Inner-City Teachers' Perceptions in a Lesson Study for Critiquing Mathematical Reasoning

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Abstract

This manuscript focuses on practical questions in a study that describes the perceptions of four teachers at the beginning stages of Common Core Mathematics implementation in a historically underperforming school district. The overarching goal was to understand the collaborative inquiry experience in a lesson study that focused on teaching the mathematics practice of critiquing reasoning. Focusing the lesson plan on the mathematics practice of critiquing reasoning required teachers to first develop their own understanding and definition of this practice. This article describes the planning, debriefing, and revision of the lesson, while examining teachers' perspectives and rationales for instructional decisions during the lesson study. Different teaching mindsets affected teachers' understanding of the learning goal as more product oriented (students will define critiquing in mathematics; students will make a mathematical critique) or more process oriented (students will develop their understanding of critique; students will work toward making a mathematical critique). Teachers engaged in a continued process of disequilibrium and reflection as they clarified learning objectives and determined effective instructional strategies.

This study describes the perceptions of four teachers at the beginning stages of Common Core Mathematics implementation in a historically underperforming school district. The overarching goal was to understand the collaborative inquiry experience in a lesson study that focused on teaching the mathematics practice "Construct viable arguments and critique the reasoning of others" (CCSSI 2010). The recently adopted Mathematics Common Core Standards in California have been challenging for many teachers who have changed their approach to instruction to emphasize a deeper level of rigor and intensity required by the standards. These standards emphasize conceptual understanding of mathematical processes and problem solving in non-routine situations (CCSSI 2010). In addition to the content standards, eight mathematical practices address the reasoning and communication skills that should be developed across each grade level. These practices represent key processes and proficiencies in problem solving (see Figure 1).

Standards for Mathematical Practice
CCSS.MATH PRACTICE.MP1 – Make sense of problems and persevere in solving them.
CCSS.MATH PRACTICE.MP2 – Reason abstractly and quantitatively.
CCSS.MATH PRACTICE.MP3 – Construct viable arguments and critique the reasoning of others.
CCSS.MATH PRACTICE.MP4 – Model with mathematics.
CCSS.MATH PRACTICE.MP5 – Use appropriate tools strategically.
CCSS.MATH PRACTICE.MP6 – Attend to precision.
CCSS.MATH PRACTICE.MP7 – Look for and make use of structure.
CCSS.MATH PRACTICE.MP8 – Look for and express regularity in repeated reasoning.

Figure 1. Common Core Standards for Mathematical Practice

For many teachers more accustomed to using traditional direct-instruction models that focused on more procedural methods, these mathematics practices may be uncharted territory: *How do I teach this? How do I assess this? What is this practice standard really expecting students to do?* Some teachers may have simply assumed they were already teaching the mathematics practices, without a clear way to assess their students' achievement. Or perhaps some just generally approached the practices randomly throughout instruction, without a clear plan or focus on a specific practice. Oftentimes, teachers may not prioritize these practices over content knowledge and grade-level number skills.

Four teachers in an inner-city school grappled with how they could teach these mathematics practices to students who typically struggled in number skills and problem solving. This paper will describe how their perceptions of student learning and understanding of the specific practice standard developed through a collaborative learning process.

Conceptual Framework

Teacher Learning

From the situative perspective, teachers learn in particular social and physical contexts in interactive systems with "authentic activities" (Putnam, & Borko, 2000). Authentic activities are defined by Brown, Collins, and Duguid (1989) as the "ordinary practices of a culture" (p. 34), or as "authentic practices" that are not separated from the situations where they are developed (Clayden et al., 1994). Key findings of effective professional development include more teacher ownership of learning, meaningful application to their unique contexts (Darling Hammond, 1995), content knowledge focus, active learning with meaningful discussion, planning and practice, and learning activities that are connected with classroom goals/activities (Garet et al., 2001; Schon, 1991). Along with the importance of clear learning goals and attention to student thinking (Fennema et al., 1996; Hiebert, 1999), teacher learning is sustained and self-generative when engaged in practical inquiry (Franke, Carpenter, Levi, & Fennema, 2001; Kazemi & Franke, 2004). Critical discussions challenging teachers' pre-existing assumptions and intentionally "provoking disequilibrium" are also important to teacher learning and professional growth (Ball, 1996).

Urban School Reform

Many urban schools statistically have more teacher shortages, higher percentage of poor and minority students, higher rates of mobility, and lower standardized achievement scores (Jacob, 2007). For many teachers in underperforming urban schools, daily teaching activities may be compounded by students' disruptive behaviors, lack of parent participation, learning gaps, and chronic stress due to poverty, hunger, or surrounding gang activities (Jensen, 2009; Predmore, 2004). Also prevalent in many failing urban schools is organizational irrationality, demoralization, and endemic frustration (Payne, 2008). However, teacher persistence and reform in challenging schools have been richly documented in studies that describe stubborn commitment, high expectations, and improvements. Many schools have beaten the odds (Chenoweth, 2007; 2009; Strahan, 2003). Reform requires understanding the process of change, which includes risk taking and learning, with investment of resources to realistically be able to implement the necessary changes (Fullan & Miles, 1992). Leading these reforms are principals who shift the emphasis from individual teacher performance to helping teachers collaboratively focus on student learning (DuFour, 2002).

Professional Learning Community

The professional learning community (PLC) is generally understood as committed educators who collaboratively work in an ongoing processes of collective inquiry to increase student achievement (DuFour, DuFour, Eaker, & Many, 2006). The development of strong professional learning communities (PLCs) that link ongoing assessment and instruction consistently characterize many successful schools (DuFour & Marzano, 2009; Fullan, 1999). Schools as learning communities are valued as the basis for profound, generative improvement (Mitchell & Sackney, 2011) and development of capacities such as "motivation, skill, positive learning, organizational conditions and culture, and infrastructure of support" (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006, p. 1). Skillful teacher leaders with a shared vision for the school are essential to carry out this work of reform (Lambert, 1998).

Lesson Study

"Lesson study" is a form of collaborative inquiry that originated in Japan to improve instruction and has become more widely accepted in the United States as a model of professional development to develop new knowledge for teaching (Fernandez, Cannon, & Chokshi, 2003; Hart, Alston, & Murata, 2011; Lewis, Perry, Hurd, & O'Connell, 2006; Perry & Lewis, 2009). This model has gained prominence as it embodies many key features of effective teacher development, such as teacher ownership of learning within an authentic context (Fonzi & Borasi, 2002). The features of a lesson study cycle are investigation, planning, observation of the research lesson, and debrief/reflection. (Hashimoto, Tsubota, & Ikeda, 2003).

