One-Week Module on Stochastic Groundwater Modeling

David C. Mays[1]

ABSTRACT

This article describes a one-week introduction to stochastic groundwater modeling, intended for the end of a first course on groundwater hydrology, or the beginning of a second course on stochastic hydrogeology or groundwater modeling. The motivation for this work is to strengthen groundwater education, which has been identified among the factors contributing to the lack of stochastic groundwater modeling in professional consulting practice. The educational objectives are for students to (1) define key terminology, (2) explain spatial correlation, (3) produce realizations of groundwater flow, and (4) critique deterministic groundwater models. This one-week module includes a reading assignment, a class presentation, a guided computer exercise, and a homework assignment. The module introduces students to a few basic terms and concepts, then gives them experience through hands-on computer exercises. This article includes a detailed lesson plan and homework assignment, and complete model inputs and solutions are provided. The guided computer exercise and the homework assignment are performed using the freely available software Processing Modflow for Windows (PMWIN). Submitted homework assignments demonstrate that students were able to transfer skills from the module to a new application, and through an assessment survey, students reported significant improvement in their ability to perform three of the four educational objectives.

INTRODUCTION

Students of groundwater hydrology enjoy several benefits from an introduction to groundwater modeling software, such as MODFLOW (Harbaugh and McDonald, 1996). From a practical perspective, students gain a skill that is useful in consulting practice. From an educational perspective, students gain insight into the geometry and dynamics of groundwater flow, since results are presented visually. In addition, learning groundwater modeling provides an opportunity to develop higher-level cognitive skills, as students critically evaluate their model in terms of its conceptual basis, assumptions, and intended use. These cognitive skills allow students to appreciate the well-known notion that "all models are wrong but some are useful" (Box, 1979).

Perhaps the most important assumption in elementary groundwater models is that of homogeneous hydraulic conductivity. This assumption is a major simplification, meaning that predictions of production rates, power requirements, and water quality may suffer from major errors. Deterministic models lack the ability to quantify these errors. Stochastic groundwater models, in contrast, allow the modeler to calculate the uncertainty of model results, most commonly by modeling the heterogeneous hydraulic conductivity as a spatially-correlated random variable. In academic research, stochastic groundwater modeling is a standard approach, dating back to the 1970s (Freeze, 1975), for which standard textbooks are available (Rubin, 2003; Zhang, 2002). However, stochastic groundwater models are not widely used in consulting practice (Schwartz and Ibaraki, 2001), and several explanations have been proposed (e.g., Pappenberger and Beven, 2006; Zhang and Zhang, 2004). One explanation is the lack of information to quantify the heterogeneity of typical aquifers; this concern has been partially addressed by a published summary of the mean, standard deviation, and spatial correlation length of hydraulic conductivity at numerous sites (Tables 10.7 and 10.8 in Rubin et al., 1999), and by the World Wide Hydrogeological Parameters Database at wwhypda.org (Comunian and Renard, 2009). A second explanation is a lack of software to implement stochastic groundwater models; this concern has been partially addressed by MODFLOW-STO (Liu et al., 2006). A third explanation is that while stochastic groundwater modeling is standard in academic research, it is not yet standard in groundwater education (Neuman, 2004; Renard, 2007; Winter, 2004). The objective of this article is to partially address this third explanation.

Li and Liu (2004) present a compelling argument for the need for a constructive approach when teaching groundwater hydrology, in which students first learn specific concepts, then integrate them into a general understanding. Since then, several constructive methods for teaching groundwater have appeared in the geoscience education literature. Neupauer (2008) presents a semester-long containment design project that integrates various groundwater topics. Neupauer and Dennis (Neupauer and Dennis, 2010) describe an in-class demonstration that allows students to discover Darcy's law. All of these studies are based on the premise that active learning—in which students manipulate physical objects, build computer models, or participate in classroom discussions—can be more effective than traditional lecturing (*e.g.*, Light, 2001; Lowman, 1995). However, articles on active learning methodology for stochastic groundwater modeling appear to be absent from the geoscience education literature.

This article describes a one-week introduction to stochastic groundwater modeling, intended for the end of a first course on groundwater hydrology, or the beginning of a second course on stochastic hydrogeology or groundwater modeling. This brief module is not intended to create expert stochastic modelers. Instead, the goal is for students to learn the qualitative concepts of stochastic groundwater modeling through a guided computer exercise, in order to gain insight into the limitations inherent in deterministic groundwater models. The educational objectives for this one-week module are listed in Table 1. Each objective begins with a verb, indicating an

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[1] Department of Civil Engineering, University of Colorado Denver, Campus Box 113, P.O. Box 173364, Denver, CO 80217-3364; david.mays@ucdenver.edu

action that the student should be able to perform after completing the assigned reading, attending class, and finishing the homework assignment. The objectives are intended to cover the spectrum from basic to advanced cognitive skills described by Bloom's taxonomy (Lowman, 1995). Because each objective is a specific action, it can be assessed through a homework assignment, quiz, or exam.

LESSON PLAN

This module includes a reading assignment, a 75-minute classroom presentation, a 75-minute guided computer exercise, and a homework assignment. This module could also be presented as a 50-minute classroom presentation, a 25-minute classroom presentation followed by a 25-minute computer demonstration on generating spatially correlated random patterns of hydraulic conductivity, and then a 50-minute guided computer exercise on stochastic groundwater modeling. Since 2006, this module has been presented during week 14 of a first course on groundwater hydrology that previously covers aquifer hydraulics (weeks 1-8), well hydraulics (weeks 9-10), and contaminant transport (weeks 11-13).

