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Teacher Knowledge and Learning *In-situ*: A Case Study of the Long Division Algorithm

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Abstract: The aim of the study reported in this paper was to explore and enhance experienced school mathematics teachers' knowledge of students' thinking, as it is manifested in practice. Data were collected from records of classroom observations, interviews with participating teachers, and weekly teacher-researcher meetings organized in the school. In this paper, we discuss the mathematical challenges faced by a primary school teacher as she attempts to unpack the structure of the division algorithm, while teaching in a Grade 4 classroom. Through this case study, we exemplify how a focus on mathematical knowledge for teaching 'in situ' helped in triggering a change in the teacher's well-formed knowledge and beliefs about the teaching and learning of the division algorithm, and related students' capabilities. We argue that in the context of educational reform, an analysis of knowledge demands placed on the teacher helps in understanding and supporting teachers' work.

Key words: learning *in situ*, long division algorithm, mathematical knowledge for teaching, professional development, students' mathematics, teacher challenges.

Introduction

The claim that teachers need specialized knowledge in order to teach school subjects effectively, has had a widespread influence on education research as well as on the design of interventions in teacher development (Edwards, Gilroy & Hartley, 2005). Sustained efforts have been made by researchers to develop characterizations of specialized teacher knowledge that remain close to the actual work of teaching (Petrou & Goulding, 2011). The design of teacher education curricula or professional development interventions is founded on a conception that individual teacher's knowledge of mathematics teaching impacts their practice. Curricular reform efforts in India, as in the other countries across the world, have called for changes in teaching practice, which place new demands on teachers' knowledge required for teaching (NCTE, 2009; ARC, 1990; Tatoo et al., 2012). There is a need to identify both the form and the content of teacher knowledge that is most likely to translate into changed classroom practice.

Existing frameworks of teacher knowledge attempt to identify its components, especially those components that are missing from typical trajectories laid out by formal teacher preparation programmes (Ball, Thames & Phelps, 2008). Teacher knowledge is characterized by focusing on the teacher and the knowledge that the teacher brings to the classroom. Such frameworks have been criticized for at least two reasons. First, the existing frameworks view teacher knowledge as static (Hodgen, 2011). Hodgen argues, "teacher

knowledge is *embedded* in the practices of teaching and any attempt to describe this knowledge abstractly is likely to fail to capture its dynamic nature" (p. 29, emphasis in original). Second, the notion that the teacher acts as an individual in the process of teaching and learning, and therefore that teacher knowledge is uniquely the province of a teacher, needs to be problematized. Takker & Subramaniam (2017) have argued that knowledge required for teaching is a complex amalgamation of students' and teacher's knowledge which unfolds itself in classroom discussions. Thus, there is a need to go beyond the individualistic assumptions about teacher knowledge and engage with the dynamic system in which teachers' work is located (Rowland & Ruthven, 2011).

These criticisms have several implications for the design of professional development interventions. Brodie (2011) argues that there is a need for textured descriptions of the difficulties faced by teachers when implementing the reformed curriculum. Further, Cobb & Jackson (2015) suggest that a study of teachers' existing practices can be used to identify aspects which can be leveraged to design support for learning. Taking these two arguments, we present an approach of engaging with the work scenarios of teachers to develop an understanding of the challenges faced by them *in situ* and design appropriate support structures. We believe that an approach, which takes the realities of teachers' work into cognizance and engages deeply with the practice of teaching, has the potential for the formation of learning communities involving teachers and researchers (Takker, 2015).

In this paper, we report a case study of a primary school mathematics teacher. The larger study aimed to explore teacher's knowledge of students' mathematical thinking as it is manifested in their practice and ways in which this knowledge could be enhanced *in situ*. We report a change in the teacher's noticing of 'mathematical' aspects of students' utterances, as the study progressed. We exemplify this change by discussing ways in which the teacher dealt with the meaning and structure of the division algorithm in two years of her teaching. We do this through an analysis of significant moments that arose in the teacher's practice and contributed to a change in her knowledge and practice. We argue that a situated approach to working with teachers and a deeper engagement with their practice provides opportunities to challenge teachers' knowledge and beliefs in order to create possibilities for reformed practices. The analysis, we hope, will reveal the situated dimension of teachers' specialized knowledge of mathematics.

Teachers' Struggles in a Reform Context

Recent reforms in the Indian curriculum emphasize students' construction of knowledge while learning in a classroom (NCERT, 2005). The aim of teaching mathematics has shifted from a focus on procedures to processes involved in *doing* mathematics. These processes include problem solving, approximations or looking for intelligent solutions, systematic reasoning, mathematical communication, and making connections (NCERT, 2006). The changes in the textbook, particularly at the primary level, align with these new goals of teaching mathematics.

The ways in which teachers make sense of reforms (often communicated to them through changes in textbooks) is varied. In an attempt to accommodate the reforms without modifying the larger structure of thinking and understanding mathematics, teachers might combine aspects of the old and the new curriculum, without critically challenging the existing practices. Teachers who are unwilling to accept the reforms completely, but have an obligation to follow them and teach accordingly, tend to create a blend of open-ended activities with traditional procedural practice (Ebby, 2005). This recalls the case study of Ms. Oublier (Cohen, 1990), a teacher who believed that she had revolutionized her teaching

following the educational reforms, but her practices were observed to be largely traditional. Such practices have been identified as 'hybrid practices' (Brodie, 2011) or 'instructional hybrids' (Cuban, 2007) in the literature.

The major shifts in the mathematics curriculum, which emphasise the processes of doing mathematics, place demands on teachers who are struggling with their content knowledge (Rampal & Subramanian, 2012). Listening and responding to different student ideas, evaluating these responses, generating mathematical meanings from these statements, and using appropriate tools to build connections with the content of mathematics, all of these pose challenges to teachers' mathematical work (Ball, Thames & Phelps 2008; Takker & Subramaniam, 2017). Due to the lack of spaces and opportunities for teachers to engage with and discuss the envisioned reforms in the teaching of mathematics, significant changes in the existing teaching learning practices following the reform initiatives are uncommon.

Teachers' struggles with the reformed curriculum involve a re-configuration of the relation between their beliefs, knowledge, and attitude towards teaching mathematics. The textbook changes alone are insufficient in changing teachers' beliefs and practices (Batra, 2005) that gain legitimacy from teachers' own experiences of schooling. Such experiences also serve as a fallback in case of challenges arising from uncertainty in the classroom (Takker, 2011). Lack of subject-specific support makes it difficult for teachers to understand and make efforts towards teaching conceptually, an experience that they need to have for themselves (Rampal & Subramanian, 2012; Takker, 2015). Teachers need knowledge, resources and support to tackle everyday struggles in the classroom. The current study is an attempt to support teachers in their school setting with the aim of unpacking and analysing the problems arising in teaching mathematics and creating a collaborative space for addressing these problems *in situ*. We argue that the intervention centered on knowledge situated in practice has potentials for bringing a change in well-formed teacher beliefs and practices. The questions we ask in this paper are:

a. How do knowledge, beliefs and practice interact as a teacher in transition struggles to implement curricular reform in the classroom?
b. How does knowledge of "why an algorithm works" lead to productive ways of engaging students' thinking in the classroom?

