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Comparisons of Mathematics Intervention Effects in Resource and Inclusive Classrooms

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Abstract

In this article, we describe results of a reanalysis of two randomized studies that tested the effects of enhanced anchored instruction (EAI) on the fractions computation performance of students in special education resource rooms and inclusive mathematics classrooms. Latent class analysis and latent transition analysis classified students according to error subtypes and tracked their performance patterns. Results indicated that EAI was more effective than business as usual in reducing combining errors (e.g., adding denominators) and denominator errors (e.g., not finding common denominator) of students with disabilities (SWD) and students without disabilities in both settings. SWD in inclusive classrooms scored higher on the pretest than SWD in resource rooms, but EAI reduced the disparity on the posttest. An important additional finding revealed that the SWD who received more support from special education teachers in inclusive classrooms scored higher and made fewer errors than the SWD who were provided only limited support.

Results of the National Assessment of Educational Progress (National Center for Education Statistics, 2015) suggest that the mathematics performance of eighth-grade students with disabilities (SWD) has not improved. The percentages of SWD who scored below the basic level were 68%, 65%, 64%, and 64% in the test administration years 2015, 2013, 2011, and 2009, respectively. In those same years, the percentages of students without disabilities (SWOD) scoring below basic were 29%, 26%, 27%, and 27% (National Center for Education Statistics, 2016). Students who perform at the basic level are able to compute whole and rational numbers; solve simple problems with the help of charts, diagrams, and graphs; and understand informal algebraic concepts. Students who score below basic have a limited conceptual understanding of rational numbers and weak fractions computation skills.

These low performances have prompted investigators to study the key prerequisite skills and understandings that students need to acquire for computing fractions (e.g., Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013; Siegler & Pyke, 2013; Ye et al., 2016). These include the ability to compare and represent relative magnitudes of whole numbers and fractions, multiply and divide whole numbers, and engage in proportional reasoning. Other factors that affect some students but which are not directly related to disability include attention deficits and working memory (L. S. Fuchs et al., 2014; Hansen et al., 2015). Addressing these issues in elementary and middle school is particularly important because longitudinal studies have shown that students' competencies with fractions at early grades predict their

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performance years later, controlling for whole number skill, intelligence, and working memory (Bailey, Hoard, Nugent, & Geary, 2012; Siegler et al., 2012).

Especially confusing for many students, not just those with disabilities (Behr, Lesh, Post, & Silver, 1983; Kieren, 1980), are the subconstructs associated with fractions (i.e., part-whole/partitioning, ratio, operator, quotient, measure, equivalence). The long history of trying to find the most effective ways of helping students to understand these subconstructs has led researchers to look closely at the errors that students make as they attempt to compute fractions (e.g., Radatz, 1979). More recently, investigators (e.g., Charalambous & Pitta-Pantazi, 2007; Clarke & Roche, 2009; Pitkethly & Hunting, 1996) have used error analysis to identify students' misunderstandings to help teachers anticipate when they are apt to occur and how to correct them.

Despite the well-documented need, relatively few studies have empirically tested interventions designed to improve the fractions computation skills of SWD (Geary, 2006; Siegler, Thompson, & Schneider, 2011). For example, Misquitta (2011) located only 10 studies published between 1998 and 2008 that compared instructional methods designed for the purpose of improving the fractions computation skills of SWD, and Shin and Bryant (2015) found only 17 studies between 1975 and 2014.

Brief History of Enhanced Anchored Instruction

Over the past 20 years, our research teams have tested versions of an instructional package called enhanced anchored instruction (EAI), which we designed for improving the problem-solving performance of SWD (e.g., Bottge, Heinrichs, Chan, & Serlin, 2001; Bottge, Rueda, LaRoque, Serlin, & Kwon, 2007). Our work has been driven by what we consider to be the negative properties of word problems (e.g., artificial contexts, accessibility issues) and the new potential for technology to deliver more realistic and motivating problems. Researchers (e.g., Bruner, 1960; Schoenfeld, 1989) have suggested that the quality of a problem should be judged by its ability to generate interest, strengthen procedural skills, and deepen conceptual understanding of learners. These qualities align closely with the philosophy of the Common Core State Standards in Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), which state that all students should have the opportunity to learn. To help ensure success in students' lives outside school, they should be afforded the accommodations necessary to participate fully in the learning activities in school.

All students should be afforded the accommodations necessary to fully participate in the learning activities in school, thereby helping to ensure success in their lives outside school.

Early versions of EAI included problem-solving lessons in the form of video-based problems (called *anchors*) and related hands-on projects (e.g., building and riding hovercrafts). Although satisfied with the results of the problem-solving components of EAI, we found that too many SWD made only minimal improvement in computing with fractions. In building our first iterations of EAI, we hypothesized that embedding fractions instruction within the engaging problem-solving lessons would result in SWD becoming more motivated and receptive to learning rational number concepts. However, we quickly realized that despite students' motivation to learn how to compute fractions, our lessons fell well short of helping them understand and compute fractions. To address the complexities of rational number concepts and procedures, we developed a new fractions module called Fractions at Work (FAW) for teachers to use with SWD prior to teaching the problem-solving modules. The lessons contained much more explicit instruction and practice with fractions than what our previous lessons had provided. Our findings indicated that the FAW module led to better results in fractions computation and enabled students to more accurately compute the answers to the anchored problems (Bottge, Rueda, Grant, Stephens, & LaRoque, 2010).