This lesson plan employs the software Processing Modflow for Windows (PMWIN), which provides a graphical user interface for the standard groundwater modeling software MODFLOW. PMWIN 5.3 is available for free download from ETH Zurich (2009) or pmwin.net (Chiang, 2006), and is also available on the CD-ROM that accompanies Chiang and Kinzelbach (2001) or Chiang (2005). Earlier in the semester, the class meets in a campus computer laboratory in order to run through the PMWIN tutorial step-by-step, which appears as Chapter 2 in the online manual by Chiang and Kinzelbach (1998) or Section 4.1 in the monograph by Chiang (2005). Then, after completing the tutorial, students complete two PMWIN-based homework assignments, so by week 14 they have some experience with PMWIN. Although the details presented below are specific to PMWIN, the educational objectives are not, and could be accomplished through any groundwater modeling software that includes a graphical user interface and the ability to generate several spatially-correlated random patterns of hydraulic conductivity, such as Interactive Groundwater (Li and Liu, 2004).

Reading Assignment

For the module on stochastic groundwater modeling, the assigned reading includes two components. First, students read Section 5.3 in Chiang and Kinzelbach (1998), corresponding to Section 2.7.3 in Chiang (2005), which explains how the Field Generator generates spatially correlated random fields. Second, students read Section 6.7.2 in Chiang and Kinzelbach (1998), corresponding to Section 5.6.2 in Chiang (2005), which presents a stochastic modeling example that is similar to the guided computer exercise described below.

Classroom Presentation

The goal of the classroom presentation is to introduce the terminology, key concepts, and modeling approach that will be used in the subsequent class meeting in the computer laboratory. The remainder of this section is presented as a lesson plan that can be directly presented to students.

Terminology - The instructor can start by writing "One Step Toward Reality" on the board, which serves as a title for this lesson. Remind students about the standard assumptions used in groundwater models, both analytical and numerical, *viz* the aquifer is homogeneous, isotropic, arbitrarily large, and has a flat impermeable base. Define the term *deterministic* to identify such a model, containing neither randomness nor unknowns. Contrast this with the definition of *stochastic*, which includes randomness or unknowns. The one step toward reality is to adopt a stochastic model of the hydraulic conductivity, while leaving other variables—such as porosity, dispersion coefficients, sorption coefficients, boundary conditions—as deterministic constants. Explicitly highlighting the fact that most of the model inputs are still deterministic emphasizes the important point that stochastic groundwater models, like deterministic models, still require simplifying assumptions.

Key Concepts - This section introduces the concept of a space random variable, using an analogy to real estate prices, with part of the presentation performed by a student volunteer. Define the hydraulic conductivity, K, with three parameters, (a) the *mean* $\overline{K}$ (b) the *standard deviation* s_K, and (c) the *spatial correlation length*. For a numerical model with N grid blocks, $\overline{K}$ and s_K are defined by

$$\overline{K} = \frac{1}{N}\sum_{i=1}^{N} K_i \tag{1}$$

and

$$s_K = \left[\frac{1}{N-1}\sum_{i=1}^{N}(K_i - \overline{K})^2\right]^{\frac{1}{2}} \tag{2}$$

which can be implemented by the Microsoft Excel functions =average() and =stdev(), respectively. These two

TABLE 1. EDUCATIONAL OBJECTIVES FOR THE ONE-WEEK MODULE WITH THE CORRESPONDING COGNITIVE LEVEL

Objective	Cognitive Level[1]
Define stochastic, spatial correlation, stationary, and realization.	1 – Recall
Explain spatial correlation in aquifers (or house prices).	2 – Comprehend
Produce three stochastic realizations of groundwater flow.	3 – Apply
Critique standard deterministic groundwater models.	6 – Evaluate

[1] From Bloom's taxonomy of educational objectives (Lowman, 1995)

parameters are defined explicitly, because they will be familiar to most students. In contrast, the explicit definition of the spatial correlation length as an integral scale requires the concept of spatial covariance, which is not familiar to most students. Instead, spatial correlation is introduced by analogy to real estate prices, a concept that most students intuitively understand. Using an analogy allows the instructor to introduce the new concept in a familiar context, and then to transfer the new concept to the intended application (Liu, 2004).

The real estate analogy for spatial correlation is as follows: The median sale price of a residential property in Denver during the 4th quarter of 2005 was approximately \$225K, with an assumed minimum of \$50K and maximum \$10M. Instructors should adjust these figures as appropriate for their local municipality. Students generally understand that there is a certain degree of variability between the prices of neighboring houses, which is the concept of a space random variable. Now sketch a map of your local municipality, and randomly pick a \$50K property. Ask a student to estimate the price of the house next door. The student will probably name a price below the median of \$225K, and will probably not name a price near \$10M. This is because students know that inexpensive houses are generally located in inexpensive neighborhoods, which is the concept of spatial correlation. Next, pick another property far away from the first \$50K property, and ask a different student to predict its price. With a little prompting, the student will likely guess the median price of \$225K. This is the concept of spatial correlation length, which indicates the distance over which spatial correlations persist. In housing the spatial correlation length might be three blocks. If that is true, then knowing that a certain property is within three blocks of the \$50K house tells us that the property is probably inexpensive. On the other hand, knowing that a certain property is more than three blocks from the \$50K house tells us nothing about its price, such that the best guess would be the median price.