Teachers' Knowledge of Arithmetic

Mastery of the four basic operations of arithmetic is considered central to the primary school mathematics curriculum. Students are expected to "know" the algorithm for each operation and use it fluently to solve problems. Kamii & Dominick (1997) probed students' understanding of arithmetic operations and found that excessive emphasis on the teaching of conventional algorithms (a part of social-conventional knowledge of mathematics) was constraining students in developing an understanding of relationships between numbers (logico-mathematical knowledge). Further, it has been noted by Khan (2004) that an over-emphasis on techniques of memorisation of algorithm makes it difficult for students to reflect on the problem and check the appropriateness of their solutions. Despite such criticisms, the knowledge and successful application of the learnt algorithms is considered an important goal of school mathematics. The performance of students in displaying algorithmic knowledge satisfies the dominant societal conceptualisation of what it means to do mathematics (Ebby, 2005).

The significance of teaching the operations using only algorithms has been challenged recently in the Indian mathematics curriculum. The change in the curriculum, however, has not changed the parental or school expectations that accord primacy to fluency with

algorithms. The knowledge of algorithms and ability to manipulate symbols is considered to be a mark of school learnt mathematics and is often used as a differentiator between the out-of-school knowledge of mathematics and school mathematics (Khan, 2004).

Students find the division algorithm difficult as it builds on their knowledge of number facts learnt during addition, subtraction, and multiplication (Anghileri & Beishuizen, 1998). Subramaniam (2003) discusses an error frequently made by students as well as some teachers in solving the division problem 981÷9, obtaining the quotient as 19. Such difficulties with long division arise from an emphasis on the inflexible procedural way of solving the problem (Windsor & Booker, 2005). The procedure of division involves remembering each step, forgetting any of which leads to errors. The misplaced emphasis on rote memorisation does not support students' understanding. Thus, even those students who use the division algorithm correctly to solve problems may not understand the meaning of the algorithm and why it works.

Anghileri, Beishuizen, & van Putten (2002) conducted a comparative study of written solutions to division problems of Grade five students from England and the Netherlands. In England, students were being taught the division algorithm from an early age. An over-reliance on the procedures did not allow students to see the structure underlying the procedure or take the numbers into account. Evidences, such as these, can be found in the Indian mathematics classrooms, where students often multiply, for instance, 40 with 10 using the standard algorithm without considering the numbers or evaluating the need to use the algorithm. In contrast, the Dutch approach based on realistic mathematics education focused on eliciting students' intuitive strategies and building progressively on them. This meant beginning from repeated subtraction to increasing the number and size of chunks and flexible use of multiplication facts. The study concluded that it is meaningless for students to reproduce the taught methods mechanically while being unaware of the links between the procedure and meaning of the division operation. The approaches of the two countries roughly correspond to the ways in which the division algorithm is dealt in the old and the new NCERT textbooks in India. We will take a closer look at these textbooks in the next section.

In a study with Grade six Government school students of rural Madhya Pradesh in India, Khemani & Subramanian (2012) reported a lack of understanding of the process of division. In their teaching experiment, the students were introduced to division as equal distribution or sharing. Students were taught to represent the process of equal distribution in a way that was visually similar to the division algorithm. The teaching trajectory for division included the physical act of distribution, using partial quotients to represent the stages in the process of distribution, and then moving to the long division algorithm. The principle of choosing an interpretation that is intuitive for students makes this approach similar to the Dutch approach.

Informal strategies used by students in equal sharing or division contexts invite multiplicative thinking. Such contexts frequently call for chunking objects into equal sized groups and keeping track of the number of groups as well as the number of items accumulated, which involves multiplicative reasoning. Thus, as Lampert (1992) argues, division can be used as an opportunity for 'cognitive reorientation' from additive structures to multiplicative structures and proportional reasoning. Development of multiplicative thinking is cognitively demanding but a valuable goal of learning mathematics (Subramaniam, 2003).

In summary, the literature on teaching and learning of the long division algorithm raises two important issues: formulation of a teaching approach for long division that focuses both on conceptual and procedural understanding of the algorithm, and the importance of using the context of learning the division algorithm as an opportunity to develop multiplicative thinking in students. In this paper, we discuss the challenges faced by an experienced

mathematics teacher while trying to unpack the structure of the division algorithm by relating it with multiplicative thinking involved in using the 'chunking method' of solving division problems.

Division in the Textbooks

In this section, we analyse the way division has been dealt with in the old and new national level textbooks of Grade four. These textbooks are designed by the National Council of Educational Research and Training (NCERT), an apex body which holds the responsibility of designing national level school textbooks to be followed by all central government run and affiliated schools. Discussion of the division trajectory in the two textbooks is necessary to understand the perspective of the teacher, whose case study is being discussed in this paper. The analysis indicates the differential nature of knowledge demands placed on the teachers when using textbooks written with different perspectives.

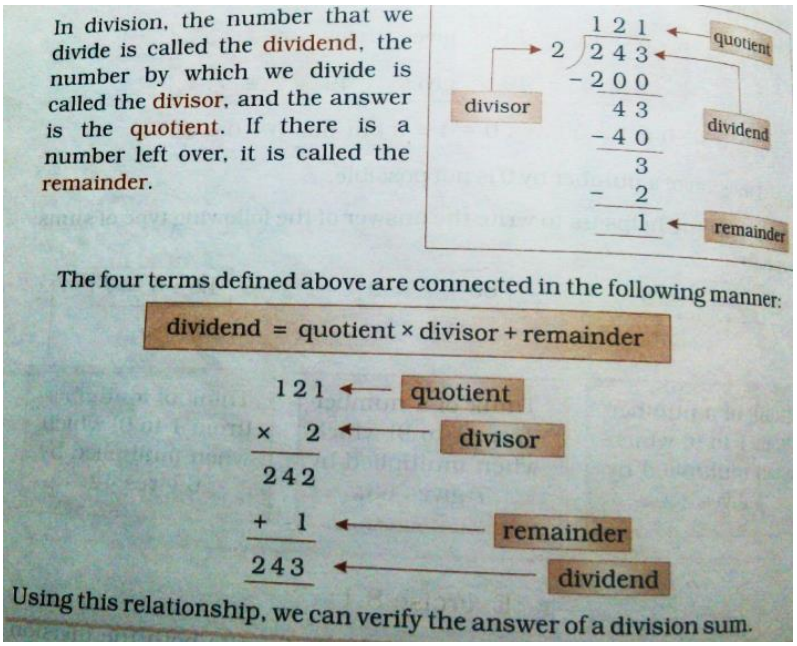
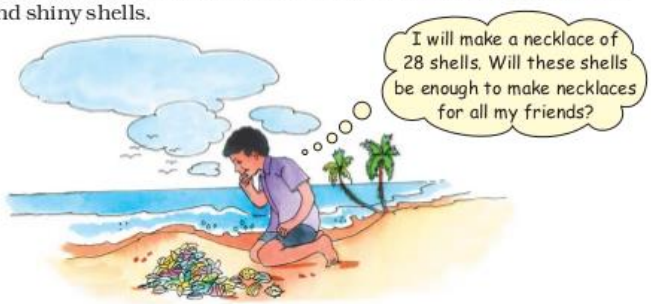


