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"ELIP-MARC" Activities via TPS of Cooperative Learning to Improve Student's Mathematical Reasoning

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Abstract

The purpose of this study is to describe and generate interaction model of learning through *Elip – Marc* activity via *TPS* cooperative learning in order to improve student's mathematical reasoning who have valid, practical and effective criteria. "*Elip - Marc*" is an acronym of eliciting, inserting, pressing, maintening, reflecting and confirming, an activity in the learning process have aim to improve student's mathematical reasoning. So there are three types product are developed, namely: (1) The interaction learning model, as a conceptual framework and a reference in developing the supporting device. (2) The learning device, for carrying out teaching methods and to measure the practicality and effectiveness of the interaction model and (3) The research instrument to assess the quality of the model and the device. All products are *Elip-Marc* activity oriented and supported by worksheet with RAT (*Reasonable Answer Test*) format. All products are declared valid as constructs and content by expert validator. Practicality and effectiveness criteria of the product conducted in even cemester at 2014-2015 by implementation interaction model on eight grade students of 'SMP Negeri 3 Kepanjen" Malang East Java, Indonesia on the material "Problem solving about three dimension spaces". By Analysis of the teacher's Elip-Marc activity and the response of student's in the learning process obtained conclusions that interaction model is practical and effective, so can improve student's mathematical reasoning as the intervention results in desired outcomes. So, If (1) the teacher is pleased to be patient and persistent engage students in the learning process through Elip-Marc activities, (2) and supported by the task of "word problems" challenging (3) and motivated by the question that digging (Prompting Questions) and guided (Probing Question), (4) and provide students the necessary scaffolding, (5) and provide space whorksheet adequate workplace, (5) and following the *RAT* steps, students were motivated to improve mathematical reasoning. However, the authors suggest that the results of this study can be adapted to the other materials in mathematics learning, and hope a lot of teacher willing to implement this model interaction.

Keywords: activities, Elip-Marc, learning, mathematical, reasoning

1. Introduction

Mathematical understanding depends on how the person reasoning (Ball & Bass, 2003; Mueller, 2009; Maher & Mueller, 2009). If the students are given an opportunity early on, supported with adequate facilities for reasoning, then they will be accustomed to construct their own knowledge very well (Maher & Mueller, 2009; Emerson, 2010; Brodie, 2010; Alajmi, 2010). The culture of reasoning in the mathematics learning process, it should be emphasized at every mathematics learning process (Mueller: 2009; Brodie: 2010; Carpenter dkk:1999). Because, a person's mathematical reasoning can be improved simultaneously the development of a person's schemata structure (Rohim, 2011; Marpaung, 2006). In the learning process, mathematical reasoning cannot be separated from the other five standards process. Mathematical reasoning, problem solving, mathematical communication, connection and mathematical representation are standards of interrelated processes (NCTM, 1989, 2000). Problem Solving and Reasoning are two standards that support each other. Reasoning as a generator in problem solving, and problem solving as a logical ground of reasoning.

The results of the preliminary study and the results of a previous research study, concluded that students' mathematical understanding and mathematical reasoning in Indonesia tend to be low (Mukhtar at al., 2013; Sukayasa, 2012; Priatna, 2003; Ardana, 2007). Similar conditions occur in other developing countries. Student reasoning tends to be low, when solving the problem only emphasizes the final answer, is attracted by the correct answer without explanation and clarification (Alajmi, 2010; Yang, 2005; Choy & Cheah, 2009; Choy & Pou, 2012; Emerson, 2010).

There are four elements that must be considered in the learning process, namely (1) planning, (2) implementation, (3) evaluation, and (4) follow-up evaluation results (Kemendikbud, 2013). If the teacher wishes to emphasize mathematical reasoning in the process of mathematics learning, it is necessary to design instructional devices that emphasize mathematical reasoning. Furthermore, the learning device is applied in the learning process, to be able to know the achievement of learning objectives need a proper evaluation tool. In order for the learning objectives to be achieved effectively, it is necessary to support the learning experience, learning materials and the types of tasks that are adequate.