This study investigated how four inner-city elementary teachers in a cross-grade level math PLC grappled with teaching their struggling students how to critique in mathematics in the context of a lesson study. The following research questions were addressed:

1. How do four inner-city teachers develop the mathematical practice of critiquing reasoning in a lesson study?
2. What perceptions influence teachers' lesson planning in this collaborative inquiry process?

Method

Setting

All four teachers at Hernandez Charter School (pseudonym) worked in a California school district with approximately 39,000 students, which was located in a city that struggles with 24% of the residents living in poverty (U.S. Census Bureau, 2013) and was identified by Forbes Magazine as one of the "Ten Most Dangerous U.S. Cities" (Fisher, 2012). A severe housing market crash and strapped city finances left scarce resources for libraries and schools.

Due to consecutive years of underperformance on statewide progress objectives, most schools in this district were designated "Program Improvement" (PI) by the California department of education (Educational Data Services, 2014). PI designation requires Title 1-funded schools to follow corrective actions for school improvement, such as professional development for teachers and restructuring of internal organization (CDE, 2014). However, at the time of this study, Hernandez Charter School was one of only three elementary schools (out of 45) in the district not in PI status.

Located in the inner city with a high concentration of English learners and socio-economically disadvantaged students, Hernandez Charter School was composed 74% Latino, 11% African American, 7% Asian, and 5% White, with a total of 573 students at the time of this study. The majority (94%) of the students qualify for free or reduced-price meals, and 45% of Hernandez students were designated English Language Learners (Educational Data Services, 2014).

Participants

Four experienced teachers from Hernandez Charter School participated in this lesson study PLC. Their years of teaching and grade levels varied. At the time of this study, Kalinda (Caucasian female) taught 7th and 8th grade mathematics, with 18 years of teaching experience; Kelly (Caucasian female) taught 4th grade, with 6 years of teaching experience; Joe (Caucasian male) taught 6th grade, with 16 years of teaching experience; and Manuel (Hispanic male) was the Spanish-bilingual 5th grade teacher, with 7 years teaching experience. The teachers volunteered to participate in this lesson study PLC. The author served to facilitate the lesson study process and focus the discussions by taking notes for the group and summarizing their ideas. Throughout the meetings, the author/facilitator asked questions to probe their thinking and check for clarification. The author/facilitator also shared online lesson mathematics units from New York City Department of Education website (NYC Department of Education, 2014) as possible resources for the lesson study.

Lesson Focus

Hernandez Charter School teachers used Common Core-based units of study written by the district to supplement their mathematics curriculum and prepare students for the state performance tasks. Teachers also had some flexibility to adapt the curriculum as needed. Prior to this lesson study PLC, the author met with the teachers to discuss possible topics for the lesson study. All four teachers expressed concerns about limited and shallow student reasoning in mathematics, regardless of the content or grade level. Despite current emphasis on conceptual understanding in each teachers' classroom, students' explanations of reasoning mainly consisted of procedural steps, without convincing or mathematically based justifications. After reading over the Mathematics Practice Standards carefully, the teachers felt that the standard "CCSS.MATH PRACTICE.MP3 – Construct viable arguments and critique the reasoning of others" most strongly addressed their concerns and should be the focus of the lesson study.

Data Collection

Each of the PLC meetings, which included the lesson study planning and debrief meetings, were audio recorded and transcribed. Then, each teacher was individually interviewed after the lesson study cycle. The questions were designed to minimize reactivity and control for the effect of the researcher conducting the interview by keeping the questions open-ended and avoiding leading questions. Occasionally, the researcher probed for further elaboration by asking, "Can you tell me more about that?" or, "Why do you think this?" The key questions in the interview are in Figure 2. The post-lesson study interviews focused on teachers' perceptions of the lesson study experience. Each interview was approximately 40 minutes in length and transcribed.

Post-Lesson Study Interview
Thoughts About the Lesson Study • How do you think the lessons went? • What did you notice about students' behaviors that made you feel that way? • As you reflect on the goals of the lesson, what can you say about students' achievement of them? *Thoughts About the Lesson Study Collaboration* • What did you think about the collaboration times? • How was your teaching knowledge engaged in this process? • What do you think the ideal teacher collaboration would be like? *Future Implications* • What ideas or insights from the lesson study would you apply in your teaching? • Some people might say that the expectations of the Common Core Mathematics standards are not achievable for students who are already struggling in school. What are your thoughts on that?

Figure 2. Interview Questions

Data Analysis

Teacher interview transcripts and meeting transcripts were reorganized into meaningful units of data, meeting criteria set by Lincoln and Guba (1985) as the smallest piece of singularly interpretable information within a general knowledge of the context, and also heuristically meaningful to the study. Each unit was coded, but side-talk not related to the study was not coded, nor were individual phrases that did not represent a complete thought, such as "Right, right, I think," or phrases that did not contribute an idea. For example, phrases such as, "You know what I mean?" or "Did I answer the question?" were not coded because teachers were seeking confirmation rather than contributing an idea. The researcher coded the meeting transcripts to examine teachers' perspectives and rationales for instructional decisions during the lesson study. Interview transcripts provided individual data from each teacher's perspective.

All codes were generated in a systematic process in which the raw data was first chunked, then coded while constantly comparing each code with the preceding codes. If two different codes seemed to identify the same chunk of data, the data was reanalyzed to determine if it could be broken down further. After each pre/post-interview was coded, all the codes were combined to construct categories. Within each category, codes were color-coded to identify the teacher's interview. This provided another way to organize raw data and identify patterns from different data sets. Initial categories from each interview were constantly compared to the next interview in the same set. The final categories were determined by identifying recurring regularities in one interview to the next. The lesson study planning meeting was coded and categorized similarly. The two lesson study debrief meetings were coded and categorized as another set of data. The categories reflected the context and purpose of the research, which was to describe and explain how these particular teachers developed a lesson plan to address critiquing in mathematics with inner-city students who typically struggled with mathematics. Categories exhaustively included all of the codes from the data and were refined to be mutually exclusive as possible. The next level of content analysis was formulation of broader conceptual themes that cut across the data (Merriam, 1998).

Results and Discussion

In this section, results from this study will be presented in three parts: the lesson study planning, the lesson debriefs, and the post-interviews.