Figure 1: Introduction to the division algorithm (NCERT, 2003, p.130)

The earlier Grade four NCERT (2003) mathematics textbook introduced division using multiplication facts, which involved division of a single digit number by a single digit number. The text gave a few examples and then introduced the algorithm for long division. As depicted in Figure 1, the long division algorithm was introduced using the terms associated with it and the procedure to verify the answer (quotient and remainder) using multiplication. The description of the procedure was followed by an exercise where students were asked to solve the numerical problems (called "sums") using the algorithm. The algorithm was extended to the division of two, three and four-digit numbers by a single digit number. The successive exercises included the use of algorithm for division by 10, 100, 20, and other multiples of 10. Then, students were taught the algorithm for division by a two-digit number. The old textbook provided several numerical problems for students to practice the long division algorithm. Except the long division algorithm, no other method or ways of solving were suggested or exemplified in the text. Further, there were no word (or contextual) problems included in the chapter on division.

Sea Shells

Dhruv lives near the sea. He thought of making necklaces for his three friends. He looked for sea-shells the whole day. He collected 112 sea-shells by evening. Now he had many different colourful and shiny shells.



He took 28 shells for one necklace.

112 – 28 = 84

Now he was left with 84 shells. Again he took 28 more shells for the second necklace.

* How many shells are left now? _____

Then he took shells for the third necklace.

* So he was left with _____ shells.
* How many necklaces can Dhruv make from 112 shells? _____
* Are the shells enough for making necklaces for all his friends? _____

Try these

A) Kannu made a necklace of 17 sea-shells. How many such necklaces can be made using 100 sea-shells?

Encourage children to solve questions based on division with large numbers, for which they do not know multiplication tables, using repeated subtraction. More problems based on real life contexts can be given.

Figure 2: Division using repeated subtraction (NCERT, 2007, p.125)

In the Grade four NCERT (2007) textbook, which is currently in use, the chapter on division begins with making a rectangular array arrangement for 18 plants. Students are expected to identify different ways in which 18 plants can be arranged. This is followed by an exercise on creating multiplication tables using the distributive property. Students are shown how to use the table of 2 and 5 to create a table of 7. The reason for why these two tables combine to give a table of 7 is not discussed. The contexts used in the text suggest the method of repeated addition, repeated subtraction, making groups, and sharing to solve division problems. Each of the methods suggested by the textbook is appended with a note to the teacher (refer Figure 2). The note mentions the ideas to be emphasized, suggests further exercises that teachers can design, and sometimes provides the justification for the activity or method discussed by the textbook writers.

The note for the teacher, at the bottom of the page in Figure 2, suggests the use of large numbers to make the shift from using multiplication facts to repeated subtraction. Similarly, other methods are introduced using a real-life context and problems are given to practice the method. The textbook expects the teacher to know different methods and make students use these methods as well as the algorithm, which is given at the end of this chapter. However, teachers struggle to understand the significance of teaching different methods and handling students' responses navigating between these methods while the goal remains teaching the long division algorithm. The knowledge of 'why' the division algorithm works, connecting different strategies of solving a division problem, and identifying links between the algorithm and these strategies constitute an important part of teacher knowledge required for teaching the long division algorithm. These are also the areas where teachers might need support and have been addressed in the study, reported in this paper.

The Study

The study reported is a part of a larger research, which aimed to explore and enhance teachers' knowledge of students' mathematical thinking, and ways in which it is manifested in practice. The study was carried out in two years, 2012 and 2013, in three phases. The first phase included understanding teachers' practices through classroom observations, two semi-structured interviews, and task-based interviews before and after the lesson observed. In the second phase, weekly meetings between participating teachers and researchers were organised in the school. The aim of the teacher-researcher meetings was to build on teacher's mathematical sensitivity and responsiveness to deal with students' questions and explanations. Initially the researchers designed tasks for reflection in these after-school meetings; gradually teachers started bringing artefacts from their classrooms and using this time for discussion and planning. The third phase, which overlapped in time with the second phase, included classroom observations and task-based interviews of teachers. In this phase, the researcher (the first author) also provided some support to the teachers in planning and initiating ideas or practices in their classroom teaching.

Sample and Setting

Four experienced school mathematics teachers participated in a two year long research study. The participating teachers belonged to a school, which is a part of a nation-wide network of schools spread across 14 locations in the country and funded by the Government of India. The students in this particular school were from mixed socio-economic backgrounds. Since the students came from different parts of the country, their mother tongues were different. The medium of instruction in the school was English, but students and teachers used Hindi as well as English while talking in and outside the classroom. All participating teachers had an experience of more than 15 years, in teaching mathematics at the school level. Two of these teachers were primary school teachers, teaching mathematics and environmental studies from Grades one to five (approximate age 6-10 years). The other two teachers were middle school teachers, teaching mathematics and physics from Grades six to ten (age 11-15 years). In this paper, we discuss the case of a teacher teaching the long division algorithm in Grade four classrooms in two consecutive years 2012 and 2013.

Pallavi (pseudonym) was a primary school mathematics teacher who had been teaching Grades one to five for 25 years. She had a Bachelor of Science (B.Sc.) degree with mathematics as a major subject and a Bachelor of Education (B.Ed.) degree. She had been working in this school for 19 years. We use Pallavi's case study to exemplify the change in her teaching, knowledge and beliefs, *in situ*. For the purpose of this paper, we use only those discussions with Pallavi, which focused on the teaching of long division algorithm. We discuss how Pallavi's teaching decisions were guided by her knowledge of the division algorithm, and by her beliefs about the revised textbook, students' capability and what constitutes the goals of mathematics teaching at the primary school level.