The learning experiences, teaching materials, task design, supporting should be designed in such a way as to support a learning process that emphasizes mathematical reasoning. Therefore, the need for a conceptual framework as a foundation for the implementation of the learning process. This conceptual framework is a model of learning (Parta, 2009).

From the reasons described above, it would be necessary to think of a conceptual framework of mathematics learning processes that have characteristics: (1) giving students opportunities to mathematical reasoning, (2) allowing teachers to motivate students to accommodate information, ideas, Knowledge, old concepts that have actually been learned with new knowledge, (3) facilitating learning processes that support the interaction that builds mathematical reasoning. Therefore, the authors are interested in developing a learning interaction model that emphasizes activities that encourage students to improve their mathematical reasoning.

On this occasion, the writer focuses on developing the learning interaction model which emphasizes on improving mathematical reasoning in Think - Pair - Share (TPS) one type of cooperative learning. This learning model is labeled “Elip-Marc Activity via TPS Co-operative Learning to Improve Student Mathematical Reasoning”. Elip-Marc is an acronym for Eliciting, Inserting, Pressing, Maintaining, and Confirming, by adding Reflecting before confirming. The need for added reflecting activities, reasoned because doing reflection at the end of each learning activity is considered to be a center for improving mathematics learning (Artzt & Armor, 1999; Van Es & Sherin, 2008). By reflecting at the end of each learning process students will be aware and control their learning actively, assess what they know, what they need to know, and what the consequences are if they do not know it. As it concluded (Sezer, 2008; Boody, 2008; Rudd, 2007) that reflection bridges the gap in learning situations.

The learning models that teachers often apply tend to cooperative learning models (Wisulah, 2009). The author wants to combine cooperative learning model as a forum to facilitate students to improve their mathematical reasoning in the mathematics learning process. The choice of TPS cooperative model (Lie: 2004) as a forum of this learning model, because the TPS cooperative model is easy to apply.

Therefore, the problem formulation the main study in this research is: “How is the process and result of learning interaction model development through Elip-Marc activity via TPS cooperative learning that is valid, practical, and effective to improve students’ mathematical reasoning?

2. Theoretical Frameworks

2.1 Rational Development Model

Effective learning models allow students to learn, Freely students to be more powerful, eventhough actualy learning models are often only appropriate for certain types of learning (Joyce, Weill & Calhoun 2009, p. 14). In fact, learning is a dynamic situation because learning is a social process, so it will be influenced by environmental factors, community demands on graduate competence, changes in government policy on passing standards, paradigm shift learning and the entry of technology in learning (Parta, 2007). Therefore, it is appropriate that Joyce, Weill, and Calhoun (2009, p. 3) suggest to us as always dynamic teachers do research and develop learning models to improve students’ abilities and intelligence.

2.2 Learning Based Questions and Tasks in Elip-Marc Activities

The basic principle of all effective learning is to ask questions in the classroom. In the classroom the teacher asked a question for various reasons (Freiberg & Driscoll, 2000), among others: (1) examine students’ understanding of learning, (2) evaluate the effectiveness of learning, (3) improve the mindset of a high level. Asking questions is one of the basic learning strategies that can be applied to almost all areas of the subject matter and grade levels. If done effectively, these strategies can encourage engagement, improve learning, motivate students, and provide feedback on the progress of learning, both for teachers and students (Jacobsen, at al., 2009). The characteristics of effective questions is (Cook, 1999) short, clear, focused, relevant, constructive, neutral, and open. The author agrees with Henniger (2004) that the implementation of the question strategy would effectively increase the interaction of the learning process, and engage students actively.