Having introduced the concepts of space random variables, spatial correlation, and spatial correlation length in the context of real estate prices, placing these concepts into the context of hydraulic conductivity is now fairly straightforward. This can be illustrated by drawing a 3x3 grid on the board and establishing a legend for sand (dots), silt (hatch), clay (crosshatch), and indicating a correlation length of one grid block. Fill in one cell with sand, then ask a student volunteer to come forward and populate the rest of the grid, one cell at a time. Remind the student that one is more likely to find sand or silt next to sand, and less likely to find clay. Getting the student involved will increase her or his comprehension, and the other students will be more likely to pay attention, since there is an element of drama when a student works at the board.

Next, define the log hydraulic conductivity, or simply log conductivity, which is frequently used in stochastic groundwater models. Explain this by noting that hydraulic conductivity varies over orders of magnitude. This is analogous to the hydrogen ion concentration $[H^+]$ in chemistry, which also varies over orders of magnitude. In that case, the pH is defined as $-\log_{10}[H^+]$. We perform a similar transformation to define log conductivity as $Y = \log_{10}(K)$. So, for example, when $K = 0.000010$ m/s, then $Y = -5.0$ (m/s); when $K = 0.0010$ m/s, then $Y = -3.0$ (m/s), where the parentheses indicate the units of K from which Y was calculated. Point out that some authors use $\ln(K)$ rather than $\log_{10}(K)$. Having defined Y, we can now define its mean $\overline{Y}$ standard deviation s_Y, and correlation length I_Y. Point out that s_Y and I_Y do not depend on the units of K, unlike Y. In many aquifers, K is log normally distributed, which makes Y normally distributed. According to Rubin et al. (1999), for aquifers 100-1000 m in size, a typical range of s_Y is 0.4-2.1, and a typical range of I_Y is 6-40 m. In contrast, it is useful to note the *de facto* assumptions used in deterministic models with homogeneous hydraulic conductivity, namely that $\overline{Y} = \log_{10}(K)$ and either $s_Y = 0$ with an arbitrary I_Y, or alternately s_Y is arbitrary with $I_Y = \infty$. These *de facto* assumptions are clearly unrealistic. In the guided computer exercise and the homework assignment, we will assume $s_Y = 1$ and $I_Y = 10$ m, which are within the normal ranges reported by Rubin et al. (1999).

One final concept is the definition of a *stationary* model. We are now assuming that $K(x,y)$ and therefore $Y(x,y)$ are spatially-correlated random variables. If $\overline{K}$ s_K, I_K (or $\overline{Y}$ s_Y, I_Y) are all constants, then the stochastic model is stationary. This can be a difficult concept for students to grasp. Having just been introduced to the concept of space random variables, students are now asked to consider whether the parameters describing the spatial variability, namely $\overline{K}$ s_K, I_K (or $\overline{Y}$ s_Y, I_Y), are space random variables in their own right. Generally we assume the answer is no. Considering that the assumption of stationary random variables is a known limitation of many stochastic groundwater models, it is important to mention this definition.

Modeling Approach - For global learners, before working through the details, it is useful to place new ideas into a larger framework. This framework is the *Monte Carlo* approach (Rubin, 2003), whose implementation is straightforward within PMWIN: (1) Create a MODFLOW model in the usual fashion. (2) Use a spatially-correlated random hydraulic conductivity field to populate each grid block with a different value. (3) Run MODFLOW to calculate the hydraulic head at each grid block. (4) Run the particle tracking subprogram PMPATH to examine pathlines visually. (5) Run a contaminant transport simulation, such as MT3D or MOC3D, to calculate the concentration of contaminants at each grid block. (6) Repeat steps 2-5 until the histogram of the output variable stabilizes, which could be production rate, power requirement, or water quality. Explaining step 2 requires two more definitions. This step creates a single *realization*, which is defined as one configuration of the hydraulic conductivity that is possible, but no more likely than another configuration. Furthermore, this is an *unconditional* realization, since there is no guarantee it will match the hydraulic conductivity measured at a specific point in the aquifer. The Monte Carlo approach is simple, but computationally intensive, since it generally takes hundreds or thousands of realizations before the histogram of the output variable stabilizes. However, because time is limited in this one-week module, students are not expected to produce full Monte Carlo simulations. Instead, students complete just a few realizations, which is sufficient to illustrate the crucial point that groundwater flow and transport includes a great deal of spatial variability and predictive uncertainty.

