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A Simple Modeling Tool and Exercises for Incoming Solar Radiation Demonstrations

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ABSTRACT

We present a MATLAB script INSOLATE.m that calculates insolation at the top of the atmosphere and the total amount of daylight during the year (and other quantities) with respect to geographic latitude and Earth's obliquity (axial tilt). The script output displays insolation values for an entire year on a three-dimensional graph. This tool facilitates classroom discussion of broadly based concept questions regarding the effect of insolation on seasonality and climate. It can be used to demonstrate insolation patterns and changes that apply to a wide range of sciences, including climatology, biology, ecology, planetary science, and green engineering. Students who used the script in class assignments in an introductory level Earth science course showed an overall 32% improvement in scores on associated test and quiz questions compared with students who did not use the script to demonstrate the effects of obliquity and latitude on insolation.

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INTRODUCTION

The Earth's climate is controlled by changes in intensity and duration of incident solar radiation (insolation) at the Earth's surface (Meech, 1856). Insolation at a given geographical location is governed by Earth's daily rotation, Earth–Sun distance, and the Earth's axial tilt relative to the Sun (obliquity). The insolation equation is as follows:

$$\begin{aligned} W &= S_0 r^{-2} = \cos z \\ &= S_0 r^{-2}(\sin\theta \sin\delta + \cos\theta \cos\delta \cos H), \end{aligned} \tag{1}$$

where W = insolation (energy flux, in W/m^2), S_0 = the solar constant (1370 W/m^2), r = Earth–Sun distance, and z = solar zenith angle, which is a function of latitude θ, solar declination δ, and hour angle H (Fig. 1). Solar declination varies throughout the year, with maximum and minimum values set by the Earth's axial tilt (obliquity); the hour angle tracks the Sun in the local sky as the Earth rotates. Normalized Earth–Sun distance r varies throughout the year as a function of the Earth's orbital eccentricity.

At times of the year when the Sun is at higher angles in the sky (altitudes closer to 90°), insolation strikes the Earth at a more direct angle, consequently increasing the intensity of insolation at a given location. Although this seems like a simple concept, in large lecture-based courses filled with slides containing snapshots of planets and angles of incidence, the concept can quickly become lost in a sea of arrows and numbers. In smaller lectures or laboratory sessions, the use of exercises and examples often use enough trigonometry to cause students to become more focused on the procedure for getting the right answer than what that answer actually means. This has become evident in climate history courses when seemingly simple questions such as "why do we have seasons?" leave students at a loss.

Textbooks and online demonstrations (e.g., Lutgens and Tarbuck, 2006) typically provide a series of pictures or diagrams with associated text to show how the Sun's angle changes with respect to the Earth's surface through the course of a day to explain how the Earth is heated in a 24 h period. In other discussions, concepts are introduced to show how solar declination changes through the course of a year to explain how the Earth is heated on an annual basis (e.g., Robinson and Henderson-Sellers, 1999). Although the units of time in these discussions may vary, the fundamental mechanism for heating the Earth is exactly the same: the higher the Sun's angle in the sky (solar altitude), the higher the insolation at a given place for a given time.

After attempts to explore this subject through lectures and homework problems, we were faced with less than enthusiastic expressions and complaints regarding "how long the math took" for various assignments. To facilitate these problems, we decided to develop a tool to calculate insolation at any geographic latitude at very high time resolution (minutes to hours). We developed a basic MATLAB script to carry out the calculations and for graphical display. The script, INSOLATE.m, can be used as is, or it can be developed for other more advanced tasks, depending on classroom objectives and the skills of the students (see Online Supplement).

Presently, there are a number of useful online applications that explore various aspects of the Earth–Sun relationship. For example, websites run by the U.S. Naval Observatory, the University of Nebraska-Lincoln Department of Astronomy, and the School of Engineering at the Australian National University (see Appendix B) provide basic information and animations that calculate the angle of the Sun in the sky on particular dates throughout the year and provide visual tools to aid students in the understanding of why this solar angle changes over time. The objective of INSOLATE.m, however, is not only to demonstrate the changing solar angle but also to investigate the incoming solar energy that results from these changing angles, and to provide snapshots of the amount of heating for any location throughout the year. In INSOLATE.m, the calculations of the changing solar angle are done "behind

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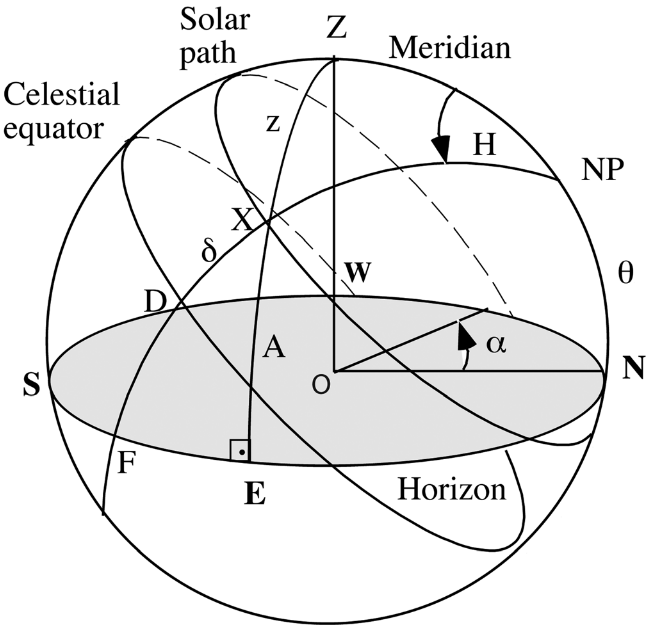

