ACTA DIDACTICA NAPOCENSIA

Volum 10, Number 2, 2017

CONSOLIDATION OF SIMILARITY KNOWLEDGE VIA PYTHAGOREAN THEOREM: A TURKISH CASE STUDY

Hatice Kübra Güler, Çiğdem Arslan

Abstract: Understanding how students learn is important for both researchers and teachers. In terms of mathematics, constructing a new mathematical structure depends on conceptual understanding and connection with previous construct and they needs to be consolidated. A construct can be consolidated when the construct is recognized and used in the further activities or while new structure is being constructed. In this paper, our aim is to observe consolidation of similarity knowledge by the help of Pythagorean Theorem in terms of characteristics of consolidation; *immediacy*, *self-evidence*, *confidence*, *flexibility*, *awareness*. We would like to observe consolidation in different ages. Participants of this study are one master, one undergraduate and one high school students from Turkey. The consolidation of similarity knowledge was observed when students tried to prove Pythagorean Theorem. The data were analyzed by using RBC+C-model's characteristics. As a result, it was seen that characteristics of consolidation were useful to analyze consolidation. However, there is a need for posterior experiments to investigate whether "*immediacy*" is an essential characteristic of consolidation. It can be said that *awareness* and *flexibility* are seen essential and easily observable characteristics of the consolidation process.

Key words: Abstraction, RBC+C, Consolidation, Pythagorean Theorem, Similarity

1. Introduction

Teaching and learning mathematics are processes which begin from concrete thinking and end with abstraction. Hershkowitz, Schwarz & Dreyfus (2001) defined abstraction as an activity (in the sense of activity theory), a chain of actions undertaken by an individual or a group and driven by a motive that is specific to a context and vertically recognizing previously constructed mathematics into a new mathematical structure. According to activity theory, outcomes of previous activities naturally turn to mathematical artefacts in further ones (Dreyfus, 2012). Tabach, Hershkowitz & Schwarz (2006) stated that the previous construct serves as a cornerstone in the new constructing since it fuels the new actions of recognizing and/or building with.

Considering from the constructivist view, emergence of new mathematical constructs depends on the fact that previous constructs are understood exactly and the relationships between constructs are linked well. It is important to understand how students construct abstract knowledge (Dreyfus, 2012). To observe this process, Hershkowitz et al. (2001) stated abstraction in context which is a model included the observable stages Recognizing (R), Building-with (B) and Constructing (C). The model is called RBC. The authors mentioned the importance of consolidation of the newly emerged structures and by adding the consolidation process the model called RBC+C model. The new constructed knowledge is weak and it needs to be consolidated. A construct can be consolidated when the knowledge is recognized and used in the further activities (Dreyfus, 2012; Hershkowitz et al., 2001).

Dreyfus & Tsamir (2004) showed that consolidation can be identified by means of the psychological and cognitive characteristics of self-evidence, confidence, immediacy, flexibility and awareness. They proposed to take the combination of five characteristics as definition for consolidation. Tsamir & Dreyfus (2005: p. 16-17) explained the characteristics of consolidation as follows:

Received March 2017.

> Immediacy refers to the speed and directness with which a structure is recognized or made use of in order to achieve a goal; self-evidence refers to the obviousness that the use of a structure has for the student; obviousness implies that the student feels no need to justify or explain the use of the structure, though (s)he is able to justify and explain it. Self-evidence is directly related to the confidence or certainty with which a structure is used. Confidence refers to be sure about activity and not to be in doubt. Frequent use of a structure is likely to support the establishment of connections, and thus contribute to the flexibility of its use. A student may be quite proficient in using a structure, even using it flexibly, but without being consciously aware that s(he)is doing so. The awareness of a structure enables the student to reflect on related mathematical and instructional issues, add to the depth of her or his theoretical knowledge and power and ease when using the structure.

Although consolidation is of importance, most of researches are interested in recognizing (R), building-with (B) and constructing (C) stages of the model (Dreyfus, 2012; Kouropatov & Dreyfus, 2013; Sezgin Memnun & Altun, 2012; Yesildere & Turnuklu, 2008). There are also several research efforts investigating the consolidation process (Monaghan & Ozmantar, 2006; Tabach et al., 2006) and in terms of characteristics of consolidation (Anabousy &Tabach, 2015; Dreyfus & Tsamir, 2004; Tsamir & Dreyfus, 2005). Anabousy & Tabach (2015) carried out an experiment on construction and consolidation of Pythagorean Theorem by using GeoGebra. They observed confidence and immediacy which were the characteristics of the consolidation but via the trajectory of the experiment, they concluded that the students consolidated the structure.