Guided Computer Exercise

The second 75-minute presentation takes place in a computer laboratory, in which each student has a individual computer pre-loaded with PMWIN, and the instructor's computer is displayed on a projection screen at the front. Earlier in the semester, this facility was used to run through the PMWIN tutorial step-by-step. The specific exercise is based on Example 6.7 in Fitts (2002), which considers steady flow to an extraction well in a confined aquifer near a lake. Model units are meters and days. When homogeneous conditions are assumed, the transmissivity is specified as T = 20 m^2/d, modeled as a single layer of thickness 20 m and a hydraulic conductivity of K=1.0 m/s. The extraction rate is 295m^3/d. The exercise is simply to plot the streamlines from the lake to the well, first assuming homogeneity, and then assuming heterogeneity with s_Y = 1 and I_Y = 10 m. After completing the deterministic model, students find it very simple to run the stochastic model by resetting the hydraulic conductivity matrix and then running the model. Random field generation in PMWIN uses the subprogram Field Generator (Frenzel, 1995), which implements the algorithm of Mejía and Rodríguez-Iturbe (1974). Field Generator produces stationary realizations (constant $\overline{Y}$ s_Y, I_Y) taken from a lognormal distribution. Field Generator output is repeatable for a given set of input specifications, such that all students should generate identical results, which facilitates homework grading.

Model inputs are provided in Table 2, following the intuitive order used by PMWIN, in which model inputs follow the structure of the menu headings (File, Grid, Parameters, Models, Tools, Help). The demonstration neglects the slight error introduced by loading Field Generator results, which assume stationary block-block

TABLE 2. PMWIN 5.3 MODEL INPUTS FOR THE GUIDED COMPUTER EXERCISE AND HOMEWORK ASSIGNMENTS

Menu Option	Input Value	
	Demonstration	Homework
Grid → Mesh Size (1st time)	37 columns, 37 rows, size 5x5 m	52 columns, 27 rows, size 2x2 m
Grid → Mesh Size (2nd time)	use CTRL-arrow keys to subdivide cell (19,19) twice in x- and y-directions	(not required)
Grid → Layer Type	confined	unconfined
Grid → Boundary Condition → IBOUND	constant head (-1) at left, no flow (0) otherwise	constant head (-1) at left and right, no flow (0) otherwise
Grid → Top of Layers	reset matrix to 20 m	reset matrix to 5 m
Grid → Bottom of Layers	reset matrix to 0 m	reset matrix to 0 m
Parameters → Time	1 period, active, length 1, unit days, steady-state	1 period, active, length 1, unit days, steady-state
Parameters → Initial Hydraulic Head	reset matrix to 40 m	reset matrix to 3 m, then manually set 3.17 m at left and 2.83 m at right
Parameters → Horizontal Hydraulic Conductivity	reset matrix to 1 m/d	reset matrix to 8.64 m/d
Parameters → Effective Porosity	reset matrix to 0.25	reset matrix to 0.33
Models → Modflow → Well	set recharge in cell (20,20) to -295 m^3/d	(not required)
Models → Modflow	run	run
Models → PMPATH	prepare output graphics (described in the main text)	prepare output graphics (described in the main text)
	→ *this completes the deterministic simulation*	
Tools → Field Generator	Accept default file name "field" with no extension. Make 3 realizations with mean value 0, standard deviation 1, correlation length/field width 0.054 in the i- and j-directions, and number of cells 39 in the i- and j-directions.	Accept default file name "field" with no extension. Make 3 realizations with mean value 0.937, standard deviation 1, correlation length/field width 0.20 in i-direction and 0.10 in j-direction, number of cells 27 in i-direction and 52 in j-direction.
Parameters → Horizontal Hydraulic Conductivity	load file "field.1"	load file "field.1"
Models → Modflow	run	run model
Models → PMPATH	prepare output graphics (described in the main text)	prepare output graphics (described in the main text)
	→ *this completes the first stochastic realization*	

correlation, into a PMWIN model with non-uniform grid blocks. Experience has shown the following tips to be useful:

- The path and filename together must be less than 120 characters including spaces.
- After setting the mesh size, use Options → Environment to reset X_2 and Y_2 in order to make the model fill the screen for easier visualization.
- To assign a single value to every grid block, use Value → Reset Matrix.
- Output graphics were prepared using PMPATH as follows: Drag a small square to select the cell containing the extraction well, then set $N_i = N_j = 3$ and $N_k = 1$. Next, choose Options → Environment. On the Cross Sections tab, uncheck visible. On the Contours tab, check visible, then click on Level, set the minimum to 20 m, the maximum to 40 m, and the contour interval to 2 m. Click on Label Height to set it to 3 m. Click on Label Spacing to set it to 50 m. Click on Label Format, check fixed, set the decimal digits to zero, and the suffix to " m". Close the Environment screen, then click ◄ to trace particles from the well back to the lake. Finally, choose File → Save Plot As to output a bitmap file.
- When entering data into the Field Generator, using 5 m cells, the Field Width is 37(5 m) = 185 m. Given the correlation length of 10 m, the correlation length per field width is (10 m)/(185 m) = 0.054 in both directions.
- After generating three realizations of hydraulic conductivity, check at least one realization to confirm $\bar{Y}$ and s_Y using Microsoft Excel or equivalent. To perform this check, import "field.1" into Excel, convert from K to Y using the =log10() function, and then confirm $\bar{Y}$ and s_Y using the functions =average() and =stdev(), respectively.