The key of effective question strategy is to ask questions that allow us to achieve the learning objectives *(Eliciting)* (Jacobsen, at al., 2009, p. 173). Teacher's ability to listen and respond student's responses through questions to sharpen his thinking *(Pressing)* allowing them to find their own mistakes (Fravillig et al., 1999; Nicol, 1999; 2003, Emerson, 2010) *(Inserting)*. When students were asked to answer and follow up questions (*Pressing*), encouraging them to think more deeply about their response, the accuracy increased significantly. Question higher level focused on the essential elements to create a conducive environment for students to think critically about mathematical concepts (Henningsen & Stein, 1997; Fraivillig at al., 1999). While directing or guiding questions (Prompting Questions), questions asked to give direction to the students in his thinking process (*Inserting*). This is usually done when there is a certain part in the discussions that are considered important, or in a way that is done by providing additional questions to answer the main question. Questions digging (Probing Question), questions that will encourage students to learn more about its own answer (*Maintening*), students are encouraged to increase the quantity and quality of previous answers. Most teachers according to McMillan (2004) to ask questions to the five main objectives: (a) to engage students in learning; (b) to encourage students' thinking and understanding; (c) to assess student progress; (d) to control the students; and (e) to review the contents of important lessons (*Confirming*).

2.3 Reflecting in Learning Process

Facilitating the development of students' skills of reflection is an important task for mathematic teachers, for reflection at the end of each learning activity is considered to be central to increasing the learning of mathematics (Artzt, 1999; Van Es & Sherin, 2008). Reflecting at each end of the learning process students will be aware of and actively control their learning, assess what they know, what they need to know, and what will happen if they do not know it. Sezer (2008) said that the reflection does bridge the gap in learning situations. Boody (2008) and Rudd (2007) concluded that the reflection is an important part of the process to analyze and articulate the problem and make connections with what they do in the classroom.

For teachers, reflecting is the practice in which teachers have to learn from their own teaching practice and gradually increased over time. Teachers who practice reflective thinking using inquiry as a tool to engage critically with the key questions and issues in their teaching practices (Jaworski, 2006). To reflect as a systematic means to achieve a broader understanding of the situation of teaching and improve the quality of learnin (Krainer, 2006). While **(**Scho¨n, 1987) suggest that in order to answer the question in resolving the problems faced by students one way is motivate them to reflect on what they have done in the learning process and looked back matter what comes up, analyze it and make decisions specifically, However, as evidenced by a study (Choy & Cheah, 2009; Rudd, 2007; Black, 2005; Vaske, 2001), students may not be able to think critically because their teachers are not able to integrate critical thinking enough in their daily practice that requires some reflection.

2.4 Elip-Marc Activities in TPS Cooperative Learning

Cooperative learning is a term for a set of teaching strategies that are designed to teach teamwork and interaction between students. At least there are three main goals of cooperative learning, which is the result of academic learning, acceptance of diversity, and the development of social skills. *Elip-Marc* Activities in the *TPS* cooperative learning process oriented (Jacobsen, at al., 2009; Lie,2004) to teach teamwork and interaction between students based on learning theory Vygotsky that emphasizes social interaction as a mechanism to support cognitive development *(*Jacobsen, at al., 2009) with learning activities adapting lessons step that emphasize mathematical reasoning (Brodie, 2010). In each of these learning activities give students opportunities to collaborate in pairs to solve existing problems in the student's worksheet that has been provided. the student's worksheet format adopts the term RAT (*Reasonable Answer Test)* of (Alajmi, 2009) and modify the steps Reasonable Evaluating Test (Dragonsky, 2012).

2.5 A Brief Description of the Elif-Marc Learning Model

The applied learning model is based on constructive learning theory and cooperative learning. The steps in the learning process pay attention to aspects of cognition and various real world issues. This learning model seeks to improve students' critical thinking by motivating mathematically reasoned thinking to motivate students to respond to Elip-Marc activities in the learning process. Students are required to work a word problem following the steps contained in RAT (Reasonable Answer Test), by providing a reasonable explanation. The explanations based on practical matters and / or mathematically based mathematical knowledge.

“Elip-Marc” as an acronym of: Eliciting, Inserting, Pressing, Maintaining, Reflecting and Confirming. At Eliciting activity, with teacher’s question and answer motivate students to come up new ideas related to the subject matter that will be discussed. In Inserting activities, teachers guide students include: adding an idea, information, facts or other knowledge that encourages improving students’ reasoning through questions, feedback on student answers, and make connections with prior knowledge and so on. While in Pressing activities, teachers motivate students to emphasize, clarify, justify or explain more clearly from the idea that comes up, by asking the questions “why...”, “how..., mention other examples, and so on. In maintaining activities, teachers motivate students to emphasize the ideas that come up, by repeating them, or asking other students to explain in another way according to their own language or ways. In Reflecting activities, teachers give students opportunities to reflect by writing essays about what they have learned, what the uses of the concepts have been learned, what the consequences are if the concepts are lacking or not mastered. At the end of the activity is done Confirming the right ideas, giving evidence, giving conclusion and affirmation.