FIGURE 1: N, S, E, and W are the north, south, east, and west points of the horizon, O = observer on Earth, NP = celestial north pole, Z = zenith, X = location of sun at some instant, z = distance XZ to zenith, A = altitude of the Sun (π/2-z), δ = declination of Sun DX, α = azimuth of Sun; H = hour angle of Sun measured from the Meridian (midday). The Sun's declination δ is referenced to the Celestial Equator and varies between +23.5° throughout the year. Solar altitude A describes the angle of the Sun above the local horizon; this quantity, over H hours of the Sun's daily transit, determines the effective total daily insolation at any geographical latitude θ (here depicted for θ = 40°N). Adapted from Smart (1965).

the scenes" in order to focus the student on the resulting energy output with the assumption that the concept of changing solar angle has already been presented using other resources. Because INSOLATE.m calculates insolation values over the course of an entire year, it is also able to provide calculations of total annual daylight hours for any location. For introductory Earth science courses, it is useful for instructors to utilize both available online tools in conjunction with INSOLATE.m to demonstrate both why solar angle changes with time at a given location, and how and when the amount of heat changes at that location.

THE INSOLATION MODEL: INSOLATE.m

INSOLATE.m computes Eq. (1), with an important simplification that Earth–Sun distance r is kept constant throughout the year, here set to unity. (See further information in the section on Extensions to INSOLATE.m, *Orbital eccentricity effect*.) No programming skills in MATLAB are necessary to execute INSOLATE.m and to complete basic exercises (Appendix A).

INSOLATE.m can be downloaded from the eGuide to Paleoclimates website http://www.jhu.edu/~lhinnov1/paleoguide/tutorial.html and clicking on "Tutorial 1." To run the script, open MATLAB, and in the "File" menu, choose "Open m-file" and browse for INSOLATE.m. Once INSOLATE.m is loaded, the user can either select the "Debug" menu and select "Run" from the options, or simply select F5 on the keyboard. The user will then be prompted with questions on the MATLAB command line.

When running INSOLATE.m, the user is prompted to input two parameters. The first input is the desired latitude, in degrees, on the Earth, positive for northern latitudes and negative for southern latitudes. The second input is the obliquity angle of the Earth, which is currently 23.5°. We included the option to change the obliquity angle, as this is known to vary from 22.2° to 24.5° through time (Berger *et al.*, 1993). The script calculates Eq. (1) for every minute of every day through the course of one year.

The results of the calculations are displayed graphically on an x–y–z coordinate system, where x is hours in the day, y is the day of the year, starting on December 21 (Winter Solstice in the Northern Hemisphere), and z is the calculated insolation at the x–y (day and time of day) point. In addition to the graph, the maximum insolation for the year, the total annual insolation, and the total number of annual daylight hours and fraction of the year with daylight are also displayed in the window. To run a new calculation using the program, return to the "m-file" window and select "Debug" and "Run" again or press F5. Users will not need to close one output to create a new output.

While the current version of INSOLATE.m delivers a three-dimensional graphical output and contains several useful data outputs, it does not save the data in numerical form for the user. When utilizing INSOLATE.m for in-class exercises, we have generated standard forms with the name for a data value and blanks for students to fill in while running the script. This allows the students to more easily manage and recall the data in follow-up assignments. Another potential issue for students is saving the output files in a usable format if MATLAB is unavailable in some computer labs on campus, which was an initial hurdle to running this exercise at Winthrop University. On the output, users can also select "File" and "Export" to save the output as an image and place in the desired folder on the computer. The instructor may also provide a quick demonstration of a "screen capture," in which the contents of any open window can be copied as a picture by depressing the "alt" and "print screen" keys (Windows) or "command-control-shift-4" (Mac). Students can then simply paste output into any standard word processing program and save all of their outputs in one file.

As an example of comparisons that can be made using INSOLATE.m, Fig. 2(a) shows output for minimum obliquity of 22.2°, at 75° North, the latitude of Axel Heiberg Island, Canada. During the Eocene ($\sim 50 \times 10^6$ years ago), this region was a highly productive forest system located far northward from the Arctic Circle (Jahren, 2007). Figure 2(b) was calculated for the same latitude but with an obliquity angle of 24.5° to represent the maximum possible obliquity as presently known (Laskar *et al.*, 2004). INSOLATE.m calculates no appreciable difference in the total number of daylight hours for the two obliquities. On the other hand, there is a substantial difference in the total annual insolation: for 24.5° obliquity the Sun rises higher above the horizon and provides more insolation to the area, compared to 22.2° obliquity. In this comparison, we see that lower obliquity results in reduced insolation to the Arctic region. This may lead to overall lower Arctic

