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RESEARCH REPORT

A Learning Progression for Geometrical Measurement in One, Two, and Three Dimensions

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As part of the *CBAL*® learning and assessment initiative in mathematics, we developed a hypothesized learning progression (LP) for geometrical measurement in 1, 2, and 3 dimensions based on a synthesis of empirical literature in this field and through expert review. The geometrical measurement LP is intended to represent a developmental progression of students' understandings and learning for 1-, 2-, and 3-dimensional measurement in terms of transitions along levels within a dimension and connections across the 3 dimensions. In addition, we designed cognitive laboratory tasks associated with the levels of the geometrical measurement LP. The development process of the geometrical measurement LP and cognitive laboratory tasks presented here provide guidance for future development of LPs and task design that would provide the evidence supporting the proposed LPs.

Keywords Geometrical measurement; learning progression; task design; mathematics

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At Educational Testing Service (ETS) the *CBAL*® learning and assessment research initiative aims to construct "a model for an innovative K–12 assessment system that documents what students have achieved (*of learning*); facilitates instructional planning (*for learning*); and is considered by students and teachers to be a worthwhile educational experience in and of itself (*as learning*)" (Bennett, 2010, pp. 71–72). This model comprises domain-specific competency models (for reading writing and mathematics), summative and formative assessments, and professional development supports. Underlying these system components is a set of learning progressions (LPs) or hypothesized trajectories of acquisition of competency 18 of which have been developed in K–12 mathematics (see Arieli-Attali & Cayton-Hodges, 2014, for rational numbers; Graf, 2009, for an initial competency model and its relationship to LPs).

As part of the CBAL initiative in mathematics, we developed a hypothesized LP for geometrical measurement in terms of one-, two-, and three-dimensional measurement (hereafter referred to as the *geometrical measurement LP*), based on a synthesis of empirical literature in this field and through expert review. The hypothesized geometrical measurement LP is aimed to represent a developmental progression of students' understanding and learning for one-, two-, and three-dimensional measurement regarding transitions along levels within a dimension and connections across the three dimensions. A set of cognitive laboratory tasks is designed to support the collection of data on the geometrical measurement LP. We are in the process of collecting validity evidence for the geometrical measurement LP via a cognitive laboratory study and analysis of data coming from a pilot of *WINSIGHT*™ assessments, a K–12 assessment system in development at ETS.

Our goal for this paper is to present the development process for the hypothesized geometrical measurement LP and for the cognitive laboratory tasks that draw on this geometrical measurement LP. The geometrical measurement LP connects measurement understanding of one, two, and three dimensions to construct a developmental progression of geometrical measurement across the three dimensions. Our purpose is to advance the research field because there are no previous studies that address this idea (see Battista, 2012). Cognitive laboratory tasks designed to reflect this aspect of the geometrical measurement LP contribute to instruction and assessment development in this field by drawing attention to possible connections across one-, two-, and three-dimensional measurement.

We organized this paper as follows. First, we identify our conception of geometrical measurement, a more specified domain model of the general model for geometry. Second, we present the conceptual foundation of this geometrical measurement LP in terms of the underlying concepts of geometrical measurement. Third, we give a brief review of two

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existing LPs for geometrical measurement. Fourth, we present our geometrical measurement LP, and lastly, we provide six cognitive laboratory tasks, as well as their design goals, that can be used to collect validity evidence for the hypothesized geometrical measurement LP. We expect that this paper may serve as a guide for future LP development and cognitive task design associated with the levels of a proposed LP (see Graf & van Rijn, 2016).

Definition of Geometrical Measurement

Measurement is a geometry content domain of K – 12 school mathematics curricula (see the *Common Core State Standards for Mathematics*, Common Core State Standards Initiative [CCSSI], 2010). In the *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) defined measurement as "the assignment of a numerical value to an attribute of an object, such as the length of a pencil" (p. 44). According to the CCSSI (2010), mathematical learning of measurement begins with describing and directly comparing different measurable attributes of objects, such as length, area, volume, time, or weight. According to Bright (1976), a measurable attribute is "a characteristic that can be quantified by comparing it to some standard unit" (p. 88). Compared to the measurement of time or weight, we consider the measurement of length, area, and volume as geometrical measurement in one, two and three dimensions, respectively. Thus, this domain analysis and LP do not include the measurement of time or weight; we propose constructing the geometrical measurement LP with regard to LPs for length, area, and volume measurement.

Concepts Underlying Geometrical Measurement

Drawing upon the review of literature in the domain of geometrical measurement, we identified key concepts underlying the thinking and reasoning of geometrical measurement in terms of (a) unit partition and unit iteration, (b) spatial ability, (c) composite unit and its spatial structuring, and (d) abstraction. Because these ideas are necessary for understanding students' development of geometrical measurement and for using the geometrical measurement LP, we detail these ideas below.

Unit Partition and Unit Iteration

The activity of measuring is a synthesis of taking a part of an object as a unit and then placing the unit alongside the object in terms of unit subdivision and unit iteration, respectively (Piaget, Inhelder, & Szeminska, 1960). Distinct from this view of measuring as a physical activity, Stephan and Clements (2003) formulated subdividing and unit iteration as conceptual understandings that underpin learning to measure; the researchers defined the concept of partitioning in length measurement as "the mental activity of slicing up the length of an object into the same-sized units" (p. 4). According to Lehrer (2003), "to iterate a unit of length, a child must come to understand length as a distance that can be subdivided" (p. 181); namely, the understanding of unit partition is ahead of the mental iteration of units in the progression of learning length measurement. We argue that the conception of partition and iteration is also applicable to measuring the area and volume of objects and that they are two foundational concepts underlying geometrical measurement (e.g., Lehrer, 2003; Lehrer, Jaslow, & Curtis, 2003; see also CCSSI, 2010).

Spatial Ability

In addition to students' understandings of unit partition and unit iteration, researchers have focused on students' spatial ability in terms of spatial visualization (e.g., Ben-Haim, Lappan, & Houang, 1985; Pittalis & Christou, 2010). The ability to visualize spatially relates to reading two-dimensional representations of three-dimensional objects in terms of their three-dimensional properties (Ben-Haim et al., 1985); this meaning of spatial visualization is consistent with Senechal's (1991) view of visualization as space perception, namely "the mental reconstruction of representations of three-dimensional objects" (p. 15).

In their pilot study for developing spatial visualization assessment items for middle-school students, Ben-Haim et al. (1985) observed that students have difficulties in visualizing the hidden parts of three-dimensional objects when they are presented as two-dimensional representations. In assessment items on the Middle Grades Mathematics Project Spatial Visualization Test (MGMP SVT) that ask a student to count the number of unit cubes in a rectangular prism built from the unit cubes, Ben-Haim and colleagues found that about 25% of 102 fifth graders, 40%–45% of 467 sixth and 229 seventh graders, and 50% of 180 eighth graders responded correctly. The students who responded incorrectly to the items used one of four erroneous counting strategies: (a) Strategy 1, counting the number of visible squares (as shown in the faces of each unit cube); (b) Strategy 2, counting the number of visible squares and doubling the count; (c) Strategy 3, counting the number of visible cubes; and (d) Strategy 4, counting the number of visible cubes and doubling the count. Specifically, in response to Item 10 on the pretest of the MGMP SVT (presenting a two-dimensional representation of a 2-by-4-by-3 rectangular prism), 23.5% of the incorrect responses of fifth graders used erroneous counting strategy 1; 25.5% used strategy 2; 8.8% used strategy 3; and 8.8% used strategy 4. According to Ben-Haim and colleagues, these erroneous counting strategies are rooted in students' difficulties in understanding and perceiving two-dimensional representations of three-dimensional objects. Other studies have reported similar findings (see Battista & Clements, 1996; Hirstein, 1981). These findings suggest the importance of accounting for spatial visualization ability in the assessment of geometrical measurement.

Efficient-Sized Measurement Units and Spatial Structuring

Students' spatial ability might also relate to the use of reference points as units in estimating and measuring the size of an object in space. According to Joram, Subrahmanyam, and Gelman (1998), the size of an object known to students could be used as a reference point; for instance, a person's known height of 6 feet can be used as a reference point to estimate the width of a room by iterating the established point of reference across the room. Joram and colleagues argued that using reference points that are larger than individual units reduces the number of iterations needed for measuring length/distance. This efficiency strategy may, in turn, reduce the burden on working memory during problem-solving tasks. Related to the selection of measurement units, the relative precision of measures is decided with proportional regard to the object being measured (Lehrer et al., 2003). From these views, we thus hypothesize that the use of a larger unit is more efficient than the use of a smaller unit in certain measurement contexts but that the selection of a smaller unit over a larger unit gives more precise measures. Because this is an important consideration for assessment, in the geometrical measurement LP we attend to the role of scale factors in converting between different-sized measurement units. To illustrate, if a football field is 300 feet long, it is more efficient to measure the length as a whole in yards even though the measurement in feet might be accepted as a more precise measure than the measure in yards.

