Mistakes Made By Freshman Students of Science Teaching and Their Reasons During the Proving Process

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The aim of this study was to examine the mistakes made by freshman students of science teaching during the process of proving and the reasons for these mistakes. To this aim, the study, which was conducted via the case study method, was performed with 52 freshman students who were studying at the department of science teaching in a state university. A test composed of eight open-ended questions was used for data collection, and non-structured interviews oriented towards identifying the reasons for the mistakes made by the students were conducted with eight students.The content analysis technique was utilised in data analysis. It was found that many of the students made mistakes in terms of method, in other words, they used incorrect methods. In this regard, it was detected that more than half of the students regarded assigning numerical values as a method of proof, whereas a few of them made conceptual mistakes, algorithmic mistakes, misunderstood the questions, etc. Furthermore, it was determined that these mistakes resulted from reasons such as the fact that the students did not have any previous experience related to proof; they regarded assigning numerical values as a method of proof; and their lack of knowledge.

Introduction

Thinking is a process required for understanding new situations. In other words, it is the conceptualisation, implementation, analysis and criticism of the knowledge obtained by observation, experience, sense, and many other ways (Özden, 1997: pp. 79). One of the most important tools that improve thinking is mathematics (Tural, 2005). Mathematics can be defined as a process that explains the relationship that constitutes the essence of surrounding situations and it is beneficial in making decisions on both the current situation and the future. Mathematics is a process that starts with looking for patterns, discovering relationships and ends with a formal process such as 'proof' (Dreyfus, 1991).

The proof process holds a key role in the field of mathematics (Lakatos, 1976) and it is necessary for doing and understanding mathematics. It does not consist only of finding the answers of theorems, it also provides students with logical reasoning, which is required for mathematical understanding and reasoning (Polya, 1981). Mathematical proof may be used at all levels, including elementary, secondary etc.

One of the basic levels that requires using proof is the high school level. The high school years are those during which the process of abstract thinking develops and during these years the methods of deduction and induction methods are formed. Conversely, use of geometric proofs commonly takes part in the geometry curriculum related to high school (Altıparmak & Öziş, 2005). Geometric proofs are the opportunity to educate students on the foundations of mathematical principles. Therefore, teachers tell students in the lower levels of mathematics that they will prove why the sum of the interior angles of a triangle is 180 degrees (Jones, 1994). In this framework, students should be able to understand mathematical proof within both mathematics and geometry courses at high school. However; many students have difficulty in understanding the process of proof. Consequently; these students lack proving skills when they come to university, whereas proofs are also central to university level mathematics courses such as '*General*

Mathematics' and *'Algebra'*. In this context, students in mathematics programmes should be able to understand and construct mathematical proofs. The process of proving is also seen in mathematics courses related to science programmes. Students in science programmes can also use proof to verify or explain a statement with various methods in '*General Mathematics I or II'*. While some students use the method of induction proof, some use the method of deductive proof. Conversely, some students give special examples for justifying an argument in the process of proof.

Virtually, "Methods of Proof" is an important topic that explains the process of reasoning, finding the true value and using it. Therefore, it has an undeniable function in the practice of university mathematics (Hemmi, 2010). However; many university students may not see the functions (meaning, purpose and usefulness) of proof (de Villiers, 1999) and may not completely use the methods (induction, deduction, etc.) of proof. One of the most important factors regarding using these methods of proof correctly is the foreknowledge of students. Both students in mathematics and science programmes may not make the use of proof, since they do not exactly learn the methods of proof in high school.

There are many studies on the students and prospective teachers at the Mathematics Departments in Turkey. However; with the exception of Aydın, (2011), Gökkurt and Soylu (2012) and Gökkurt, Şahin, and Soylu (2014), little research has been done on students in Science Departments. These studies have also studied science students' abilities of making proof or their views to prove it. However; in Turkey, there is not any studies on what students' mistakes are in the process of making mathematical proof and on the reasons of these errors. But the mistakes while science students are making mathematical proof and to investigate the reasons of these mistakes are important. Because General Mathematics-I and General Mathematics-II courses in the first class of the Science Teaching Program in Turkey are taught. Since these courses content mathematical proof, science students must have the ability making proof. Therefore; the main purpose of this study is to identify freshman studentsand make them aware of any mistakes they make in understanding proof and give them basic knowledge related to "proof" within *Mathematics,* and *Geometry* courses and the reasons for their mistakes.