The *key model*, as illustrated by a key and lock, has guided our theory–research–instructional development cycle (Bottge, 2001). In previous studies (e.g., Bottge et al., 2014), elements of the key model (e.g., engage students with meaningful problems, explicitly teach foundation skills) emerged as important factors in improving the math skills of SWD. The model emphasizes the equity principle of "mathematics for all" advocated by the National Council of Teachers of Mathematics (2017) and thus applies to students with and without disabilities. The model is based on theories of cognition that consider learner variables (e.g., motivation, foundation skills), contextual variables (e.g., school, community), and task variables (e.g., higher- and lower-order thinking) essential to an adequate description of teaching and learning math.

Recently, we conducted two large randomized experiments to test the revised EAI lessons. Study 1 took place in resource rooms. Results indicated that students in the EAI classrooms outscored students in the business-as-usual (BAU) classrooms on all 10 subscales of a researcher-developed Fractions Computation Test (FCT) and on the six subscales of a standardized test that involved adding and subtracting fractions (Bottge, et al., 2014). We found that students in both instructional conditions often made the same mistakes of combining denominators. Some students knew that they needed to find common denominators to compute the fractions, but they were unable to figure out what they were.

The next year, we conducted Study 2 in inclusive mathematics classrooms (Bottge et al., 2015). Similar to Study 1, posttests showed that students in the EAI condition outscored their peers in the BAU classrooms. Of particular interest were the co-teaching roles that the mathematics teacher and the special education teacher assumed in each classroom. Co-teaching is typically defined as two teachers sharing the instructional responsibilities for all students in the inclusive classroom (Hourcade & Bauwens, 2001). Also known as *collaborative* or *cooperative teaching*, Cook and Friend (1995) defined *co-teaching* as "two or more professionals delivering substantive instruction to a diverse, or blended, group of students in a single physical space" (p. 1). Co-teaching practices have several versions, such as one teach/one assist, station teaching, parallel teaching, alternative teaching, and team teaching (Vaughn, Shay Schumm, & Arguelles, 1997). The most common approach is one teach/one assist, where the content-area teacher does most of the formal teaching and the special education teacher provides students assistance as needed (Solis, Vaughn, Swanson, & McCulley, 2012). Our 191 whole-class period observations revealed that the one-teach/one-assist model was employed most often (44%), followed by team teaching (17%). The other co-teaching arrangements were seldom, if ever, observed.

We also distinguished high- from low-support classrooms and tested possible differences in performance between the groups. In high-support classrooms, the special education teacher took a good share of the responsibility for teaching the lessons with the mathematics teacher. In low-support classrooms, the special education teacher taught very little, if any part, of the mathematics lessons. Prior to scoring the mathematics achievement measures, the five classroom observers reached consensus over which classrooms were high support or low support. Our findings showed that SWD in the high-support classrooms outscored the SWD in the low-support classrooms. SWD and SWOD made errors, and their error patterns were similar to what we found in Study 1.

Research Questions

In this article, we describe the errors that SWD and SWOD made in computing fractions in Study 1 and Study 2 and provide a detailed description of their overall performance in each instructional setting. Specifically, we conducted this secondary analysis to answer three main questions:

Research Question 1: What latent groups of fractions computation errors could be identified for SWD and SWOD, and what were their pretest-to-posttest transition patterns?

Research Question 2: How did the fractions computation scores and error patterns of SWD who were taught with EAI in special education resource rooms compare with those of SWD who were taught with EAI in inclusive mathematics classrooms?

Research Question 3: How did the achievement and error patterns of SWD who were taught with EAI in high-support inclusive classrooms compare with those of SWD who were taught with EAI in low-support inclusive classrooms? How did these performances compare with those of SWOD in inclusive classrooms?

Method

Participants

Data for the follow-up analysis were drawn from two studies that we conducted in urban and rural middle schools in the Southeast. Human subjects permissions were obtained in the original studies. Both studies employed pretest-posttest cluster randomized designs. The main purpose of each study was to test the effects of EAI on students' ability to compute with fractions and problem-solve. Each school's Admissions and Release Committee had decided either (a) that the students' skills were too low for them to succeed in the general education mathematics classrooms and therefore required small group instruction in the resource room or (b) that the SWD would benefit from learning alongside their peers without disabilities.

In Study 1, 49 special education teachers delivered mathematics instruction to their SWD. All students received their total math instruction in the resource room, as documented on their individualized education programs. We randomly assigned 15 schools to the EAI condition (23 teachers, 33 resource rooms) and 16 schools to the BAU condition (26 teachers, 31 resource rooms). Four to six students attended the resource rooms, and class size did not differ between the EAI and BAU conditions. Special education teachers taught the mathematics lessons to students in self-contained special education resource rooms.