The idea of using a larger unit instead of a smaller unit in measurement is referred to as the concept of composite units, an iterable set of individual, single units (Reynolds & Wheatley, 1996; see also Battista, 2004; Battista & Clements, 1996; Stephan & Clements, 2003). In their study on elementary-school students' strategies and errors in counting the number of unit cubes in rectangular prisms built from the unit cubes, Battista and Clements (1996) found that students conceptualized a column or row of cubes and a horizontal or vertical layer of cubes as iterable composites. Battista and Clements argued that students conceptualize a composite unit when considering the set of single units as a unit; they see spatial structuring, in enumerating a three-dimensional cube array as the set of unit cubes (i.e., a composite unit), as "the mental act of constructing an organization or form for an object or set of objects" (p. 282). They described the process of spatial structuring in forming a composite unit as (a) establishing units, (b) establishing the relationships between different units as that relates to positioning units in relation to each other, and (c) recognizing that the iteration of a set of units forming a composite unit can be used to construct the whole object. In addition, Battista and Clements saw the spatial structuring in organizing the composite units into "layers" as fundamental to abstracting the enumeration procedure of layers in terms of the volume formula of length × width × height. Drawing on Battista and Clements' view, we thus hypothesize that the spatial structuring of forming a composite unit as an iterable unit is crucial in the progression from measuring with single units, to composite units, to more efficiently sized composite units for a given measurement context, as well as in the conceptualization of the spatial structure of formulas such as those for perimeter, area, and volume.

Abstraction

Battista (2004) saw abstraction as "the process by which the mind selects, coordinates, unifies, and registers in memory a collection of mental items or acts that appear in the attentional field" (p. 186). When students' thinking and reasoning of (composite) unit iteration and the spatial structuring thereof are fully developed, they reach a level of abstraction in geometrical measurement. According to Battista, students at this level are capable of working with the enumeration and structuring of arrays of squares and cubes (as dimensional units for area and volume measurement) on the internalized mental models of the arrays without the need to work with the iteration of units concretely. This leads us to hypothesize that the highest level of a developmental progression of geometrical measurement is a level of abstraction in which students are able to work with measurement in the abstract, namely, by using symbols and formulas with a sophisticated understanding of the connections between spatial structuring and numerical procedures.

Based on key concepts identified from a literature review, we thus theorize that the fundamental thinking and reasoning underlying geometrical measurement is (a) the conceptualization of measurement units in the process of measuring an object in term of unit partition and unit iteration; (b) the spatial reasoning of or about an object being measured in relation to various representations of the object; (c) the progression from measuring with single units to efficient composite units with regard to the presented measurement contexts; and (d) the transition from working with measurement concretely to understanding measurement at a level of abstraction. These key concepts become the foundation for constructing the geometrical measurement LP for length, area, and volume measurement.

Existing Learning Progressions

In constructing the geometrical measurement LP composed of three LPs of length, area, and volume measurement (hereafter referred to as *length*, *area*, and *volume measurement LPs*, respectively), we reviewed existing LPs in the domain of one-, two-, and three-dimensional measurement (e.g., for perimeter, Barrett, Clements, Klanderman, Pennisi, & Polaki, 2006; for area, Battista, Clements, Arnoff, Battista, & Borrow, 1998; for volume, Battista & Clements, 1996; for area and volume, Battista, 2004). Here we briefly introduce the LP of Barrett et al. (2006) for perimeter in terms of one-dimensional length measurement and the LPs of Battista (2004) for area and volume measurement as the most recent ones drawn from their own empirical research on students' developmental sequence of geometrical measurement understandings within each dimension.

Length Measurement Learning Progression

Barrett et al. (2006) examined 38 second through 10th graders' development of levels of measurement understanding observed during two fixed-perimeter measurement tasks (one for a rectangle and the other for a triangle), asking students to find all possible cases of a rectangle/triangle with a perimeter of 24 units. From observing the types of student understandings about length units, the researchers categorized three primary levels of thinking and reasoning about one-dimensional measurement, with the second and third levels having two sublevels.

At Level 1, students estimated length depending on visual observation alone, with no unit identification. On a fixed-perimeter measurement task, Level 1 students labeled the side lengths of a rectangle/triangle to present the given perimeter of 24 units by estimating visually, revealing incomplete understandings of the unit concept. For instance, a second grader, whose performance was classified at Level 1, wrote down the numerals of 1 – 24 around a rectangle to show its perimeter length of 24 units, but reached the number 24 before placing the numbers for all four sides of the rectangle. This result suggested the student used the number labels to count, not to identify length units.

At Level 2, students used inconsistent, uncoordinated markers as length units. Sublevels 2a and 2b are distinguished by the proper iteration of units, namely using inconsistent and uncoordinated unit identification (2a) versus using consistent, but uncoordinated, unit identification (2b). On a fixed-perimeter measurement task, students at Level 2a iterated inconsistent (i.e., not equal-interval-sized) units and labeled each side length of a rectangle/triangle improperly to present a perimeter of 24 units. For instance, a third grader at Level 2a drew a rectangle to represent a quadrilateral built from eight, four, nine, and three straw pieces and labeled the drawing 8, 4, 8, and 3 by counting the notches along two adjacent sides of the rectangle, not the notches at the vertices. This suggested that the student at Level 2a struggled to iterate the straw pieces as length units and to coordinate the side lengths properly.

Level 2b students showed sufficient unit iteration with consistent length units, but still labeled each side length of a rectangle/triangle improperly. For instance, a fifth grader at Level 2b drew a right triangle, labeling the hypotenuse 9 and the other two sides 11 and 4; this result revealed that the student identified 9 – 11 – 4 as an appropriate set of side lengths to draw a triangle with its perimeter length of 24 units but did not recognize the improper coordination of the side lengths in the drawing of a right triangle (because the hypotenuse of a right triangle is longer than the other two sides).

At Level 3, students used consistent and properly coordinated units of length and started to iterate a composite unit. Sublevels 3a and 3b are distinguished by the ways students coordinate side lengths: static, nonintegrated abstraction (3a) versus dynamic, integrated abstraction (3b). On a fixed-perimeter measurement task, students at Level 3a began to iterate "a collection of units as a unit itself" (p. 197), but had some difficulties integrating each of the side lengths in relation to the entire perimeter. For instance, a sixth grader at Level 3a tested various sets of three segments to draw a triangle with the perimeter of 24 units. As he examined the possible combinations, the student failed to reject the sets of 5 – 7 – 12 and 12 – 6 – 6, not satisfying the triangle inequality rule. Level 3b students recognized "part-whole relationships among units and groups of units" (p. 209) in iterating composite units; coordinated each side length in dynamic, comprehensive sequences; and used deductive reasoning to find all available cases of a rectangle/triangle having the perimeter of 24 units. For instance, a 10th grader at Level 3b suggested the valid and invalid cases of a triangle with the perimeter of 24 units, reasoning about the connection between an angle of the triangle and the opposite side of the angle (e.g., the angles must get larger as the opposite sides get longer). This reflects the student's understanding of the triangle inequality rule with regard to his inferential reasoning about the relation between side length and the size of the opposite angles and the relation among the three sides.

Area and Volume Measurement Learning Progressions

Battista and Clements (1996) examined 123 third and fifth graders' enumeration strategies of volume units (i.e., cubes) on measurement tasks such as finding the number of unit cubes in a rectangular prism built from the cubes. Similar to their earlier study on volume measurement, Battista et al. (1998) examined 12 second graders' spatial structuring and enumeration of area units (i.e., squares) on area measurement tasks, asking a student to find the number of squares needed to cover the inside of a rectangle completely. Drawing from the findings of these two empirical works (for volume, Battista & Clements, 1996; for area, Battista et al., 1998), Battista (2004) refined a developmental sequence of students' understandings of area and volume measurement in terms of seven levels of the enumeration of the arrays of squares/cubes (as measurement units) and the spatial structuring of the enumerated arrays.

At Level 1, students showed insufficient processes of unit locating and unit organizing. For instance, on a task that involved covering a 7-by-3-inch rectangle with inch-sized squares, a student at Level 1 partitioned the given rectangle with inconsistent (not equal-sized) squares and counted some of the squares two times. Thus, the student revealed her insufficient understandings of the concepts of unit and unit iteration with regard to the spatial structure and coordination of the enumerated squares.

At Level 2, students began to locate some equal-sized units, but the process of organizing the iterated units was not complete. For instance, on a task of covering a 6-by-3-inch rectangle with inch-sized squares, a student at Level 2 counted the top and bottom rows of squares properly as six and six, but failed to visualize and enumerate the middle row of squares in the rectangle as also six.

At Level 3, students coordinated units sufficiently and were aware of their double-counting errors. For instance, on a task that involved covering a 4-by-6-inch rectangle with inch-sized squares, a student at Level 3 counted the left and right columns of squares as six and six and then counted two and two for the top and bottom and eight in the middle. This reveals the student's sufficient understandings of unit iteration and proper spatial structuring of the iterated units.

At Level 4, students began to iterate maximal composite units (for area, rows or columns of squares; for volume, layers of cubes) but failed to coordinate the iterated composite units. For instance, on a task that asked students to estimate the number of squares covering a rectangle after being shown that five squares fit across the top of the rectangle and seven squares fit down the middle, a student at Level 4 counted squares by five, but the student failed to count them all. This result reveals that the student had the conception of a composite unit (i.e., iterating the group of five squares) but did not have sufficient coordination for iteration.

At Level 5, students demonstrated a sufficient process of coordinating units but used less-than-maximal composite units. For instance, on a task of covering a 5-by-4-inch rectangle with inch-sized squares, a student performing at Level 5 visualized the spatial structure of squares in the given rectangle and counted squares by two (rather than five or four). Thus, the student had sufficient spatial structuring of the iterated composite units, but, for example, her enumeration strategy was not efficient in terms of enumerating a maximal composite unit.