The results of this exercise, shown in Figures 1-3, provide an immediate stage on which to dramatize the qualitative difference between deterministic and stochastic models. In some respects, the results are similar for both approaches: The water flows from the lake to the well, with hydraulic head contours perpendicular to the streamlines, which converge at the well, where the hydraulic head is minimum and the drawdown is maximum. Clearly, the deterministic model does capture these aspects of the flow. However, the stochastic realizations make it clear that the deterministic model is a simplification, particularly with regard to two key points. First, the stochastic realizations illustrate more realistic flows—asymmetrical and non-uniform. Second, the qualitative differences between realizations visually demonstrate the concept of uncertainty. For most students, grasping the concept of uncertainty in groundwater modeling will be facilitated by comparing Figures 2 and 3. Then, armed with a conceptual understanding of modeling uncertainty and its geologic origin, the student will be prepared to learn quantitative expressions of this uncertainty, such as calculating the standard deviation of the design variable of interest after a full Monte Carlo simulation. Comparing Figures 2 and 3 is certainly less work. Importantly, since the student will have just created these figures personally, their impact will be deeper than identical figures illustrated in a text. It is also possible to analyze the results of these realizations quantitatively (See Discussion Section).

Homework Assignment

After the reading assignment, class presentation, and guided computer exercise, the students complete a homework assignment based on their previous PMWIN-based solution to Problem 10-17 in Fitts (2002). The problem statement is shown in the Appendix; model inputs are shown in Table 2; results are shown in Figures 4 -6. To complete the assignment, students apply knowledge from the reading assignment, class presentation, and guided computer exercise to a new problem, using their textbook, class notes, and the PMWIN manual, but without the benefit of step-by-step guidance from the instructor. As such, the assignment reinforces the learning objectives, in particular by asking each student to evaluate the relative merits of the deterministic and stochastic groundwater models.

ASSESSMENT

This one-week module has been included in the University of Colorado Denver's graduate course Groundwater Hydrology since 2006. Two approaches have been used to assess its instructional effectiveness. First, submitted homework assignments (see Appendix) demonstrate that students were able to transfer skills from the module to a new application. Specifically, homework grades for the module-based assignment were analyzed and compared to homework grades for the remainder of Groundwater Hydrology, with details provided in the electronic supplement. For the module-based assignment, submission rates and mean grades for submitted assignments, respectively, were 79% and B+ in 2006; 75% and A- in 2007; 100% and A- in 2008. These grades are not significantly different from the mean homework grades for the remainder of Groundwater Hydrology. Since the homework assignment addresses objectives 3 and 4, corresponding to cognitive levels 3—Apply and 6—Evaluate from Bloom's Taxonomy (Table 1), the fact that the majority of students were able to complete the assignment supports the instructional effectiveness of the one-week module.

Second, students of Groundwater Hydrology between 2006 and 2008 were asked to complete a voluntary self-assessment survey, which was reviewed by the university's Human Subjects Research Committee, and which is included in the electronic supplement. The course rosters and admission files for the 33 students in this time period indicate that the student population included 24 men and 9 women; 27 Caucasians, 4 Asians and 2 Hispanics; 29 domestic students and 4 international

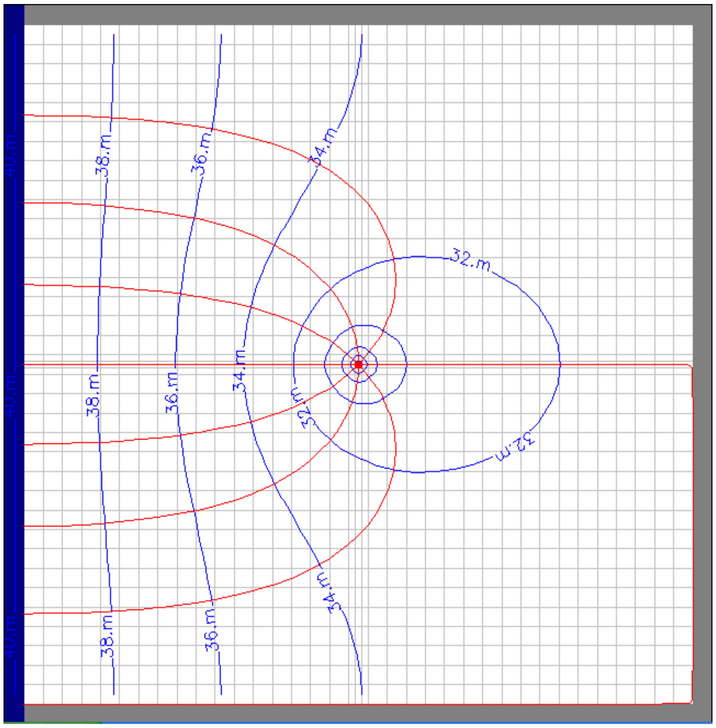

FIGURE 1. PMWIN output for the deterministic solution in the guided computer exercise.