Data

The data was collected in the form of field notes and audio and video records of (a) classroom observations[1], (b) interviews with individual teachers, and (c) teacher-researcher meetings. Records of teacher's plans, writings, students' notebooks, and worksheets were also collected. Classroom observations, though largely unstructured, were detailed since the researchers wanted to get a sense of the practices adopted by different teachers while

responding to students during teaching. There was no protocol that was followed during classroom observations or while preparing field notes. Care was taken to record students' and teacher's mathematical questions, explanations or justifications, and written work on the chalkboard and in students' notebooks. The two semi-structured interviews in Phase 1 (refer to the phases in the section on The Study) focused on exploring the teacher's perception of her mathematical practices. These interviews were designed on the basis of the observations made by the researchers over a period of time. The task-based interviews in Phase 1 and 3 included discussions with the teacher before and after their teaching of a lesson, and happened whenever the teacher's time permitted. The pre-lesson interviews aimed at encouraging the teachers to explicate considerations for planning lessons. The post-lesson interviews involved discussions on student utterances (questions, responses, reasons) and teaching decisions made by the teacher. In another kind of interviews, data from which is not used in this report, teachers were requested to anticipate their students' responses to the problems posed in the worksheet designed by the researcher. After the students had solved these worksheet problems, teachers were asked to reflect on the student responses. Each of these interviews was audio recorded and the records of written work by the teachers were collected. The data used in this paper is from classroom observations of the division lessons, long interviews, and task-based interviews with Pallavi teacher. The transcript[2] of each lesson was classified into distinct episodes where a specific sub-topic was being dealt. The episodes dealing with the same sub-topic, from the two years of the study, were paired. In this paper, we use paired episodes which show maximum variation in teaching from the first to the second year. Through our analysis, we explore the nature and reasons for this change.

Nature of Intervention

During the task-based interviews, which were carried out in the first phase of the study, Pallavi was reluctant to talk to the researcher (the first author) due to lack of time and not feeling the need for such an interaction. By the third phase, we found Pallavi initiating interactions with the researcher before and after the lesson, as and when her time permitted. The discussions during these interactions included detailing topic-specific errors faced by students, unpacking the division algorithm, formulating problems for students by going beyond the textbook content, examining the appropriateness of the representations, and anticipating students' responses to some of her teaching decisions. The increased demand from Pallavi (and other participating teachers) to have these interactions with the researcher, before and after the lessons, suggests that the task-based interviews acquired an interventionist character during the course of the study.

More systematic intervention was planned in the second phase of the study through teacher-researcher meetings. The participating teachers and researchers met during these after-school meetings, which ranged from 60 to100 minutes. Although the major topic of discussion was decimal numbers, there were brief discussions on division of whole numbers and fractions. Initially, researchers designed tasks for engaging teachers during these meetings, using artefacts collected in the first phase of the study. These artefacts included common student errors, atypical student responses or questions, rationale underlying procedures, consistency of representations, and use of contexts. Other discussions included analysing the textbook problems, drawing connections between the topics taught in the primary and middle school, and the importance of examples and non-examples. Gradually, teachers began to initiate discussions during these meetings. Pallavi actively participated in these discussions by being explicit about her teaching decisions, stating her disagreements with the researchers and other teachers, and sharing anecdotes from her classroom teaching

and her views about the changed textbooks. The aim of these meetings was to challenge some of the existing beliefs held by teachers, initiate dialogue between researchers and teachers on the use of research based knowledge on students' thinking, and reflect on the knowledge in play in classroom.

Analysis and Findings: Episodes of Teaching Division at Grade 4

In this section, we discuss the episodes from Pallavi's classroom teaching of the division algorithm and interactions related to the topic in the two years of the study. Pallavi's initial resistance as well as the process of change in her teaching through constant dialogue about the issues of practice is noted. We analyse the reasons for change in Pallavi's teaching through this process.

Year 1: "Different methods confuse, students should be 'taught' the division algorithm"

The new textbook expects a teacher to consider different strategies like repeated addition, repeated subtraction, use of multiplication facts, and partial quotients for solving division problems with sharing (partitive) and grouping (quotitive) interpretation. For instance, consider the problem of Gangu's sweets shown in Figure 3. In the problem context, the grouping meaning is indicated by the image of 80 sweets in a box, and small boxes with 4 sweets each. The question posed is whether 23 boxes are sufficient to pack all the sweets. The problem can be solved using multiplication facts (taking products with convenient numbers 10, 5, 20), repeated addition or subtraction. The note to the teacher suggests encouraging students to use their own methods – making groups in the tray, using multiplication, or repeated subtraction, etc. The selection of a strategy by the student can indicate his or her understanding and use of additive or multiplicative thinking.

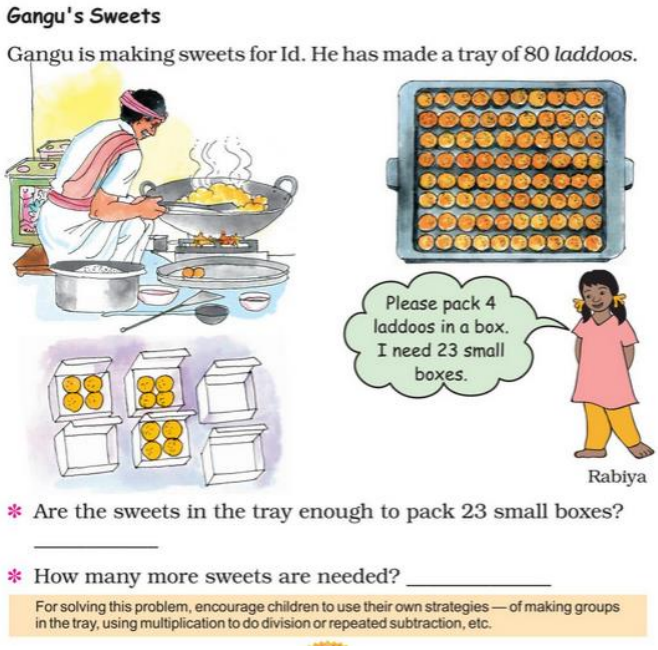


Figure 3: Grouping of Gangu's sweets (NCERT, 2007, p.126)

Pallavi's interpretation of dealing with different strategies as proposed in the new textbook was to 'teach all the methods' to students. In Excerpt 1, Pallavi indicated that the burden of teaching all these methods was on the teacher and consequently her concerns were guided by the difficulty of teaching them to students.