At Level 6, students fully developed the understandings about the processes of unit locating and unit organizing; thus, they can work with the enumeration of the arrays of units built upon fully incorporated spatial structuring of the enumerated arrays (for area, a row-by-column structuring; for volume, a layer structuring).

At Level 7, students reached a level of abstraction with respect to enumeration strategies and spatial structurings of the iterated (maximal composite) units. Thus, the students made the connection between numerical procedures by applying formulas and spatial structurings of the formulas (i.e., for length multiplied by width, a row-by-column structuring; for length times width multiplied by height, a layer structuring).

In summation, across the three LPs from Barrett et al. (2006) and Battista (2004), we found a common sequence of the development of students' understandings of geometrical measurement: namely, the progression from students' measurement thinking and reasoning from concrete and experiential to abstract with regard to unit iteration and spatial structuring of the iterated units, as well as with the use of efficient-sized composite units for presented measurement contexts. This review of existing LPs provided us a foundation for hypothesizing a developmental sequence across length, area, and volume measurement LPs within the geometrical measurement LP.

Hypothesized Geometrical Measurement Learning Progression: Distinctive Features

As part of the CBAL research initiative, Deane, Sabatini, and O'Reilly (2012) defined a LP as:

> a description of qualitative change in a student's level of sophistication for a key concept, process, strategy, practice, or habit of mind. Change in student standing on such a progression may be due to a variety of factors, including maturation and instruction. Each progression is presumed to be modal—i.e., to hold for most, but not all, students. Finally, it is provisional, subject to empirical verification and theoretical challenge. (para 1)

Drawing on Deane and colleagues' definition of LP, we conceptualize the geometrical measurement LP as a developmental progression of the levels of students' understandings and learning for geometrical measurement.

The geometrical measurement LP presented here represents an effort to incorporate our reading of the empirical research on the development of students' understandings of length, area, and volume measurement, as well as the review of the related literature in the domain of geometry and measurement, into a LP for geometrical measurement. Our goal in creating this geometrical measurement LP is to show how length, area, and volume measurement LPs connect to each other, providing a more comprehensive picture of the development of students' understandings of geometrical measurement across one, two, and three dimensions.

We are not the first to undertake such a task. Battista (2012) organized three LPs for length, area, and volume measurement in a coherent developmental sequence that constituted a LP of geometrical measurement. In this geometrical measurement LP, however, Battista did not address connections across the three LPs. Connecting understanding of measurement across one, two, and three dimensions advances the field of geometrical measurement in considering how understandings of and about length, area, and volume measurement can be associated with one another in students' development of geometrical measurement knowledge and learning. Making these connections across length, area, and volume measurement LPs also may inform mathematics educators about how length, area, and volume measurement can be taught and learned relative to one other. To date, there has been limited empirical research on concurrent development of length, area, and volume measurement (e.g., Curry, Mitchelmore, & Outhred, 2006; Curry & Outhred, 2005). In the proposed geometrical measurement LP, we attend not only to the transitions along levels within a single LP but also to the connections across the three dimensions. This distinguishes our geometrical measurement LP from other currently published LPs that consider length, area, and volume measurement separately, with no attention to potential connections across the three dimensions (see Battista, 2012).

Hypothesized Geometrical Measurement Learning Progression

We propose a hypothesized geometrical measurement LP composed of five levels corresponding to key aspects that emerged from the review of the literature and existing LPs for length, area, and volume measurement. The approximate grade span of this LP is Grade 2 through Grade 8. The five primary levels are defined below.[1]

- Level 1: Intuitive/holistic/visual comparison. The student compares size as a whole or counts parts of an object at the holistic level but with no iteration of measurement units;
- Level 2: Early unit concept (experiencing stage). The student iterates measurement units but insufficiently coordinates and/or structures the iterated units;

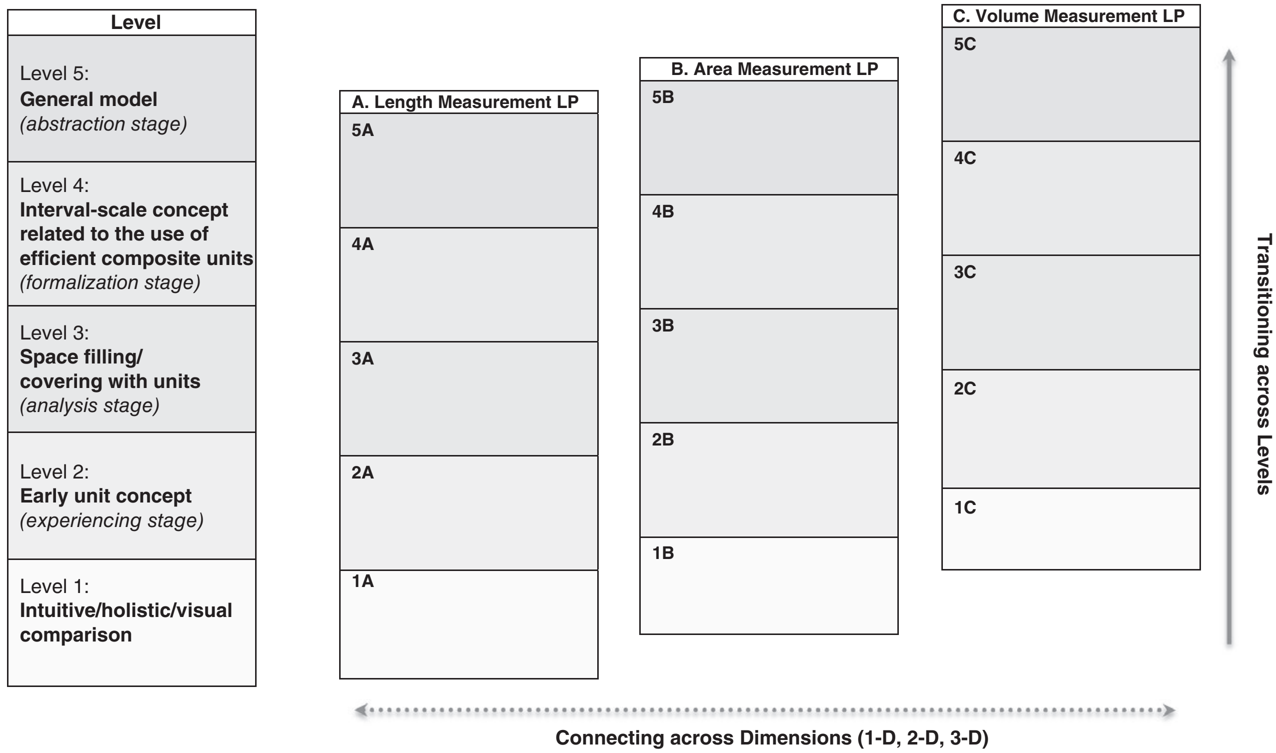


Figure 1 Geometrical measurement learning progression for length, area, and volume measurement.

- Level 3: Space filling/covering with units (analysis stage). The student structures iterated measurement units correctly and begins to use composite units;
- Level 4: Interval-scale concept related to the use of efficient composite units (formalization stage). The student measures with an efficient composite unit for each dimension and visualizes its spatial structure; and
- Level 5: General model (abstraction stage). The student operates in the abstract regarding the concepts of measurement.

The underlying proposition of this geometrical measurement LP is that students' understanding of geometrical measurement may progress across five levels within a dimension (vertical progression) and across one, two, and three dimensions (horizontal progression). Figure 1 is an overview representation of our geometrical measurement LP in terms of (a) three dimensions in one progression, (b) transitions across levels within each dimension, and (c) connections across three dimensions.

Three Dimensions in One Progression

To propose one LP for geometrical measurement, we first specified three LPs for length, area, and volume measurement within the larger geometrical measurement LP (see Wilson, 2009). The sequencing of the length, area, and volume LPs within this geometrical measurement LP is based on prior empirical work by Curry et al. (2006).

To compare the development of students' understandings of length, area, and volume measurement, Curry et al. (2006) interviewed 96 third to fifth graders using a set of three tasks for length, area, and volume measurement. They found "a steady and almost parallel progression from Grade 1 to Grade 4, with length slightly ahead of area and area far ahead of volume" (p. 382). In addition, they pointed to "the relative similarity between length and area measurement and the much larger gap between area and volume measurement" (p. 383). Drawing on these findings, we placed the area measurement LP behind the length measurement LP and the volume measurement LP following the development of the other two LPs. This structural aspect is represented as the positional structure of length, area, and volume measurement LPs within the geometrical measurement LP (i.e., area measurement LP is positioned slightly higher than length measurement LP, and volume measurement LP is relatively higher than area measurement LP), as shown in Figure 1.