The following year, we conducted Study 2 in 25 inclusive mathematics classrooms in 24 middle schools. We selected schools to participate in the study based on interest of the personnel (principals, special education teachers, and mathematics teachers) and the assurance that they had in place at least one inclusive mathematics classroom. From the pool of 24 schools, we randomly assigned 12 schools to each instructional condition. One mathematics class participated in each school with the exception of one school that had two. Pairs of mathematics and special education teachers taught students in the EAI or BAU condition. The average class size ranged from 17 to 21 students and did not differ by type of instruction. In the EAI and BAU conditions, 56 (26%) and 67 (29%) of the students had a disability, respectively, and four to six SWD were included in each mathematics class.

Teacher and student demographics for Study 1 and Study 2 are presented in the online supplemental materials. Most teachers were Caucasian, female, and experienced, and had earned master degrees. Teachers in both studies were comparable across instructional conditions in gender, ethnicity, education level, and teaching experience. Most students in Study 1 were receiving special education services for a disability in one of three categories: specific learning disability, mild mental disability, or other health impairment. Most of the students in Study 2 were receiving special education services for other health impairment or specific learning disability. Detailed descriptions of each disability are available from the National Dissemination Center for Children With Disabilities (2010). Students in both studies were comparable across instructional conditions in gender, ethnicity, subsidized lunch status, and disability area. Instructional sessions in each study were 55 to 60 min, although one school scheduled the school day in 90-min blocks.

Instructional Conditions

Teachers assigned to EAI in each study attended a 2-day summer workshop to learn how to teach with EAI. In Study 1, only special education teachers assigned to EAI attended the training, whereas participants of Study 2 consisted of pairs of mathematics teachers and special education teachers assigned to the EAI condition. A mathematics teacher who had taught the EAI lessons for several years conducted the workshop activities, which were the same in both years.

EAI summary. Five EAI units targeted several of the middle school Common Core State Standards in Mathematics (i.e. Ratios and Proportional Relationships, Number System–Fractions, Statistics and Probability, and Geometry–Graphing) and included a mix of computer-based interactive activities, video-based anchored problems, and hands-on applied projects. The researchers provided teachers with all instructional materials. The first unit, FAW, is a computer-based application designed to build conceptual understanding of and procedural skills with fractions. Concrete materials, such as fraction strips and number lines, were used frequently alongside the lessons to help students understand the important but difficult concept of equivalence. For example, students used their fraction strips to solve problems such as "If you have 1/2 of a stick of gum and your friend has 1/4 of a stick, who has more?" and "How much more does she have?" The first chapters of FAW were devoted to basic concepts, such as the purpose and function of fractions, the idea of equivalence, and the role of numerators and denominators. An interactive tape measure displayed in the software showed that the value of fractions depends on the number of parts into which an inch is divided (i.e., denominator) and the number of these parts available (i.e., numerator). In successive chapters, students were shown how to add simple fractions with like and unlike denominators, how to add and subtract mixed numbers, and how to rename and simplify fractions. The software also provided formative checks of students' skills with 10 practice problems matched to the instructional content. This unit took classes an average of 15 instructional days to complete.

Following the explicit instruction on fractions with FAW, teachers taught two problem-solving anchors that required students to work with fractions to solve the problems. The first anchor, called Fraction of the Cost, consisted of an 8-min problem-solving video portraying three friends who want to build their own skateboard ramp. The task for the students was to help the characters in the video figure out how they could build the ramp under budget. To solve the problem, students had to apply what they had learned in FAW to add and subtract fractions. Teachers taught Fraction of the Cost for an average of 11 instructional days.

The next unit was a hands-on problem called the Hovercraft Challenge. Students had to apply the concepts and skills that they learned in the first two units to build a hovercraft. Students drew schematic plans for the "rollover cages" of their hovercraft and then built their scale models out of plastic straws. On subsequent days, they built full-size cages out of PVC pipe. The payoff for students was "race day," when they rode their hovercrafts. Like the Fraction of the Cost unit, the project required students to figure out how to use materials in the most economical way. This unit is complex and took classes 22 instructional days to complete.

BAU summary. Classroom observations and close inspection of the mathematics objectives and curriculum indicated a close parallel to the EAI condition. The school district curriculum was aligned with the Kentucky Core Academic Standards and Combined Curriculum Document from the Kentucky Department of Education. Teachers and students used technologies such as computers, interactive whiteboards, and manipulatives to teach several lessons.

Typically, teachers began lessons with Calendar Math (Gillespie & Kanter, 2000). In most classrooms, instruction focused on strategy instruction for solving fractions computation problems. Teachers showed how to solve mathematics problems, and students were encouraged to follow the step-by-step procedures. For example, each student was assigned a certain number of fractions (1/2, 2/5, 3/10, etc.) and then asked to arrange them from smallest to largest. To complete this activity, students needed to review the previous lesson, work with partners, or use additional support (e.g., calculators).