Review of Literature

Proof is an important subject of mathematics within the mathematics community (Healy & Hoyles, 2000). Proof broadens students' mathematical proficiency, because proof is "involved in all situations where conclusions are to be reached and decisions to be made" (Fawcett, 1938: p.120). According to Smith and Henderson (1959), proof is one of the pivotal ideas in mathematics. Therefore, it is the basis of mathematical understanding, and it is essential for establishing, communicating mathematical knowledge (Kithcer, 1984; Polya, 1981). It provides students with a better understanding of the concepts and a good belief in the results (Gökkurt & Soylu, 2012). At this point, mathematical proof plays an important role in the development of mathematical thinking and reasoning ability (Knuth, 2002; Tall, 2002). In addition, the critical thinking ability of students develops in the process of proving (Fawcett, 1938) and it provides students with new methods, tools and strategies for problem solving (Rav, 1999). Hanna and Barbeau (2008) claim that the teaching of proof plays an important role in conveying mathematical elements to students. Therefore, students obtain mathematical comprehension through their own experiences in the process of proving (Hanna, 1990).

Owing to the importance of proof in the development of mathematical thinking, many studies have focused on the mathematical proof (Vargese, 2009). These research studies have been concerned with students' or prospective teachers' understanding of proof, their views on proof, their level of proving, the role of proof in mathematics education and different perspectives of the teaching of proof (Gökkurt & Soylu, 2012; Güven, Çelik, & Karataş, 2005; Hanna, 1990; Harel & Sowder, 2007; Healy & Hoyles; 2000; Hersh, 1993; Miyazaki, 2000; Raman, 2003; Stylianides, 2007; Yang & Lin, 2008; Weber, 2005).

Conversely, some studies highlight the recurrent difficulty that students face with proof. That is, they report that students have a poor understanding of proof and have difficulties in constructing their own proofs. Many students do not understand exactly the concept of proof. It is commonly considered by many students to be a challenging, formidable, and unpleasant process (Almeida, 2003). Therefore, they have many difficulties in the process of proving. It has been observed thatthere are many things that cause these difficulties such as: lack of knowledge about the definitions of proof and how to use them (Edwards & Ward 2004; Knapp, 2005; Moore, 1994; Weber, 2006), not understanding the nature of proof, mathematical rules and proof strategies (Gibson, 1998; Weber, 2006) and not to use mathematical language correctly (Baker & Campbell 2004; Edwards & Ward 2004; Knapp, 2005; Moore 1994). Meanwhile, Anapa and Şamkar (2010) and Jones (2000) state that students have difficulties in understanding proof theorems and lack self-trust in proving. The authors of these studies claim that students could not use proof methods and techniques sufficiently and could not form proof at an expected level.

The purpose of the Study

This study has two aims. The first aim is to identify the mistakes made by the students of science during the process of proving. The second aim is to determine the reasons for these mistakes. In the context of these aims, the problems shedding light on the study are as follows:

- What are the mistakes made by the first-year students, who were studying at the Department of Science Teaching, during the process of proving?
- What are the reasons underlying the mistakes made by the first-year students, who were studying at the Department of Science Teaching, during the process of proving?

Method

The case study method, which is based on the qualitative research approach, was used in the study. In a case study, a system with specific boundaries or a case is deeply discovered with the help of different data collection tools (Creswell, 2007; McMillan & Schumacher, 2010). The fact that more than one data collection tool (written statements from the students, non-structured interviews and audio recording) were used and the fact that the case study method allows for obtaining rich data to examine the mistakes made by the students regarding mathematical proof can be considered as the reasons for preferring the case study method.