Transitions Across Levels

As we are also interested in how understanding of geometrical measurement progresses across levels, we incorporated previous research on thinking and reasoning at the transition between levels. For example, Gutiérrez, Jaime, and Fortuny (1991) argued that the van Hiele levels of geometrical reasoning are not discrete, assuming students in transition between levels show different levels of thinking at the same time. To support their assumption, they tested 50 eighth graders and preservice teachers using a test of three-dimensional geometry. To assign participants to a specific degree of acquisition within a van Hiele level, the researchers classified their responses into one of five qualitatively different degrees: (a) no acquisition, (b) low acquisition, (c) intermediate acquisition, (d) high acquisition, and (e) complete acquisition. Gutiérrez and colleagues found that in response to one item in a three-dimensional geometry assessment, some participants revealed two consecutive van Hiele levels of geometrical reasoning simultaneously, although the lower level acquisition was more complete than the upper level acquisition. In addition, most participants in the study engaged in several levels of reasoning simultaneously rather than using a single level of reasoning. This finding led us to hypothesize that the thinking and reasoning of geometrical measurement may occur continuously in terms of transitions across levels of LPs.

Drawing on the findings of Gutiérrez et al. (1991), we proposed that students' thinking and reasoning of measurement in one, two, and three dimensions progresses vertically within a dimension and up the five levels and therefore includes transition points between each level of the geometrical measurement LP (as shown in Figure 2). The five transition points for the geometrical measurement LP are characterized below:

- Transition points from Level 1 to Level 2: Early unit iteration begins to emerge and continues to develop in upper levels;
- Transition points from Level 2 to Level 3: Sufficient unit iteration and initial composite unit conception, as well as accumulation and initial interval-scale conception, emerge and continue to mature into upper levels;
- Transition points from Level 3.5 to Level 4: Efficient composite unit conception appears, and its spatial structuring carries through to upper levels;
- Transition points from Level 4 to Level 5: Sufficient conceptualization of symbolic representations of geometric properties and formulas for measurement and unit conversion has developed and is carried forward; begins to assign meaning to numerical quantities (i.e., numbers) for the lengths in three dimensions of an object with regard to the symbolic representations.

Connections Across Three Dimensions

Thus far, although some research has been conducted connecting understanding of measurement across length, area, and volume (e.g., Curry et al., 2006; Curry & Outhred, 2005), there is no research on the horizontal progression of students' understanding of geometrical measurement across one, two, and three dimensions (as shown in Figure 2).

Because spatial visualization ability is related to reading the two-dimensional representations of three-dimensional objects (Ben-Haim et al., 1985), we attend to spatial visualization ability in connecting understanding of measurement across one, two, and three dimensions. In addition, we think that the measurement of perimeter and surface area offers important opportunities to examine the role of spatial visualization in making connections across the three dimensions (e.g., for perimeter, see Barrett et al., 2006; for spatial visualization, see Ben-Haim et al., 1985). Although measuring of perimeter is a length measurement, perimeter exists only when a (two-dimensional) shape is present; similarly, surface area can be also placed between two and three dimensions.

We also propose horizontal connections across the length, area, and volume measurement LPs based on a generalized mistake in length to area measurement. For instance, in Dickson's (1989) study of students' strategies for finding the area of a rectangle, one elementary-school student determined the area of a 4-by-6 rectangle to be 3 multiplied by 5 by counting tick marks on the length and width of the rectangle rather than spaces between the tick marks. The use of this strategy may suggest that the student's insufficient conception of length unit led to similar incorrect area measurement. Since no previous study has connected understanding of measurement across the three dimensions in terms of a horizontal progression, further empirical study is needed to investigate this idea.

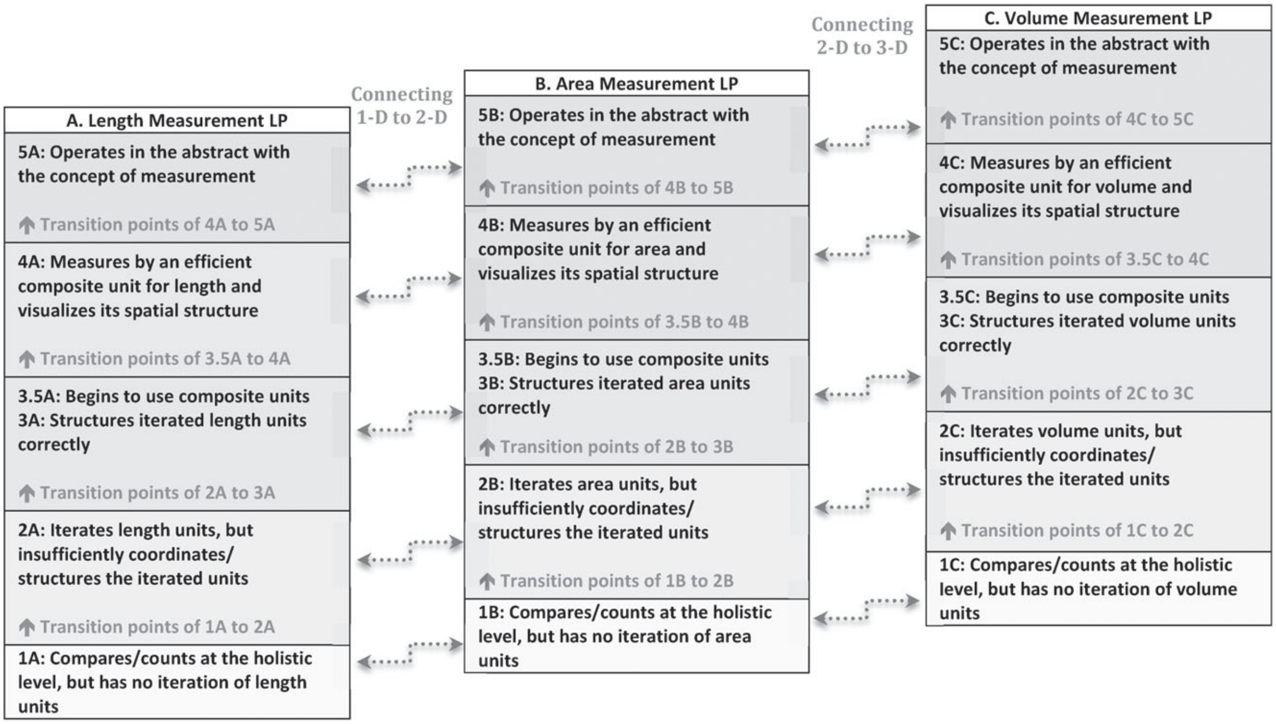


Figure 2 Geometrical measurement learning progression for length, area, and volume measurement with possible transitions within each dimension and connections across the three dimensions.

Panel Review of the Hypothesized Geometrical Measurement Learning Progression

This hypothesized geometrical measurement LP was reviewed by two external experts in the field of geometrical measurement and mathematics education and two internal ETS experts in spring 2016. Reviewers participated in two consecutive panel meetings and were asked to provide a detailed written review on the geometrical measurement LP in response to a set of guiding questions. In particular, we asked the reviewers to focus on (a) the accuracy and clarity of the content of each level of the geometrical measurement LP, (b) the transition points across five levels in each dimension of the geometrical measurement LP, and (c) the idea of the horizontal connections across the length, area, and volume measurement LPs.

We received positive feedback on the idea of having transition points between levels, as well as the description of each transition point, within a single LP and the idea of the horizontal connections across the three dimensions of our geometrical measurement LP. The reviewers suggested revising and enhancing the proposed LP in the following ways:

- including particular content in a certain level, such as adding the idea of space filling and measurement estimation with inconsistent/incorrect units to Level 2, or proportional reasoning related to unit conversion to Level 5;
- presenting the idea of correct unit iteration and use of composite units as two sublevels of Level 3, such as Level 3 in which correct iteration of measurement units appears and Level 3.5 in which composite unit iteration appears; and
- for Level 4, reframing the meaning of efficient composite unit for linear measurement in comparing to the concept of maximal composite units for area and volume measurement (i.e., for area, rows or columns of squares; for volume, layers of cubes, Battista, 2004).

According to each suggestion, we revisited the related literature and made several subsequent changes to the geometrical measurement LP. At the final panel meeting, all the changes that we made were discussed with and confirmed by the reviewers who commented on them initially. Appendix A contains the full version of the geometrical measurement LP.

At this panel meeting, the reviewers suggested conducting an empirical study to obtain evidence supporting the five levels of the geometrical measurement LP and to formulate the contents for the horizontal connections that occur within each level of the geometrical measurement LP and across the length, area, and volume measurement LPs. To allow for an empirical study, we designed a set of tasks to be given in a cognitive laboratory study.

Cognitive Laboratory Tasks for Geometrical Measurement Learning Progression

The purpose of task development is to examine students' understandings of geometrical measurement within each dimension of length, area, and volume measurement and across the three dimensions, mainly looking for evidence of vertical transitions along the five levels of the geometrical measurement LP and horizontal connections across the length, area, and volume LPs.

In considering the panel reviewers' comments on our proposed LP, the use of efficiently sized composite units for a presented measurement context is of particular interest for the development of cognitive laboratory tasks. The use of (efficient) composite units in measuring the length, area, and volume of a geometrical object is the main characteristic of Level 3.5 and Level 4 of the geometrical measurement LP (across the length, area, and volume measurement LPs; see Appendix A for the full version of the geometrical measurement LP). We believe that students' understanding of measuring the length, area, or volume of a geometrical object through the use of an efficient composite unit for the given object is crucial to the progression from iterating single units (Levels 2 and 3) to iterating (efficient) composite units (Levels 3.5 and 4) and to fully understanding measurement conceptually in the abstract (Level 5).