In another lesson, teachers brought grocery flyers from local grocery stores (e.g., Kroger) and encouraged students to identify which store offered a better deal for grocery items. For example, Walmart had three dozen eggs for $4.50 versus Kroger, which sold two dozen eggs for $2.50. Students calculated "unit rate" to answer which store offered a better deal (e.g., $4.50/3 = $1.50 at Walmart vs. $2.50/2 = $1.25 at Kroger). Teachers gave formative assessments to check students' mastery of computation procedures and understanding of the mathematics concepts. Hands-on activities, web-based instruction, and drill-and-practice were also used throughout the study period. Another project-based example involved calculating unit rate (e.g., ratio and proportional relationships) from several grocery ads.

Fidelity of Implementation

Project personnel serving as primary observers conducted a total of 290 whole-class observations in Study 1 (173 EAI, 117 BAU) and 285 whole-class observations in Study 2 (191 EAI, 94 BAU). Interobserver agreement was 94% in Study 1 (43 secondary observers) and 90% in Study 2 (41 secondary observers). In both studies, the observations showed that teachers taught EAI with a high level of treatment fidelity (94%).

Mathematics Measures

The mathematics assessments were the same in Study 1 and Study 2. Two researcher-developed tests and two standardized achievement subtests were administered over 3 consecutive days immediately prior to and following the instructional period. A second rater independently scored 20% of the pretests and posttests. For the purpose of this secondary analysis, we focused our attention on the FCT. The FCT is 20-item (14 addition, six subtraction), 42-point researcher-developed test that measures students' ability to add and subtract simple fractions and mixed numbers with like and unlike denominators. The test also includes items that require students to add three fractions. Students were told to simplify their answers and to show all their work. Calculator use was not allowed. On 18 items, students earned 1 point for showing correct work and 1 point for the correct answer. On two items with mixed numbers that required renaming prior to subtracting, students were awarded an additional point. Internal consistency estimates were .96, 95% CI [.95, .97], at pretest and posttest for SWD and .97, 95% CI [.96, .97], at pretest and .96, 95% CI [.95, .97], at posttest for SWOD. Interrater agreement was 99% on the pretest and 97% on the posttest. In this study, items were dichotomized into binary responses of 0 for incorrect responses and 1 for correct responses. Partial credit on an item was scored as a correct response.

Latent Class Analysis and Latent Transition Analysis

Latent class analysis (LCA; Lazarsfeld & Henry, 1968) is a statistical method for classifying individuals into latent subtypes (referred to here as *latent groups*). The method detects latent groups of individuals who are homogeneous on some latent (i.e., unobserved) characteristics. The latent groups in this study were formed per their responses to the questions on the FCT. This use of the LCA provided a way of capturing latent (i.e., unobserved) heterogeneity in the sample. Latent transition analysis (Collins & Lanza, 2010) is an extension of LCA, which accounts for changes in latent groups over time.

LCA and latent transition analysis were used in this study to investigate qualitative and quantitative changes in students' errors on fractions computation items from the pretest to the posttest. Students were classified into latent groups depending on the types of errors they made.

The numbers of latent groups in the pretest and posttest data were detected via the bayesian information criterion index (BIC; Schwartz, 1978), the bootstrapped likelihood ratio test (BLRT; McLachlan & Peel, 2000), the proportions of the students in individual latent groups (mixing proportions), and the interpretability of the results. We used BIC and BLRT because they help identify the correct number of latent classes in mixture models (Nylund, Asparouhov, & Muthén, 2007). The model that yields the smallest BIC value among the LCA models suggests the best-fitting model. For BLRT, a statistically significant *p* value indicates that the *k* group model provides a better fit than the $k-1$ group model (i.e., the LCA model with one less group). Nonsignificant *p* values indicate failure to reject the null hypothesis and that the *k* group model is better than $k-1$ group model. In that case, the $k-1$ group model is considered to function neither worse nor better than the *k* group model.

Changes in students' membership in a latent group from pretest to posttest were then examined to determine the effects of the EAI intervention. Mplus (Version 7) was used in calculating model fit indices for selecting a best model. After the better-fitting model was determined, all subsequent analyses were implemented via SAS PROC LCA/latent transition analysis (Lanza et al., 2015).

Results

Types of Errors

Based on a first review of students' item responses, we found two specific mistakes that accounted for most errors. Students who made the combining error consistently applied the same operation of adding or subtracting numerators and denominators:

$$1/3 + 1/3 = 2/6 \qquad 7/8 - 1/4 = 6/4$$

Students who made the denominator error attempted to find a common denominator but made the mistake of either not selecting a denominator from one of the fractions that would make the two fractions equivalent or not finding a unique number to serve as the common denominator:

$$1/2 + 3/16 = 4/161$$

Because the other types of errors were far less common, we combined them into a single miscellaneous category for purposes of this analysis. An example of miscellaneous errors was an Add All procedure, where the student computed the sum of all the numbers in both fractions.