Excerpt 1

	[Researcher[3] notes: I was observing Pallavi's lesson in Grade 4, where she was teaching the division algorithm. The lesson was about to end. She came to me with the textbook and started talking about it. I think what she said is linked to the question I asked her yesterday about the difference between the old and new math textbook.]
TP	You can't expect them [students] to learn so many methods like the new textbook gives. It says you teach this method also, that method also. It is very confusing for students and then when you ask a question, which method do you want them to use? They should use long division. It is what we have been doing for ages. I did it when in school. And it is the systematic way. (Y1TPLI[4])

Pallavi did not seem to associate the choice of 'method' with the problem context. Her emphasis on teaching all the methods over-rides the discussion on the choice of method. Observations over several lessons show that she explicitly taught students each of the methods and then gave practice problems to use the same method repeatedly. She did not allow for students to use their own strategies or discuss why some strategies are more efficient than the others. Consider an excerpt from the classroom teaching of the division problem shown in Figure 2, where the focus was on using repeated subtraction as a method to divide.

Excerpt 2

	Pallavi writes the question on board and students' copy.
Board	1. Dhruv lives near the sea. He thought of making the sea shells. He took 28 sea shells for one necklace. How many necklaces can he make using 112 sea shells?
TP	Read the problem.
	Students read aloud the problem in chorus.
TP	Total?
G St	112 shells.
TP	Method?
G St1	Division.
TP	One necklace is equal to?
G St2	28 shells.
G St4	Number of necklaces is 112 ÷ 28.
TP	Here comes the problem how will you divide? Okay you know how to divide. Tell.
	Teacher points to a girl student to come to the board.
G St3	$28\overline{)112}$ (*She writes this on the board and pauses.*
TP	For this type of division I already told you the method.
S Sts	Minus.
TP	What is it called?
S Sts	Subtraction.

TP	We have to do minus minus minus.
G St6	Repeated subtraction.
TP	Okay so you do. All of you do it by repeated subtraction. Don't do long division. Do repeated subtraction. Don't think anything else. Just do repeated subtraction. (Y1TPDvL10)

Pallavi's decision to break down the problem context into procedural steps (classifying the given information, stating the operation and method, using the method to find the unknown), and emphasising the use of one method at a time was consistent across problems and lessons. We note Pallavi's concern (Excerpt 1) that the teaching of several methods leads to confusion among students. Pallavi explicitly discouraged students in relating this method to the other methods. Pallavi's belief that students should not experience confusion is a strong one, also evidenced in Excerpt 2, where she says, "Don't think anything else. Just do repeated subtraction". We also find a similar concern expressed in Excerpt 3 and 4 below. Moreover, the cause of confusion is seen to lie in the varied and multiple responses from students. Pallavi prefers students to be clear about which method to adopt when faced with a problem, which essentially forecloses any variation in student responses. If students are allowed freedom to think about a problem, then it is inevitable that multiple approaches will arise. It is not clear at this point whether Pallavi is against allowing variability in the students' response *per se*, or whether she feels ill confident about dealing with such variability.

Further, although problems were solved using each of the methods: repeated subtraction, grouping, and multiplication with convenient numbers, these methods were not connected with each other or the algorithm. The teaching of the long division algorithm, at the end, was given more attention and practice. Pallavi taught different methods following the textbook but held a strong belief that students must know the algorithm. The teacher's emphasis on the learning of the algorithm is a reality of Indian classrooms, as it is considered to be an important goal of 'school' learnt mathematics and is used as a differentiator from the 'out of school' mathematical knowledge. The legitimacy of the algorithm comes from the authority of the content in the school textbook and the experience of learning and teaching the same method for several decades. When Pallavi was probed about the teaching of justification of an algorithm in class, she stated the following reasons for avoiding it while teaching.

Excerpt 3

	[Researcher notes: I had one of my general conversations with Pallavi. I wanted to know the reason for her emphasis on teaching the algorithm and her views on why the algorithm works. I also intended to know about her thoughts on using different methods.]
R	There must be a reason for why an algorithm works. Don't you think it is important for students to know why this method works?
TP	They are very young. Telling them what lies behind this concept or you had done that remember, we [teachers] can't do that. Their [students'] brains are not that developed. When they grow up, go to class 7 or 8 you can tell them see this is why we did that but not now. They are too young. They will just get more and more confused. (Y1TPPI)

Pallavi attributed the decision of not teaching the justification of the method when discussing the algorithm to the developmental incapacity of students. She consistently maintained that young students are incapable of handling multiple methods and representations, independent problem solving, and reasoning about why something works. Like other participating teachers, she believed that students face difficulty in understanding

the justification of why an algorithm works. This led to lowering the cognitive demand of the task by demonstrating the procedure (also noted by Jackson, Gibbons & Dunlap, 2014).

We note that although Pallavi believes that all methods proposed by the textbook need to be taught, she does not pay attention to the connections between these methods and their relation to the problem situation. Pallavi could not anticipate the possibility that students might use these strategies or methods when given an opportunity to solve problems by themselves. She seemed to be underestimating student capabilities by thinking that they cannot deal with different methods. We note that placing a low cognitive demand in problems and methods is done to avoid confusion in students, which in turn is not considered as contributing to their learning.

Year 2: "I don't understand how this method works, why don't you teach?"

In the second year, after teaching and providing practice on solving division problems using repeated addition, repeated subtraction, and use of multiplicative facts, Pallavi intended to teach the *chunking method,* identified in literature as working with partial quotients. In this method, convenient multipliers are chosen and the multiple is subtracted from the dividend. In other words, in a quotitive interpretation where the divisor is interpreted as the fixed size of a group or share, one has to reach the maximum number of groups/shares of divisor that can be taken away from the dividend. (Alternatively, in a partitive interpretation where the divisor indicates the fixed number of equal groups, one needs to arrive at the maximal size of a group.) The number of groups may be decided by the ease of arriving at multiples using doubling, multiplication with ten and its multiples, etc. For example, Figure 5(a) shows how the chunking method is used to solve 585÷16. Literature (Anghileri, Beishuizen, & van Putten 2002; Khemani & Subramanian, 2012) suggests that partial quotients builds on students' intuitive strategies and allows for greater flexibility in the choice of chunks unlike the standard division algorithm. Although the partial quotients method is described in the textbook, and Pallavi was following the textbook closely, she had avoided introducing this method in her class in the previous years. In Year 2, Pallavi worked with the researcher to understand the partial quotients method before teaching it in the classroom. She struggled to use the method with different numbers and while trying she remarked that the method is confusing. In the excerpts below, we notice the process of Pallavi's gradual negotiation with the method and it's teaching.