In addition to using composite units in measurement, spatial structuring and measurement estimation are also attended to in this task design. At the panel review, the reviewers proposed adding the idea of space filling and measurement estimation with inconsistent or incorrect units to Level 2 of the geometrical measurement LP; now these two abilities are anticipated to begin to emerge at Level 2 and carry through to the upper levels of these progressions. This change, which was made to the geometrical measurement LP according to the reviewers' suggestions, points to the need for empirical verification.

Thus, the idea of the vertical and horizontal progression of students' understanding and reasoning about length, area, and volume measurement, regarding the use of (efficient) composite units, spatial structuring, and measurement estimation, is the measurement goal of the cognitive laboratory tasks. To meet that goal, we created six measurement tasks targeting (a) measuring height and perimeter, area and surface area, and volume of a given geometric object (Tasks 1, 2, and 3); (b) estimating length, area, and volume of a box (Task 4); (c) filling a box with blocks (Task 5); and (d) estimating volume by using an everyday object (Task 6). See Appendix B for the full version of cognitive laboratory tasks.

Three Tasks for Horizontal Connections

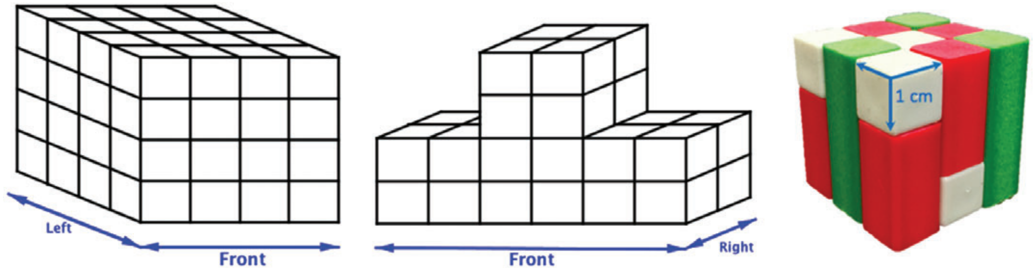
Three measurement tasks (Tasks 1, 2, and 3) are designed to examine students' understandings of (efficient) composite unit iteration and the horizontal connections across the length, area, and volume LPs in terms of perimeter and surface area measurement as well as spatial visualization.

Since the task design aims to examine individual students' concurrent understandings about length, area, and volume measurement, these three tasks use similar formats, although at different levels of difficulty. All three tasks present a two-dimensional representation of a three-dimensional object (i.e., a cuboid built from unit cubes, a stack of cubes having an irregular shape, and a stack of blocks of three different sizes, as shown in Figure 3). The tasks ask students to measure the height and perimeter and area of a face of the object, and the surface area and volume of the object. Levels of the geometrical measurement LP that are targeted by these tasks are Levels 2 to 3.5 (Task 1: cuboid), Levels 3 to 4 (Task 2: stacked cubes), and Levels 3.5 to 4 (Task 3: stacked blocks).

Each figure used in a task is designed to present an iterable set of single units in measurement (as a composite unit; Reynolds & Wheatley, 1996). In the context of a cuboid presented in Task 1, for example, a student may iterate a set of four length units (as a side length) to measure the perimeter of a square face of the given cuboid, a column or row of four squares to measure the area of the square face of the given cuboid, and a layer of 16 cubes to measure the volume of the cuboid.

Measurement Estimation Task

To examine students' understandings and reasoning about measurement estimation of length, area, and volume, in Task 4 we present a figure of a box in which the length and width measurements are given but the height measurement is not (see



(a) (b) (c)

Figure 3 Two-dimensional representation of a cuboid (a), stacked cubes (b), and stacked blocks (c).

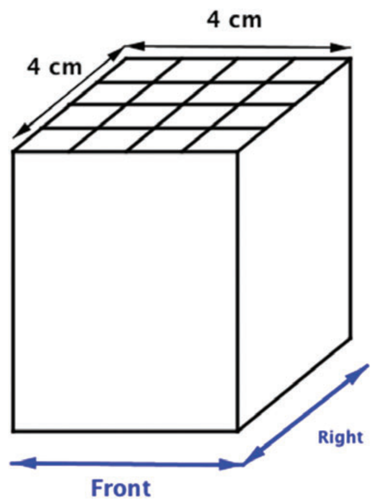


Figure 4 Two-dimensional representation of a box with height missing.

Figure 4). Levels 2 to 5 of the geometrical measurement LP are targeted for this estimation task, requiring visual inference in measurement estimation of height, area, and volume simultaneously.

The box figure given in this task is designed to present geometrical properties of a rectangular prism-shaped object, such as the congruence of each pair of opposite sides of a rectangular face of a box and the congruence of each pair of opposite rectangular faces of the box. The student is expected to use the properties of rigid transformations in the context of measurement estimation. In the figure of the box, for example, a student may obtain an estimate of the height of the box in terms of the known side lengths, and estimates of the area of the front face of the box and the volume of the box in terms of the known area of the top-face.

Manipulation Activity Related to the Use of Composite Units

Task 5, a manipulative activity, is designed to examine students' enumeration strategies of volume units (i.e., cubes) in measurement in structuring given blocks concretely. For this activity, we present a box (the inside dimensions of the box are 10 cm by 10 cm by 3.5 cm) and a set of Cuisenaire rods with lengths from 1 cm to 10 cm, asking students to find the number of cubes (1 cm by 1 cm by 1 cm) needed to fill the given box completely (see Picture 1). Levels 2 to 4 of the geometrical measurement LP with regard to the proposed progression from the iteration of single units to (efficient) composite units in volume measurement are targeted for this activity.

In the design of this manipulation task, the Cuisenaire rods are expected to play the role of composite units in volume measurement; for example, a student may recognize the equivalence relationship between 10 1-cm-long Cuisenaire rods and one 10-cm-long Cuisenaire rod, then figure out the number of 1-cubic-cm-sized objects needed to fill the given box drawn on the recognized equivalence relationship (see Picture 1 for a white 1-cm-long Cuisenaire rod and an orange 10-cm-long Cuisenaire rod).

Picture 1 Box of 10 cm by 10 cm by 3.5 cm with a set of Cuisenaire rods.

Task of Estimating Volume With an Everyday Object

Task 6, asking students to estimate the number of soup cans that can be put on a storage shelf, as well as the remainder, is designed to examine students' strategies of measurement estimation by using everyday objects (i.e., soup cans). Level 5 of the geometrical measurement LP, with regard to making visual inferences based on known numerical information in estimating volume with an everyday object and structuring the enumerated (set of) everyday objects spatially, is targeted for this estimation task in volume measurement.

In this task, we present a figure of a storage shelf 16 in. long by 12 in. wide by 12 in. high with the description of a given measurement context: "At an online store, Kim found her favorite brand of canned soups were selling at a discount price for a pack of four soup cans. Kim ordered 16 packs of different flavors of canned soups, and they are delivered today. Kim wants to stack the soup cans in a shelf of her pantry storage." To make students enumerate and structure the soup cans spatially, the numerical information about the soup cans is also given, such as "The dimensions (diameter and height) of each soup can are 3 in. by 4 in." For example, because the diameter of each can is 3 in. and its height is 4 in., a student may spatially place five cans along the 16-inch length of the shelf and four cans along the 12-inch width of the shelf, and then structure three layers of 20 cans in the 12-inch high shelf (see Figure 5).

We anticipate that the findings from our cognitive laboratory study using this set of six cognitive tasks will inform us about the vertical and horizontal progression of students' understanding and reasoning about the measurement of length, area, and volume and will contribute evidence to validate the levels of our geometrical measurement LP that feature the use of composite units, spatial structuring, and measurement estimation. This verified geometrical measurement LP will then be applied in the development of Winsight's geometrical measurement items in both summative and formative tasks.

Summary

We began this paper with the description of our conceptualization of geometrical measurement and the review of key concepts and existing LPs of geometrical measurement that provide the foundation for constructing one LP of geometrical

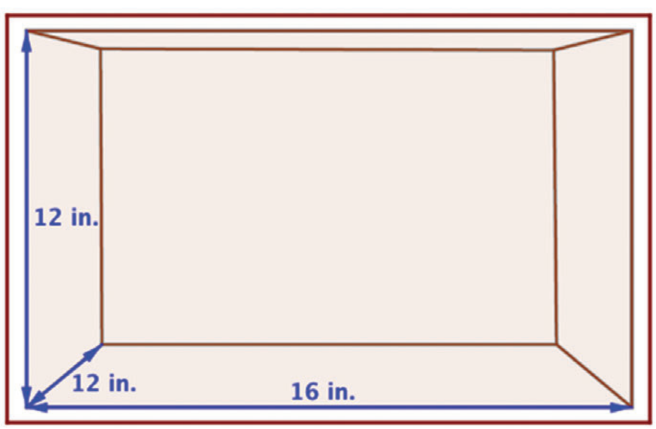


Figure 5 Two-dimensional representation of a storage shelf 16 in. long by 12 in. wide by 12 in. high.

measurement in terms of length, area, and volume measurement. Then, we presented our hypothesized geometrical measurement LP composed of the length, area, and volume measurement LPs and reported feedback given from the review panel on our LPs. Finally, we provided a set of cognitive laboratory tasks that were designed based on the levels of the geometrical measurement LP.