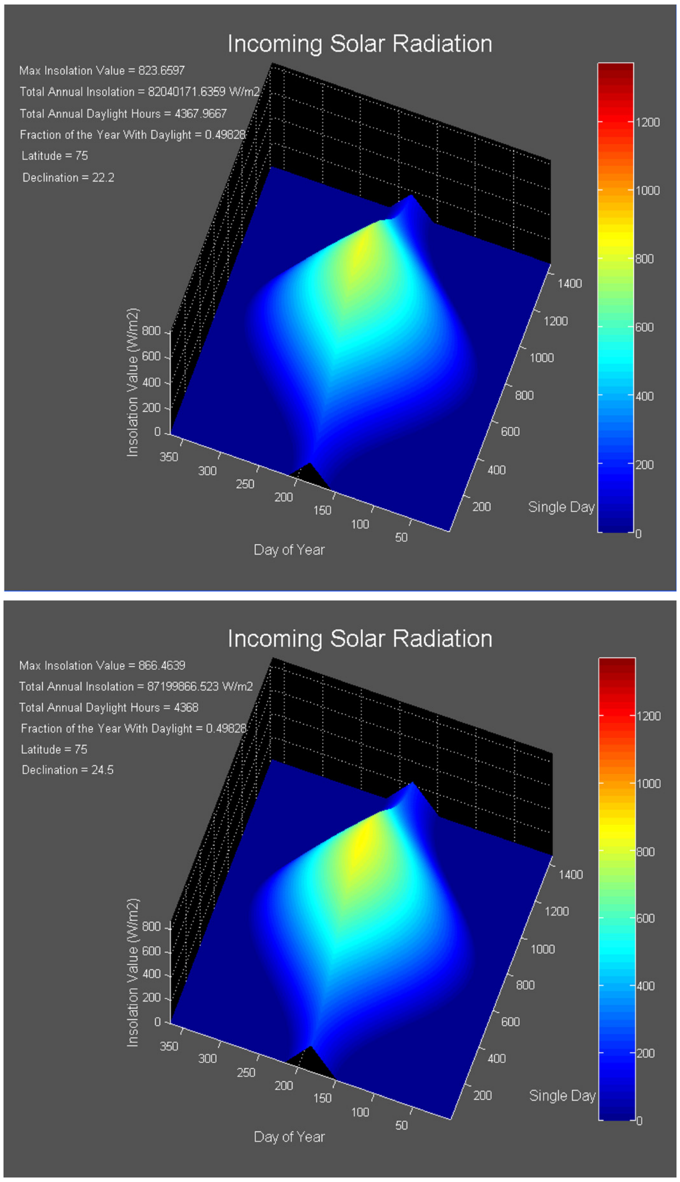

FIGURE 2: (Color online) INSOLATE.m calculation of insolation at 75°N latitude using (a) minimum obliquity of 22.2° and (b) maximum obliquity of 24.5°, the currently known extremes (Laskar *et al.*, 2004). The units for the single day axis are in minutes. During the Eocene (55–40 Ma), highly productive forests existed at this latitude, which is located well northward of the Arctic Circle, at 75° North.

temperatures during times of low obliquity and, conversely, elevated temperatures during times of high obliquity. Comparisons such as this can aid students and instructors in exploring different models or ideas regarding ancient climate systems. It should be noted that INSOLATE.m does not model other important climate variables, such as greenhouse gas concentrations or effects from atmosphere–ocean circulation.

INSOLATE.m can provide other exercises for different obliquity values and latitudes for a variety of class uses. Figure 3 shows a series of outputs from the script arranged in columns and rows; each column depicts a different obliquity angle (13.5°, 23.5°, and 33.5° from left to right). Each row depicts a different latitude (0°, 20°, 40°, 60°, and 80° from top to bottom). The following section that describes how insolation and daylight hours vary with respect to these parameters will be in reference to this diagram. All outputs were calculated for every minute of every day during the year. Notable aspects of script output are shown in Table 1.

VARIATION OF OBLIQUITY/SOLAR DECLINATION

Two insolation maxima per year occur at the Equator (Fig. 3, center output in the top row). This effect is due to the Sun passing directly overhead twice per year, once at the Spring Equinox and once at the Fall Equinox. Peak insolation (the maximum attainable) of 1370 W/m^2 occurs at these times (y axis). When obliquity is lowered from 23.5° to 13.5°, the intensity of the double maximum is lowered, as solar declination will not deviate as far from the Celestial Equator (Fig. 1). The maximum value of insolation is unchanged since the Sun still passes directly overhead twice a year. The values at midday, however, are otherwise lower for higher obliquity. This is also reflected in the total annual insolation. In other words, since the Sun stays closer to the Celestial Equator throughout the year, the total annual insolation is higher for lower obliquity.

For latitudes above the Tropic of Cancer or below the Tropic of Capricorn, i.e., poleward, maximum insolation increases as obliquity increases. This is because the maximum solar altitude gets closer to 90° (at midday) during the year with higher obliquity. Total annual insolation also increases as obliquity increases. During the spring and summer months in the Northern Hemisphere, the Earth's surface experiences progressively more insolation at higher obliquities than at lower obliquities. During fall and winter at the same location, the Earth's surface receives less insolation at a higher obliquity because solar altitude is lower. The timing of these effects is the opposite in the Southern Hemisphere, but the magnitude of the effects is identical.

The increase in total annual insolation with increasing obliquity is also interesting in that the total number of daylight hours simultaneously decreases. This effect is due to a lower solar in the winter at higher obliquities (and closer to the horizon). Due to the nature of the Sun's path through the sky (Fig. 1), the Sun spends less time above the horizon in the winter than the time added at the higher altitudes in the summer. This effect is balanced at the Equator where the Sun spends an equal amount of time on either side of that latitude. This effect is also balanced at latitudes above the Arctic Circle. For any given location on the Earth, there cannot be less than 4367 h of daylight per year (just less than ½ of the year). Although sunlight reaches locations within the Arctic Circle sooner at lower obliquity values, the amount of time spent at 24-h daylight is higher at higher obliquity values. These two effects appear to balance each other in this calculation. The real effect of decreased insolation above the Arctic Circle is due to the decreased solar angle, not decreased daylight hours.