Lesson Study Planning

All four teachers had already participated in lesson studies with inquiry-based science in the past year. They were very familiar with the process and valued the insights they gained from the lesson study experience but had never conducted a lesson study in mathematics. Also, in contrast to previous lesson studies, teachers in this PLC had more ownership of the process. Due to other professional development obligations and very busy teaching schedules during the time of this study, they wanted to conduct a lesson study in a more condensed timeframe of two full days. This satisfactorily addressed the challenge of scheduling substitute teachers to cover their classrooms, minimalized the time they would be away from their students, and did not add on to their afterschool responsibilities. After the brief introductory meeting, the mathematics lesson study began with 5 hours of planning the following week. All four teachers were provided with release time during regular school hours, with substitute teachers covering their classrooms. Figure 3 presents an overview of the lesson study procedures.

Introductory Meeting (after school)	40 min.	• Shared general concerns about students' learning in mathematics • Scheduled interviews and meetings
Lesson Study Planning	5 hours	• Discussed students' current levels of mathematics reasoning • Discussed students' challenges related to math and language • Shared individual teaching strategies • Selected demonstration teacher • Determined lesson objective • Planned lesson outline • Predicted student responses • Prepared assessment task and lesson materials
Lesson Teaching and Observation #1	50 min.	• Kalinda implemented the lesson to the first class section • Other teachers observed and monitored student groups during the lesson
Debrief #1	1.5 hours	• Discussed students' learning outcomes • Analyzed students' assessment task • Discussed improvements in lesson design • Adjusted lesson plan and materials
Lesson Teaching and Observation #2	50 min.	• Kalinda implemented the revised lesson to the second class section • Other teachers observed and monitored student groups during the lesson
Debrief #2	2 hours	• Discussed students' learning outcomes • Analyzed students' assessment task • Discussed to what extent the lesson objective was achieved

Figure 3. Lesson Study Activities

We began the planning with sharing ideas about the problem they wanted to address in the lesson study and their knowledge of students. Teachers described their students' current level of mathematics reasoning and why their difficulties may have occurred: Students were not used to explaining their thinking and needed more practice; there was a disconnect between their understanding and ability to communicate it; they were not looking deeply enough; there was a lack of engagement.

> I think it's like a translation. They cannot take what's happening and see in their mind the steps they're taking and translate it into language. There's a disconnect in that link in there, and I think a huge part of it is just practice. It's vocabulary, not having a language and just not having a practice of speaking about mathematics and math processes, because I've noticed the same thing. It's almost like they say surface-level things like, "Well, I just added it because I needed to." (Kelly)

Teachers shared strategies they were already using to engage students in mathematics and the importance of conceptual understanding through problem-based learning. Students needed to be able to "access" the task and relate to the context. Mathematics needed to be purposeful and built from the "bottom up." Teachers described how they were pressing their students to justify mathematics reasoning during classroom discussions but admitted that this expectation was not consistent. Kalinda and Manuel shared their experiences of utilizing meaningful contexts such as planning a vacation and planning a meal to engage students in mathematical justification for their financial decisions.

After deciding to focus on "CCSS.MATH PRACTICE.MP3 – Construct viable arguments and critique the reasoning of others," the teachers grappled with their own understanding of this mathematics practice. *Are these really two separate practices? Should one build upon the other? What defines a viable argument? What are the aspects of a mathematical critique?* We looked closely at Mathematics Practice 3 to determine how students should demonstrate critiquing and make a mathematical argument. Then we defined the learning goal we wanted students to ultimately achieve, knowing this would take more than one lesson.

Kalinda volunteered to teach the lesson, since she had different sections of mathematics students (Grades 8 and 7) throughout the day, and she could reteach the revised lesson to another group on the same day. Since all the teachers in the PLC were from different grade levels, having Kalinda teach both lessons would provide more consistency and control over the variables from one lesson observation to the next. The first group of students was 8th graders learning pre-algebra, and these students were substantially behind their grade-level peers. The second group of students was 7th graders who were generally higher in mathematics skills than the 8th grade class, making both groups on comparable levels.

Then we selected a 5th grade performance task problem and student work sample from NY.gov, since adding fractions was already a familiar concept for both groups of students. We hoped this would allow students to focus more on the mathematics reasoning of the sample solution. To elicit prior knowledge, students would complete a quick-write addressing the question: "What is critiquing?" Then they would watch a humorous video clip portraying obviously incorrect mathematics reasoning. The purpose of the video was for students to form a general understanding of "critique" in a more comfortable context before critiquing in a more complex context. For the main task, students would work in table groups to look closely at the reasoning that was given to solve another mathematics problem. Their task would be to critique the reasoning and justify their conclusions. Afterward, Kalinda would ask students to share their critiques and ask the prompting questions to refine their understanding of critiquing. Figure 4 provides a more detailed outline of this lesson plan.

Lesson Plan: Critiquing Mathematical Reasoning

Learning goal: Students will be able to critique and make a viable mathematical argument, demonstrated by

- *Analyzing situation*
- *Logical thinking*
- *Supporting argument with concrete references*
- *Seeing flaws and explaining them*
- *Asking useful questions*
- *Justifying conclusions*
- *Recognizing and using counterexamples*

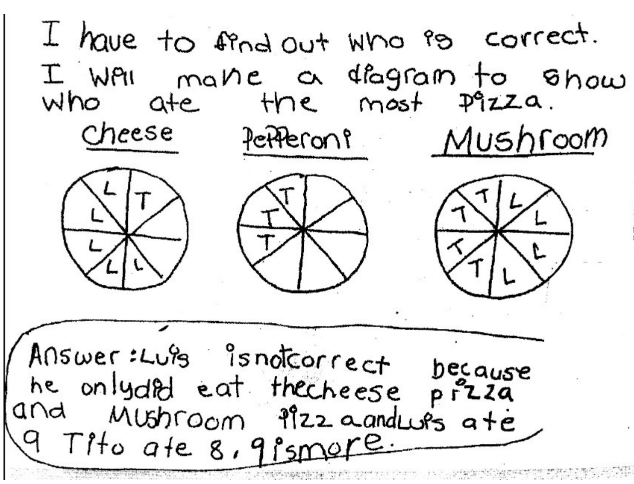
1. Quick-write: "What is critiquing?" Students write on half sheets. Collect papers. Quick sharing of randomly selected ones.