Participants

The participants were 52 freshman students who were studying at the Department of Science Teaching in a state university in Turkey. They are selected using the purposive sampling method. When it is considered that the study was held towards the end of the period, freshman students have taken General Mathematics-I and General Mathematics-II.

Data Collection Tools

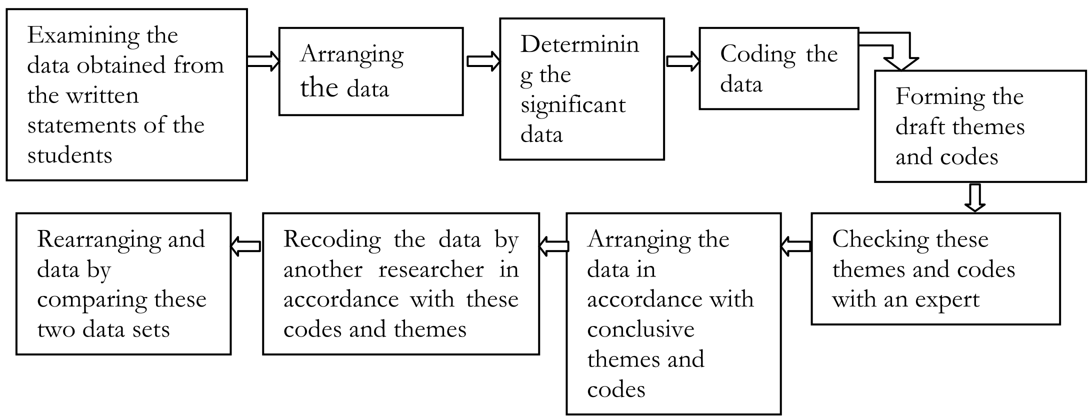
A test, which was composed of 21 open-ended questions that required basic proof skills primarily in the field of geometry and numbers, was prepared in the study. The questions, which were featured in the test, were prepared for the first problem of the study. The questions were prepared by considering the literature (Altıparmak & Öziş, 2005; Baki, 2008; İmamoğlu, 2010; Gökkurt, Şahin, & Soylu, 2014; Ören, 2007; Özer & Arıkan, 2002) and geometry course sources for the reliability of the study. An expert and three researchers were consulted in terms of content, language and level to maintain the validity of the questions. The number of questions was reduced to eight after performing the necessary revisions. The fact that there were questions with a high level of difficulty; that there were more than one question that measured the same proof skills; and that there was not enough time for implementation were given as the reasons for omitting the questions.

The non-structured interview technique was used as the data collection tool regarding the second question of the study. Interviews were conducted individually with eight participants. By doing so, an attempt was made to obtain detailed data via maintaining a flexible environment during the interviews. The purposeful sampling method was used in selecting these eight participants. It has been given priority to the selection of students making different errors in order to obtain rich data in the specifying of eight students. The interviews were conducted at the hours deemed suitable by the students in accordance with the requests of the students in order to perform the interviews. Each conducted interview lasted for approximately 15-20 minutes and was audio-recorded with the consent of the students.

Data Analysis

An answer key, which contained the correct answers that might be given to each question in the test, was prepared in analysing the data. Then, incorrect answers from the students were identified in accordance with this answer key, and a content analysis was conducted on these answers. The steps taken in analysing the data that belonged to the first problem of the study are given in a diagram in Figure-1.

Figure 1. The scheme of data analysis



As seen in Figure 1, the obtained data was recoded by another researcher. Then, these codes were compared with the codes that were made by the researcher. The reliability percentage was found to be 92% at the completion of coding. The resulting different codes were rearranged by coming to an agreement. These themes and codes were divided into significant sections as follows:

Themes	T_1	T_2	T_3
Codes	Making an algorithmic mistake	Assigning numerical values	Writing the question as is
		Benefiting from special triangles	
	Making a conceptual mistake	Showing $q \Rightarrow p$ instead of $p \Rightarrow q$ by assigning values	Blank
	Making algorithmic and conceptual mistakes	Trying to generalise by assigning numerical values	
		Performing operations by accepting the assertion as true	
		Incorrect use of mathematical concepts and notations	
		Unrelated answers	

T_1: Making mistakes in implementing the method in spite of the correct method selection
T_2: Making mistakes in the method selection
T_3: Not selecting any method

First of all, audio records were transferred to computer environment in order to analyse the data that was obtained from the interviews conducted for the second research question. The interview records were transcribed in a computer and were checked by the participants. Then, these records were examined by the researcher, and significant data was extracted. The content analysis was used in analysing the data. The codes obtained from the content analysis were featured in the results section.