THE LATITUDE EFFECT

Insolation and daylight hours with respect to latitude (at a constant obliquity) are far greater than the effect of changing obliquity at the same latitude. In Fig. 3, a

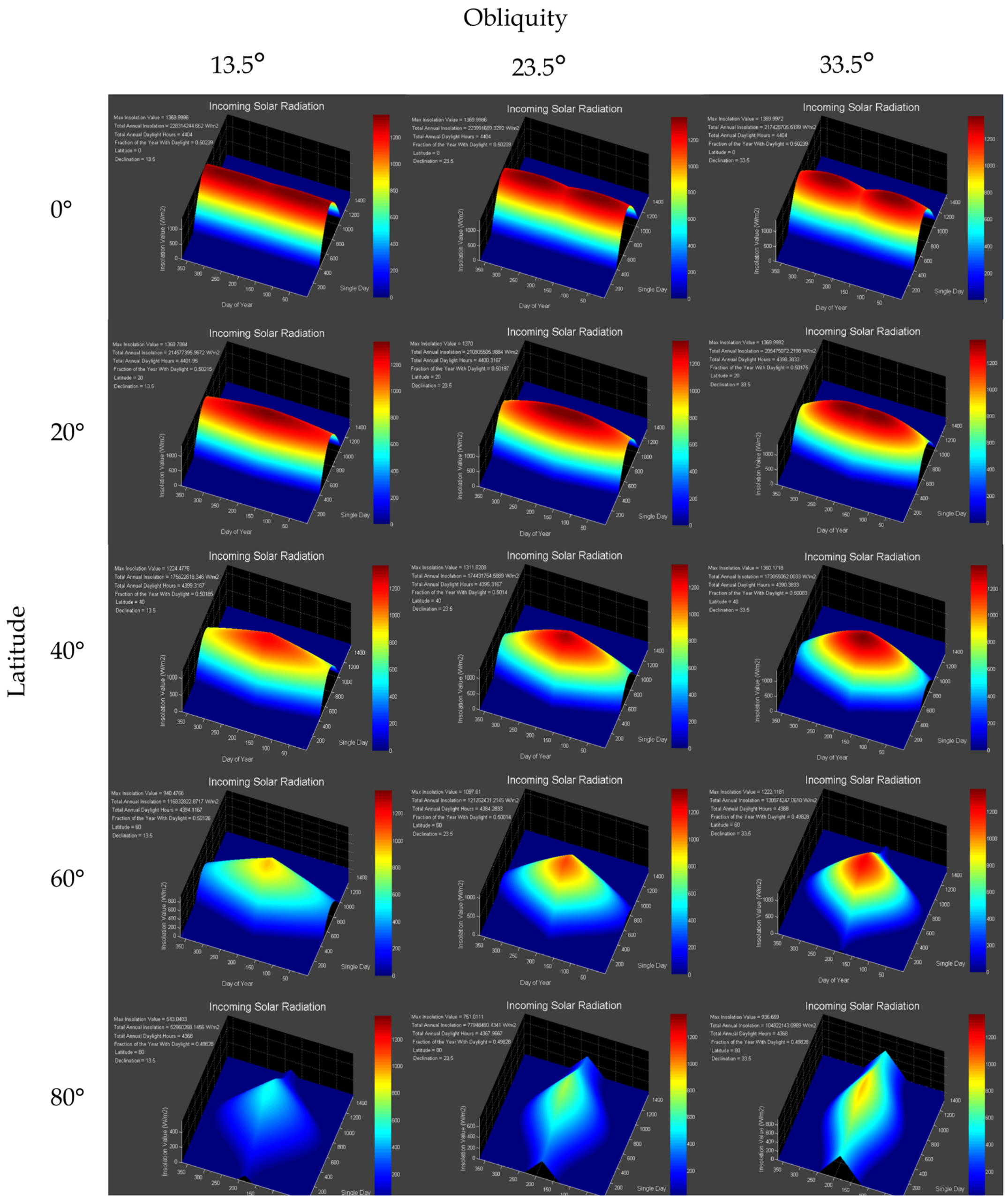

FIGURE 3: (Color online) A collage of INSOLATE.m outputs showing trends in insolation values for increasing latitude (vertically in increments of 20° with 0° at the top and 80° at the bottom) and increasing obliquity (horizontally in increments of 10° with 13.5° on the left and 33.5° on the right).

reduction in all insolation and daylight hour values is observed for increasing latitude. This is not surprising given that lower solar angles allow for lower maximum insolation and shorter days (as described above). Insolation drops off at a faster rate as the latitude approaches the pole.

An interesting effect occurs near the Arctic Circle (66.5°N). The big "hump" in insolation during the day (Fig. 3) seen at lower latitudes switches to a yearly hump with the advent of polar days and nights (24 h of total light or dark, depending on time of the year). With increasing latitude, the transition of the shape of the graph occurs gradually. As Summer Solstice is approached in the Northern Hemisphere, at progressively higher latitudes, the graph begins to widen at that location until the latitude lies just within the Arctic Circle (the edges of the graph). This can be best seen at 60° latitude at 33.5° obliquity. Here we see a sharp ridge beginning to form, which is very prevalent in all outputs at 80°.

USING INSOLATE.m IN SCIENCE AND ENGINEERING COURSES

The Earth–Sun relationship appears in many forms in many earth science classes. At present there are many effective tools available for some of these courses that are useful in explaining changing solar angles, axial tilt, and several other components. However, in many discussions,

TABLE I: Corresponding output from INSOLATE.m graphs shown in Fig. 3. The information contained in this table will be shown for each output run on the program. These data refer to the models in Fig. 3 and are listed in the same order as the displays.