We believe this proposed geometrical measurement LP will contribute to the field of geometrical measurement by articulating the developmental sequence of students' understanding of geometrical measurement in terms of a vertical progression in each dimension and a horizontal progression across one-, two-, and three-dimensional measurements. The geometrical measurement LP that is investigated through cognitive laboratory study and analysis of Winsight pilot data may also help to strengthen instruction and assessment in geometric measurement. In particular, the six measurement tasks designed for a cognitive laboratory study are expected to guide future cognitive task design associated with the geometrical measurement LP (see Graf & van Rijn, 2016).

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Notes

1 An early draft of the five levels of this geometrical measurement LP was initially presented for linear measurement concepts in the *2014 End of Year Report to the* CBAL™ *Initiative* (Cayton-Hodges et al., 2014). We revised these five levels and also adapted them for area and volume measurement.

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Appendix A

Geometrical Measurement Learning Progression

Level	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
Level 1: Intuitive/holistic/visual comparison[a]	↑ **Transition points of 1A to 2A:** Early unit iteration for length begins to emerge and continues to develop in the upper levels of this progression. **1A: Compares/counts at the holistic level but has no iteration of length units**[b] (1) Perceives length/distance as an attribute of an object. [hypothesized] (2) Judges by appearance alone (van Hiele Level 1, as cited in Gutiérrez et al., 1991; see also Mason, 1998); may recognize only part of the visual cues of an object (e.g., longer/shorter, Clements, 1999; Clements & Battista, 1992). (3) Has no apparent conception of unit iteration but instead uses more of a holistic, visual comparison. [hypothesized from Barrett et al.'s, 2006, Level 1 for perimeter] Misconceptions and difficulties (1A): In comparing two equal-length strips, views one strip as longer, for example, by judging the left strip to be longer because it has longer pieces or the right strip to be longer because it has more pieces (Carpenter & Lewis, 1976, as cited in Clements, 1999).	↑ **Transition points of 1B to 2B:** Early unit iteration for area begins to emerge and continues to develop in the upper levels of this progression. **1B: Compares/counts at the holistic level but has no iteration of area units**[b] (1) Perceives area as an attribute of an object. [hypothesized] (2) Judges by appearance alone (van Hiele Level 1, as cited in Gutiérrez et al., 1991; see also Mason, 1998); may recognize only part of the visual cues of an object (Clements & Battista, 1992). (3) Has no apparent conception of unit iteration but instead uses more of a holistic, visual comparison. [hypothesized from length] ↗ **Connecting 1A to 1B:** N.A.	**1C: Compares/counts at the holistic level but has no iteration of volume units**[b] (1) Perceives volume as an attribute of an object. [hypothesized] (2) Judges by appearance alone (van Hiele Level 1, as cited in Gutiérrez et al., 1991; see also Mason, 1998); may recognize only part of the visual cues of an object (Clements & Battista, 1992). (3) Has no apparent conception of unit iteration but instead uses more of a holistic, visual comparison. [hypothesized from length] ↗ **Connecting 1B to 1C:** N.A.

[a]This level may be related to Clements and Battista's (1992) van Hiele Level 0 for measurement: pre-recognition and Mason's (1998) van Hiele Level 1: visualization. [b]Levels 1A, 1B, and 1C are hypothesized related to Clements and Battista's (1992) van Hiele Level 0 for measurement: pre-recognition, Mason's (1998) van Hiele Level 1: visualization, and Gutiérrez et al.'s (1991) van Hiele Level 1: recognition.

Level	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
Level 2: **Early unit concept** *(experiencing stage)*[a]	↑ **Transition points of 2A to 3A:** Sufficient unit iteration (with an equal-interval sized length unit) and initial composite unit conception, as well as length accumulation conception and initial interval-scale conception, emerge and continue to mature into the upper levels of this progression.	↑ **Transition points of 2B to 3B:** Sufficient unit iteration (with an equal-sized area unit) and initial composite unit conception, as well as spatial structure for area units, emerge and continue to mature into the upper levels of this progression.	**2C: Iterates volume units, but insufficiently coordinates/structures the iterated units**[b] (1) Perceives volume to be measured. [hypothesized from length] (2) Has insufficient unit conception (i.e., partitioning with inconsistent units) and is not using a fixed (equal-sized) volume unit (Battista, 2004; Battista & Clements, 1996). (3) Improperly coordinates/structures iterated volume units relative to each other (i.e., by leaving gaps and overlapping volume units; Battista, 2004; Battista & Clements, 1996). (4) Conceptualization of space filling may begin to emerge but may not use correct volume units (Battista, 2004). (5) Measurement estimation abilities may begin to emerge (Cayton-Hodges et al., 2014; Joram et al., 1998).
	2A: Iterates length units, but insufficiently coordinates/structures the iterated units (1) Perceives length/distance to be measured. [hypothesized from Barrett et al.'s, 2006, Level 1 for perimeter] (2) Has insufficient unit conception (i.e., partitioning with inconsistent units) and is not using a fixed (equal-interval sized) length unit (Stephan & Clements, 2003). (3) Improperly coordinates/structures iterated length units relative to each other (i.e., by leaving gaps and overlapping length units; Stephan & Clements, 2003). (4) Conceptualization of space filling may begin to emerge but may not use correct length units. [hypothesized from area and volume] (5) Measurement estimation abilities may begin to emerge (Cayton-Hodges et al., 2014; Joram et al., 1998).	**2B: Iterates area units, but insufficiently coordinates/structures the iterated units**[b] (1) Perceives area to be measured. [hypothesized from length] (2) Has insufficient unit conception (i.e., partitioning with inconsistent units) and is not using a fixed (equal-sized) area unit; Battista, 2004; Curry et al., 2006; Outhred & Mitchelmore, 2000). (3) Improperly coordinates/structures iterated area units relative to each other (i.e., by leaving gaps and overlapping area units; Battista, 2004; Outhred & Mitchelmore, 2000). (4) Conceptualization of space filling may begin to emerge but may not use correct area units (Battista, 2004; Outhred & Mitchelmore, 2000). (5) Measurement estimation abilities may begin to emerge (Cayton-Hodges et al., 2014; Joram et al., 1998).	Misconceptions and difficulties (2C): (a) Uses inconsistent unit iteration; makes double-counting errors (e.g., around the corners of a rectangular prism) and conceptualizes a set of cubes in terms of its faces (Battista, 2004) and (b) may apply measurement estimation strategies but may be inefficient or may not be fully correct. [hypothesized]
	Misconceptions and difficulties (2A): (a) Mixes/uses inconsistent units (e.g., in measuring the length of an object, mixes paper clips and pen tops or uses different-length paper clips as length units, Stephan & Clements, 2003; Clements, Battista, & Sarama, 1998; Lehrer, 2003); makes double-counting errors (e.g., around the corners of a path; Battista, 2006); (b) counts ticks using a ruler (Stephan & Clements, 2003) and counts or measures from the mark of 1 on the ruler instead of from the mark of 0 (Kamii, 1995; Lehrer, 2003) because students do not perceive the space between the mark of 0 and the mark of 1 as the amount of space covered by one unit (Stephan & Clements, 2003); and (c) may apply measurement estimation strategies but may be inefficient or may not be fully correct. [hypothesized]	Misconceptions and difficulties (2B): (a) Uses inconsistent unit iteration (Battista, 2004) and (b) may apply measurement estimation strategies but may be inefficient or may not be fully correct. [hypothesized] ↗ **Connecting 2A to 2B:** Carries over the mistakes in 1-D measurement to 2-D measurement. In a problem of calculating the area of a rectangle, counts the dimensions of a figure given with tick marks as 3×5 instead of 4×6 (counting only the segments); thus, calculates the area as $15\ cm^2$ and not $24\ cm^2$ (Dickson, 1989).	↗ **Connecting 2B to 2C:** N. A. ↑ **Transition points of 1C to 2C:** Early unit iteration for volume begins to emerge and continues to develop in the upper levels of this progression.

[a]This level may be related to Gutiérrez et al.'s (1991) van Hiele Level 1: recognition, and Mason's (1998) van Hiele Level 1: visualization and Level 2: analysis. [b]Levels 2B and 2C may be consistent with Battista's (2004) Level 1 for area and volume, respectively; and Level 2C may be consistent with Battista and Clements' (1996) Level C in terms of iteration of units and its insufficient spatial structuring.