2. Hook: Show video clip with funny, bad math reasoning (25% Divided by 5: Ma and Pa Kettle: YouTube)

   Ask, "What was wrong with this reasoning? Connect with what critiquing is. Discuss: "What do you think critiquing is?" (Expected responses: Looking at something closely, explaining the good/bad, support their argument.)

3. Give students example of "Stuffed with Pizza" (with level 2 student work sample). Read the task as whole class.

   Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis, because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking. (http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm)



4. Ask them to critique the student's reasoning in small groups: "What did you notice about how this student showed her mathematical thinking?" Monitor students and ask probing questions to groups as needed: How do you know?

5. "Random" calling of students to share (try to get many different views): Teacher will facilitate the discussion: What critiques do you have about this?

What we hope students will say: (Expected student response)	**Probing questions**
They stated what they needed to solve.	Did they know what the task was?
They showed us a diagram of the pizza slices.	How did they try to solve it? What were the effective things they tried?
The cheese pizza diagram had the wrong amount for Tico.	How do you know their answer is correct/incorrect?
They're not talking about the fractions, only the numbers of slices.	What mistakes did they make in their math?

6. **"What did just we do?" "We just critiqued this math problem. What was involved with that?"** Chart students' responses:

   (Critique means to analyze the situation, see flaws/strengths in the argument and explain them with concrete evidence, ask useful questions.)

   "Why is it important to know how to critique in math? In life?"

   (Give purpose for this skill: to make smart decisions, looking for evidence and giving evidence for your argument, knowing how to make things better.)

7. **Exit ticket:** What do you think critiquing is? (Same question as above)

Figure 4. Original Lesson Plan

Analysis of this group lesson planning discussion revealed several dynamics at work: teaching theories (less teacher guidance verses more teacher guidance), purpose of learning (general verses specific), and perception of students (students will need more support verses students will need less support) (see Figure 5).

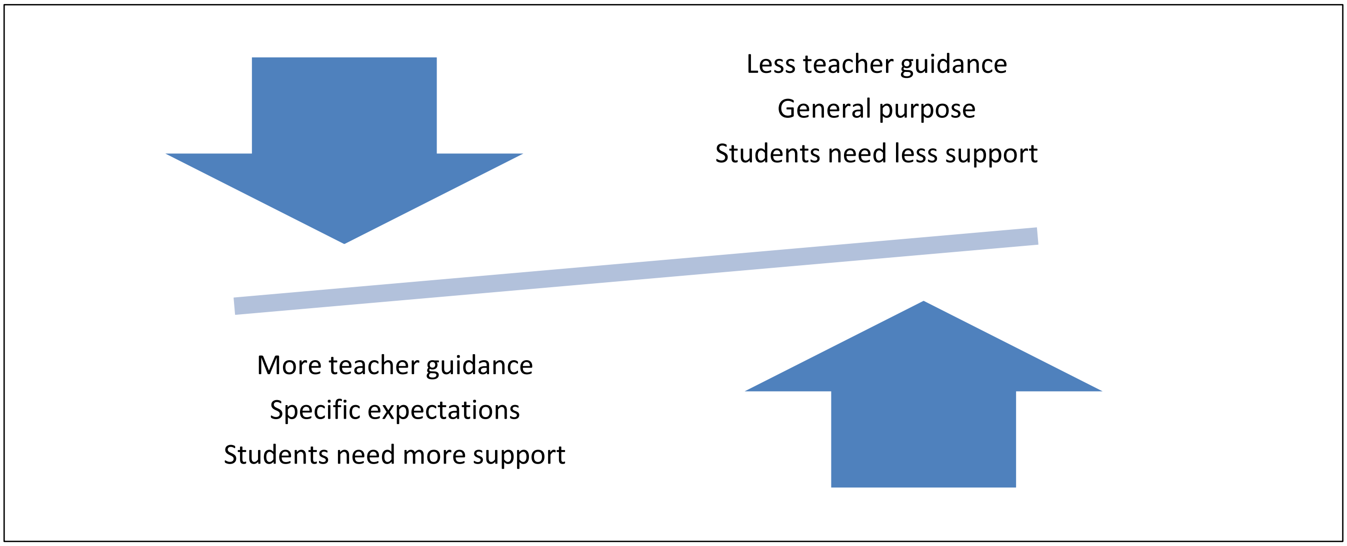


Figure 5. Lesson Planning Dynamics

Although all the teachers believed that students would learn more by working collaboratively, Joe expressed that in order for students to know how to critique the sample work, they needed to be told exactly what to critique, while Kalinda and Kelly wanted to keep the task more open.

> But if we want a certain thing from them, we're leading them in a certain direction. We have to give them the things that we expect them to do. (Joe)

> We have to allow for that creativity, you know, the inquiry with the students, and then we can scaffold as needed. We can add some probing. Some kids are going to get there on their own and have some pretty cool observations. (Kelly)

> I tend to just throw things out there and see where they are, and then I'll probe as needed. (Kalinda)

Throughout the lesson planning meeting, there was constant juxtaposition between a more behaviorist position, in which teaching is assumed to cause learning, with outside stimulus primarily responsible for learning (Gredler 1997; Marshall, 1992) and the cognitive-constructivist position, where learning is generated through student engagement and mental processes within each student (Wink, 2002; Woolfolk, 1998).

These differing perspectives could also be framed in terms of Vygotsky's Zone of Proximal Development (Vygotsky, 1986), which is defined as the distance between the "actual developmental level" and "potential development level," and determined by interactive problem solving with assistance from more capable peers or by an adult. As seen in Figure 3, all teachers agreed that students should develop an understanding of mathematical critiquing through peer interaction and analysis of a sample solution. Each student group was arranged heterogeneously to ensure a balanced interaction with more advanced peers. However, teachers disagreed *when* the assistance of the adult should come in and *how* the assistance should be provided (see Figure 6).