Consolidation is characterized by reorganization of previous constructs, which are recognized, with higher confidence while capitalizing on previous constructs in the course of a new similar activity, or by a further elaboration. Because of this reason, structure and sequence of activities should provide opportunities of consolidating knowledge; which should include relevant activities. Relevance may be attained through similarity of tasks that invites recognizing and building with previous constructs (Tabach et al., 2006).

A task may offer opportunities for consolidation if it is structurally analogous to an earlier task but appears in different context (Dreyfus & Tsamir, 2004). While preparing the tasks for the current study, these opportunities were taken into consideration. Since most of the proofs of Pythagorean Theorem are based on similarity (Ispir, 2000), in the current study the process of consolidating similarity knowledge was observed while students were trying to prove Pythagorean Theorem.

In this study, our aim is to observe consolidation of similarity knowledge by the help of Pythagorean Theorem in terms of characteristics of consolidation. By this way, it can be seen whether the mentioned characteristics of consolidation can be used effectively in order to analyze the process and they need to be modified or not.

2. Methodology

Education is a cultural issue and generally teachers educate students in accordance with their beliefs and cultural structure. Because of this reason it can be said that constructing and consolidating knowledge vary from one culture to another culture. The current research is a case study and its participants are three students from different education levels from Turkey. Consolidation was investigated while students were working individually through two tasks. Participants' backgrounds, tasks and their analysis are explained below.

2.1. Participants

The study was carried out with three students from different education levels. One of them was a successful student (Gul) who was educated in a high-achieving school at 9th grade. According to mathematics curriculum in Turkey, the 8th grade students take the courses on Euclidean Theorems metric relations in a right triangle: Proportional Mean Theorem, Product of Sides Theorem, Altitude to the Hypotenuse Theorem), Pythagorean Theorem and similarity. Also they do exercises on these subjects. So, it was assumed that Gul learnt mentioned topics and constructed these knowledges one year ago.

The second participant was a preservice mathematics teacher (Elif) who was junior at the education faculty. She had taken the courses which included special teaching methods of mathematics before the study. In these courses, it was taught how they could teach mathematical concepts, how a student constructed a mathematical knowledge and etc. So, it was assumed that Elif knew the mentioned topics and how she could teach them.

And the third participant was a master student (Ahmet) who was enrolled in middle school mathematics education program. He had taken all the courses which Elif took and in addition as a master student, he had also taken the courses on research fields about mathematics education. So, it was assumed that Ahmet knew the mentioned theorems and their proofs, and also he became aware of the researches on these issues.

The aim of choosing participants from different education levels is to reveal how and to what extend different students in different ages consolidate a construct and whether we can observe different characteristics of consolidation.

2.2. Tasks

The following two tasks were carried out by the participants during one by one interview. The studies were videotaped by one researcher and the other one carried out the interview.

Task 1

We gave students a worksheet on which a right triangle was drawn a height belonging to hypotenuse. The students were asked to "*State the height (h) of the given right triangle by using the legs (a, b)*". ($|AE| = e, |EB| = c, |AC| = b, |BC| = a, |CE| = h$ and $|CE| \perp |AB|$) The given drawing which belongs to question is as follows.

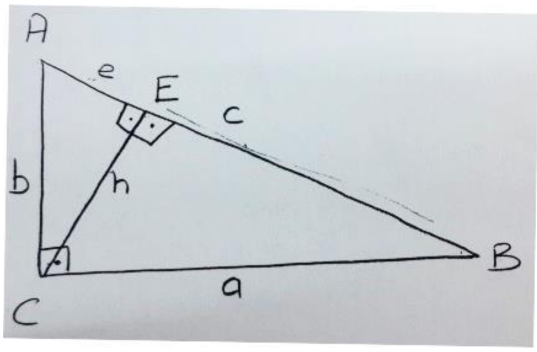

Figure 1. *The drawing of Task 1*

The desired expression was $\dfrac{1}{h^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

The First Method for Task 1

Students were expected to use Pythagorean Theorem for Task 1. In this process, they must have been recognized that there were three triangles and decided to use Pythagorean Theorem and similarity in order to accomplish the mentioned task. From this point of view, we would say whether they recognized (as a stage of RBC) the mentioned knowledges.