Level	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
Level 3: Space filling/ covering with units *(analysis stage)*[a]	↑ **Transition points of 3.5A to 4A:** Efficient composite unit conception for length (efficient-sized/scaled composite units for a given length measurement context) appears, and its spatial structuring carries through to the upper levels of this progression. **3A: Structures iterated length units correctly** (1) Has conservation conception for length (Hiebert, 1984; Stephan & Clements, 2003). (2) Transitivity conception for length is present (Hiebert, 1984; Stephan & Clements, 2003). (3) Has sufficient unit conception (i.e., using equal-interval sized, consistent length units) and uses standard length units (rather than nonstandard units; Cayton-Hodges et al., 2014). [the first part is hypothesized from area] (4) Spatial position/structure of iterated length units is interiorized (i.e., by not leaving gap or overlap). [hypothesized from area and volume] (5) Sufficiently uses space filling/covering with length unit techniques (i.e., by using either physical or virtual objects/units to fill in line segments to determine length; e.g., given 4 units and a 6-unit length, a student lays the 4 units end to end and reuses some of them to fill/cover the full length; Lehrer, 2003). (6) Has conception of accumulation of length (Stephan & Clements, 2003). (7) Has sufficient measurement estimation abilities (Cayton-Hodges et al., 2014; Joram et al., 1998).	↑ **Transition points of 3.5B to 4B:** Efficient composite unit conception for area (efficient-sized/scaled composite units for a given area measurement context in terms of row-by-column structuring; Battista, 2004) appears, and its spatial structuring carries through to the upper levels of this progression. **3B: Structures iterated area units correctly**[b] (1) Has conservation conception for area (Stephan & Clements, 2003). (2) Transitivity conception for area is present. [hypothesized from length] (3) Has sufficient unit conception (i.e., using equal-sized, consistent area units; Battista, 2004) and uses standard area units (rather than nonstandard units). [the second part is hypothesized from length] (4) Spatial position/structure of iterated area units is interiorized (i.e., by not leaving gap or overlap; Battista, 2004). (5) Sufficiently uses space filling/covering with area unit techniques (i.e., by using either physical or virtual objects/units to fill in 2-D figures to determine area; Battista, 2004). (6) Has conception of accumulation of area. [hypothesized from length] (7) Has sufficient measurement estimation abilities. [hypothesized from length] **3.5B: Begins to use composite units** (1) Has the initial conception of composite units for area (i.e., recognizing a set of units as iterable units to generate the whole set; Battista, 2004; Stephan & Clements, 2003). (2) The conception of conversion between area units (i.e., unit conversion) is emergent (from larger to smaller units). [hypothesized from length] (3) As iterating composite units for area, uses (skip) counting (Battista, 2004). (4) Conceptualizes the inverse relationship between the size of the area unit and the number of units. [hypothesized from length]	**3C: Structures iterated volume units correctly**[b] (1) Has conservation conception for volume. [hypothesized from length and area] (2) Transitivity conception for volume is present. [hypothesized from length] (3) Has sufficient unit conception (i.e., using equal-sized, consistent volume units) and uses standard volume units (rather than nonstandard units). [hypothesized from area and length, respectively] (4) Spatial position/structure of iterated volume units is interiorized (i.e., by not leaving gap or overlap; Battista, 2004). (5) Sufficiently uses space filling/covering with volume unit techniques (i.e., by using either physical or virtual objects/units to fill in 3-D objects to determine volume; Battista, 2004; Battista & Clements, 1996). (6) Has conception of accumulation of volume. [hypothesized from length] (7) Has sufficient measurement estimation abilities. [hypothesized from length] **3.5C: Begins to use composite units** (1) Has the initial conception of composite units for volume (i.e., recognizing a set of units as iterable units to generate the whole set; Battista, 2004; Battista & Clements, 1996). (2) The conception of conversion between volume units (i.e., unit conversion) is emergent (from larger to smaller units). [hypothesized from length] (3) As iterating composite units for volume, uses (skip) counting (Battista, 2004; Battista & Clements, 1996). (4) Conceptualizes the inverse relationship between the size of the volume unit and the number of units. [hypothesized from length] Misconceptions and difficulties (3C): (a) Not counting hidden cubes (Battista, 2004; see also Ben-Haim et al., 1985; Hirstein, 1981); (b) after recognizing a layer composite, fails to visualize the position of each layer (Battista, 2004); and (c) when iterating composite units for volume, uses inconsistent-sized composite units (Battista, 2004).

Level	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
	3.5A: Begins to use composite units (1) Has the initial conception of composite units for length (i.e., recognizing a set of units as iterable units to generate the whole set; Stephan & Clements, 2003). (2) The conception of conversion between length units (i.e., unit conversion) is emergent (Cayton-Hodges et al., 2014, from larger to smaller units). (3) As iterating composite units for length, uses (skip) counting. [hypothesized from area and volume] (4) Conceptualizes the inverse relationship between the size of the length unit and the number of units (Cayton-Hodges et al., 2014; Hiebert, 1981).		
		↗ **Connecting 3A to 3B:** N. A.	↗ **Connecting 3B to 3C:** N. A. ↑ **Transition points of 2C to 3C:** Sufficient unit iteration (with an equal-sized volume unit) and initial composite unit conception, as well as spatial structure for volume units, emerges and continues to mature into the upper levels of this progression.

[a]This level may be related to Mason's (1998) van Hiele level 2: analysis. [b]Levels 3B and 3C may be consistent with Battista's (2004) Levels 2, 3, 4, and 5 for area and volume, respectively; and Level 3C may be consistent with Battista and Clements' (1996) Level B in terms of iteration of units (involving the use of composite units) and its spatial structuring.

Levels	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
Level 4: Interval-scale concept related to the use of efficient composite units *(formalization stage)*[a]	↑ **Transition points of 4A to 5A:** Sufficient conceptualization of symbolic representations of geometric properties and formulas for perimeter measurement and unit conversion has developed and is carried forward. Begins to assign meaning to numerical quantities (i.e., numbers) for the lengths of line segments and the side lengths of 2-D figures, such as the number of units iterated alongside the line segments; thus, relates the numbers to symbolic representations of geometric properties and formulas for perimeter measurement and unit conversion (Barrett et al., 2006).	↑ **Transition points of 4B to 5B:** Sufficient conceptualization of symbolic representations of geometric properties and formulas for (surface) area measurement and unit conversion has developed and is carried forward. Begins to assign meaning to numerical quantities (i.e., numbers) for the length and width of 2-D figures, such as the number of units iterated alongside each side; thus, relates the numbers to symbolic representations of formulas for (surface) area measurement and unit conversion (e.g., students can think about 3 and 5 of a 3×5 card as the number of inch measurements and reason that the card is composed of 15 in^2-sized squares, Reynolds & Wheatley, 1996; and they recognize that L and W represent the length and width of a rectangle in the area formula of $L \times W$; Battista, 2004; Huang & Witz, 2013). **4B: Measures by an efficient composite unit for area and visualizes its spatial structure**[b] (1) After iterating individual, single units within an efficient composite unit, successfully operates unit iteration with the composite units (e.g., on a task of measuring the area of a 5×7 rectangle, counts 5 squares across the rectangle and then iterates the row of 5 squares to determine the area of the rectangle in terms of the number of squares in the rectangle; Battista, 2004). (2) Visualizes the spatial structure of iterated efficient composite units for area (e.g., row-by-column structuring; Battista, 2004). (3) Has sufficient unit conversion conception for area. [hypothesized from length]	**4C: Measures by an efficient composite unit for volume and visualizes its spatial structure**[b] (1) After iterating individual, single units within an efficient composite unit, successfully operates unit iteration with the composite units (e.g., on a task of measuring the volume of a $4 \times 3 \times 3$ cube building, counts 12 cubes in the top layer and then iterates the layer of 12 cubes to determine its volume, the number of cubes in the building; Battista, 2004). (2) Visualizes the spatial structure of iterated efficient composite units for volume (e.g., layer structuring; Battista, 2004; Battista & Clements, 1996). (3) Has sufficient unit conversion conception for volume. [hypothesized from length]

Levels	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
	4A: Measures by an efficient composite unit for length and visualizes its spatial structure (1) After iterating individual, single units within an efficient composite unit, successfully operates unit iteration with the efficient composite units. [idea of efficient composite units is hypothesized from Barrett et al.'s, 2006, Levels 3a and 3b that incorporate "part-whole relations" (p. 195) for path length measurement of perimeter or routes on a grid/map, e.g., when iterating a side length of a polygon to measure around its entire perimeter, the side length can be efficient/maximal composite unit] (2) Visualizes the spatial structure of iterated efficient composite units for length. [hypothesized from Barrett et al., 2006, Level 3b for perimeter] (3) Has sufficient unit conversion conception for length (Cayton-Hodges et al., 2014). Misconceptions and difficulties (4A): May think the length of a line segment cannot be measured when the beginning of the line segment is not aligned with the mark of 0 of a ruler. [hypothesized from Lindquist, 1989].	↗ **Connecting 4A to 4B:** N. A.	↗ **Connecting 4B to 4C:** N. A. ↑ **Transition points of 3.5C to 4C:** Efficient composite unit conception (efficient-sized/scaled composite units for a given volume measurement context in terms of layer structuring; Battistia, 2004 ; Battista & Clements, 1996) appears, and its spatial structuring carries through to the upper levels of this progression.

[a]This level may be related to Gutiérrez et al.'s (1991) van Hiele Level 3: informal deduction. [b]Levels 4B and 4C may be consistent with Battista's (2004) Level 6 for area and volume, respectively; and Level 4C may be consistent with Battista and Clements' (1996) Level A in terms of iteration of maximal composite units and spatial structuring.