Latent Groups and Transition Patterns

The first research question asks for the characteristics of the latent groups and transition patterns of all students participating in both studies. Table 1 shows the number and types of errors that students made on individual FCT items. Overall, students tended to make fewer errors on the posttest than the pretest, which suggests that students in both instructional settings (resource room and inclusive classroom) profited from EAI and BAU.

Latent groups. To determine the number of latent groups in the data, we fit nine LCA models with one to nine latent groups to the pretest and posttest data. Based on BIC, BLRT, and the mixing proportions, a four-group model was the best fit. The dominant error type that students made in each latent group was used to characterize each latent group. Students in Latent Group 1, Latent Group 2, and Latent Group 3 had a high probability of making a combining error, denominator error, or other miscellaneous error, respectively. Students in Latent Group 4 had a high probability of making no errors.

Table 2 displays the numbers of students in each latent group and their pretest and posttest means and standard deviations on the FCT. On the pretest, the combining error group was the largest latent group (42%), but on the

Table 1. Frequencies of Errors on Individual Items on the Pretest and Posttest (n = 756).

			Pretest				Posttest			
Item[a]	Description[b]		Combining	Denom	Misc	No error	Combining	Denom	Misc	No error
1	Simple	Like	207	3	118	428	97	0	64	595
2	Simple	Like	201	4	112	439	91	2	63	600
3	Simple	Unlike	342	89	191	134	192	78	158	328
4	Simple	Unlike	321	82	216	137	187	78	174	317
5	Simple	Unlike	337	77	235	107	194	78	209	275
6	Simple	Unlike	327	91	216	122	196	97	171	292
7	Mixed	Unlike	305	89	245	117	186	85	206	279
8	Mixed	Unlike	308	70	271	107	186	84	225	261
9	Mixed	Unlike	286	99	260	111	174	94	231	257
10	Mixed	Unlike	298	75	280	103	186	78	239	253
11	Simple	Unlike	312	72	240	132	192	70	206	288
12	Simple	Unlike	294	104	239	119	188	90	213	265
13	Mixed	Unlike	282	99	272	103	170	94	261	231
14	Mixed	Unlike	291	101	272	92	185	100	260	211
15	Simple	Like	166	5	165	420	71	3	117	565
16	Simple	Unlike	312	91	237	116	181	102	205	268
17	Mixed	Like	169	8	281	298	80	4	229	443
18	Mixed	Unlike	13	1	676	66	11	1	604	140
19	Mixed	Unlike	258	76	325	97	152	89	296	219
20	Mixed	Unlike	261	51	404	40	142	43	486	85

Note. Denom = denominator error; misc = miscellaneous error.
[a]Items 1–14, addition; Items 15–20, subtraction. [b]Simple or mixed number fractions, like or unlike denominators.

Table 2. Frequencies and Mean of Fractions Computation Test for Each Latent Group on the Pretest and Posttest (n = 756).

	Pretest			Posttest		
Latent group	*n*	*M*	*SD*	*n*	*M*	*SD*
Combining	316	1.65	1.65	187	2.30	2.00
Denom	111	3.77	1.04	102	4.62	2.33
Misc	196	1.76	2.38	173	3.36	3.12
No error	133	16.93	3.09	294	17.77	2.35
Total	756	4.68	6.09	756	8.87	7.55

Note. Denom = denominator error; misc = miscellaneous error.

posttest, the no-error group was the largest (39%). Students' mean scores increased from pretest to posttest. Means for the no-error group were highest for both the pretest and the posttest, and means for the denominator error group were the lowest. Transitions from the combining error group to the no-error group reflected positive instructional effects.

Transition patterns. Table 3 displays students' transition patterns from the pretest to the posttest by instructional condition. The pretest results show that few students in either the EAI (n = 65) or BAU (n = 68) condition were in the no-error group. However, on the posttest, a majority of the students in the EAI condition (i.e., 205/360 = 57%) moved from one

Table 3. Transition Frequencies From Pretest to Posttest Under Each Instructional Condition (*n* = 756).

	Posttest				
Pretest	Combining	Denom	Misc	No Error	Subtotal
			EAI		
Combining	28	31	20	74	153
Denom	1	14	8	34	57
Misc	9	8	34	34	85
No error	0	1	1	63	65
Subtotal	38	54	63	205	360
			BAU		
Combining	103	15	28	17	163
Denom	9	22	17	6	54
Misc	32	10	57	12	111
No error	5	1	8	54	68
Subtotal	149	48	110	89	396

Note. Table 3 displays the numbers of students in each latent group on the pretest and the posttest in the EAI and BAU instruction conditions. For example, in the first row of the EAI panel, 28 students were in the combining group on the pretest and stayed there for the posttest, whereas 74 students started there but then moved to the no-error group. Denom = denominator error; misc = miscellaneous error; EAI = enhanced anchored instruction; BAU = business as usual.

of the latent error groups to the no-error group or remained in the no-error group. By contrast, only 89 of the 396 BAU students (i.e., 22%) transitioned to or remained in the no-error latent group on the posttest.