The learning process that is expected to occur in the classroom is inseparable from the structure of the TPS model, is (1) the learning is centered on the activity of the students’ partners which are interdependent with each other (the characteristics of cooperative learning), (2) the students are given the freedom to think about the problem, (3) teachers to train and guide students to think logically and systematically in solving problems, (4) teachers attempt to organize cooperation in learning partners, train students communicate (Pair), (5)) Strived students always try to present the work of the couple in front of the class (Share).

3. Research Approach and Development Result

3.1 Research Approach and Quality Product of Prototype

This study is in the research development of education (educational design research). Barab and Squire (2004), Van den Akker et al. (2006, p. 5) define the design of educational research as “a range of approaches, with a view to generate new theories, artifacts, and practical models that explain and potentially have an impact on learning with a natural setting (naturalistic)”. Plomp (2007, p. 13) defines educational design research as “a systematic review of designing, developing and evaluating educational interventions (such as programs, strategies and learning materials, products and systems) solutions to solve complex problems in educational practice, which also aims to promote our knowledge of the characteristics of the interventions as well as the design and development process

The study was based on problems in the field and in the implementation of participants, researchers, experts and other stakeholders with the research stages to follow (Nieven, at al., 2006: 153, Plomp 2007) with the steps: (1) Preliminary research: analysis of the context and issues for the development of a conceptual framework grounding through literature reviews, field observations and preliminary research or conduct; (2) Prototyping stage: designing user design, prototype through recycling optimize the design, formative evaluation and revision. (3) Summative evaluation: an evaluation of the effectiveness of the implementation and use of the prototype. The evaluation was conducted for the eighth grade students of 3th Kepanjen Junior High School at the end of the semester with the matter “Problems solving of volume and wide areas of three dimension geometry”. Evaluation of the prototype is 12 meetings. Half of meeting the authors as a model teacher observed by two colleagues and an expert mathematic education, while the other half of the meeting as a teacher model is a colleague of the author as an observer accompanied by two colleagues and an expert mathematic education.

3.1.1 Product Quality Criteria

Product quality developed, following advice (Nieven et al., 2006; Plomp, 2007), have criteria: validity, practicality, and effectiveness. The description for each of the criteria is presented below. **Validity of product**. Nieven et al. (2006) and Plomp (2007) distinguish the prevalence of the product into two, namely the content validity and construct validity. A product is said to have content validity if judged valid by an expert (based on state-of art knowledge). The model has content validity when based on strong theoretical rationale. If each component in the model is consistently interconnected then the model is said to have construct validity. In relation to the development of the learning model, the model is said to have constructive validity, if syntax, social systems, reaction principles, support systems, and associated accompanist impacts are consistent. If a product meets these two criteria of validity then the product is said to be valid. **Practicality of Product**. The second characteristic of the quality of a product is (1) teachers and experts view the product as usable, (2) it is easy for teachers and students to use it, and (3) have a high degree of conformity with the developer’s purpose. In relation to the development of the learning model, it implies that there must be consistency between the learning objectives and the developed model. If there is consistency between the two objectives, then it is said that the model developed is practical. **Effectiveness of Product**. This third characteristic measures the quality of the product from the perspective of the student, ie the student enjoys or appreciates the learning program and the desired learning goal is achieved. Both aspects of this quality are representations of the situation experienced and the results achieved by students. If the product or material is effective, then there must be a consistency between the situation experienced and the results achieved.

3.2 The Outcome Research

Preliminary Studies. The activities undertaken by the researcher at this stage are: (1) reviewing the literature (2) looking at applicable curriculum policies and demands, (3) reviewing previous research and (4) conducting preliminary study of empirical conditions about mathematical learning taking place in the field. Preliminary studies focused on observing, reflecting, analyzing and making statements related to research focus. From the preliminary study of the literature review results, the results of the reflection of the learning process and the outcomes of the preliminary research are found, among others: (a) The mathematical reasoning of students in our country as well as the neighboring countries tends to be low, (b) However, if teachers are persistent facilitate the learning process with supportive tasks, the students' mathematical reasoning can increase.