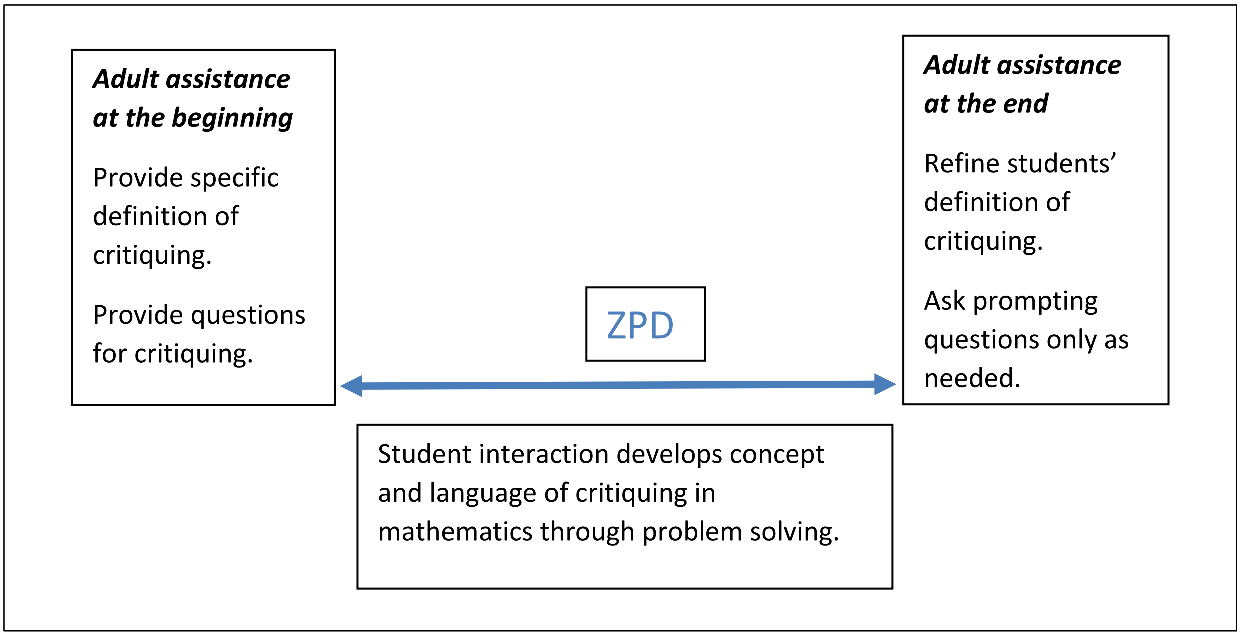


Figure 6. Adult Assistance Within the Zone of Proximal Development

Joe shared his discomfort with the lack of clear guidance at the beginning of the lesson, with his belief that students would need more support to achieve a more specific goal of critiquing the mathematics reasoning:

> The thing says critique the reasoning of others. For me, what exactly does that mean to do? If it's really open-ended, then they can go in 50 million directions… If this is what we're doing, how are we going to get them to this point and able to do that on their own?

Kalinda supported her position with the more general goal of developing students' thinking:

> I think when you do leave it somewhat open, it's so much easier for them to transfer it… I mean because critiquing and providing evidence and all of that is in science, it's in language arts, it's in everything. If we give them a list of, "You need to do this, this, this" I think it's harder for them to transfer that to something else.

Manuel was quieter during the lesson planning. He mainly asked clarifying questions and agreed that students needed to make relevant life connections with critiquing in general. As a compromise, we developed specific probing questions to ask students, such as, "What were the effective things they tried? How do you know their answer is correct/incorrect? What mistakes did they make in their math?" But students would first have the opportunity to critique the mathematics reasoning on their own, and probing questions would only be used if they did not come to the expected conclusions. It was the general consensus to provide the adult support at the end of the lesson, rather than at the beginning.

This lesson study planning process was strongly influenced by pedagogical beliefs of how students would best learn, the purpose of the lesson, and the perceptions of how students would respond in the lesson. Through collegial conversation, these experienced teachers comfortably moved back and forth between these dynamics of lesson planning, until a general consensus was reached for the final lesson design.

Lesson Teaching

The following week, the PLC met another full school day for the lesson study. We briefly met in the morning to review the lesson plan and prepare instructional materials. Kalinda taught the lesson to her 8th grade students, while the other PLC teachers and author observed. She was determined to follow the original lesson plan closely, but we agreed that changes could be made mid-flow if she deemed it necessary. Kalinda taught the original lesson to her first group of 8th grade students during a 50-minute time block. During students' small-group discussion time, the PLC teachers monitored and prompted students as needed to support their maximum participation and engagement. The PLC met immediately afterward to debrief the lesson for approximately 90 minutes. After some revisions to the lesson plan, Kalinda taught the revised lesson to her 7th grade mathematics group, during a 50-minute time block. The lesson was again debriefed immediately afterward.

The Lesson Debriefs

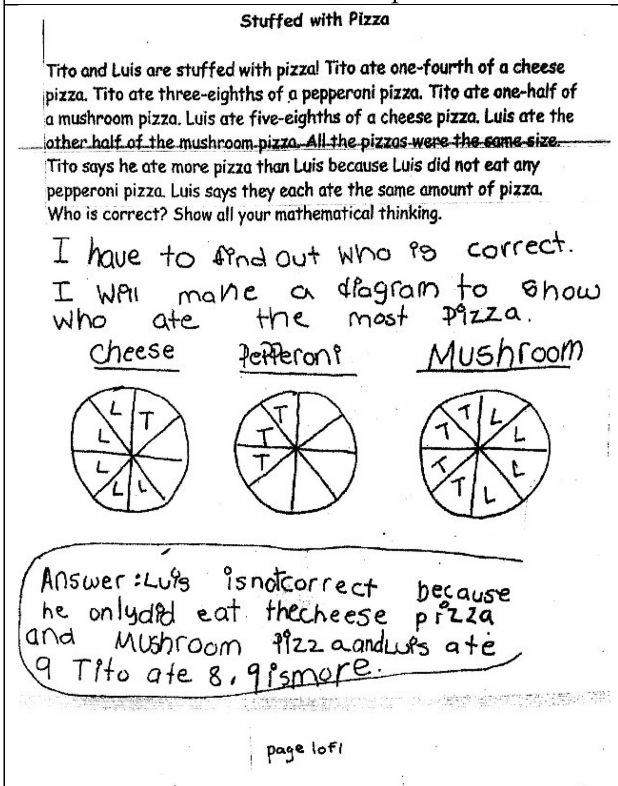
There were two lesson study debriefs, each immediately following lesson observations 1 and 2. Each debrief was approximately 90 minutes. This section describes what occurred during both debrief meetings and discusses two main themes that were emphasized in both analyses of the lesson observations: 1) thoughts about teaching, and 2) students' response to the lesson. Discussion around these themes determined teachers' decisions to improve the lesson.