Of the students in the lowest-ability latent group (i.e., combining error group), 48% in the EAI classrooms (i.e., 74/153) transitioned to the no-error group, as opposed to 10% of the students in BAU classrooms (i.e., 17/163). Results from a multinomial logistic regression analysis for predicting latent group membership on the posttest found this difference to be significant. The odds were 22 to 1 that SWD in the EAI condition would transition from the combining-error latent group to the no-error latent group on the posttest.

Table 4 shows transition patterns for SWD in EAI and BAU conditions. For SWD students, the EAI effect was clearly evident. On the pretest, SWD in both EAI and BAU classrooms tended to make combining errors (i.e., 99/202 = 49% in EAI, 110/231 = 48% in BAU). On the posttest, 45% of SWDs in EAI classrooms transitioned to or remained in the no-error latent group (i.e., 90/202), as compared with only 10% of SWD in BAU classrooms (i.e., 22/231). Overall, SWD in the EAI classes committed fewer errors and improved their skills in computing with fractions when compared with SWD in BAU classrooms.

Fractions Computation in Two Settings

The second research question asked how the fractions computation scores and error patterns of SWD who were taught with EAI in special education resource rooms compared with those of SWD who were taught with EAI in inclusive mathematics classrooms. Table 5 and Figure 1 present comparisons of FCT mean scores and error patterns for SWD in each EAI classroom setting. Pretest scores showed that SWD in the inclusive classrooms scored higher on the pretest than SWD in the resource room settings ($t = 4.15$, $df = 753$, $p = .001$, ES = 0.30). On the posttest, SWD in the inclusive classrooms also scored higher than SWD in the resource rooms, but the effect size was about half of what it was on the pretest ($t = 2.25$, $df = 753$, $p = .025$, ES = 0.16). Results also indicated that only one student in EAI resource classrooms was in the no-error

Table 4. Transition Frequencies From Pretest to Posttest for SWD in EAI and BAU (n = 433).

Pretest	Posttest: Combining	Denom	Misc	No error	Subtotal
			EAI		
Combining	20	20	16	43	99
Denom	1	5	6	14	26
Misc	9	6	29	22	66
No error	0	0	0	11	11
Subtotal	30	31	51	90	202
			BAU		
Combining	81	8	17	4	110
Denom	6	11	10	1	28
Misc	26	6	40	3	75
No error	2	1	1	14	18
Subtotal	115	26	68	22	231

Note. Table 4 displays the numbers of SWD in each latent group on the pretest and the posttest in the EAI and BAU instruction conditions. For example, in the first row of the EAI panel, 20 students were in the combining group on the pretest and stayed there for the posttest, whereas 43 students started there but then moved to the no-error group. SWD = students with disabilities; EAI = enhanced anchored instruction; BAU = business as usual; denom = denominator error; misc = miscellaneous error.

Table 5. Mean Scores for Enhanced Anchored Instruction Students With Disabilities on the Fractions Computation Test: Resource Versus Inclusive Classrooms (n = 202).

	Class					
	Pretest			Posttest		
Latent group	*n*	*M*	*SD*	*n*	*M*	*SD*
			Resource room			
Combining	75	1.21	1.58	20	1.90	2.13
Denom	16	3.38	1.02	21	5.52	3.08
Misc	54	0.43	1.11	46	3.83	3.33
No error	1	10.00	—	59	16.47	2.78
Subtotal	146	1.22	1.76	146	8.92	6.95
			Inclusive classroom			
Combining	24	1.92	1.50	10	2.90	1.60
Denom	10	4.40	1.17	10	4.60	1.07
Misc	12	2.42	1.98	5	7.00	7.00
No error	10	16.30	2.87	31	17.35	2.52
Subtotal	56	5.04	5.67	56	11.57	7.13

Note. Denom = denominator error; misc = miscellaneous error.

latent group on the pretest, but this number increased to 59 students on the posttest.

High and Low Support in Inclusive Classrooms

Table 6 and Figure 2 present comparisons of FCT mean scores of SWD in high-support EAI mathematics classrooms, SWD in low-support EAI inclusive mathematics classrooms, and SWOD in all the inclusive mathematics classrooms. To answer Research Question 3, we first compared the performance of SWD in high-support inclusive classrooms with the performance of SWD in low-support inclusive classrooms. On the pretest, SWD in high- and