The operations expected from students are as follows:

From ΔECB, it can be written by using Pythagorean Theorem; $a^2 = h^2 + c^2$ (i)

From $\Delta CEB \sim \Delta ACB$, it can be written by using similarity; $\dfrac{c}{h} = \dfrac{a}{b} \rightarrow c = \dfrac{ah}{b}$ (ii)

From (i) and (ii);

$$a^2 = h^2 + \frac{a^2h^2}{b^2}$$
$$a^2b^2 = h^2b^2 + a^2h^2$$
$$\frac{a^2b^2}{a^2+b^2} = \frac{h^2(b^2+a^2)}{a^2+b^2}$$
$$h^2 = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{1}{h^2} = \frac{a^2+b^2}{a^2b^2}$$
$$\frac{1}{h^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

The Second Method for Task 1

Another method expected from the students was using metric relations in a right triangle for Task 1. The expressions are as follows:

$$b^2 = e(c+e) \quad \text{(i)}$$
$$a^2 = c(c+e) \quad \text{(ii)}$$
$$h^2 = ce \quad \text{(iii)}$$

The expressions (i) and (ii) are from Proportional Mean Theorem and (iii) is from Altitude to the Hypotenuse Theorem. If (i) and (ii) multiplied, $a^2b^2 = ce(c+e)^2$ is obtained; and from (iii) it can be written as $a^2b^2 = h^2(c+e)^2$. From Pythagorean Theorem this expression can be written as $a^2b^2 = h^2(a^2+b^2)$. If the operations are done as below, the desired expression is obtained.

$$\frac{a^2b^2}{a^2+b^2} = \frac{h^2(a^2+b^2)}{a^2+b^2}$$
$$h^2 = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{1}{h^2} = \frac{a^2+b^2}{a^2b^2}$$
$$\frac{1}{h^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Task 2

After we determined whether they recognized the Pythagorean Theorem and similarity, we would like to see if they consolidated similarity knowledge. The students were asked to "Prove the Pythagorean Theorem". As for the Task 2, it was expected to prove the mentioned theorem by using similarity. Thus, it is possible to observe consolidating the similarity knowledge.

There are many kinds of proofs for Pythagorean Theorem and while proving it, similarity can be used. Two of them can be as follows:

Proof 1:

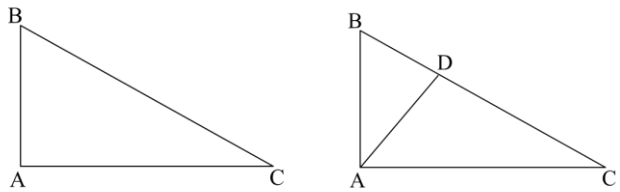

Figure 2. *Similar right triangles for Proof 1*

The original right triangle is BAC. The triangles BAC, BDA and ADC are similar right triangles. And the ratios are;

$$\frac{BA}{BC}=\frac{BD}{BA} \text{ and } \frac{AC}{BC}=\frac{DC}{AC}$$

$$BA^2 = BC.BD$$
$$AC^2 = BC.DC$$
$$BA^2 + AC^2 = BC(BD + DC)$$
$$BA^2 + AC^2 = BC^2$$

Proof 2:

The original right triangle is ABC. The triangles ABC, C'AB, A'CB and B'AC are similar right triangles. In addition, ABB' and ABC'; and also ABC and A'BC are congruent.

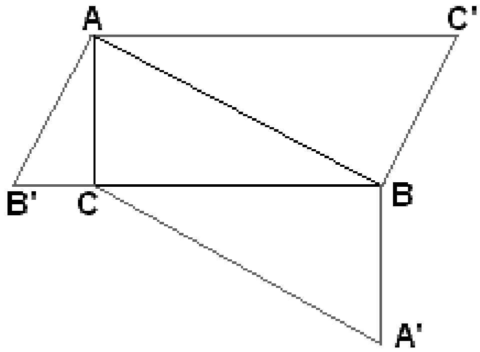

Figure 3. *Similar right triangles for Proof 2*

If ratios are written as Proof 1, it can be seen that the expression is $AC^2 + BC^2 = AB^2$. We expected participants to prove by using similarity. We analyzed the process depending on the characteristics of consolidation which were stated by Dreyfus and Tsamir (2004).

2.3. Analysis

The interviews were transcribed and analyzed in terms of abstraction. For the first task, we only expected to see whether the participants could recognize the Pythagorean Theorem and similarity. For the second one, we tried to observe characteristics of consolidation which are awareness, flexibility, self-evidence, confidence and immediacy. Their signs are stated below:

Immediacy: When we ask how they can prove the theorem, they should directly state "similarity" and start to use it.

Flexibility: When we ask if they can prove the theorem by using another way, they should try it and find another way but it should be related with similarity again.

Self-Evidence: When they start to prove the theorem, even if they are able to justify similarity knowledge, they shouldn't need to prove it. They should directly use it.

Confidence: When they directly start to use similarity, they shouldn't be in doubt.

Awareness: They shouldn't be in doubt about the fact that they do. From the point of theoretical knowledge, they should be sure and conscious about what they do.

3. Results

The results were presented below separately for each student. The accomplishing process of tasks was reported. And also, in order to emphasize the characteristics of consolidation, only the related parts of the transcripts were given below.