Debrief 1

After lesson observation 1, the first task was to look through students' prewrite responses to compare students' initial knowledge of critiquing with their understanding at the end of the lesson. At the beginning of the lesson, students completed the quick-write: "What is critiquing?" We sorted the pre-writes into those that had no understanding of critique (19) and a somewhat clear understanding of critique (9). Group 1's initial understanding of critique was very limited to synonyms such as "judge" or "give opinion," and no responses were connected with mathematics. As expected, the concept of critiquing was new for most students. We also noticed that students' prior knowledge of critique connected to restaurant critics and talent show judges, focusing on the negative aspects of someone's performance or work. This was when we realized that we should try to focus students on critiquing in the mathematics context, and we slightly altered the pre- and post-write question.

We then shared our observations of lesson 1. The main difficulty teachers noticed in the lesson was unanticipated confusions with the critiquing task. The task was for students to critique the reasoning in the sample solution to the "Stuffed with Pizza" problem. However, because the problem context itself portrayed two people's opinions of who ate more pizza, and the wording in the problem context asked, "Who is correct? Tito or Luis?" students critiqued the reasoning of Tito and Luis, rather than the reasoning in the sample solution to the problem (see Figure 7).

Problem Task with Sample Solution



Critique of Reasoning

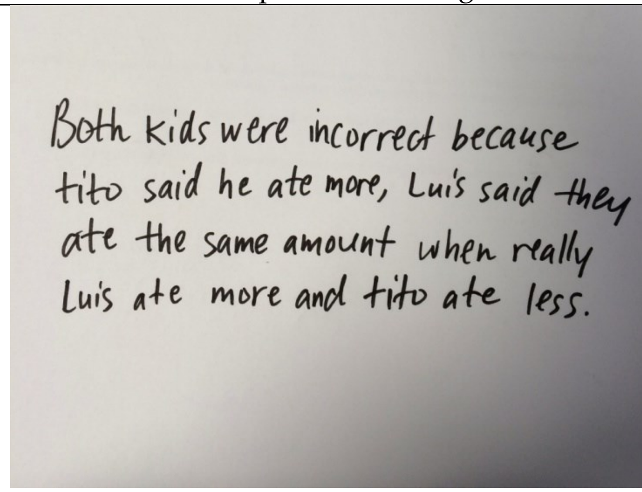


Figure 7. Problem task with sample solution and students' critique or reasoning (Group 1)

Teachers in this lesson study PLC discussed various ideas for how to improve the lesson, based on their observations of students' confusions. The discussion focused on these possible changes to the lesson: provide more specific guidance; break down the task; check for understanding; press for student evidence in their critique.

After this discussion, we looked at the students' post-write responses (exit tickets) that addressed the same question as the prewrite: "What is critiquing?" We sorted these into three groups: High understanding (6); Medium understanding (16); and Low understanding (7). These levels were determined from the aspects of constructing arguments and critiquing reasoning described in the Common Core Mathematics Practice Standard 3 – "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments… Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is…" Examples of high understanding demonstrated analysis of the flaw and included use of evidence to support their critique: "Critique means to evaluate because you have to see how it was put together and you have to look for evidence." Medium level of understanding critique focused on finding the flaw in the problem, without mention of supporting evidence: "Critique means to check someone's work and give your opinion to them about there [sic] work. Also about what you think about it." Low level of understanding critique, demonstrated little or no understanding: "Critique means to cridisise [sic] something by the way they look or taste or smell." Compared to the prewrite responses, a significant number of students demonstrated the ability to define *critiquing*.

During this debriefing, the purpose of the lesson was continually debated. Was the purpose of this lesson for students to make an effective critique of someone's mathematical reasoning? Or was it to build and develop students' own ideas as they attempted to critique? The lines between these goals blurred as teachers discussed what they wanted to accomplish through this lesson:

> What's the end goal? Do we want the kids to actually learn this and use this, or do we want them to get through a set of things that they are just going to forget there. (Kelly)

Revisions were made to the original lesson plan to address students' confusions, and make the stronger emphasis on analysis of the reasoning, rather than just confirming the right/wrong answer: Alter the pre- and post-write question to: "What do you think it means to critique in math?" After the video, share students' responses to "What do you think it means to critique in math? Give students opportunity to solve the "Share My Pizza" problem first. Give them just the problem without the sample solution. Give the sample solution after students solved the problem, and check for understanding that they will critique the sample solution. Refer to the sample solution as, "Sarah's work." Direct students' thinking by asking these probing questions before they begin the analysis: "What did you notice about how Sarah showed her mathematical thinking? What did she do correctly? How did she try to solve it? If you were her teacher, what would you tell her about her work?"

Debrief 2

After the second lesson observation that Kalinda then taught to her 7th grade class, we met again for Debrief 2. This concluding discussion of the lesson emphasized similar themes from Debrief 1 (thoughts about teaching; students' responses to the lesson), and teachers also shared strategies to address this math practice in future lessons.

The prewrites from Group 2 demonstrated similar levels of prior knowledge to Group 1: No understanding of critique (17) and a somewhat clear understanding of critique (6). Responding to the revised question: "What does it mean to critique in mathematics?" made no significant difference to the prewrites from group 1 or group 2. Students who had a somewhat clear understanding of critique also focused on "to judge right or wrong."

The changes to the lesson were deemed effective, particularly the revision of allowing students to solve the problem on their own before critiquing "Sarah's" reasoning. Even though the skill level of the problem was low, students still benefited from time to process their own reasoning for the problem before analyzing someone else's reasoning. They needed time to develop a frame of reference for their argument. The focus on critiquing in mathematics was stronger, and it was also observed that Kalinda made an instinctive change in the lesson by making a connection to students' prior knowledge of providing "evidence" with the television show "Judge Judy" during the closure of the lesson. She explained the reasoning for this mid-flow decision:

> Someone said Judge Judy when we were talking about judging. They were like, "We should watch Judge Judy." So I thought, "What does she base her decisions on?" Because at that point, we had on the chart the word, "judge," and someone said right or wrong or good or bad. I'm thinking, "What is that based on? Where do you base your judgment on?" And they said, "Judge Judy." I go, "What does she base it on?" Then someone said, "Facts."

Kalinda' wanted to emphasize that judging, evaluating, or critiquing needed to be based on evidence to support their reasoning for a decision. Listening to students' responses during the lesson influenced Kalinda's decision to make this real-life connection in the lesson.