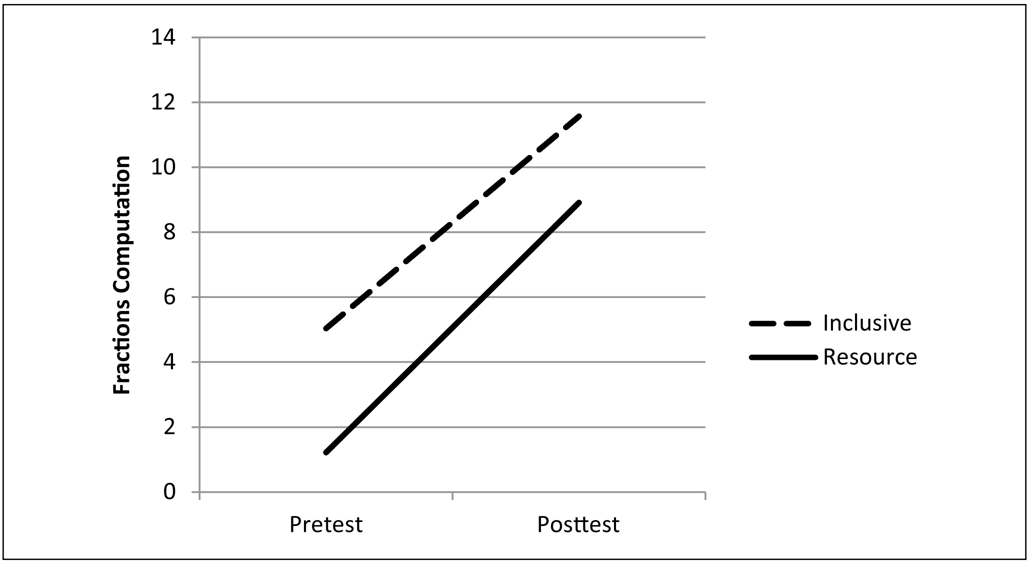

Figure 1. Mean scores of students with disabilities receiving enhanced anchored instruction in resource and inclusive settings.

Table 6. Students' Mean Score on Fractions Computation Tests in EAI Inclusive Classroom (n = 214).

	Learning disability: Support level					
	Pretest			Posttest		
Latent group	*n*	*M*	*SD*	*n*	*M*	*SD*
			Students without disabilities			
Combining	54	2.80	1.39	8	2.88	1.46
Denom	31	3.65	1.05	23	4.65	2.37
Misc	19	3.16	2.03	12	4.25	3.19
No error	54	17.26	2.94	115	18.67	1.43
Total	158	7.95	7.05	158	14.73	6.70
			Students with disabilities: High			
Combining	11	2.00	1.18	2	4.50	0.71
Denom	5	4.40	1.52	3	4.67	1.15
Misc	5	1.60	1.14	1	2.00	—
No error	7	15.29	2.81	22	17.32	2.53
Total	28	5.68	5.97	28	14.50	5.96
			Students with disabilities: Low			
Combining	13	1.85	1.77	8	2.50	1.51
Denom	5	4.40	0.89	7	4.57	1.13
Misc	7	3.00	2.31	4	8.25	7.41
No error	3	18.67	1.15	9	17.44	2.65
Total	28	4.39	5.39	28	8.64	7.10

Note. EAI = enhanced anchored instruction; denom = denominator error; misc = miscellaneous error.

low-support EAI inclusive classrooms did not significantly differ on their FCT scores (t = 0.82, df = 752, p = .41). However, on the posttest, the mean score of SWD in high-support EAI inclusive classrooms was significantly higher than that of SWD in low-support

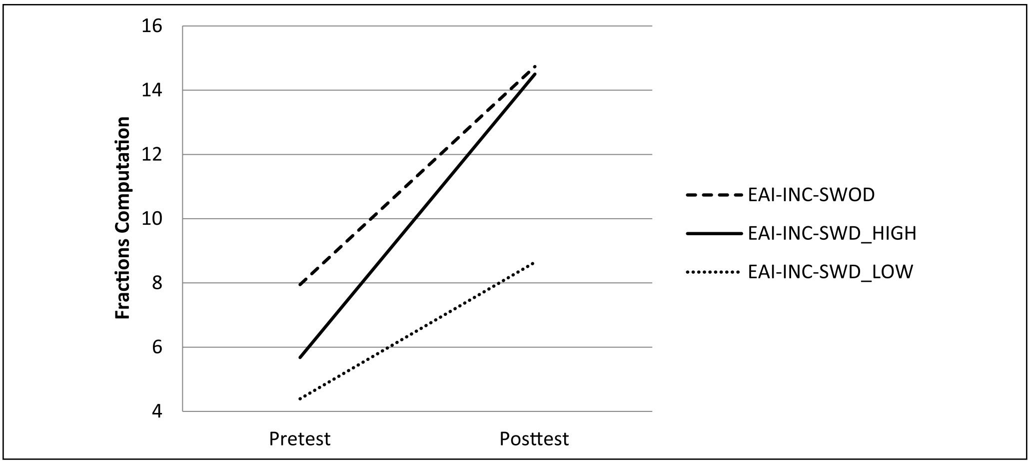

Figure 2. Mean scores of students without disabilities (SWOD) and students with disabilities (SWD) receiving enhanced anchored instruction (EAI) in high- or low-support inclusive (INC) mathematics classrooms. SWD = students with disabilities.