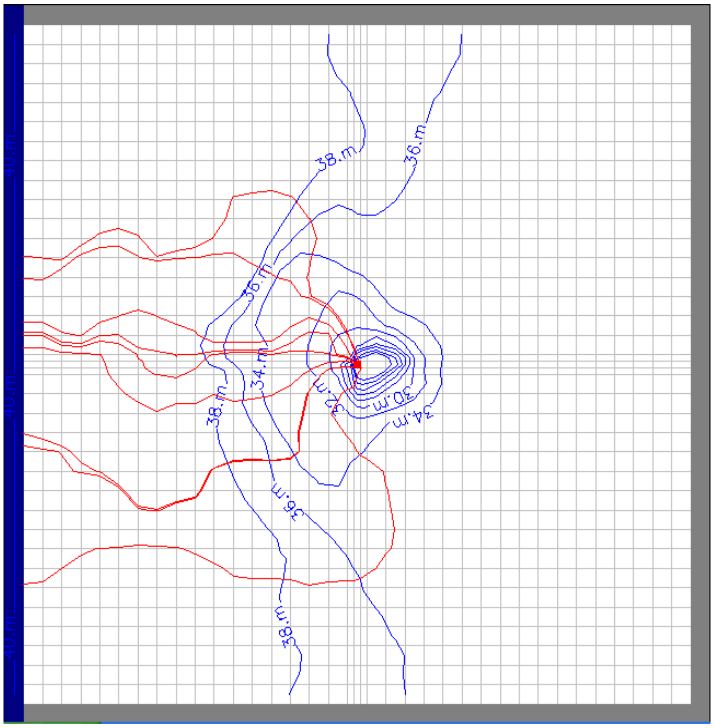

FIGURE 2. PMWIN output for stochastic realization #1 in the guided computer exercise.

students; 28 graduate students, 2 undergraduates, and 3 non-degree students; 28 civil engineering students, 2 environmental science students and 3 non-degree students. Their ages at the time of enrollment ranged from 23 to 48 years, with a median age of 31. Most of the students were employed as engineering or environmental professionals, and some had prior experience with hydrologic modeling. These demographic data do not allow us to identify possible correlations with the survey results, simply because the survey did not request demographic data. This was a deliberate choice, since the survey was designed to keep students anonymous, which was required for the survey to be approved by the university's Human Subjects Research Committee. The need to avoid collecting demographic data on the surveys, in turn, was driven by the small class sizes (14 students in 2006, 8 students in 2007, 11 students in 2008). With classes of this size, knowing that a survey respondent is—for example—female and undergraduate could be sufficient to uniquely identify the student, or at least to narrow the list down to a small subset of people. The reported demographic data was extracted from the course rosters and from information contained in the student's admission files, so it should be considered reliable, but simply uncorrelated with the assessment survey results.

In the survey, students were asked to rate the degree

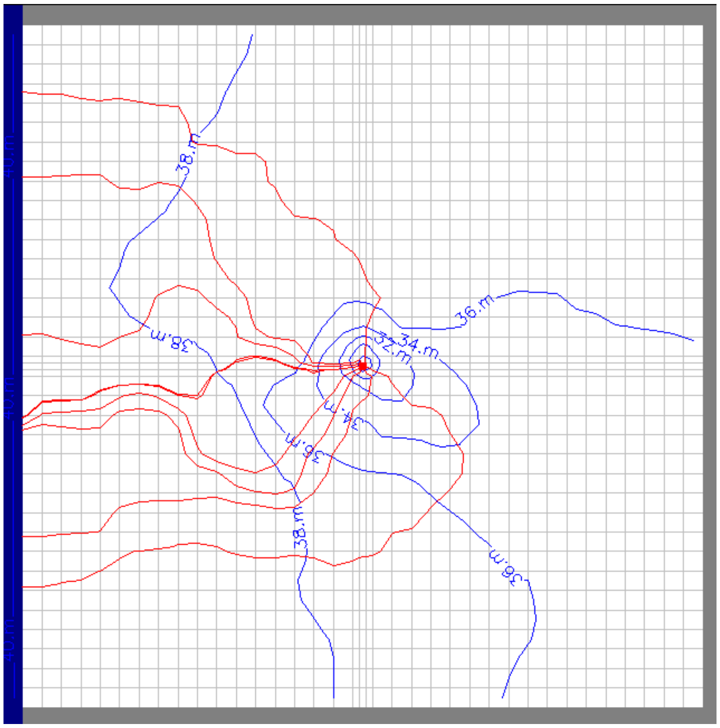

FIGURE 3. PMWIN output for stochastic realization #2 in the guided computer exercise.

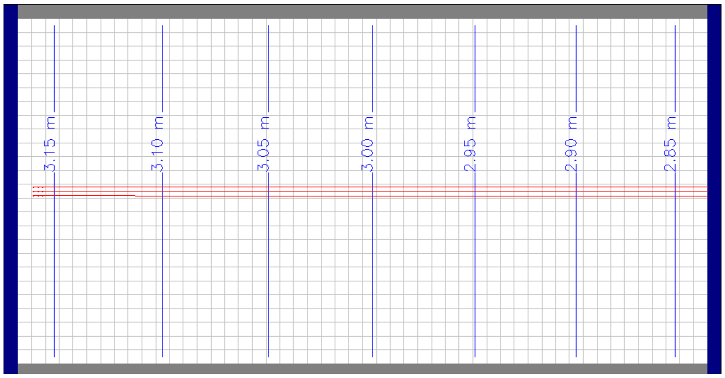

FIGURE 4. PMWIN output for the deterministic solution in the homework exercise.

TABLE 3: ASSESSMENT SURVEY RESULTS, EXPRESSED AS THE MEAN DIFFERENCE BETWEEN SELF-ASSESSED BEFORE AND AFTER RESPONSES, PLUS OR MINUS ONE STANDARD ERROR*.