We observed that breaking down the task and first allowing students to solve the problem on their own gave them more confidence to critique "Sarah's reasoning." Interestingly, the students in Group 2 who did not have "Sarah's reasoning" in front of them at the beginning, did not use any models to solve the pizza problem. With the exception of one student, everyone in Group 2 used the procedure of finding common denominators to add the fractions, rather than drawing pizzas like "Sarah" did. This was different from Group 1—almost everyone drew pizzas since they were given "Sarah's reasoning" at the beginning. We realized that looking at the sample solution prior to solving the problem on their own influenced the strategy students used to solve the problem. Also, Group 2's engagement level and mathematical skills seemed higher than Group 1. Group 2 noticed the errors in the video more quickly and seemed more confident with mathematical procedures.

Yet students in this second lesson also experienced some difficulties. After most had solved the problem on their own, they knew that "Sarah's solution" was incorrect but could not pinpoint where she had made the error. We discussed how students had difficulty analyzing "Sarah's solution" because she drew models to solve the problem, and most of them had used the procedure to solve it. Although they were adept at using the procedures, it was evident that some students had limited conceptual understanding of fractions—or a possible disconnect between models and symbolic representations. Manuel shared his observations of one particular group:

> They all critiqued that she was wrong. But they had a hard time finding out why she was wrong other than having the evidence that they were right. It's what I thought when I talked to most of the groups.

Kelly shared her observations of students who revised their thinking and seemed more successful with critiquing:

> But because they didn't do it conceptually with any sort of model or picture, when they got the problem the way she did it and start to critique it, the first thing out of someone's mouth was, "Well, they saw she's wrong, so they said she's wrong." I'm like, "How do you know?" "Well, she's wrong because she used the diagram. She used the pie chart, and you can't do that. That's why she's wrong…" And then when they looked at it and broke it down a little bit more and talked about it, they realized, "Oh, you could do it, but she just did it wrong." A lot of the kids, the tables I went to, got that.

The post-writes after the lesson showed that Group 2 had a more adequate understanding of critiquing. All the responses were sorted using the same criteria from Group 1: high (8), medium (12), and low (3). Teachers surmised that the revisions made to the lesson significantly improved students' understanding of critiquing in mathematics, despite other possible differences between the two groups of students.

> First, they get the problem solving out of the way which I think is what the first group wanted to do. So they get that problem solving out of the way, and then I think it gave them a clear lens to focus the task. They knew that they had to critique someone else's work. (Kalinda)

Teachers' Perspective on Critiquing

Debrief 2 concluded our analysis of the lesson study but continued a larger conversation about how to teach students to critique. Teachers emphasized that critiquing was a cognitive practice that applied to all content areas and was an important life skill. Students needed to understand the importance of making a fair judgment based on evidence in order to convince others of their reasoning. This type of evidence-based argument was also accentuated in the Common Core English Language Arts Anchor Standards. For example, this College and Career Readiness Anchor Standard for Reading (Integration of Knowledge and Ideas) required students to "Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence" (CCSS.ELA-Literacy.CCRA.R8). Use of evidence was also addressed in various cross-cutting concepts in the Next Generation Science Standards. For example, use of evidence was mentioned as early as kindergarten in the Cross-Cutting Concept of Patterns (K-ESS2-1): "Patterns in the natural world can be observed, used to describe phenomena, and used as evidence." Evidence was also addressed in the Cross-Cutting Concept of Cause and Effect in science (K-PS2-1), (K-PS2-2): "Simple tests can be designed to gather evidence to support or refute student ideas about causes. (NGSS Lead States, 2013).

Overall, teachers felt that the students were still at the starting point for understanding how to critique in mathematics and shared strategies for future lessons:

- Push students to critique beyond the right/wrong answer.
- Continually press students to expand their reasoning by asking, "Why? What makes you think that?" to provide evidence for critiques.
- Isolate the practice of critiquing within the context of easier mathematics.
- Provide powerful life examples of critiquing to make this skill more relevant.
- Have a discussion focused on counter-examples of critiquing to identify what makes an effective critique.
- Eventually make critiquing more explicit—what to do, what it looks like.

The Post-Interviews

After this lesson study cycle, post-interviews were conducted individually, and teachers were asked to share their perceptions of the lesson study experience and their ideas about future teaching implications for the Common Core Mathematics Standards. Interview transcripts emphasized two main themes: 1) teaching perspectives, and 2) thoughts about collaboration.

Teaching Perspectives

Each teacher called attention to different teaching perspectives. Joe shared how expectations for learning mathematics had really changed and students needed to be "retrained" to think beyond right or wrong. Also for him, teaching practices changing to a more student-directed approach was not always realistic:

> Really, I've so many hours in a day, and there are so many days a year we teach. So, if they take a long time to get there… if we missed three weeks… it's hard to figure out time-wise what exactly needs to be done, and there are so many things we have to do. I was like, "Oh, boy. How much do we need to spend waiting for them to come up with this information?" (Joe)

Joe also acknowledged that students were still at the starting point for learning how to critique. He ascertained that breaking down the lesson task and specifying expectations improved the lesson plan and more clearly addressed the learning goal. Joe reiterated a main concept from Interview 1—that teaching should adapt to students' needs, within meaningful and purposeful mathematics contexts.

Kelly's interview echoed many of Joe's perceptions of students' struggles in the lesson but also noted that in order for students to really focus on critiquing, the mathematics in the lesson should not be a barrier. Therefore, the context of the lesson was critical to this learning process:

> If they could do it, they could understand it and conceptually work with this math to be able to then create that context of critique around it. Because if this fails, if they can't do the math, then, we're not really isolating critiquing itself. So, we needed to create a problem they could critique that they really understood the math, and they could just focus on the critique. (Kelly)

Kelly also emphasized that learning math practices did not have to be dependent upon students' mathematics skills. She shared her belief that students who typically struggle in mathematics would benefit from learning the Common Core Mathematics Practices by making them "better thinkers in math."

Manuel's perspective on the lesson was influenced by his position as a bilingual teacher. He was more intrigued by the differences in student participation and who was excluded from collaboration due to what he interpreted as social-class bias:

> We have tremendous differences in the school population, and some of them you were able to see. I was able to see that their status in the classroom is not high. Some students actually were not invited to be part of the conversation. It was obvious. It was open. It was palpable.

Manuel had observed this pattern for many years in his own classroom:

> And, almost without failure, all the students I've seen that have low status in the classroom are usually dark-skinned Mexicans. You rarely see a light-skinned Mexican who has low status in a classroom.