3.1. Gul's Results

Gul was able to recognize the right triangle but she couldn't tell its properties. It can be understood from the following dialog.

5 Researcher (R): What do you see here? (Researcher showed the triangle BCE in Figure 1).
6 Gul (G): A triangle.
7 R: Which type of triangle is it?
8 G: Right triangle.
…
11 R: Could you tell us something about right triangle? Which properties occur in your mind when you think about it?
12 G: I can't remember.

As seen from the dialog, she had a lack of self-confidence in general. She didn't prefer to think for a while, because she believed that she couldn't remember. After a while, we started to talk on sides of the right triangle and their relation. It can be noticed from the following dialog that she couldn't use Pythagorean Theorem exactly. After we led her with questions, she could decide to use it.

25 R: You said hypotenuse. Do you know anything about relationship between hypotenuse and the legs?
26 G: Yes, it must be.
27 R: What kind of relationship?
28 G: (Silence)
…
31 R: If I give lengths of these two legs (showing a and b), can you find the length of hypotenuse?
32 G: Square of this and this equal to square of this. (She intended addition of legs' square equal to square of hypotenuse.)

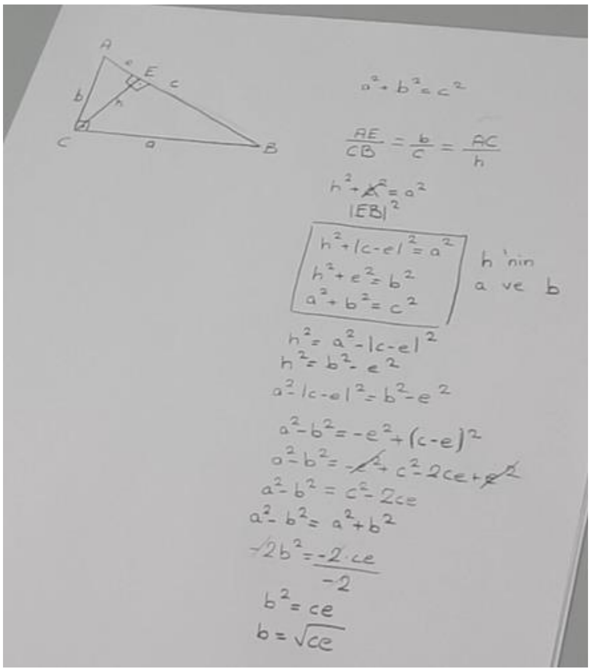

Figure 4. *Gul's paper for the answer to the question*

Gul couldn't name Pythagorean Theorem but after the above mentioned dialog, she tried to use it to accomplish the Task 1. Although she tried, she couldn't find the desired expression. It wasn't a problem about the mentioned theorem but it was about algebraic knowledge. She wrote the equations which required to the mentioned theorem as seen in Figure 4.

After her trials, she gave up trying to accomplish Task 1. The study went on with researcher's question on Pythagorean Theorem and its proof.

118 R: How did you learn this theorem last year?
119 G: There were triangles like 3-4-5 and 6-8-10. Special right triangles.
…
122 R: Did you learn any proofs of it?
123 G: At school no, but I suppose at private lesson I had learnt.
124 R: What kind of proof was it?
125 G: I couldn't remember exactly.

As can be understood from the dialog, she knew Pythagorean Theorem as "formal" knowledge; but she couldn't construct it exactly. She didn't want to talk about it more. As a result it was understood that she memorized the expression but she didn't consolidate it, and also she didn't consolidate the similarity because she couldn't tell anything about similarity except the beginning. The dialog is as follows:

10 G: It is a right triangle, do I use the similarity?"
…
19 R: How can you use?
20 G: We learnt it last year but this year I don't remember anything.

She didn't aware of why she needed to use similarity. Also, at a first look it seemed that she had an immediate decision on using similarity. But when focusing, it could be understood that this attempt wasn't conscious. It can be said that she knows the name of "similarity", not more.

3.2. Elif's Results

As soon as Elif saw the figure, Pythagorean and Euclidean Theorem occurred to her mind. She used both Pythagorean Theorem and similarity for Task 1. The related dialog is as follows.