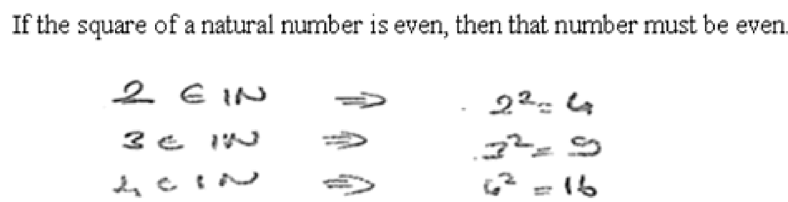
Results

The results, which were reached as a result of qualitative analyses, are featured in this section. The results regarding the mistakes detected from the written statements of the students are featured in the first section, whereas the results obtained from the interviews, which were conducted in order to identify the reasons for the students' mistakes, are featured in the second section.

Table 1. Frequency and Percentage Distributions of the Wrong Answers Given to the First Question

Themes	Codes	f	%
T_1	Making a conceptual mistake	2	4
T_2	Assigning numerical values	28	54
Total		30	58

When Table 1 is examined, it is observed that more than half of the participants selected assigning a numerical value as a method of proof. In their written statements, these participants stated that the expression $a^2 + b^2$ was an odd number by assigning an even number value to a whileassigning and odd number value to b. Two of the participants were not able to represent the proof in a fully correct manner or they made a conceptual mistake by using the expression $(a^2 + b^2)$ instead of $a^2 + b^2$, although they both used correct methods. However; 42% were able to prove the given theorem in a correct manner. When Table 2 is examined, it is observed that nearly all participants (92%) made mistakes in selecting methods (T_2) during the process of proving. It was found that approximately half of them (48%) assigned numerical values as a proof method; one fourth tried to show the opposite of the given expression; and 13% tried to reach a generalisation. Four remaining participants tried to prove by either accepting the given claim as true or writing the question as is. It is considerably interesting that none of the participants were able to give a fully correct answer to this question. Instead of showing that the number, whose square was an even number, was itself an even number in the expression *"If the square of a natural number is even, then the number* $p \Rightarrow q$ *itself is even"*, the participants, who tried to show $q \Rightarrow p$ instead of by assigning numerical values, tried to show the opposite of this expression by assigning values.In respect to this, the answer of participant



S_{43} is given below as is:

Figure 2. S_{43} student's wrong answers to question no. 2

Table 2. Frequency and Percentage Distributions of the Wrong Answers Given to the Second Question

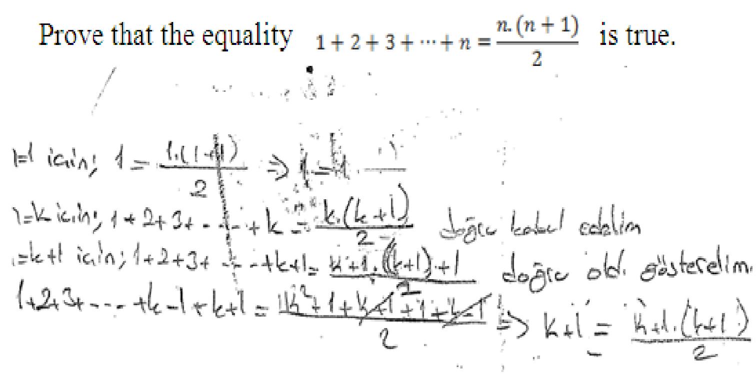
Themes	Codes	f	%
T_1	Making a conceptual mistake	2	4
T_2	Assigning numerical values	25	48
	Showing $q \Rightarrow p$ instead of $p \Rightarrow q$ by assigning values	13	25
	Trying to generalise by assigning numerical values	7	13
	Performing operations by accepting the assertion as true	3	6
T_3	Writing the question as is	1	2
	Blank	1	2
Total		52	100