Prototyping Stage. At this stage there are two important things to do, namely: (a) made prototype component learning model and its supporting devices and (b) perform validation test. All prototypes in the form of learning model components and supporting tools along with research instruments are calibrated and tested by expert validators. Qualitatively and quantitatively all components of the model, supporting devices and research instruments have been declared to meet the validity of the content and the validity of the construct. Below is shown the profile of prototype component of learning model and its supporting device.

Table 1. Elements of the model component of interaction through the Elip-Marc activity

Sintak Model	Social System	Supporting Principles	System Reactions
Introductions Core activities:: • TPS Phase I Classical Elip-Marc Activity • TPS Phase II In pairs Elip-Marc activity Covered • Reflecting • Confirming	• Implementation of Ki Hajar Dewantara's Basic Principles ➢ *Ing Ngarso Sung Tulodho Ing* ➢ *Madyo Mangun Karso* ➢ *Tut wuri Handayani* • Personal and/or spousal scaffolding	• TPS Cooperative Learning • Student centered • Teacher's role as: facilitator, motivator and mediator	• RAT (Reasonable Answer Test), • Student Books, with the RAT format provide sufficient work space • Master's book, contains instructions on implementing learning models

Evaluation Stage. This stage is to test the practicality and effectiveness of the learning model. **The Practicality of Learning Model**. The practicality of the model in each evaluation is seen from the observation of teacher activity in the learning process through the teacher activity observation sheet which already available supported by the result of joint reflection after each learning process ended. At the end of evaluation stage, model teachers fill out a questionnaire about the practicality of the model and respond to interviews about the practicality of the model. In Table 2, a summary of all teacher activity analyzes.

Table 2. Summary of teacher activity analysis all evaluation stage results

Kind Of Activity		Average analysis results				Average	information
		1st evaluation	**2sd evaluation**	**3th evaluation**	**4th evaluation**		
Introduction		2.83	3.56	3.7	4	3.5	High
Cored	TPS	2.77	3.67	3.2	3.83	3.4	High
	In pair Elip-Marc activity	2.77	3.50	3.0	3.7	3.2	High
	Classical Elip-Marc activity	2.42	3.88	3.2	3.0	3.1	High
Covered	Reflecting	2.62	2.88	3.0	4	3.1	High
	Confirming	3.25	3.76	3.6	3.9	3.6	High
Average of the all results		2.91	3.54	3.2	3.74	3.3	High

From Table 2 it can be seen that the average teacher activity reached the high category of 3.3 more than 3. The results of teacher questionnaire responses and interview results from model teachers support that the learning model through Elip-Marc activities oriented cooperative learning TPS can practically be applied in an effort to increase students' mathematical reasoning.

Top of Form.

Implementation of Elip-Marc Activities in TPS cooperative learning

In the following, we presented snippets example of the transcription of learning process in the classroom involving Elip-Marc activity.

Through the slide showed "word problems" as a first observation of learning materials

Teacher: *Okay. Now, please consider the problems that exist on the slide, and each person to try sketching the picture. You should colaborete cooperate with less seatmate. Student*. (The students trying to translate this into a representation of the sketch)

Teacher: Walk around checking student work. The teachers provide scaffolding personally. (Having considered there are several pairs of students who have sketched the picture correctly, the teacher pointed to one couple pouring sketch drawings on the whiteboard)

Students (Bayu): (sketched picture on the whiteboard)

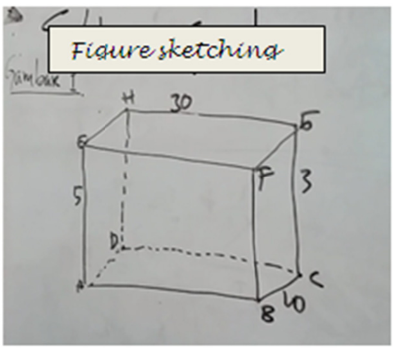

Figure 1(a).

