is calculated as ((Shapley Value of Student X)/((100/n))) X Assessment Score.

Conclusions and Implications

This study has applied the Shapley-value concept in the arena of class group based assessments. Previous applications of the Shapley value from the review of the literature were focused on fields as diverse as internet service providers in their profit sharing decisions. No study to the best of our knowledge has attempted to apply the Shapley value in group based assessments as none have treated it as a cooperative game and therefore have not considered the effect of coalitions in these groups where peer assessments have been practiced.

This study has shown the use of the Shapley value in two scenarios in our discussion and the mark each member is allocated, based on his or her contribution. The major issue for any group based assessment has been the free-rider problem and when these free-riders can form coalitions in groups where peer assessments are practiced. Therefore, to combat this, this application of the Shapley Value is proposed. This method can be applied to any group settings where there is a Peer assessment or Peer evaluation such as group assessments in education to team assessments to allocate commission or bonuses in businesses. The motivation to use Shapley value is that members of a group can form coalitions and drive a member to get lower marks and Shapley Value considers this and gives fairness to all the members of the group.

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