The consideration of language and status of students influenced Manuel's teaching perspective. He also addressed the need for more reteaching and use of models to develop students' conceptual understanding with fractions.

As the demonstration teacher, Kalinda's perspective on the lesson study addressed her feelings of uncertainty while teaching the lesson plan. She was not sure if it would be acceptable for her to make changes to the lesson mid-flow if she felt that the lesson was not working, or how closely she was expected to follow the lesson plan created by the group:

> When you're teaching yourself, you're able to make those little switches mid-stream, but when you have the lesson that you all designed, I feel like I have responsibility to stay on that lesson until I get to the point where it's like, "Okay, we need to scrap this." But there is that accountability piece, because other people were in on the critiquing of it.

Similar to Kelly, Kalinda also emphasized the importance of an approachable context for students to focus on learning a specific mathematics practice. She shared that when students who struggled with math have a familiar, real-world context, they "have better access to the math in general." Kalinda highlighted the power of understanding when she mentioned a specific student in her class who hated math at the beginning of the school year and later became the strongest student in her math class because he could "make sense of it."

Thoughts About Collaboration

Teachers mentioned many different facets of collaboration, highlighting perceptions about trust, benefits to professional growth, participation dynamics, and barriers to collaboration. Joe, Kelly, and Kalinda each mentioned collegial familiarity and expressed a high level of comfort with the PLC group. They each felt comfortable speaking their minds and sharing their ideas. Kelly and Kalinda emphasized how they benefited from collaboration:

> It's good for me to have to explain my thinking to other people, and I think that stretches me. But explaining my ideas to someone else forces me to slow it down a little bit and make sure that I explain my purpose. I think any time we are forced to explain things and communicate the purpose behind what it is that we're doing, I think that's good for us. (Kalinda)

> Through dialogue, all of us have grown in many ways—we get a variety of perspectives and cultures and experiences. For example, having [Manuel] there is key, because you walk your own path, and when you have others that have walked different paths, you see those paths where you may not see them otherwise. So [Manuel] constantly taking the language learners into consideration and thinking about how this experience is for them, what may be impacting what they say and what they do, that was constantly bringing in that lens that may not have been there otherwise. (Kelly)

Manuel also valued different perspectives in collaboration:

> Even though we had our differences, I think that's good because we had different perspectives. We don't want to work with people who have the same theories about education and pedagogy. (Manuel)

Joe shared that the school culture was conducive to collaboration and described the impact of the school's administration allowing teachers to have more freedom. According to Joe, trust was also an important factor of successful collaboration and was necessary to overcome barriers of fear or judgment:

> Sometimes, people get offended when you throw them together and work with somebody else, whether it's out of fear that they're going to feel they don't know what's going on or somebody's critiquing and judging them the whole time. So, really it has to be a lot of trust in those situations… but I think the culture of the school kind of lends itself to doing that here. (Joe)

Teachers in the lesson study PLC shared these views on what made collaboration work:

- Clear goal or task for collaboration (Joe, Kalinda)
- Consistent "norms" for participation (Joe)
- Culture of trust (Joe)
- Familiarity and comfortable to share (Joe, Kalinda, Kelly)

- Value different perspectives (Manuel)
- Everyone's voices heard (Kelly)

Conclusions and Implications

This study looked closely at the perceptions of four inner-city teachers engaged in a mathematics lesson study. Focusing the lesson plan on the mathematics practice of critiquing required teachers to first develop their own understanding and definition of this practice. They could not begin planning the lesson until a common understanding was established. Teachers themselves need time to process and interpret applications for the mathematics practice before they can plan instruction that will develop these practices in their students. Offering teachers more discussion time to deepen and develop their own knowledge of each mathematics practice may be an important component of professional development. Conversations with teachers across different grade levels can provide further insight into how these practices could progress and overlap.

Also, focusing on a mathematics practice still required application of mathematics content knowledge and skills. Practice could not be separated from content but should be integrated with content knowledge regularly. All those involved with mathematics preparation of teachers are encouraged to explicitly connect mathematics practices within the development of content knowledge.

Determining the pedagogical sequence for instruction and lesson strategies was a combined effort and dynamic struggle between providing students with more/less directed guidance to achieve the learning goal. However, the learning goal seemed to be in constant flux as different teaching philosophies emerged through the lesson study. Different teaching mindsets affected teachers' understanding of the learning goal as more product oriented (students will define critiquing in mathematics; students will make a mathematical critique) or more process oriented (students will develop their understanding of critique; students will work toward making a mathematical critique). Future studies could address the effectiveness of different teaching approaches for the same mathematics practice, as well as how these practices should be assessed.

Similar brief lesson studies focusing on practices (Common Core Mathematics), anchor standards (Common Core English language arts), or cross-cutting concepts (Next Generation Science Standards) could be implemented in schools that engage as professional learning communities. Vertical grade-level teams may provide further insight into the progression and development of these practices, which span K-12.

Implicit in the Common Core Mathematics standards was a more student-centered approach with emphasis on communication and reasoning. Similarly, this lesson study exemplified a teacher-centered approach to professional development with emphasis on communication and reasoning. As many students must now adjust to learning mathematics differently, many teachers may also struggle with different approaches to lesson planning and delivery to help their students achieve these newly defined practices, and they need to develop more deepened conceptual understanding mathematics. Lesson study could be a powerful vehicle for mathematics professional development at schools, as teachers develop different approaches and priorities for instruction.

There are many factors that affected these teachers' participation and perceptions of this lesson study experience, but it is important to note that these teachers participated in a professional learning community that was established in a school culture of collaboration. School culture at Hernandez was described with a high level of teacher autonomy, collaboration, and expectation for continued professional growth. School cultures that support reform recognize that change is a collaborative learning process (Lieberman, 1995).

Additionally, effective PLCs develop trust and relationships as part of collective and continuous learning (Hord, 1997). At Hernandez, teachers have the freedom and flexibility to try different instructional strategies found from outside resources and adapt curriculum to meet the needs of their students:

> You know, [Principal] Annabeth expects a lot from us, but she also supports us in everything that we do. And she lets us do a lot of things… I don't think other schools… have the kind of freedom that we have here… (Kalinda)

It may seem ironic to expect students to increase their reasoning and communication with any kind of scripted curriculum or professional development that limits teachers' own reasoning and communication. Teachers must also engage in a continued process of disequilibrium and reflection as they clarify learning objectives, determine effective instructional strategies, and plan lessons that are relevant for their students.

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