Excerpt 4

	[Researcher notes: This is one of Pallavi's Grade 4 classes where she teaches regularly. When I asked her about her plan for the lesson she showed me the textbook and started talking about the partial quotients method.]
TP	Now I have tried this method given in the book but see it is confusing… have always done long division only with children. So I am not sure how to introduce it, how to actually do it in class. I am confortable in long division and it is shorter you know. It is a step-by-step process, take one digit at a time so they [students] can easily divide. (Y2TPPI1)

Pallavi was struggling to use the partial quotients method to solve division problems. Her difficulty seemed to stem from the fact that the partial quotients method lacks the procedural clarity that is found in the long division algorithm. The standard algorithm works implicitly with place value, dividing one digit at a time. Each step of the algorithm repeats the same logic consistently. Pallavi's comfort with the long division algorithm came from her confidence in using the method for a long period of time, following the steps sequentially, and its efficiency.

The division algorithm has an underlying structure. It looks at the place value of the digits in the number to be divided. The dividend is not operated as a whole but by breaking it into parts according to place value units and the left overs are transformed into the next unit (Lampert, 1992). To keep track of the place value of digits in the quotient, students are often given a clue, i.e., to write the digit of the quotient just above the dividend over the same place value. Although the visual clue helps in identifying the quotient correctly, it does not explain why such an orientation must be maintained. Deconstructing the division algorithm would mean understanding the implicit place values in the number to be divided, finding the chunks of the divisor that are closer to the dividend, and distributively dividing the dividend.

In contrast, in the partial quotients method the number as a whole is taken and chunks are identified that can be safely taken away from the whole number, recording the number of chunks taken each time (called partial quotients), and finally adding the number of chunks to obtain a quotient. Structurally, partial quotients can be seen as intermediary between students' intuitive strategies and the division algorithm (van Putten, Brom-Snijders & Beishuizen, 2005; Khemani & Subramanian, 2012).

Pallavi's motivation to explicate the difficulty in using partial quotients and in seeking support from the researcher probably arises from the pressure of teaching the method, being a part of the textbook. She approached the researcher to seek support in teaching of the method to the students.

Excerpt 5

TP	Why don't you [researcher] take this [division by chunking] in my class? Tell them what this method is. [After a pause] Yes we can see how they [students] pick it and decide then only which method. I don't know if they will understand. I tried around 8 to 10 numbers, dividing them using that method. The bigger the number, the more confusing it was. I think it can confuse. But you try and let me see how they try to do it.
	[Researcher notes: Pallavi asked me to teach in her class today. I was thinking about several things - whether I should teach because my role is to do classroom observations, what will I teach which will encourage students to think about chunks, how will the change in the teacher affect students' response, how will Pallavi observe and interpret the classroom interaction.] (Y2TPPI2)

Pallavi's suggestion of switching the role of the teacher and researcher marks an important event in the research. She suggested that the researcher take a more 'active' role in teaching a difficult topic. The goal of the researcher (who became the teacher) changed to thinking about a problem context that would elicit the meaning of division and will provide students with an opportunity to build on their own strategies. Along with the identification of problem context and learning goal for students, Pallavi's understanding of the method also needed scaffolding.

Year 2: "I understand why the algorithm works!"

In the second year, Pallavi introduced the researcher as a teacher in one of the division lessons. The researcher posed the following problem to the students in the class.

Problem: Grandpa wants to distribute Rupees[5] 75 among three of his grand children equally. Can you help him in doing this? Explain your reasoning.

The rationale for beginning with a sharing context was that students might relate to this meaning of division intuitively. The money context offers a potential to see the place value structure in the denominations of powers of ten. As soon as the problem was posed, students began to propose how to distribute the money to arrive at the share of each grandchild. With some guidance from the researcher on how to record the amount to be distributed to each

grand child at every step, students were encouraged to come up with different ways in which the money could be distributed. They began with distributing “10 to each grand child”, to which another student suggested “20” and a third student “25” or, the student said, “10, 10, and 5”. When all students solved the problem, the next problem posed was, “what if there were 5 grandchildren?”. And before the whole problem was restated, several students responded that the share of money would reduce. When asked why, students responded by saying that the money was the same but the number of grand children had increased, so each of them would get less money when compared with the previous distribution. Noticing the relation without solving the problem or finding the quotient for x/a and x/b, and comparing marked an important step towards thinking proportionally (Lampert, 1992). To justify their responses, students used the sharing interpretation to find the exact share of each grand child in the second case. In this situation, students were able to see that x/a > x/b when b > a.

As the lesson progressed, Pallavi took over the teaching and gave students the problem of distributing Rupees 127 among 5 friends equally. The choice of these numbers by Pallavi is interesting because 127 is not evenly divided by 5. We also note that Pallavi preferred to retain the number 5 as the divisor. As students proposed chunks of 10, 10 and 5; she recorded these on the blackboard labeling the number of friends as the divisor, the total amount as the dividend, and pointing to the partial quotients as the share of each friend. After the money context, students were asked to divide 89 with 4.

Pallavi’s decision to switch the roles while the lesson was in progress was an in-the-moment decision. Her choice of numbers 127 and 5 seemed deliberate as she intended that students focus on the act of distribution and discuss convenient combinations. The decision to shift from a contextual problem to a bare number problem indicates the shift from dependence of students’ reasoning on the context of sharing, while it still acted as a reference or an anchor.

Excerpt 6

	[Researcher notes: Pallavi gave students the bare number problem 89 divided by 4. She gave students time to think and solve the problem. And during this time she came to me and started talking about the way of recording partial quotients.]
TP	This way of grouping works, as it tells you each time what you are distributing. In [old] textbook all of this was at the top. In fact this way (horizontal) of writing is better than (writing on the top of the division symbol, refer Figure 4a) because they cannot keep track and the place value is there. (Y2TPDvL2)

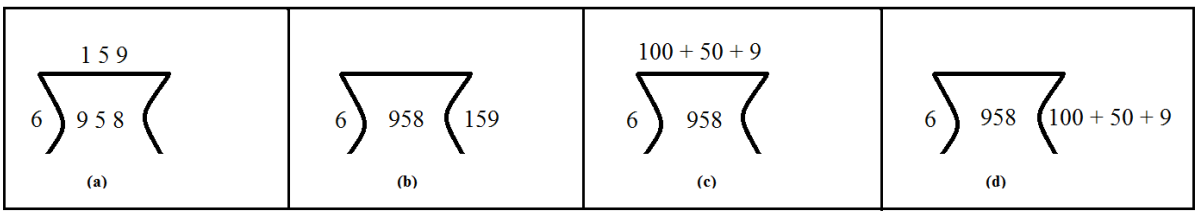


Figure 4: Quotient at the top and right of the dividend in the long division algorithm (a,b) and partial quotients (c,d) respectively.