Table 3. Frequency and Percentage Distributions of the Wrong Answers Given to the Third Question

Themes	Codes	F	%
T_1	Making a conceptual mistake	3	6
	Making an algorithmic mistake	2	4
	Making algorithmic and conceptual mistakes	2	4
T_2	Assigning numerical values	36	69
	Trying to generalise by assigning numerical values	3	6
	Unrelated answers	3	6
T_3	Blank	2	4
Total		51	99

It is understood from Table 3 that 14% of the participants developed correct strategies (T_1) during the process of proving, and they tried to use the induction proof method. However; these participants made algorithmic mistakes and conceptual mistakes since they experienced some difficulties during the process of proving. In respect to this, participant S_{40}wrote the $(k+1)$ term instead of*n* and made a conceptual mistake, and performed the operation by considering the term, which came before$(k+1)$,as $(k-1)$instead of *k*.That participant incorrectly implemented the induction method in the remaining part of the operation and made an operational mistake. The answer from that participant is given below as is:

Figure 3. S_{40} student's Wrong Answers to question no. 3



It is again understood from Table 3 that many of the participants fell short of developing strategy and method, and they resorted to incorrect methods. A total of 69% of them proved by assigning a single numerical value, whereas 6% tried to make a generalisation by assigning more than one numerical value. Only one participant was able to faultlessly complete the proof given in this question. It is considerably interesting that the participants were so inadequate in the induction proof method that requires a simple proof and holds an important place in both high school mathematics and university mathematics.

When Table 4 is examined, it is observed that the participants made fewer mistakes in proving the expression "the sum of the interior angles of triangles is $180°$", which is basic knowledge in the field of geometry, than they did in the previous questions, and half of them expressed this proof in a fully correct manner. Seven of the participants, who selected an incorrect method, performed the operation by accepting that the sum of the interior angles of a triangle is $180°$, whereas four of them gave answers that were not related to the solution of the proof. Seven students tried to prove by using special triangles. In respect to this, the answer of participant S31 is given below as is:

Figure 4. S_{31} student's wrong answer to question no. 4

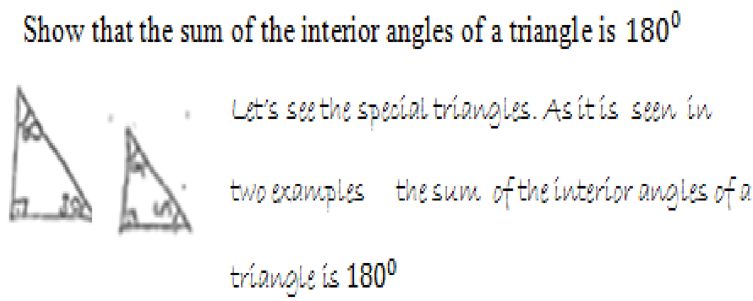


Table 4. Frequency and Percentage Distributions of the Wrong Answers Given to the Fourth Question

Themes	Codes	f	%
T_1	Making an algorithmic mistake	2	4
T_2	Benefiting from special triangles	7	13
	Performing operations by accepting the assertion as true	7	13
	Unrelated answers	4	8
T_3	Writing the question as is	3	6
	Blank	3	6
Total		26	50

It is observed from Table 5 that nearly all participants (91%) failed in the fifth question and were unable to fully prove the correctness of the theorem. When the answer papers of the participants are examined, it is observed that half of them used the Pythagorean relation by accepting the correctness of this rule instead of proving the rule that shows the relationship among the side lengths in a

$30° - 60° - 90$ ‘triangle.In respect to this, the answer of participant $S_1$is given below as is:

Figure5. S_1 student’s wrong answer to question no. 5

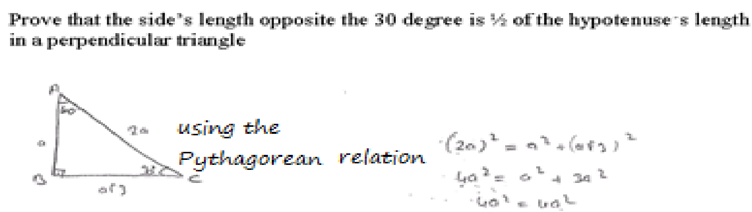


Table 5. Frequency and Percentage Distributions of the Wrong Answers Given to the Fifth Question

Themes	Codes	f	%
T_2	Assigning numerical values	14	27
	Performing operations by accepting the assertion as true	26	50
	Unrelated answers	4	8
	Incorrect use of mathematical concepts and notations	1	2
T_3	Blank	2	4
Total		47	91

When Table 6 is examined, it is observed that all students made mistakes during the process of proving, whereas half of them used incorrect strategies and methods (T_2) during the process of proving. As seen in Table 6, approximately one fourth of the students experienced difficulty in proving that *“an inscribed angle is half of a central angle that subtends the same arc”*, and they were unable to make any written statements. Conversely, 21% of students incorrectly used mathematical concepts and notations in showing the relationship between the inscribed angle and the central angle. One of these participants incorrectly used mathematical concepts by using expressions such as *"The more an angle decreases, the more its intercepted arc increases; the central angle and the inscribed angle are equal since they subtend the same arc.* It is considerably interesting that the participants experienced difficulty in proving this expression, which is frequently used by teachers in high school geometry courses.

Table 6. Frequency and Percentage Distributions of the Wrong Answers Given to the Sixth Question

Themes	Codes	f	%
T_2	Assigning numerical values	6	12
	Performing operations by accepting the assertion as true	5	9
	Unrelated answers	4	8
	Incorrect use of mathematical concepts and notations	11	21
T_3	Writing the question as is	5	10
	Blank	21	40
Total		52	100

Table 7. Frequency and Percentage Distributions of the Wrong Answers Given to the Seventh Question

Themes	Codes	f	%
T_2	Assigning numerical values	27	52
	Performing operations by accepting the assertion as true	3	6
	Trying to generalise by assigning numerical values	12	23
	Unrelated answers	1	2
	Incorrect use of mathematical concepts and notations	1	2
T_3	Writing the question as is	5	9
	Blank	3	6
Total		52	100

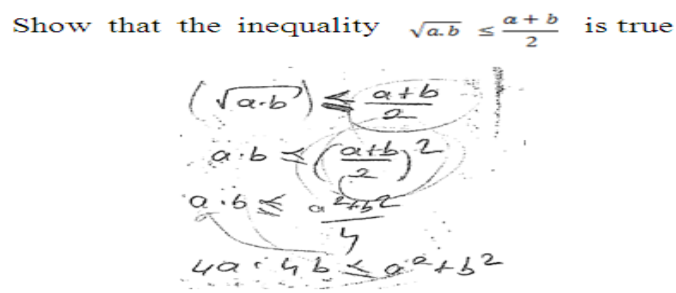
Table 7 shows the difficulties experienced by the students in the proof regarding the divisibility by nine that is among the divisibility rules. Just like in the second and the sixth questions, all participants experienced problems in showing the correctness of the given expression, and they preferred making a generalisation by assigning numerical values as the proof method. One of the participants gave an insignificant answer that was not related to the question: *"If we multiply a number with the same number and then divide it by the same number, the result is the original number..."*

Table 8. Frequency and Percentage Distributions of the Wrong Answers Given to the Eight Question

Themes	Codes	f	%
T2	Assigning numerical values	33	63
	Performing operations by accepting the assertion as true	14	27
	Unrelated answers	2	4
	Incorrect use of mathematical concepts and notations	1	2
	Writing the question as is	1	2
T3	Blank	1	2
Total		52	100