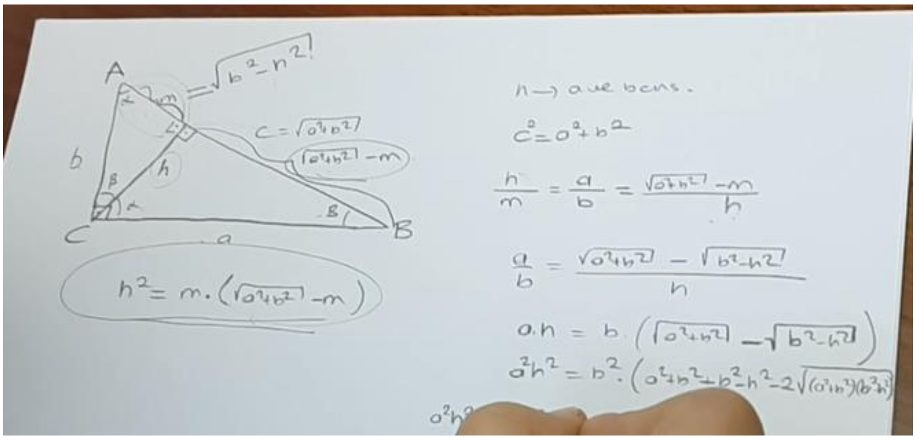

Figure 5. *Elif's solution for Task 1*

5 R: Which expression are you writing?
6 Elif (E): Pythagoras.
…
9 R: What are you doing now?
10 E: I am trying to use similarity.
…
22 R: (After Elif told about Euclidean Theorem) Didn't you think to use Euclidean Theorem?

23 E: I thought but when I used similarity, I found the h^2 *is equal to this (she showed the expression:* $h^2 = m.(\sqrt{a^2+b^2} - m)$*, where m is the projection length of leg b on the hypotenuse as seen in Figure 5).*

After the dialog and operations given above were presented in Figure 5, she went on doing operations as follows (Figure 6) and reached the desired expression at Task 1.

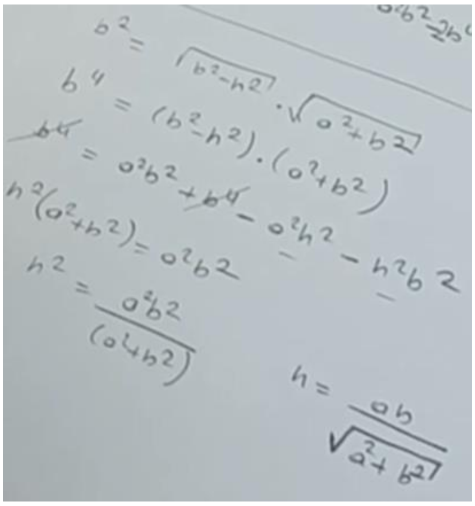

Figure 6. *Elif's solution for the desired expression*

It can be said that she recognized Pythagorean Theorem, Euclidean Theorem and similarity. After she had obtained the desired expression, she started to accomplish Task 2. We asked her to prove Pythagorean Theorem. She drew a circle and a triangle into the circle (Figure 7). After her drawing, researcher asked her what she did. It can be followed from the given below.

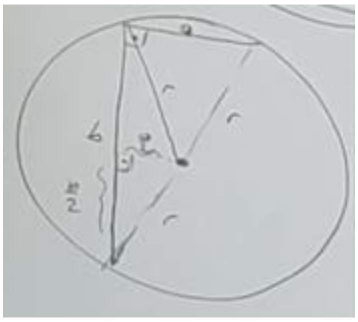

Figure 7. *Elif's first trial for Task 2*

48 R: What do you think to do now?
49 E: I think that I can use the similarity.
50 R: What do you mean about similarity?
51 E: Among the triangles.
52 R: Which triangles?
53 E: The big triangle and this small one. (When she was working on the triangles seen in Figure 7.)
54 R: What can you do except similarity?
55 E: Nothing.

She used similarity of triangles' to prove the Pythagorean Theorem. While she was proving the theorem, she made a mistake about operations, but didn't give up using the same circle and the same way for proof. She couldn't finish the proof because of trying to write the same algebraic expressions.

But she never gave up using similarity. According to previous dialog (55E), it can be said that she consolidated the similarity knowledge in terms of characteristics (instead of flexibility) which revealed by Dreyfus & Tsamir (2004).

