Rethinking The Role Of Counting In Mathematics Learning

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Abstract

This paper challenges the emphasis on counting in New Zealand's Numeracy Development Project (NDP), arguing that subitizing provides an alternative pathway to quantification. Longitudinal data is presented showing that children's subitizing skills at the age of five years were a strong predictor of their later success in mathematics at the age of nine years. Sophian's work of comparison of continuous quantities is explored. Data from students whose teachers participated in NDP professional development programmes are compared with the expectations documented in the Mathematics Standards. The analysis shows that the percentages of students who reach the expected level for their year group is well short of the Standards. It is suggested that the large number of micro-stages at the lower end of the Number Framework together with the positioning of part-whole strategies as the fifth stage on the framework may give the impression that teachers should not focus on the relationship between a whole and its parts until students are able to count on (stage 4). The paper concludes by suggesting that a dual focus on subitizing and counting right from the beginning might help students to develop a deeper understanding of cardinality and of the relationship between a whole and its parts, resulting in them reaching expected levels earlier.

In the last few decades, mathematics education has undergone major reform worldwide. New Zealand, along with other western countries responded to its relatively poor results on the Third International Mathematics and Science Study (TIMSS) by developing an initiative designed to strengthen students' understanding of mathematics and numeracy. Most initiatives in mathematics focused initially on the early years of school (Bobis et al., 2005; British Columbia Ministry of Education, 2003; Commonwealth of Australia, 2000; Department for Education and Employment, 1999; Ministry of Education, 2001; National Council of Teachers of Mathematics, 2000). These initiatives have several features in common, including professional development programmes for teachers to enhance their capacity to teach mathematics, the construction of developmental frameworks that describe progressions in the learning of mathematics, individual task-based interviews to assess students' thinking and reasoning in mathematics, and a constructivist/ socioconstructivist view of mathematics teaching and learning, so that instruction builds on the existing knowledge of the learner (Bobis et al., 2005).

At the core of many numeracy initiatives are the learning frameworks, consisting of progressions of increasingly sophisticated strategies in particular mathematical domains (Bobis et al., 2005). Most initiatives include a sequence of stages outlining progressions in number, reflecting the perceived importance of number in the curriculum, and the comprehensive foundation of research that is available to support this (Kilpatrick, Swafford, & Findell, 2001). The work of Steffe has been extremely influential in the development of progressions in the domain of number (Steffe, 1992).

Many of the learning frameworks constructed as part of reform in mathematics education begin with the development of counting skills (e.g., Bobis et al., 2005; Ministry of Education, 2008). Such frameworks have taken the work of Gelman and Gallistel (1978), who documented the progression from saying number names in order (rote counting), to assigning number names to items while maintaining one-to-one correspondence between the number name and the item (object counting), and finally recognition of the cardinal principle—the idea that the last number name used when counting a collection of objects tells how many items are in the collection in total. Young children who do not understand cardinality typically respond to a question about "how many altogether?" by recounting the objects they have just counted. Once children have mastered object counting, they can use it to solve problems involving addition and subtraction (see Figure 1 for the Number Framework).

Stage	Description
0	**Emergent** Cannot count
1	**One-to-One Counting** Can count a small collection up to 10, but cannot use counting to add or subtract collections.
2	**Counting from One on Materials** Can add two collections by counting, but counts all the objects in both collections
3	**Counting from One by Imaging** Adds two collections by counting all, but counts mentally by imaging objects
4	**Advanced Counting** Recognises that the last number in a counting sequence stands for all the objects in the collection, so counts on for the second collection
5	**Early Additive Part-Whole Strategies** Recognises that numbers are abstract units that can be partitioned (broken up) & recombined (part-whole thinking). Uses known number facts to derive answers

6 **Advanced Additive Part-Whole Strategies**
Chooses from a range of different part-whole strategies to find answers to addition and subtraction problems

7 **Advanced Multiplicative Part-Whole Strategies**
Chooses from a range of different part-whole strategies to find answers to multiplication and division problems

8 **Advanced Proportional Part-Whole Strategies**
Chooses from a range of different part-whole strategies to find answers to problems involving fractions, proportions, and ratios