	Obliquity		
	13.5 deg	**23.5 deg**	**33.5 deg**
0° Maximum isolation value (W/m^2)	1370	1370	1370
0° Total annual insolation (W/m^2)	2.271×10^8	2.228×10^8	2.164×10^8
0° Total annual daylight hours	4380	4380	4380
0° Fraction of the year with daylight	0.4996	0.4996	0.4996
20° Maximum isolation value (W/m^2)	1361	1370	1370
20° Total annual insolation (W/m^2)	2.134×10^8	2.098×10^8	2.045×10^8
20° Total annual daylight hours	4378	4376	4374
20° Fraction of the year with daylight	0.4994	0.4992	0.499
40° Maximum isolation value (W/m^2)	1224	1312	1360
40° Total annual insolation (W/m^2)	1.747×10^8	1.735×10^8	1.721×10^8
40° Total annual daylight hours	4375	4371	4367
40° Fraction of the year with daylight	0.4991	0.4987	0.4981
60° Maximum isolation value (W/m^2)	940.5	1098	1222
60° Total annual insolation (W/m^2)	1.162×10^8	1.205×10^8	1.291×10^8
60° Total annual daylight hours	4370	4360	4344
60° Fraction of the year with daylight	0.4985	0.4974	0.4955
80° Maximum isolation value (W/m^2)	543	751	936.7
80° Total annual insolation (W/m^2)	5.251×0^7	7.718×10^7	1.038×10^8
80° Total annual daylight hours	4344	4344	4344
80° Fraction of the year with daylight	0.4955	0.4955	0.4955

understanding insolation and the time-space distribution of heating on Earth's surface is crucial. This aspect of the Earth–Sun relation is not generally available in most educational tools. Below we describe how INSOLATE.m can be used in a broad selection of science and engineering courses.

Climatology

A challenging concept for nonscience majors to visualize is not only how the Sun changes position in the sky throughout the year but also the associated patterns of increasing and decreasing heat that are associated with these changes. One advantage of INSOLATE.m is that students can obtain a visual output of insolation at a proscribed location throughout the year. An instructor in an introductory or survey course in climatology, for example, may have students choose locations at several different latitudes, run INSOLATE.m at our current obliquity angle (23.5°) and have the students describe the trends in intra-annual insolation, daylight hours, maximum insolation, etc. A comparison of the timing of peak insolation at specific Northern and Southern hemisphere locations may be helpful in discussing seasonality in the different hemispheres. Sample exercises that illustrate this are in Appendix A (*Problem 1*).

Astronomy

Concepts similar to those described above can be presented in introductory astronomy courses through laboratory or take-home exercises. An additional use in astronomy (planetary sciences) might be to compare how the Earth is heated differently at different declination values. An exercise comparing heating in the polar regions, midlatitudes, and equatorial regions for different declinations may prove useful in discussing other planets in the Solar System and their various solar declinations. Mars with its highly unstable obliquity would be an especially interesting case. Caution should be taken in this discussion, however, since INSOLATE.m utilizes insolation customized for the Earth only. The trends should be analogous but maximum and minimum insolation values would be quite different.

Paleoclimatology

INSOLATE.m was originally designed as a project in a paleoclimatology class. There is much discussion in paleoclimatology regarding the range of particular biozones in the past in relation to the locations of the continents at the time. Temperature ranges and greenhouse gas concentrations are often discussed for these time periods. INSOLATE.m allows an additional component of climate analysis to be included in these discussions. For example, instructors can select a relatively well-studied time in Earth's history, such as the Paleocene/Eocene Thermal Maximum ($\sim 55 \times 10^6$ years ago), and discuss the various climatic influences and evidence at a particular geographic location where sample data exist. INSOLATE.m can facilitate the design of assignments and projects in which students attempt to match or fit an obliquity range to simulate the appropriate environment. Another useful exercise could explore the feasibility of an increased obliquity during the Eocene (e.g., Fig. 2), as suggested by the presence of the high Arctic forest and absence of any tree line during that time (Appendix A, *Problem 2*).

Biology and Ecology

Concepts of climatic zones and biomes often arise in introductory ecology and biology courses with discussion of seasonal temperature tolerance of plant and animal species. The use of INSOLATE.m may be coupled with maps of the distribution of specific forest communities in order to test the influence of latitudinal temperature variation on these distributions. Care must be taken, however, not to confuse the influence of changes in latitude with changes in altitude. In recent years, research has focused on the influence of obliquity cycles on biological productivity in particular communities over time (Kitamura, 2004; Wu *et al*., 2001). INSOLATE.m can be used in upper level biology courses to explore the influence of different obliquity angles on the heating of certain environments and effect on biological productivity.

EVALUATION OF INSOLATE.m

INSOLATE.m has been used in two different courses at Winthrop University for four semesters. The first is an introductory earth sciences course in which the script was used in an in-class exercise related to seasonality and heating of the atmosphere, very similar to Problem 1 in Appendix A. This course is a required course for early childhood and elementary education majors and covers aspects of geology, astronomy, meteorology, and climatology on an introductory level and is usually taken in the sophomore or junior year. The average class size throughout the study period for this course was 32 students. Because Winthrop's student population is dominated by women (>66%) and the population of females in education majors at Winthrop is even higher, the student population in this course was over 95% females. It should be noted, however, that it is not the intent of this study to make a statement regarding the improvement in learning for female students in using this script, since there is no control population of male students available in this course. In addition to this course, the script was also used in a physical hydrology course as a tool to generate discussion of the influence of varying levels of incoming solar radiation on energy balance models. However, there were no exam questions or assignments built exclusively around this script for evaluation in this course.