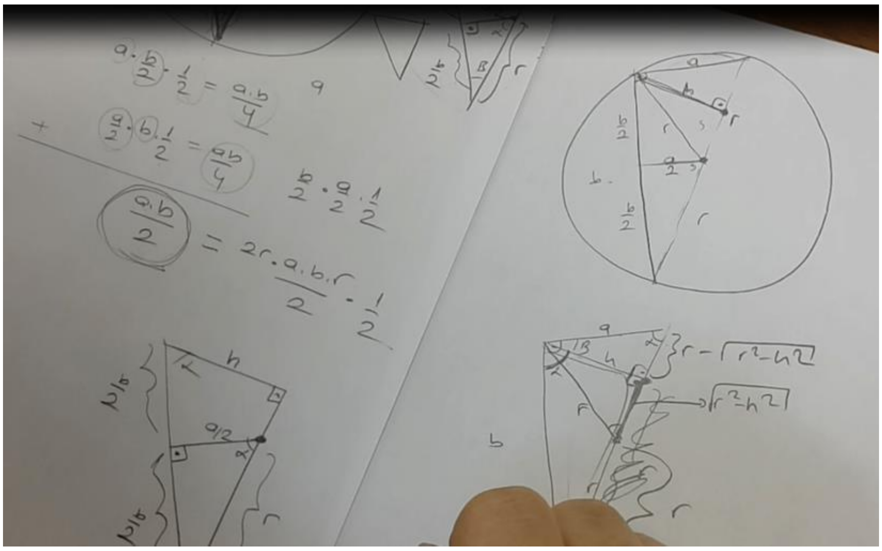

Figure 8. *Elif's second trial for Task 2*

3.2. Ahmet's Results

As soon as Ahmet saw the figure of the Task 1, he stated that he could use Euclidean and Pythagorean Theorem and he used these two theorems to find the desired expression despite our expectations about the fact that the participants would use Pythagorean Theorem and similarity. Different from other participants he named the right legs of the triangle as a and c; and the hypotenuse of the triangle as b (Figure 9).

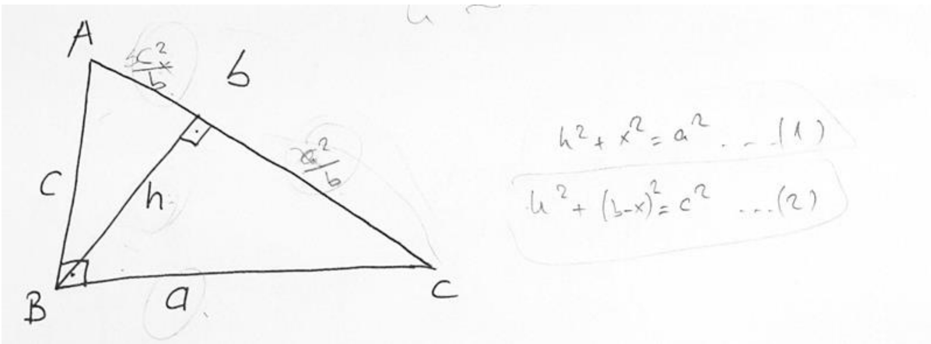

Figure 9. *The triangle which was drawn by Ahmet*

During the operations he frequently used Pythagorean Theorem instead of Euclidean Theorem. It can be seen from Figure 10 that he stated the Pythagorean Theorem as "P.T" in parenthesis.

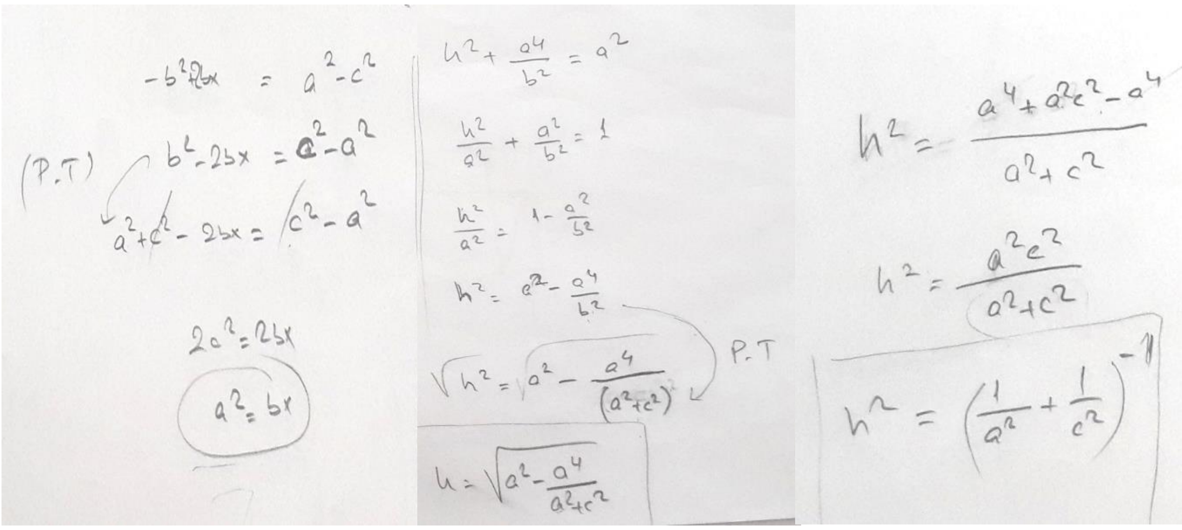

Figure 10. *The usage of Pythagorean Theorem by Ahmet*

After that we asked him to prove the mentioned theorem. The dialog was as follows:

> *46 R: How can you prove the Pythagorean Theorem?*
> *47 Ahmet: I think that I can prove it by the help of similarity. (…)*

The figure which was drawn by Ahmet is as follows:

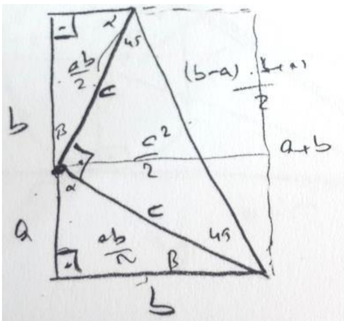

Figure 11. *Ahmet's drawing*

As it is seen in the figure, he drew two similar right triangles. Then he completed them as a right trapezium. He calculated the areas of triangles by using similarity. After the operations he proved the theorem.

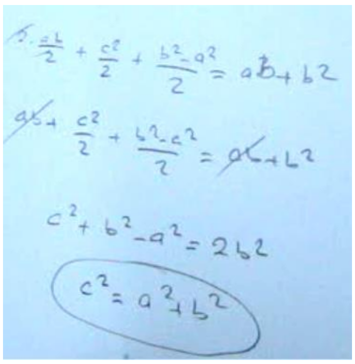

Figure 12. *Ahmet's calculation*

After that, we asked him if he could prove it in another way. The dialog was as follows:

48 R: Can it be proved by another way?
49 A: Yes, it can. But it occurred to my mind like this.
50 R: Another way? If we do like this… We draw our right triangle and then draw the incircle of it. Could we prove from this way?
51 A: I suppose, we draw perpendiculars here.
52 R: Why?
53 A: Because this is an incircle. If we draw lines from centre of circle to tangents, they must be perpendicular to each other.

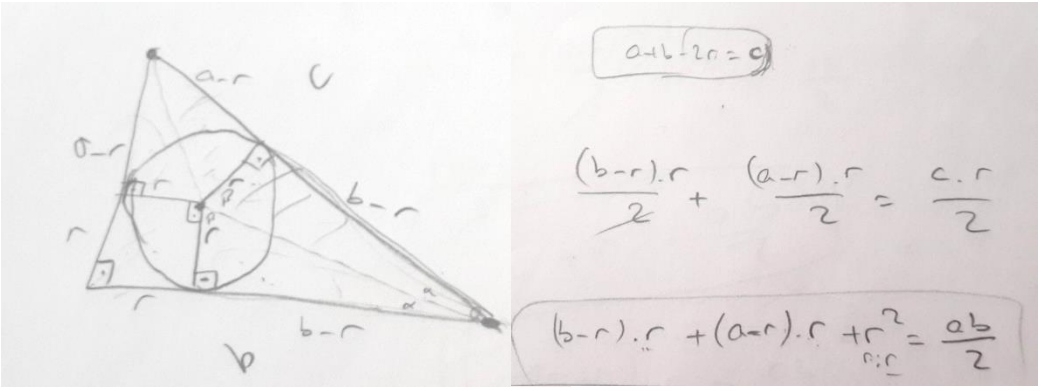

Figure 13. *Ahmet's attempt to accomplish Task 2 by another way*

Then he started to draw and tried to prove the theorem by the help of circle. Even if he couldn't prove it by this way, during this process he tried to use similarity knowledge again and also congruence of triangles. He changed his drawing to prove but never gave up using similarity. From these two tasks, it can be said that he consolidated the similarity knowledge completely in terms of characteristics which revealed by Dreyfus & Tsamir (2004).

4. Conclusions and Discussion

In this study, our aim was to determine if the tasks regarding Euclidean and Pythagorean Theorems could be used effectively or they needed to be modified in order to analyse consolidation process of similarity knowledge. This process was observed by the consolidation characteristics that were proposed by Dreyfus & Tsamir (2004), while students were trying to prove Pythagorean Theorem.

It was expected the students in Turkey at the age of Gul (age-15) to construct both similarity and Pythagorean Theorem. Gul didn't exhibited characteristics of consolidation for similarity; because of the lack of previously encountered knowledge structures about similarity and Pythagorean Theorem knowledge. The reason of Gul's failure of consolidation might be due to mathematical topic under consideration. On the other hand, Gul answered the question regarding the proof of the theorems as 123 G. It is a defective issue for Turkish education system. In general students need to take private lessons in addition to school lessons in Turkey. Each teacher educates students in accordance with his/her own beliefs and this situation can be mixed students' mind. For this reason, constructing and consolidating knowledges cannot be completed as desired at Gul's situation.

As to Elif and Ahmet, the mentioned characteristics could be observed while they were studying on the tasks. They were aware of what they were doing and they had confidence about their doings. They started with similarity directly as the pre-knowledge and accepted it as right. These behaviours refer to existence of immediacy and self-evidence characteristics of consolidation.