EAI inclusive classrooms ($t = 3.23$, $df = 752$, $p = .001$). Figure 2 shows that on the posttest, mean fraction computation scores for SWD improved in the high-support classrooms to the point of being comparable with the performance of SWOD. In fact, the difference between the posttest means of SWD in the high-support group classrooms and SWOD was only 0.23 and not significant ($t = 0.17$, $df = 752$, $p = .87$).

Discussion

The purpose of this study was to report the results of a fine-grained analysis of error patterns and performance outcomes from two randomized studies funded by the Institute of Education Sciences. Teachers in both studies taught the same units of the EAI mathematics intervention, targeted the same Common Core objectives, and employed the same randomized pretest-posttest design. The major difference between them was the instructional setting: Study 1 (Bottge et al., 2014) was conducted in resource rooms, whereas Study 2 took place in inclusive mathematics classrooms (Bottge et al., 2015). Overall results showed that EAI was effective in improving the fractions computation performance of SWD by reducing two common errors: adding or subtracting denominators and computing fractions with unlike denominators.

Descriptive statistics show that the pretest scores of SWD in Study 1 are significantly lower than those of SWD who were taught in inclusive settings. This was expected because SWD who receive their mathematics instruction in pullout settings typically require more assistance than do SWD who attend regular education classes. Posttest scores also favored SWD in the inclusive classrooms versus SWD taught with EAI in the resource rooms. However, the posttest difference was much less than the pretest difference, which can be explained by the fact that EAI eliminated more errors of SWD in the resource rooms than in the inclusive settings.

Close inspection of the results also detected differences in the performance of SWD depending on the way that special education teachers and content area teachers structured their co-teaching arrangement. Results show that the SWD who received more individualized instructional support from special education teachers in the inclusive classrooms improved their skills much more than SWD who received little support. In fact, the posttest scores showed no significant difference between the high-support SWD and the SWOD overall. By contrast, the SWD in mathematics classes where the special education teacher provided little, if any, supplemental instruction made minimal, if any, performance gains.

This latter finding is especially important considering special education legislation (i.e., Individuals with Disabilities Education Act, 2006) that mandates schools to include SWD in general education classes when appropriate. Movements such as the Regular Education Initiative suggested that pullout settings should in most instances be eliminated. Advocates of inclusive practices have cited a range of educational benefits for SWD, such as more time on task, better grades, and higher scores on standardized tests (Rea, McLaughlin, & Walther-Thomas, 2002; Salend, 2011) However, recent studies (Kurz et al., 2014) and position papers (e.g., D. Fuchs & Fuchs, 2015) have called into question the benefits of inclusionary practices, asserting that many SWD who are included in general education classes actually spend less time on standards-related lessons, engage in more noninstructional time, and are exposed to less content than the rest of the class. Our findings support the more recent stance that SWD who are taught in small groups with a specialized curriculum that targets their learning deficits can make larger academic gains than SWD who are taught with the general school curriculum in the inclusive mathematics settings.

Positive academic outcomes are possible for SWD in both settings depending on the amount and quality of the special education teachers' instructional involvement.

Our studies suggest that positive academic outcomes are possible for SWD in both settings depending on the amount and quality of the special education teachers' instructional involvement. In Study 2, the reason for teachers not taking on more instructional responsibility in inclusive classrooms cannot be attributed to not knowing the content of the lesson plans or the teaching procedures, because the special education teacher and cooperating mathematics teacher attended the EAI training together. All teachers were provided with detailed day-by-day lesson plans. Although just a guess based on 285 whole-class period observations (191 and 94 in EAI and BAU, respectively) conducted by project personnel and poststudy discussions with teachers, our conclusion was that the teaching relationships had been formed long before our involvement in their classrooms, and would be difficult to change.

Limitations

We are encouraged by these results, but, unfortunately, they also demonstrate the large number of students who needed more time during the intervention phase of these studies to develop a sound conceptual understanding of and computation skills with fractions. The first lessons of the fractions unit were designed to help students understand the purpose of fractions and the functions of the numerator and denominator. The next activities helped students to recognize when a common denominator was needed and how to find one. Teachers in both EAI classroom settings devoted much of their instruction to explaining why adding or subtracting denominators is not appropriate. We are quite sure that the next version of our fractions lessons will engage the students in more systematic reviews of the FAW instructional modules based on their performance on daily computation checks. We will also provide students with additional examples of concrete manipulatives (e.g., fractions strips) to help more SWD and SWOD attain greater mastery of fractions.

Implications for Practice

Our findings align closely with the Opportunity to Learn factors, which include allocated instructional time, content of the intended curriculum, and quality of instruction. EAI as a specialized curriculum can reduce students' errors in computing fractions in resource room and inclusive classrooms. However, as is usually the case, the quality of instruction made an important difference in the performance of SWD, especially in the inclusive settings. In the mathematics classes where the special education participated with the mathematics teacher in teaching the concepts, the posttest scores of SWD approximated those of SWOD. In other words, EAI was effective in raising the achievement level of the SWD in inclusive classrooms but only when the special education teachers contributed meaningful instructional support.

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Supplemental Material

The supplemental material is available in the online version of this article.

Authors' Note

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