Figure 1. New Zealand's Number Framework

Subitizing

While counting is an important process that leads to quantification, it is not the only way for students to begin developing numerical thinking. An alternative method of quantification is subitizing—recognising the number of items in a small collection without counting (Clements, 1999; Sarama & Clements, 2009, 2010), or "instantly seeing the quantity" (Kling, 2011, p. 85). Clements and Sarama distinguish between two types of subitizing: perceptual and conceptual subitizing. Conceptual subitizing involves combining several small quantities that have been identified initially by perceptual subitizing to calculate a total sum (e.g., recognizing a pair of dice displaying identical patterns of five as signifying simultaneously two groups of "five" and one group of "ten"). Children who play dice games and dominoes quickly learn to name the stylised patterns presented on the dice without needing to count all the dots (see Young-Loveridge, 1991, 2004). According to Sarama and Clements (2010), subitizing "is most children's first method of quantification" (p. 117). They also point out that subitizing the number of items in a collection "encourages and reinforces understanding of the cardinal principle" (p. 117).

A longitudinal study of students from school entry at five until the end of Year 4 identified subitizing skill at age five the second strongest predictor of their later success in mathematics at the age of nine years, explaining 41 percent of variance in overall performance at the end of Year 4 (Young-Loveridge, 1991). Only the task asking students to construct groups was a better predictor, explaining 45 percent of variance at the end of Year 4. It was for this reason that subitizing (referred to as pattern recognition) was included in Checkout/Rapua, the supermarket game developed to assess young children's mathematics/numeracy on entry to school as part of the School Entry Assessment kit (see Ministry of Education, 1997). The longitudinal study showed that more than two-thirds (70%) of new five-year-olds could subitize a pattern of "three", and more than half could subitize patterns of "four" and "five" (58% and 52%, respectively).

There is immense potential for students to learn basic facts for addition and subtraction of small quantities using subitizing as the foundation. For example, Buchholz (2004) used doubles as a foundation with her Grade 2 class, and extended the combinations to "Doubles Plus One" (e.g., seeing 5 + 6 but thinking 5 + 5 + 1). The "Doubles Minus One" strategy emerged as students recognised it as an alternative strategy for the same problem (e.g., seeing 5 + 6 and thinking 6 + 6 – 1). "Doubles Plus Two" and "Doubles Minus Two" emerged as strategies where one addend was two more (or less) than the other (e.g., seeing 5 + 7 and thinking 5 + 5 + 2 or 7 + 7 – 2). In the case of subitizing, the important idea is the notion of "One More Than" or "One Less Than" the quantity that has been subitized. Although the answer to these questions is the same as "The Number Just After" and "The Number Just Before," the posing of the question in terms of a cardinal value rather than asking about number sequence orients the students towards cardinality and quantification instead of the counting process. Clements and Sarama (2009) suggest the use of games that help students to take mental "snapshots" of dot patterns so they can match them (or identify a mismatch) with other patterns showing the same (or different) quantities in different spatial arrangements (see Figure 2 for an example of "The Odd One Out").

Comparing Continuous Quantities

Support for the idea that counting may not be the only starting point for the development of numerical knowledge comes from the work of Catherine Sophian (2007, 2008), who argues that mathematical thinking begins not with counting, but with comparisons of continuous quantities. Such quantities are defined as continuous because they are measurable rather than countable (discrete), and include things like sand, water, and dough. Sophian's view is based on several lines of evidence, including studies showing that very young children develop an understanding of very general properties of quantities long before they are able to articulate counting words. Sophian (2008) points out that counting presupposes a quantity, whereas quantity comparison does not presuppose number. She argues that numbers are arbitrary symbols that are used to represent measured quantities, whereas quantities are physical properties that can be measured directly. According to Sophian, there are some concepts that are even more fundamental than number, including the concept of set (a group or collection), the idea of equivalence, and the concept of unit. She draws on the work of Soviet psychologists who have noted that a fundamental aspect of the concept of unit is the idea of equivalence between units. Also, because different units can be applied to the same quantities, "numerical values are essentially representations of the relation between the quantity they represent and a chosen unit" (Sophian, 2008, p. 25).

Sophian (2004) suggests that one possible reason that students experience so much difficulty learning about fractions in the later primary years is that the initial emphasis on counting and whole-number quantities may be counterproductive, and may in fact inhibit students' learning about part-whole relationships in the context of fractional quantities where the size of the parts is critical to understanding fractional concepts. She cites a number of studies showing that young children often centre (focus narrowly on a single attribute) on the number of objects while ignoring the size of the objects, and that a focus on counting encourages this narrow view.