As students were engaged in the problem context of distributing money, Pallavi came up to the researcher and made two observations about the partial quotients method (refer Excerpt 6). First, she noticed that the horizontal recording of the partial quotients is important to keep track of the number of chunks that have been taken away from the whole and the changing whole (“what you are distributing”). And second, she observed how the place value of each digit plays a role in the division algorithm. When Pallavi remarks that the horizontal way of writing is better, she may have been referring to the practice of writing the quotient

digits to the right of the dividend rather than above the dividend. The textbook uses both ways of recording partial quotients (refer Figure 4c,d), and Pallavi may have been concerned about this inconsistency. After working individually on the problem, students suggested different combinations for dividing 89 with 4. Pallavi listened to these variations, each of which allowed students to arrive at the correct answer, and then closed the day's lesson. Pallavi and the researcher continued the discussion about the partial quotients after the lesson.

Excerpt 7

	[Researcher notes: Today I did not have to ask Pallavi about the lesson. She was excited to talk about it with me. So as soon as she finished teaching, she started talking to me about the method.]
TP	I think the method is good. They [students] can use different ways to get it [answer]. Also it is very clear, this vertical arrangement of numbers. And grouping by tens they are aware also. Then slowly they can move to choosing bigger numbers. Actually you know the number of steps increases if you take small numbers [multiples]. But it doesn't matter because they anyway get it. They can use 8 directly or if not 4 and 4 or 5 and 3, it doesn't matter. This method is better and they picked it up faster also. As a teacher, I can see how they are liking it. Taking it as a full number [number as a whole] is clear to them. They find it more easy. Easy only, no? They can make as many groups and how much they want. This also tells us about the multiplication knowledge. But you know one more difference is there. In long division, I have to teach them for each increasing digit like dividing by one digit, then two [digit number] and three, all are different. But in this they have to use the same method for big numbers, by themselves and they can do also. (Y2TPTI2)

While reflecting on use of partial quotients, Pallavi seemed to be unpacking the structure underlying the division algorithm and related student capabilities (Excerpt 7). She noticed that the method revealed students' multiplication knowledge expressed through their choice of convenient numbers for chunking. Different students used different sequences of partial quotients, while arriving at the correct answer. As indicated in Excerpt 7, she noted the flexibility in the choice of the size of chunks as well as the relation that smaller chunks lead to a larger number of partial quotients. She made an interesting distinction between the way she taught the long division algorithm and partial quotients. It was the difference between a digit-based approach versus treating numbers as a whole. The reliance on the face value of the digits of a number takes away the attention from the place value. Pallavi also remarked that she does not need to teach the partial quotients method separately for one-digit, two-digit or three-digit divisors. In contrast, she mentioned that earlier she needed to teach the standard algorithm differently for divisors of different digit lengths, a view that suggested again the highly prescriptive, step-wise approach to teaching a procedure.

The data is not sufficient to conclude that Pallavi's belief about the lack of students' ability to discover methods by themselves has been challenged. But it was evident that she had begun thinking about building on students' prior knowledge. In this case, she considered that students used their knowledge of multiplication with convenient numbers to solve a division problem using partial quotients. She was beginning to engage with the aspects of multiplicative thinking involved in the process of chunking.

In the lessons that followed, Pallavi explicitly dealt with the relation between using partial quotients and the long division algorithm. She gave students the following division problems to solve: 115÷3, 236÷11, 427÷13 and 585÷16**.** She noticed that a majority of students used chunking to solve these problems by themselves. She found that students were extending the chunking to numbers for which they had not memorised the tables (for instance, division by 13 and 16). She was excited to notice this and shared the observation with the researcher. Later in the lesson, she brought students' attention to the relation between chunking and the long division algorithm. While teaching in class, she gave a division problem and asked students to solve it using both methods: partial quotients and long division algorithm (refer Figure 5).

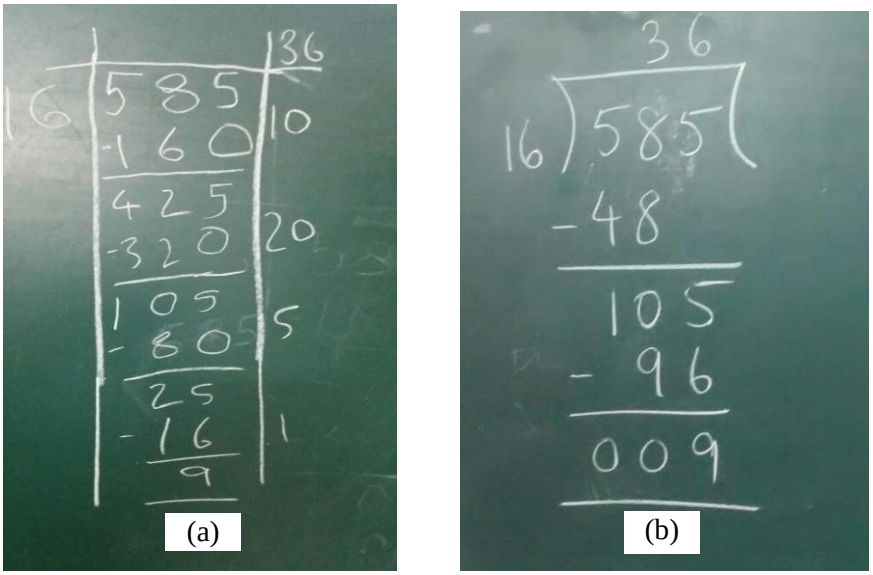


Figure 5: A number problem solved using (a) partial quotients and (b) the long division algorithm

Excerpt 8

	Teacher asked the students to solve 585÷16. *After giving students some time to solve this problem, she starts talking. She asks students how they have solved the problem and records it on board.*
TP	Now same thing let's try to do using long division. You have to tell me what's happening?
Board	Refer Figure 5.
TP	So what do you see? What is the difference?
G St	In long division, we are multiplying the number.
TP	Here [pointing to chunking] also we do.
B St	In long division we don't have to plus [add] the tens.
G St	Teacher we are not taking the full number for division.
TP	Good. In long division we are not taking the number as a whole but the digits. In grouping method, we take the whole number together. Since in long division we take one digit at a time, the number of steps is less as we look for biggest multiple.
G St	We take 10, 20, 30 in [long] division also.
TP	Yes you can reduce the number of steps in grouping also. If you are thorough with your multiplication you can take bigger multiples. (Y2TPDvL4)

Through the presentation of both the methods, Pallavi tried to engage students with the links between finding partial quotients and the long division algorithm (refer Excerpt 8). While teaching in the class, she figured that the place value structure is implicit in the division algorithm. The contrast between taking a digit based approach and the number as a whole was triggered by a student's explanation. It was during teaching that Pallavi noticed and explicated that the underlying structure of the division algorithm is in finding the greatest partial quotient or with the highest place value. Although not all students could explicate the relation between the two methods sufficiently well, Pallavi reported in the post-lesson interview that the conceptual knowledge of 'why division algorithm works' must be included as an important part of the teaching of division and she would like to henceforth discuss the link between the two methods when teaching division.