When Table 8 is examined, it is observed that the participants were considerably inadequate in proving the inequality that represents the relationship between geometric mean and arithmetic mean. The participants did not know how to begin the proof and they resorted to incorrect methods. As seen in Table 8, the majority of the participants (90%) either tried to show the inequality as correct by assigning numerical values or attempted to reach a result by regarding the given inequality as correct and performing algorithmic operations on this inequality. The answer of participantS_{48}, who performed the operation by regarding the given inequality as correct, clearly shows this:

Figure6. S_{48} student's wrong answer to question no. 8



Results obtained from the interviews

In this section, an answer was sought to the question "What are the reasons underlying the mistakes made by the students during the process of proving?". In view of the data that was obtained from the interviews conducted with eight students for that purpose, it is observed that the reasons for making mistakes are connected with many different factors. These factors are classified below in three different themes as follows: "The reasons for the mistakes made by the students while implementing the method in spite of correct method selection", "The reasons for students' selecting incorrect methods" and "The reasons for students' not selecting any method".

Students' Opinions on the Reasons for the Mistakes Made by the Students While Implementing the Method in Spite of Correct Method Selection
Carelessness
Weak operation skill
Deficient or wrong learning in previous experiences

As mentioned above, students made mistakes resulting from different reasons, although they selected correct methods during the process of proving in the research. In view of the conducted interviews, it is observed that these mistakes resulted from *carelessness, weak operation skill and deficient or wrong learning in previous experiences*. The data obtained from the interviews conducted with the students clearly shows this condition. Students' opinions on this issue are as follows:

> ...I was careless while proving. When I examined my paper, I understood that I had made an operational mistake. I think my answer would have been correct if I had not made an operational mistake..."
> "...We solved a similar question like this in high school. From what I learned in high school, I think we would give value k instead of n. Then, we would give value $k+1$ value instead of n and show its correctness. I did so, but I cannot remember the name of the method....

When the above statement is considered, it is observed that the student is not aware that the method he used is the induction method, and he knows that he must assign value 1instead of while implementing this method.

Students' Opinions on the Reasons for Students' Selecting Incorrect Methods
Seeing its correctness more easily
Using assigning value as a method in high school
Thinking that assigning numerical values is an easy method of proof
Not knowing any method other than assigning numerical values

When the interviews conducted with the students are examined, it is observed that the reasons for students' selecting incorrect methods while proving is related to four factors. As can be understood from the wrong answers of the students, nearly all students used assigning numerical values as a method of proof. In respect to this, many of the students who gave statements explained the reasons for assigning numerical values as follows: they see the correctness of the given expression more clearly with the help of numbers; they used this method in high school; and they consider assigning values as a method of proof. In this regard, the opinions of two students are given below.

> Assigning values is useful in terms of its correctness. I can see easily when I give values. I think the solution cannot be found otherwise...
>
> I thought that I could see more easily by assigning numerical values. In my opinion, assigning values is also a method of proof. We would reach solutions generally by assigning values in high school. I tried to find the solution using what I remembered from those days. That is because I cannot see it when I assign letters. I don't understand what will come from where....

Another student, who preferred assigning values, stated that the basic reason for his using numbers during the process of proving is the fact that no method came to his mind other than assigning values. He gave the following statement in the conducted interview:

> I wanted to assign values when no method came to my mind....

The Reasons for Students' Not Selecting Any Method
Prejudice against the geometry course
Lack of knowledge
Encountering the proof for the first time
Teachers' not giving importance to proof and their giving examination-oriented courses
Not fully understanding the question

In view of the interviews conducted with the students, it is observed that students' not being able to develop a strategy is related to many factors. A student, who had prejudice against geometry course that is among these factors, stated the following as the reason for not answering the questions that required proof about geometry.

> I was not at all good at geometry in high school at. I do not remember it well. I always accomplished certain things with mathematics. That is why I did not answer the question about geometry. I did not even feel the need to read the question because of my prejudice against the geometry course....