For the introductory course, we have analyzed exam data related to the topics investigated with the script in semesters that the in-class exercise was utilized and compared the class scores on these questions to previous semesters in which the script was not used. While these data were collected for the use of standard assessment purposes initially instead of testing the initial design and effectiveness of the exercise, the increase in student scores on this portion of the exam (Table 2) suggests that INSOLATE.m is an effective tool for these subjects.

In addition to the higher scores in questions relating to atmospheric heating and seasonality, a number of misconceptions exhibited on earlier exams were not exhibited on examinations given following this exercise. For example, on midterm exams given prior to the use of this script, 7 out of 36 students answered that the primary reason for higher temperatures in the Northern Hemisphere during the summer months was that the Earth was closer to the Sun and 3 students out of 36 repeated this answer on the final exam. During the two semesters in which the in-class exercise was used, 4 out of 72 students cited this misconception for those two semesters on all examinations combined. In addition, 10 out of 36 students indicated that the primary reason the tropics are warmer than the polar regions is due to the latter receiving much lower amounts of daylight throughout the year on the midterm examination prior to the script being used. In the following two semesters in which the in-class exercise with the script was utilized, no students included this response on their examination.

We have also received some evaluative quotes from students regarding the in-class exercise with INSOLATE.m. The most frequently commented on aspect of the program was the standardized colors of the insolation values on the graphical output. The students commented on how helpful it was to see the amount of solar energy arriving at a specific latitude throughout the year rather than just the discussions of solar angles at a few key points throughout the year. Being able to pair the graphical output with the key data points provided was also viewed as very helpful by most students. In addition to this, nearly all of the students participating in the evaluation indicated that it helped in understanding the concepts of solar heating but also indicated that it should be paired with other demonstrations or explanations of changing solar angles throughout the year. The most frequently noted complaint by the students on their evaluations was in regard to the three-dimensional layout of the graphical output appearing complicated in the outset. We have found this to be overcome in the classroom quickly by a few minutes of explanation of the axis orientation and values and allowing the students to rotate the graph inside the output box to gain a better perspective of the layout.

When utilizing INSOLATE.m in the classroom, there are several key points that should be emphasized in order to avoid confusion or misunderstandings of the output with the students. When initially discussing the graphical output, we have found there are usually several students who mistake the z axis for solar angle instead of energy. Constantly referring to the z axis as energy is usually helpful in alleviating this confusion. In recent semesters, we have found that writing statements such as "vertical axis = incoming radiation (not solar angle)" on the board next to the screen to be helpful in eliminating incorrect statements on class assignments. Another potential pitfall regarding this program is in regard to the "hot" and "cool" colors on the displays. The colors displayed do not correspond to a temperature but to radiative flux. Students may be inclined to interpret light blue colors at certain times of the day near the Equator to mean that the air is at a similar temperature as the light blue values at higher latitudes. Care should be taken to explain that insolation is absorbed by the Earth during the day and is reradiated after sunset to maintain warmer temperatures. It should also be noted that this program does not include modifications for

TABLE II: Average scores on select questions from introductory geology course midterm examinations related to the concepts investigated with INSOLATE.m. These three questions (or questions with very similar wording and concepts) were utilized in all three semesters. Partial credit was available for the two short answer questions.

		Averaged student exam scores			
Examination question	**Question type and point value**	**Semester 1: without in-class exercise (n = 36)**	**Semester 2: without in-class exercise (n = 23)**	**Semester 1: with in-class exercise (n = 36)**	**Semester 2: with in-class exercise (n = 36)**
Which of the following is true regarding the Earth/sun relationship at 80° north latitude on the winter solstice? (a) The sun will not rise on this day. (b) The sun will rise but be in the sky for less than 12 h on this day. (c) The sun will be in the sky between 12 and 24 h on this day. (d) The sun will not set on this day (24 h of daylight.)	Multiple choice (1 point)	0.72	0.61	0.86	0.92
Explain why the Earth's Northern Hemisphere experiences much warmer temperatures during the summer months.	Short answer (2 points)	1.37	1.58	1.74	1.69
What is the primary reason that the tropical regions stay much warmer throughout the year than the arctic? Explain your answer.	Short answer (2 points)	1.17	1.36	1.69	1.86

humidity or elevation. For example, INSOLATE.m would not be useful in attempting to explain the annual temperature differences between Flagstaff, AZ, and Charlotte, NC, both of which lie at 35° N latitude. In fact, temperature is not output by INSOLATE.m.

EXTENSIONS TO INSOLATE.m

INSOLATE.m can be adapted to address advanced problems in engineering, astronomy, and climatology, by manipulating variables within the script and/or by adding new algorithms. Next, we discuss three examples requiring computationally intensive solutions that can be carried out with extensions to INSOLATE.m.

Orbital Eccentricity Effect

In Eq. (1), the variable r is the normalized Earth–Sun distance, taken as a constant (unity) in INSOLATE.m. However, the Earth's orbit around the Sun is not constant throughout the year but is eccentric. To assess the effect of the orbital eccentricity on insolation, the script can be modified to estimate r as a function of day d during the year:

$$r = 1 + 2e\cos([2\pi/365]d), \tag{2}$$

where $d = 1$ is January 1, and e is eccentricity (McCullough and Porter, 1971). Today $e = 0.01675$; in the Earth's past, e has varied over 10^5–10^6 year timescales between values of ~0 and 0.07 (Laskar *et al.*, 2004).