Statement	2006	2007	2008	Pooled
1. I can define the term *stochastic*.	1.4±0.5[1] $p = 0.063$	1.6±0.6 $p = 0.13$	2.4±0.3 $p = 0.0078$	1.9±0.3 $p = 1.5x10^{-5}$
2. I can explain the concept of spatial correlation for hydraulic conductivity (or home values).	1.9±0.3[1] $p = 0.016$	2.0±0.6 $p = 0.13$	1.9±0.2 $p = 0.0078$	1.9±0.2 $p = 3.8x10^{-6}$
3. I can use software, such as MODFLOW, to simulate flow in heterogeneous aquifers.	2.7±0.3[1] $p = 0.016$	3.0±0.4 $p = 0.063$	2.2±0.4[2] $p = 0.016$	2.6±0.2 $p = 3.8x10^{-6}$
4. Standard (deterministic) groundwater models accurately represent flow in aquifers.	-0.4±0.6[1] $p = 0.69$	-0.2±0.4 $p = 1.0$	-0.1±0.6 $p = 1.0$	-0.3±0.3 $p = 0.46$

[1] Omits one student who checked only before, not after.

[2] The response from one student who checked Neutral and Agree was averaged.

*The Likert scale was 1 = Strongly Disagree; 2 = Disagree; 3 = Neutral; 4 = Agree; 5 = Strongly Agree, so larger numbers indicate greater agreement with each statement after completion of the one-week module. The exact Wilcoxon signed rank test was used to calculate *p*-values. Raw data are available in the electronic supplement.

to which they agreed or disagreed with the four statements in Table 3, which mirror the educational objectives in Table 1, both before and after the one-week module. Students were also invited to provide comments or suggestions about the one-week module. Students from 2006 and 2007 were contacted by an individual e-mail sent to their official university address, with a carbon copy to an alternative address, if available. All subjects replied by e-mail, although they were given the option of returning their survey by fax or postal mail. Students from 2008 were provided a hard copy of the survey on the last day of class. The response rate was 8 of 14 students for 2006 (57%), 5 of 8 students for 2008 (63%), 8 of 11 students for 2008 (73%), and 20 of 33 students overall (61%). Because the survey was conducted after the course was over, students completed both the before and after assessments at the same time, so the survey effectively measured the perceived change in agreement with each statement.

Results were interpreted using a 5-point Likert scale. The standard error for each question is the standard deviation divided by the square root of the number of responses. A statistical test was performed to determine whether there was significant change in the self-assessed agreement with each statement in Table 3, and this test was repeated for each class and for all three classes pooled together. The *p*-values were determined with the exact Wilcoxon signed rank test, using the statistics software R 2.7.1 (R Development Core Team, 2008). The Wilcoxon test is a non-parametric alternative to the *t*-test that is appropriate in this case because the data are ordinal and the sample sizes are small (Clason and Dormody, 1994).

DISCUSSION

When interpreting the survey results, one should bear in mind that a self-assessment such as this measures student perception, which is not equivalent to instructional effectiveness. However, some discussion of the survey results is merited, because the literature does suggest that student perception is correlated with instructional effectiveness (Felder, 1992; Lowman, 1995). As shown on Table 3, there is a significant improvement ($p \leq 1.5x10^{-5}$) for statements 1-3 when results are pooled. When specific years are considered individually, the sample sizes are smaller, and consequently the *p*-values are larger, indicating a smaller degree of significance. There was no significant change for statement 4, which asked students to critique standard deterministic groundwater models, and for which a decline had been expected. The lack of a significant change for statement 4 could indicate that the phrase "accurately represent flow" was ambiguous, or may indicate the need to articulate the limitations inherent in deterministic groundwater models more clearly in future years. On the whole, the assessment survey indicates that the students self-reported significant improvement in their ability to perform three of the four educational objectives in Table 1. Taken together, the

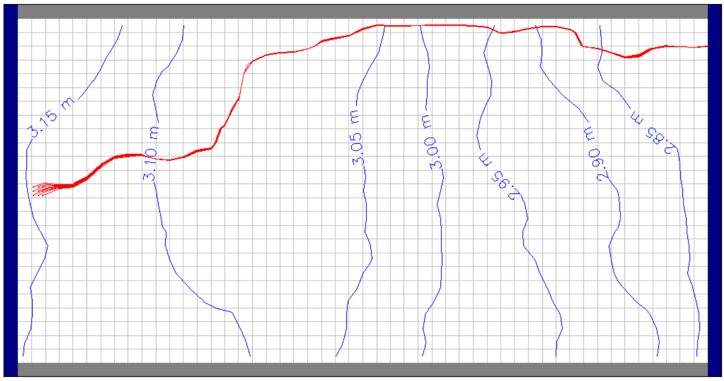

FIGURE 5. PMWIN output for stochastic realization #1 in the homework exercise.

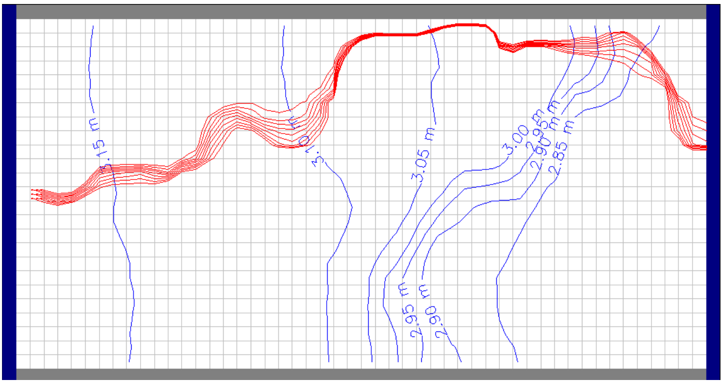

FIGURE 6. PMWIN output for stochastic realization #2 in the homework exercise.

homework performance and the survey results suggest that the module was effective.