At the end of the operations although Elif couldn't prove the theorem, she still intended to prove it by using similarity. But she didn't try to use another way to prove and use similarity so it can be said that there is a lack of flexibility. So she needs more activities about similarity to consolidate it completely.

But Ahmet used similarity again for the second way of proving the theorem and this is a sign of flexibility.

Considering the characteristics, it is quite difficult to distinguish the mentioned characteristics from each other. Although Tsamir & Dreyfus (2005) stated that self-evidence and confidence are directly related to each other, the current study showed us that not only these two characteristics are related, but also all of five characteristics are nested in each other.

All of three participants stated that they could use similarity for accomplishing the tasks. Even if immediacy can be seen from texts such as 10 G, 10 E, 49 E and 47 A, it is really difficult to say that it is a characteristic of consolidation. Because, it is clear from the dialogs that Gul couldn't consolidate similarity knowledge, although she could talk on the mentioned knowledge immediately (10 G). In contrast Anabousy & Tabach (2015), by observing immediacy we couldn't conclude for participants to consolidate the knowledge. Thus, it can be said that immediacy may be a deceptive characteristic for the process. Furthermore, there is a need for posterior experiments to investigate whether "immediacy" is an essential characteristic of consolidation.

Awareness and flexibility are seen essential and easily observable characteristics of the consolidation process. The signs of flexibility were seen from the dialogs (55 E, 49 A) clearly while it is really difficult to refer only one dialog directly to show the signs of awareness; it needs to be observed it from the trajectory of the study. As a result, it can be said that RBC+C model has observable stages as Hershkowitz et al. (2001) stated and the consolidation process of similarity knowledge through the proof of Pythagorean Theorem can be analysed by using the characteristics of consolidation.

References

[1] Anabousy, A. & Tabach, M. (2015). Constructing and consolidating mathematical knowledge in the geogebra environment by a pair of students. *Psychology of Mathematics Education (PME-39)*, Hobart, Australia.

[2] Dreyfus, T. & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets, *Journal of Mathematical Behavior*, 23, 271-300.

[3] Dreyfus, T. (2012). Constructing abstract mathematical knowledge in context. *12th International Congress on Mathematical Education*, 8-15 July, Seoul, Korea.

[4] Hershkowitz, R., Schwarz, B. B. & Dreyfus, T. (2001). Abstraction in context: Epistemic actions, *Journal of Research in Mathematics Education*, 32(2), 195-222.

[5] Ispir, N. (2000). Pisagor Teoremi ve yüzlerce ispata eklenen yenileri, *Matematik Dünyası*, 5, 9-13.

[6] Kouropatov, A. & Dreyfus, T. (2013). Constructing the integral concept on the basis of the idea of accumulation: suggestion for a high school curriculum, *International Journal of Mathematical Education in Science and Technology*, 44(5), 641-651.

[7] Monaghan, J., & Ozmantar, M. F. (2006). Abstraction and consolidation, *Educational Studies in Mathematics*, 62, 233–258.

[8] Sezgin Memnun, D. & Altun, M. (2012). Examination of the abstraction process about linear equation of two sixth grade students, *Necatibey Faculty of Education, Electronic Journal of Science and Mathematics Education*, 6(1), 171-200.

[9] Tabach, M., Hershkowitz, R. & Schwarz, B. B. (2001). The struggle towards algebraic generalization and its consolidation, *In van den Heuvel, M. (Ed.), Proceedings of Psychology of Mathematics Education (PME-25)*, Utrecht, The Netherlands, 4, 241–248.

[10] Tabach, M., Hershkowitz, R. & Schwarz, B. B. (2006). Constructing and consolidating of algebraic knowledge within dyadic process: a case study, *Educational Studies in Mathematics*, 63, 235-258.

[11] Tsamir, P. & Dreyfus, T. (2005). How fragile is consolidated knowledge? Ben's comparisons of infinite sets, *Journal of Mathematical Behavior*, 24, 15-38.

[12] Yesildere, S. & Turnuklu, E. (2008). The Investigation of knowledge construction processes of primary 8th grade students according to their mathematical power, *Journal of Uludag University Education Faculty*, 21(2), 485-510.

Authors

Hatice Kübra Güler, Duzce University Education Faculty, Department of Mathematics and Science Education, Duzce, Turkey, e-mail: haticeguler@duzce.edu.tr

Çiğdem Arslan, Istanbul University Hasan Ali Yücel Education Faculty, Department of Mathematics and Science Education, Istanbul, Turkey, e-mail: arslanc@istanbul.edu.tr

Acknowledgement

This work was supported by Scientific Research Project Coordination Unit of Istanbul University. Project number: BEK-2016-21422.