Optimum Angle for Solar Collectors

Although solar energy collection is still a minor enterprise in the United States, solar electric power markets are projected to double over the next several years. This growth is being supported by the federal government, and involves development of photovoltaic technologies and market transformation initiatives (see Appendix B).

At the foundation of solar energy production is how to set up collectors to capture the most insolation at a given location (Fig. 4). This is an active area of research for solar engineers, who grapple not only with basic variables of insolation but also with other complicating local geographic or climate conditions. The optimal situation would have a continuously adjustable collector track solar altitude exactly throughout each day. In practice, however, collectors are flat and fixed, especially those for home use. One rule of thumb is to face the collector plate toward the Equator (i.e., southward in the Northern Hemisphere) and tilted at an angle above the Earth's surface that is equal to the latitude +15° (Garg, 1982; Ulgen, 2006). This rule—and other potential angles and times of collection—can be tested with INSOLATE.m, through addition of a subfunction to calculate the angle of incidence of solar radiation on the tilted surface (the collector).

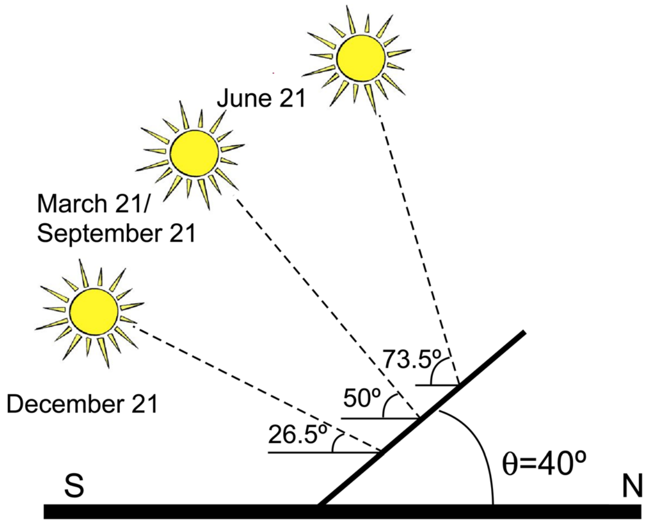

FIGURE 4: (Color online) A diagram of an optimum solar collector at θ = 40°N latitude. In the Northern Hemisphere, the solar panel should be inclined 40° from horizontal, and toward the south in order to maximize the amount of insolation as the Sun changes declination throughout the course of the year.

Atmospheric Extinction

As the Sun proceeds across the sky during the day, its rays pass through a continuously changing thickness of atmosphere before reaching the Earth's surface. The basic effect is modeled by estimating the number of "optical air masses" that insolation must travel through at a given moment (e.g., Vollmer and Gedzelman, 2006). An air mass value of 1 is given to sea level air mass at zenith (directly overhead); at the horizon ($z = 90°$), insolation passes through approximately 38 air masses. The plane-parallel formula, $M(z) = \sec(z)$, accurately defines this effect for $z = 0°$ to 60° but not for higher zenith angles (close to the horizon) due to Earth's curvature and atmospheric refraction.

Various approximations have been proposed to improve accuracy at higher z, e.g., the equation by Rozenberg (1966):

$$M(z_a) = (\cos(z_a) + 0.025\exp[-11\cos(z_a)])^{-1}, \quad (3)$$

where apparent zenith angle approximates as $z_a = (60z–864)/70$, to account for atmospheric refraction (McCullough and Porter, 1971). Thus, Eq. (1) modifies to

$$W_a = S_0 r^{-2} \cos z_a \exp[-kM(z_a)], \quad (4)$$

where k = total atmospheric extinction coefficient (evaluated for one air mass at sea level) due to scattering and absorption. Full-scale modeling of k requires detailed knowledge of frequency-dependent absorption, refraction, and scattering of solar radiation by the full atmosphere. This has been an enduring problem in astronomy, now also for remote sensing and related space-based applications. Bird and Hulstrom (1981) consider the problem in terms of transmittance modeling of solar radiation through the atmosphere; the models presented in their paper can be adapted to simulate k.

CONCLUSIONS

The MATLAB script INSOLATE.m calculates total insolation and daylight hours with the option for the user to select latitude and Earth's axial obliquity angle. In courses designed for students with a wide range of science and math backgrounds, this can be especially advantageous as the script allows for demonstrations and assignments to focus on understanding the concepts of insolation and seasonality rather than students being burdened with a lengthy series of calculations. It is apparent by looking at selected outputs in this paper that day length is not the major player in determining the total (annual) insolation at a given location. The maximum angle of the Sun, and consequently higher levels of insolation at a given time, plays a much larger role in determining the cumulative amounts of radiation per year. These observations, among others, can be easily integrated into courses ranging from climatology, earth systems based courses, plant physiology, and astronomy, as well as emerging green engineering and solar energy research courses.

APPENDIX A: BASIC EXERCISES

INSOLATE.m allows comparison of insolation for different geographic latitudes and obliquities throughout the year and for more investigative exercises that the instructor can follow in many directions. The outputs from these exercises appear in Fig. 3 and Table 1.