Of the 21 surveys returned, 11 had written comments (52%). Of these, 7 were positive and 4 were neutral, containing both positive and negative comments. Several of these comments indicate the one-week module was effective:

- "The software demonstration was most helpful for me. To run the model with [the instructor] and have questions answered on the spot aided my success in [the homework] and understanding of the groundwater model."
- "It is good to show the different paths that occurred in MODFLOW since they are both viable alternatives that still meet the parameters."
- "I think the one week module was well worth the time… From my experience I have noticed that stochastic modeling is less applied in practice. Through the one week module I am more apt to consider such modeling practice or at a minimum mention that there are more solutions available in a problem [than] the solutions presented with deterministic modeling schemes."

Among the negative comments, one concern was that this one-week module was "a tad bit rushed," which was also reflected by another student who felt that the exercise "was insufficient in length to allow students to become comfortable or competent with the modeling process." Another comment suggested the assignment could have been more ambitious, writing that "perhaps a critical thinking application could also be added."

Along these lines, a number of extensions could easily be added. One option would be to have each student create not just three realizations, but three times the number of students in the class. By assigning unique simulation numbers to each student (*e.g.*, 1-3 to student #1, 3-6 to student #2, and so on), a large number of realizations could be performed without requiring any one student to perform more than three. Then, each student could report back three values of a selected model output. For example, the deterministic model output (Figure 1) indicates that the hydraulic head at the well is in the range of 24‑26 m, while the first stochastic realization (Figure 2) indicates 18‑20 m, and the second realization (Figure 3) indicates 26‑28 m. The instructor, or a teaching assistant, could then assemble the results into a histogram to be presented at a subsequent class meeting. A second option would be to proceed along the lines of Section 6.7.2 in Chiang and Kinzelbach (1998), corresponding to Section 5.6.2 in Chiang (2005), who calculated the proportion of a contaminated area that would be captured by a certain extraction well, first in a given realization, and then in a suite of realizations. Again, this could be explored through a coordinated effort by the entire class.

CONCLUSION

When a panel of experts was asked "What must be done in order to render stochastic theories and approaches as routine tools in hydrogeologic investigation and modeling?" one response included a call to action: "The schools must acknowledge the fundamental nature of uncertainty in hydrogeology by teaching stochastic modeling. A subject must be taught if it is to have much impact" (Winter, 2004). The one-week module described above partially addresses this concern by providing a brief introduction to stochastic groundwater modeling, with student involvement in the classroom and in the computer laboratory, whose instructional effectiveness has been assessed through submitted homework assignments and a self-assessment survey. While these exercises are no substitute for a formal course in stochastic groundwater modeling, it is hoped that students who complete this module at the end of a first course in groundwater will be more likely to take a second course, and that students who complete this module at the beginning of a second course will be motivated and excited, having gained some insight into stochastic groundwater modeling before diving in to the rigorous methods provided in textbooks by Zhang (2002) or Rubin (2003). And, even if students never generate another random field, it is hoped this one-week module will help them gain a more mature appreciation for the limitations inherent in the deterministic groundwater models that continue to be the bread and butter of groundwater consulting practice.

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APPENDIX: HOMEWORK ASSIGNMENT ON STOCHASTIC GROUNDWATER MODELING

Simulate a 100x50 m unconfined sand aquifer, with constant head boundary conditions of 3.17 m upstream and 2.83 m downstream and no-flow boundary conditions along the 100 m edges. Assume an average hydraulic conductivity of $K = 8.64$ m/d. Use grid blocks 2 m square.

(a) Assume homogeneous hydraulic conductivity, then run MODFLOW. Inside PMPATH, highlight cell (3,14), then use the “Particles in Cells” box to enter $N_i = N_j = 3$ and $N_k = 1$. Then choose Tools → Presentation → Options→ Environment → Contours, then double-click on the Level heading to specify head lines every 0.05 m from 2.80 to 3.20 m. Finally, click on the ► button to show streamlines. Your results should show parallel potential lines with perpendicular streamlines. Choose File→Save Plot As to export your results to Microsoft Word or equivalent.

(b) What average $Y=\log_{10}(K)$ is equivalent to 8.64m/d?

(c) Assume the standard deviation of Y is 1.0 log unit, and that the spatial correlation length is 10 m. What is the correlation length per field width in the *i*-direction and *j*-direction? Note, the *i*-direction corresponds to rows, and the *j*-direction corresponds to columns.

(d) Use the Field Generator to generate three possible realizations of hydraulic conductivity. Import your first realization into Microsoft Excel or equivalent, and confirm that the average Y and the standard deviation of Y are correct. Note, the average simulated K will *not* be 8.64 m/d.

(e) Inside PMWIN, change the horizontal hydraulic conductivity, rerun MODFLOW, and then use PMPATH to track particles. Submit a plot showing head lines and streamlines for each of three realizations. Based on your results, briefly evaluate the relative merits of the deterministic and stochastic models.