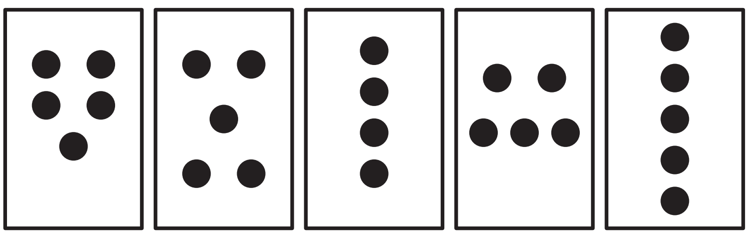

Figure 2. An example of "Find the Odd One Out" showing different arrangements for conceptual subitizing of 5 as 4 + 1, 2 + 1 + 2, 2 + 3, or 5, with 4 as the odd one out in the middle (adapted from the work of Clements & Sarama, 2009, p. 11)

She argues that children need far more experience with continuous quantity and the unitisation of that quantity in order to draw their attention to the magnitude of quantities, not just the number. One of the ways she has done this is to investigate children's understanding of the order relations of unit fractions (e.g., $\frac{1}{12}$ and $\frac{1}{3}$) and of their "complements" (the fraction formed by subtracting the unit fraction from one; e.g., $\frac{11}{12}$ and $\frac{2}{3}$). Some of the students she studied appeared to understand the idea that the larger the denominator, the greater the number of parts, and the smaller the magnitude of each part, but could not coordinate information about the numerator as well as the denominator in comparing the size of the "complements". These children tended to focus just on the denominator, judging that because thirds are larger than twelfths, $\frac{11}{12}$ must be smaller than $\frac{2}{3}$ (Sophian, 2004). Their difficulty appeared to stem from them not being able to coordinate different units of quantity (the unit and the fractional part). According to Sophian, this goes back to their lack of understanding about the iteration (repetition) of units of measurement.

In another study, Sophian and her colleagues (1997) found that young children initially had difficulty appreciating the inverse relationship between the number of shares into which a quantity is partitioned and the size of those shares. However, after only a brief instructional period, even 5-year-olds understood this connection. Sophian and colleagues (1995) have investigated children's ability to reason relationally and their understanding of numerosity (counting to determine total number). They argue on the basis of their findings that reasoning relationally develops quite independently of counting processes. In earlier work, Sophian (1987) had shown that very young children tended not to use counting to compare two quantities even though they were clearly able to count quite proficiently.

According to Sophian, the instructional implications of the "counting first" perspective are very different from those of the "comparison of quantities" perspective. Instruction is important for developing an understanding of units, learning about whole numbers, and later learning about fractional quantities, both of which involve understanding about how units are used to make sense of numerical representations. Sophian points out that in additive reasoning, the same unit is applied to both quantities, enabling a comparison that shows how much greater one quantity is than another. In multiplicative reasoning, one quantity is used as a unit to measure the other, enabling a comparison showing how many times greater one is than another. Sophian (2008) argues that "early instruction that focuses on particulars and eschews abstraction may result in ways of thinking about the particulars that are not congenial to the abstractions to be studied later" (p. 39). Moreover, "the difficulties students experience with relatively advanced topics such as fractions may derive from an inadequate grasp of much more basic concepts" (p. 40). Hence, early instruction must be designed to take into account the relationships between concepts taught early on and the mathematics that students will need to learn in later years. Sophian suggests that the "comparison of quantities" perspective can be used to build concepts of both quantity and of relative amount. She urges early years teachers to go from the observation of relationships in concrete contexts to mathematical abstraction, by asking children to consider the generality of the observed relationships: "Is that always true? How can we be sure?"

Sophian (2007, 2008) suggests that an exclusive focus on counting in the early years may put too much emphasis on discrete quantity and not enough on continuous quantity. This brings to mind a delightful video clip used to illustrate Piaget's notion of centration, the tendency by younger children to focus on just one feature of the display in a conservation of quantity task (e.g., the height of liquid in a container but not its width, or the length of a row of counters but not their spacing). In the video clip, the child is given one cookie and the interviewer keeps two cookies. The interviewer asks the child if that is fair. As expected, the child disagrees on the grounds that he has only one cookie and the interviewer has two. The interviewer then breaks the child's cookie into two pieces and asks if it is fair now. The child responds: "yes, because these are two pieces and those are two pieces". This example beautifully illustrates the concept of centration, with the child focusing only on discrete quantity (the number of pieces), and ignoring continuous quantity (the amount of cookie).