Problem #1

INSOLATE.m computes the total insolation over an entire year, called "annual insolation." For latitudes 0°N, 20°N, 40°N, 60°N, and 80°N, calculate and display the annual insolation for modern day obliquity (23.5°). Consider the following questions:

> Question 1. The graphical output at the Equator reveals two distinctive peaks in insolation during the year. Why is this? On what two days is the insolation value at its peak at this latitude?
>
> Answer. At the Equator, the Sun passes directly overhead twice through the course of one year. It will pass once moving toward its highest declination of +23.5° in the Northern Hemisphere Summer and then pass directly overhead again as the declination moves to −23.5° in the Northern Hemisphere Winter. The Sun should be directly overhead at the Equator at Spring and Fall Equinox every year.
>
> Question 2. The present tree line in the Alaska lies at about 68°N. Looking at the graphical output between 60°N and 80°N, explain why it is difficult for plants to grow in the Arctic considering that the number of daylight hours are nearly the same as in lower latitudes.
>
> Answer. While it is true that the total amount of daylight hours is nearly identical at higher and lower latitudes, the overall amount of insolation at the higher latitudes is significantly lower. This would result in much colder temperatures, possibly frozen soil and a

significantly shorter growing season. The timing of the insolation is different as well. Although plants would have some insolation 24 h/day in the summer months, they would have no insolation for much of the winter.

Problem #2

For latitudes 0°N, 20°N, 40°N, 60°N, and 80°N, calculate and display the annual insolation at declinations of 13.5° and 23.5° Consider the following questions:

Question 1. At our modern obliquity, two insolation maxima per year occur at the Equator and at 20°N. (At 20°N, you will need to look closely in order to see both maxima.) For an obliquity of 13.5° 20°N latitude does not experience this effect. Why?

Answer. At a modern obliquity of 23.5°, the Sun achieves maximum declination of +23.5° at Summer Solstice, or geographically, at 23.5°N, which is above 20°N. That is, the Sun passes directly over 20°N on its way to maximum declination, and once again on its way to minimum declination (at 23.5°S), leading to two insolation maxima per year. For lower obliquity at 13.5°, the Sun reaches maximum declination at 13.5°N at Summer Solstice, which is below 20°N; thus, the Sun does not pass even once directly over 20°N, and so this latitude experiences only one insolation maximum per year (at Summer Solstice).

Question 2. What are the "Total Daylight Hours" and "Fraction of the Year With Daylight" calculated for each obliquity angle at 80°N? What are the "Maximum Insolation Values" and "Total Annual Insolation" totals at each obliquity angle at 80°N? Explain why these two sets of parameters reveal different trends at this high latitude.

Answer. The total annual daylight hours and fraction of the year with daylight are exactly the same (4344 h and 49.5%, respectively) at each obliquity angle at 80°N. The graphical output suggests that although the Sun rises and sets more days per year at lower obliquity; higher obliquity results in more days per year with 24 h of sunlight. The total amount of sunlight is the same, but the timing of the sunlight changes. The maximum insolation and total annual insolation, however, both increase with increasing obliquity. This is because the Sun reaches higher altitudes at higher latitudes in the summer months at higher obliquity.

Question 3. Assuming a high enough average (annual) temperature, at which obliquity angle might plant life be more successful?

Answer. Lower obliquity values overall promote shorter periods of the year with the Arctic in complete darkness. The consequence of this, however, is that the overall annual insolation values are lower in the Arctic due to the Sun not rising as high in the sky. Assuming that the average annual temperature here was high enough, the lower obliquity values may be more favorable for plant life since the length of the growing season for the plants would increase as lower amounts of stored carbon would be utilized for photosynthetic processes.

APPENDIX B: SHORT LIST OF ONLINE RESOURCES

- Naval Oceanography Portal, Sun and Moon Altitude and Azimuth:
  http://www.usno.navy.mil/USNO/astronomical-applications/data-services/alt-az-us
  This website will calculate the angle of the Sun at any given U.S. city in 10 min intervals for 1 day. The data are displayed in tabular format.
- University of Nebraska-Lincoln Astronomy Applet Project, Motions of the Sun Lab:
  http://astro.unl.edu/naap/motion3/motion3.html
  This is an interactive laboratory assignment page that displays an animation of the changing angle of the sun during the day and gradually throughout the year. It allows the user to manipulate the latitude of observer and also observe the equinox and solstice paths of the Sun on a three-dimensional sphere.
- University of Oregon, Solar Radiation Monitoring Laboratory, Sun Path Chart Program:
  http://solardat.uoregon.edu/SunChartProgram.html
  This website can be used to create a Sun chart, displaying the azimuth and elevation of the sun at any given location for the entirety of one given day or for one half of the year.
- Build Solar, Solar Analysis Tools:
  http://www.builditsolar.com/References/SunChartRS.htm
  This website contains links to a wide variety of online tools designed to give information and calculations to design the proper solar panel system for a given location.
- Australian National University, Educational Sun Applets:
  http://engnet.anu.edu.au/DEpeople/Andres.Cuevas/Sun/Sun.html
  This website contains an applet that calculates and visualizes the angle and azimuth of the Sun at any given location for any time.
- U.S. Department of Energy–Energy Basics:
  http://www.eere.energy.gov/basics/
  This website contains links for basic information on most types of renewable energy resources. This includes helpful information on the design of solar panels and solar power plants.
- National Renewable Energy Laboratory, Renewable Resource Data Center:
  http://www.nrel.gov/rredc/models_tools.html
  This website contains several solar models and calculators designed to maximize the output of a given solar collector or power system.

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