Levels	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
Level 5: **General model** *(abstraction stage)*[a]	**5A: Operates in the abstract with the concept of measurement** (1) Uses reasoning about length measurement (numerically and deductively) with spatial structuring of efficient composite units; thus, meaningfully conduct measurement using numbers to find the lengths of line segments and the perimeters of 2-D figures (Battista, 2012). (2) Assigns meaning to numerical quantities (i.e., numbers) for the lengths of line segments and the side lengths of 2-D figures; thus, relates the numbers to symbolic representations of geometric properties and formulas for perimeter measurement and unit conversion (Barrett et al., 2006). (3) By using transformations and geometrical properties of shapes, makes visual inferences in measurement estimation of length (Cayton-Hodges et al., 2014; Joram et al., 1998). (4) Masters conversion between length units and other operations by using formulas (Cayton-Hodges et al., 2014), incorporating the sophisticated proportional reasoning for unit conversion. [hypothesized] (5) Connects numerical procedures of a computation of perimeter to spatial structures of 2-D figures (Barrett et al., 2006) and relates the result of the computation to the number of iterated (composite) units. [hypothesized from area and volume]	**5B: Operates in the abstract with the concept of measurement**[b] (1) Uses reasoning about area measurement (numerically and deductively) with spatial structuring of efficient composite units (i.e., row-by-column structuring); thus, meaningfully conduct measurement using numbers to find the areas of 2-D figures (Battista, 2004; Battista, 2012) and the surface areas of 3-D objects (Battista, 2012). (2) Assigns meaning to numerical quantities (i.e., numbers) for the length and width of 2-D figures (Reynolds & Wheatley, 1996); thus, relates the numbers to symbolic representations of geometric properties and formulas for (surface) area measurement and unit conversion (Battista, 2004; Huang & Witz, 2013). (3) By using transformations and geometrical properties of shapes, makes visual inferences in measurement estimation of area. [hypothesized from length] (4) Masters conversion between area units and other operations by using formulas, incorporating the sophisticated proportional reasoning for unit conversion. [hypothesized] (5) Incorporates row-by-column structuring into various situations in which complex composite units are used (e.g., measuring the area of a 12×30 rectangle with 3×5 card; Battista, 2004; Reynolds & Wheatley, 1996) or in finding the areas of irregular 2-D figures. [hypothesized] (6) Connects numerical procedures of a computation of (surface) area to spatial structures of 2-D figures and relates the result of the computation to the number of the iterated (composite) units (Battista, 2004).	**5C: Operates in the abstract with the concept of measurement**[b] (1) Uses reasoning about volume measurement (numerically and deductively) with spatial structuring of efficient composite units (i.e., layer structuring); thus, meaningfully conduct measurement using numbers to find the volumes of 3-D objects (Battista, 2004; Battista, 2012). (2) Assigns meaning to numerical quantities (i.e., numbers) for the length, width, and height of 3-D objects; thus, relates the numbers to symbolic representations of geometric properties and formulas for volume measurement and unit conversion (Battista, 2004; Battista & Clements, 1996). (3) By using transformations and geometrical properties of shapes, makes visual inferences in measurement estimation of volume. [hypothesized from length] (4) Masters conversion between volume units and other operations by using formulas, incorporating the sophisticated proportional reasoning for unit conversion. [hypothesized] (5) Incorporates layer structuring into various situations in which complex composite units are used (Battista, 2004) or in finding the volumes of irregular 3-D objects. [hypothesized] (6) Connects numerical procedures of a computation of volume to spatial structures of 3-D objects and also relates the result of a computation to the number of the iterated (composite) units (Battista, 2004; Battista & Clements, 1996).

Levels	A. Length measurement learning progression	B. Area measurement learning progression	C. Volume measurement learning progression
	Misconceptions and difficulties (5A): Determines the perimeters of 2-D figures primarily through application of the formulas, with little or no connections to the geometric meaning of perimeter relative to 2-D figures. [hypothesized from volume]	Misconceptions and difficulties (5B): Determines the areas of 2-D figures primarily through application of the formulas, with little or no connections to the geometric meaning of area relative to 2-D figures. [hypothesized from volume]	Misconceptions and difficulties (5C): (a) In a package problem made from two cubes, multiplies $3 \times 5 \times 3$ instead of $6 \times 10 \times 3$ (Battista, 2004); (b) when shown a unit that is either half or double the size of the one used for measurement task, remeasures the given object with new units (Curry et al., 2006); and (c) determines the volumes of 3-D objects primarily through application of the formula $L \times W \times H$ with no understanding of layers in 3-D objects or multiplicative reasoning (Battista & Clements, 1996). ↗ **Connecting 5B to 5C:** N.A.
		↗ **Connecting 5A to 5B:** N.A.	↑ **Transition points of 4C to 5C:** Sufficient conceptualization of symbolic representations of geometric properties and formulas for volume measurement and unit conversion has developed and is carried forward. Begins to assign meaning to numerical quantities for the length, width, and height of 3-D objects, such as the number of units iterated alongside each side and face; thus, relates the numbers to symbolic representations of formulas for volume measurement and unit conversion (see the example given for area; Battista, 2004; Battista & Clements, 1996).

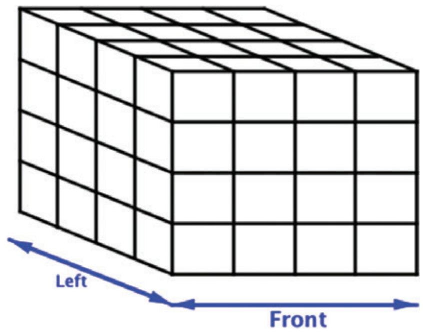
[a]This level may be related to Mason's (1998) van Hiele Level 4: deduction. [b]Levels 5B and 5C may be consistent with Battista's (2004) Level 7 for area and volume: abstraction stage of unit iteration and its spatial structuring.

Appendix B

Measurement Cognitive Laboratory Interview Tasks

Task 1: Cuboid

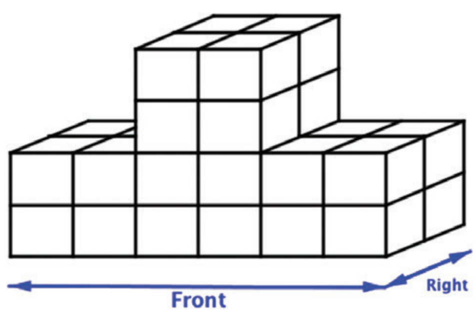
Here is a cuboid built from small cubes of the same size.



a. In the figure above, what is the height of the cuboid?
b. In the figure above, what is the perimeter of the front face of the cuboid?
c. In the figure above, what is the area of the front face of the cuboid?
d. In the figure above, what is the surface area of the cuboid?
e. In the figure above, what is the volume of the cuboid?

Task 2: Stacked Cubes

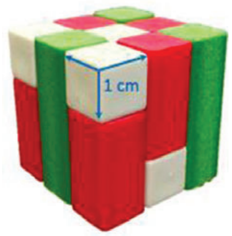
Here is a stack of cubes of the same size.



a. In the figure above, what is the height of the tallest part in the stacked cubes?
b. In the figure above, what is the perimeter of the front face of the stacked cubes?
c. In the figure above, what is the area of the front face of the stacked cubes?
d. In the figure above, what is the surface area of the stacked cubes?
e. In the figure above, what is the volume of the stacked cubes?

Task 3: Stacked Blocks

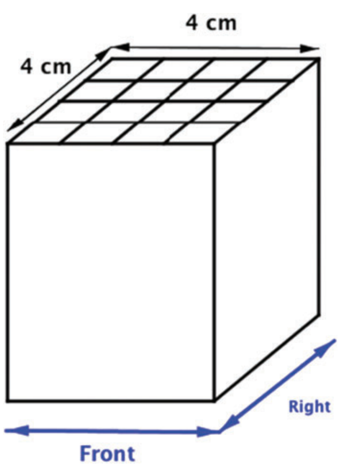
Here is a stack of rectangular blocks of three different sizes. The dimensions (length, width, and height) of all the "white" blocks are 1 cm by 1 cm by 1 cm.



a. What is the height of the stacked blocks?
b. What is the perimeter of a face of the stacked blocks?
c. What is the area of a face of the stacked blocks?
d. What is the surface area of the stacked blocks?
e. What is the volume of the stacked blocks?

Task 4: Estimating the Size of a Box

Here is a box that is 4 cm long and 4 cm wide.



a. In the figure above, estimate the height of the box.
b. In the figure above, estimate the area of the front face of the box.
c. In the figure above, estimate the volume of the box.

Task 5: Filling Cubes (Manipulative Activity)

How many 1-cm^3 cubes are needed to fill a given box completely?

Task 6: Soup Cans

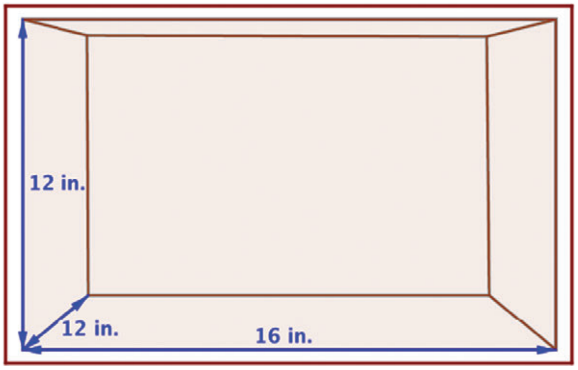
At an online store, Kim found her favorite brand of canned soups selling at a discount price for a pack of four soup cans. Kim ordered 16 packs of different flavors of canned soups, and they are delivered today.

Kim wants to stack the soup cans in a shelf of her pantry storage.

The dimensions (length, width, and height) of a shelf are 16 in. by 12 in. by 12 in.

The dimensions (diameter and height) of each soup can are 3 in. by 4 in.

If she does not want to stack the soup cans sideways, how many soup cans can Kim put in one shelf? How many soup cans are left?



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