The Numeracy Development Project

Sophian's (2004, 2007, 2008) challenge to the "counting first" perspective raises some important questions about the Number Framework used in New Zealand's Numeracy Development Project (NDP), (Figure 1). The hierarchical arrangement of stages on the framework implies that students should move from "Counting All" (stages 2-3) to "Counting On" (stage 4) before they begin thinking about the relationships among parts and wholes (stage 5 Early Additive Part-Whole thinking).

Hence the emphasis on counting, while effective initially, may makes it difficult for students to develop advanced additive, multiplicative, and proportional reasoning. Certainly efforts to increase the percentages of students reaching stages 5 through 8 in Years 5 to 9 have been somewhat disappointing (see Table 1 and Young-Loveridge, 2005, 2006, 2007, 2008, 2009, 2010). Data from students whose teachers participated in NDP professional development programmes have been compared with the expectations documented in the Mathematics Standards (Ministry of Education, 2009). The analysis shows that the percentages of students who reach the expected level for their year group is well short of the Standards. It is possible that a greater emphasis on "comparison of quantities" and on subitizing might benefit students substantially in the long term. As Sophian (2008) points out, we don't know a lot about the possible merits of alternative approaches to mathematics teaching.

Table 1 The percentages of students in Years 1–8 at or above a particular stage on the Number Framework and the corresponding curriculum expectations from the Mathematics Standards (boxes indicate expected stages)

Year	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6	Yr 7	Yr 8
Curriculum Expectations	stages 2–3	stage 4	early stage 5	late stage 5	early stage 6	late stage 6	early stage 7	late stage 7
Additive Domain								
Number of students	*24931*	*27947*	*29720*	*30576*	*31475*	*32526*	*27286*	*27998*
Stage 3+	*41*	*76*	*92*	*97*	*98*	*99*	*98*	*99*
Stage 4+	*19*	*57*	*84*	*94*	*97*	*98*	*98*	*98*
Stage 5+	*2*	*14*	*41*	*63*	*75*	*84*	*86*	*90*
Stage 6+		*1*	*5*	*14*	*25*	*38*	*46*	*58*

Until recently, the assessment of the youngest children in the NDP has focused only on the additive domain (Form A of NumPA; see Ministry of Education, 2008, n.d.). It might be useful also to consider asking some questions about simple fractional quantities (perhaps also multiplication using doubling, and sharing division). Currently, the absence of multiplication/division and proportion/ratio questions implies that information about students' understanding in the multiplicative and proportional domains is not relevant to students who are not yet able to use part-whole strategies to solve addition and subtraction problems involving whole numbers. This might also mean that many teachers with Year 1–3 students mistakenly assume that they do not need to understand the multiplicative and proportional domains. These concerns have been addressed in the recently developed Junior Assessment of Mathematics ([JAM] Ministry of Education, 2011).

The large number of counting stages at the lower end of the framework (stages 0 to 4) may lead some teachers to spend more time on counting than is absolutely necessary. For those teachers lacking in confidence about teaching mathematics, it might be all too easy to continue focusing on counting and delay progression to part-whole concepts. Aggregating the initial counting stages on the framework for the additive domain in pairs, combining stages 0–1 (emergent and one-to-one counting) and stages 2–3 (counting from one on materials and counting from one using imaging) could effectively reduce the number of different counting stages on the framework and might help make stage 5 (early additive part-whole thinking) seem more attainable for students in the early school years. Anecdotal evidence suggests that some early years teachers may have been putting more energy into building number knowledge (including number-word sequence forwards and backwards, numeral identification, basic facts, and place value) than on number strategies, in particular, part-whole strategies. However, the introduction of the Mathematics Standards has made it clear that teachers need to raise their expectations of their students (see Ministry of Education, 2009).

Conclusion

The numeracy initiatives that have developed as a consequence of mathematics reforms have played an important part in improving the mathematics learning of students and teachers. However, there are important questions to consider about whether counting should continue to be emphasised so heavily in the early years, or whether students would benefit from having a greater variety of mathematical experiences, including subitizing and comparison of continuous quantities. Evidence suggests that the challenges of sustaining the benefits of the numeracy initiatives such as the NDP are many (see Young-Loveridge, 2010). The reform process is a long and difficult one. It will take time and commitment to bring about the kind of deep and lasting change that mathematics educators envisage for the future.

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