Conversely, three students gave the following reasons for not proving the statement "an inscribed angle is half of a central angle that subtends the same arc": they did not have the knowledge to prove it; they had not encountered the proof before; and they were not able to fully comprehend the question. In this regard, excerpts from the three students who make statements are given below:

> ...I have no idea about the arc. I do not have the knowledge to prove it. Besides, I encountered the proof for the first time. We would use this rule in high school, but I did not encounter a question

> requiring its proof. That is why I was not able to solve it....
>
> I did not do anything since I did not have any idea about this question. Our teacher would sometimes show the proof in mathematics course in high school, but he would not particularly emphasise it. Besides, we would always solve questions since we were preparing for YGS-LYS examinations. We would not prove them. I did not dwell upon the proof since it was not asked in the examinations..."
>
> "Actually, I was not able to fully understand this question. That is why I was not able to do anything....

Discussion

The mistakes made by the students of science during the process of proving and the reasons for these mistakes were set forth in this study. None of the 52 students who participated in the study was able to fully and correctly prove all of the eight questions that required simple proof and that were included in secondary education mathematics curriculum. As a matter of fact, nearly all students made mistakes during the process of proving in questions, except for the first and fourth questions. It is clearly observed that many of the students tried to prove by especially assigning numerical values in this process. The fact that Knuth, Choppin, Slaughter, and Sutherland (2002) stated that students utilised numerical values and examples while proving support the result of this study. Similarly, Güler (2013) and Güler, et al. (2011) stated that prospective mathematics teachers were inadequate in performing mathematical induction, and they preferred proving generally by giving examples. In view of the conducted interviews, it was concluded that some factors – such as the fact that students used assigning values as a method in high school; that they thought that using variables is difficult; and that they thought that it was easy to prove by assigning values – were influential on the emergence of this mistake.

When other mistakes made by the students were examined, it was observed from the data of the qualitative analyses that the students made algorithmic mistakes and conceptual mistakes. It is seen that an "algorithmic mistake", that is to say, a mathematical operation mistake was dominant in the third and the fourth questions, whereas a "conceptual mistake" was dominant in the first three questions. It was observed that the students generally assigned 1, k and $k+1$ values instead of n, particularly in the third question that required the induction proof method, but they were inadequatein terms of knowing what to do with the operations that contained these symbols and on how they would reach a generalisation. This obtained result shows parallelism with the result of Harel (2001)'s study. In view of the conducted interviews, it can be stated that the reasons for these mistakes are as follows: carelessness of the students, their weak operational skills and their previous experiences. Conversely, it was observed from the data of the interviews that the following factors were influential in students' not selecting any strategy during the process of proving: they did not know the methods of proof; their teachers in high school did not attach importance to the proof; their teachers gave examination-oriented courses; and they did not have previous experiences about the proof. Furthermore, in the interviews conducted with the students, some students stated that they encountered the proof for the first time, whereas others stated that they encountered the proof in especially geometry courses in high school, although their high school teachers did not emphasise the proof and they gave an examination-oriented instruction. These reasons show similarity with Yıldız (2006)'s reasons for the difficulties experienced by students in the process of proving.

Proving is an important skill for the mathematics courses that are given in universities. Good understanding and implementation of the proof by the students is among the basic objectives of the mathematics course. The role of theorem and proof is significant, especially in the 'General Mathematics' course (Yıldız, 2006). However; proving is a process in which both high school students and university students experience problems and fail, and generally it is a disliked process (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006). As is understood from the results of this study, many of the students experienced various difficulties during the process of proving and made many mistakes during this process. In this regard, the following suggestions can be offered, which will improve students' proving skills and which will be of use to the researchers who will conduct further studies in this field.

- The students, who proved by assigning numerical values during the process of proving, think that assigning certain values is enough to prove the correctness of the given assertion. However; emphasis must be given to the fact that assigning numerical values can be used in order to show the incorrectness of an assertion instead of showing its correctness.
- Among the reasons for the difficulties experienced by the students during the process of proving is the fact that high school teachers did not attach importance to the proof and they gave examination-oriented courses. In this regard, teachers must feature adequate proof operations in mathematics courses.
- When the literature was examined, it was revealed that the students made many mistakes during the process of proving in this study. In this regard, it can be suggested that mathematics educators should be aware of the

mistakes and their reasons stated above, and they should conduct studies in order to eliminate these mistakes.

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