Conclusions and Discussion

In this paper, we have discussed the case of a mathematics teacher struggling to unpack the structure of the division algorithm while teaching in a Grade four classroom, using a textbook with a reformed curriculum. In the beginning of the paper, we had raised two questions. First, how do teachers' knowledge, beliefs, and practice interact as they attempt to teach conceptually? Second, how does knowledge of 'why an algorithm works' manifest in practice.

A careful analysis of Pallavi's teaching of specific topics over two academic years indicated the ways in which knowledge and beliefs interplay when a teacher makes decisions in the classroom. A focused engagement with the topic of division allowed us to analyse the complex character of the teacher's work. We note that Pallavi was teaching the new textbook for several years before this research study was conducted. She used the "new" methods of division, described in the textbook, in her teaching. In the first year, she explicitly taught each of these methods while being worried about the possible confusions arising from the use of multiple methods, in students' minds. However, she had omitted the partial quotients method because, as she admitted, it was confusing to her. She needed topic specific support to engage with the trajectory suggested by the textbook. In particular, she needed to understand the mathematical significance of different methods and connections between them. We also notice that working with a few examples using the partial quotients method along with the researcher was not sufficient for Pallavi to develop an understanding of the method or to convince her to teach it to her class. Pallavi's initiative of articulating her struggles with the partial quotients method and seeking support from the researcher while teaching it in the classroom, marked an important shift allowing for a re-examination of existing beliefs and practices.

Further, noticing the varied responses from students when partial quotients were introduced, Pallavi's decision to take over the teaching showed her interest in working with the method with the students and probably added to her conviction that students could make sense of the method and use it. While working with and reflecting on the students' use of partial quotients, Pallavi engaged with the conceptual structure of the division algorithm. The students' responses led Pallavi to see the possibilities inherent in using the new method. An important aspect of the knowledge-in-play was the variations in students' responses to the problem posed. As seen in Excerpt 7, this variation helped Pallavi in noticing different "correct" responses emerging from the students. The variations in the choice of chunks seemed to provide a direction to the complexity, which was difficult for her to anticipate in isolation from the classroom. The variations in examples and choice of chunks observed by Pallavi supported the insight that partial quotients allow for such variations and gives an insight into the structure of the algorithm. This may have led Pallavi to take over the teaching and to introduce her own examples by way of variation. The sequencing of examples provided the scope for students to utilize their multiplicative knowledge and make connections between different ways of solving the division problem. Students' responses to the variety of examples which go beyond the knowledge "taught" to them may have led to Pallavi designing more challenging tasks for them.

As it became a part of Pallavi's explicit knowing, she decided to include a discussion of 'why' the division algorithm works in her teaching and make the structure of the long division algorithm transparent for the students. Pallavi engaged students in the comparison of the "chunking method" with the algorithm to identify the differences and similarities in them. The design and conduct of this mathematical task contrasts with her belief that the discussion of more than one method creates "confusion" among students and is beyond their cognitive ability. The links between teacher's actions, students' engagement at different levels, and

teacher's responses to students are contingent to the classroom and are specific to the situated experience of learning from teaching. We suggest that it was the situated nature of this experience that led to the beginnings of a deeper understanding of the mathematical structure underlying the long division algorithm. The attempts made by Pallavi in linking the partial quotients and the division algorithm was a change triggered partially by discussions with the researcher about the mathematics underlying different methods of teaching division and with the students in the classroom while solving problems using the partial quotients. Additionally, the variation in student responses triggered Pallavi's imagination of a pedagogy where the straight-jacketed approach to teaching and reproducing the algorithms was challenged. Earlier, Pallavi tended to see variation as a source of confusion among students and as impeding their learning. After a deeper engagement with the mathematical structure of the algorithm in the classroom context, she remarked on the variations afforded by the partial quotients approach. Engaging with the mathematics of the algorithm and how it played out in the classroom addressed both Pallavi's knowledge and belief; knowledge about how and why the partial quotients method works and belief about the desirability of allowing variations in student responses. We claim that without the situated nature of this experience, this simultaneous addressing of knowledge and belief would have been difficult to achieve. This may explain why Pallavi resisted including the teaching of the method for several years. We acknowledge the possible role of the intervention in the form of teacher-researcher meetings focused on the topic of decimal numbers, in orienting Pallavi to be more sensitive to student responses and in priming this change.

The evidences also suggest that the teacher's knowledge of the structure or justification of the division algorithm has a bearing upon the kind of teaching decisions made in the classroom. Evidently, experienced teachers also struggle with the conceptual understanding of a mathematical procedure. We see the importance of creating a social learning space for collaboration with researchers and peer support with a focus on classrooms in enabling such an understanding. The mathematical knowledge *in situ* is grounded in the complex work of teaching and needs to be examined to analyse the challenges or demands posed on teachers when teaching in a reform context. Teachers need support in responding to these demands posed by the curriculum and teaching in practice. The nature of knowledge situated in practice allows for an engagement with the knowledge of content, teaching, and students in an integrated manner (Takker, 2015). Our research also indicates that discussions centered around *knowledge in play* (Rowland & Ruthven, 2011) invite experienced teachers to participate in active decision making and make the discourse of professional development meaningful. Further, an intervention grounded in practice has the potential for challenging teacher's existing beliefs and knowledge, and utilise the knowledge generated through research to inform practice. The engagement with a focus on teaching practice can be utilised for building and sustaining communities of practice with teachers and researchers for continuous teacher professional development.

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[1] In the first year of the study, data was collected in the form of audio records and field notes, which were transcribed for analysis. The teachers were not comfortable with video recording in the first year. In the second year, data was collected in the form of audio and video records and field notes, which were transcribed for analysis.
[2] A part of the data and analysis presented in this paper have been used elsewhere (Takker, 2015) by the first author to argue for the need to develop teacher-researcher communities in the Indian context.
[3] Researcher in the transcripts refers to the first author.
[4] The legends used in transcript expand as follows - Y1: Year 1 (2012) of observations, Y2: Year 2 (2013) TP: Teacher Pallavi, LI: mode of data collection (in this case, long interview). In the transcripts from classroom observations, the additional legends used are - G/B St: Girl/ Boy Student, S Sts: Some Students.
[5] Rupees is the official currency of India. A rupee is divided into 100 paise (singular 'paisa').