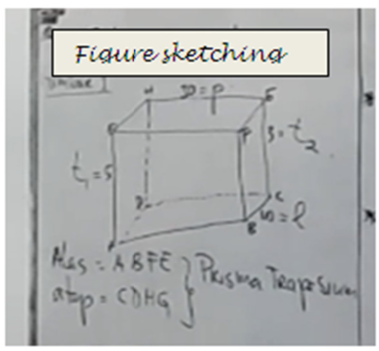

Figure 1(b).

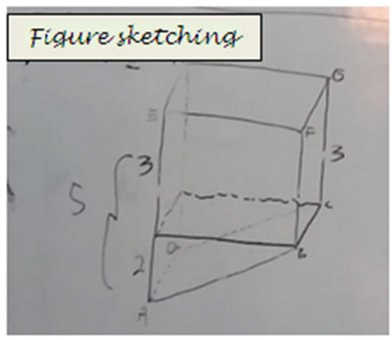

Figure 1(c).

Teacher : *Shaped whether the pool is drawn Bayu (eliciting)....... Saso?*

Saso : *Prism*

Teacher : *Please, Mention the name of the base.... Saso*

Saso (Saso pause)... with doubt he answered *ABCD*.

Teacher: *Is it true that the base of the prism ABCD, Try to remember again what the main characteristics geometry prism shaped? (Inserting)......... Fanju.*

Fanju: *The geometry shaped the base and the roof has the same shape*.

Teacher: *Ok. Fanju is true. Well now look at the pictuce, what ABCD and EFGH is equal.... Back to Saso.*

Saso: *No*

Teacher: *Why.... Try to explain! (Pressing)*

Saso: *Due to the size of the AE is not equal to BF.*

Teacher: *Try to note the information on the slide and note the size of the picture belongs to Bayu, which the shape is congruent? (Inserting)*

Bayu: *ABFE and CDHG*

Teacher: *Shaped whether ABFE ……?*

Some students: *Trapezoid*

Teacher: *Yes..... then try drawing to be addressed..... Bayu*

Bayu (Bayu fix the picture so that it looks like Figure 1(b)

Teacher: *Well, we have got trapezoidal prism whose base*

Teacher: *In addition to using the formula Volume of prism = the base area times the high mesurement, can you calculate the volume of the pool with an other ways? (Maintaining)*

Aurel: *Yes..... Mom, by making Blocking.*

Teacher: *Try to make the picture*

Saso (Saso drawing as shown in Figure 1 (c)).

Effectiveness Quality. Table 3 is a summary of student activity analysis for four evaluations stage. Similar to teacher activity, the element seen from this student activity is responding to teacher activity during learning process. Furthermore, in table 3.4 is a summary analysis of teaching material mastering test results. Mastery learning consists of the average value of worksheet and value of RAT at the end of completion one competence of learning. Mastery learning is obtained by searching for an average value of worksheet score of each test with a weight of 40% and an overall RAT value of 60% weight. The learning objective in this research is the students can increase their mathematical reasoning through process and learning outcomes.

Table 3. Summary of student activity analysis all evaluation stage results

Kind Of Activity		Average analysis results				Average	information
		1st evaluation	2sd evaluation	3th evaluation	4th evaluation		
Introduction		2,83	3.5	4	4	3.83	Very High
Cored	TPS	2,77	3.48	3.5	3.5	3.49	High
	In pair Elip-Marc activity	2,77	3.33	3.2	3.7	3.41	High
	Clasical Elip-Marc activity	2,42	3.00	3.8	3.2	3.33	High
Covered	Reflecting	2,62	3.08	3.0	3.7	3.26	High
	Confirming	3,25	3.32	3.5	3.5	3.44	High
Average of the all results		2,91	3.24	3.39	3.6	3.41	High

4. Discussion

When the teacher is motivating students to perform Elip-Marc activities in classical. This activity begins by displaying "word problem" as a preliminary observation material and distributing the initial "worksheet" to each student. The initial worksheet given to each student gives students opportunities to pour the work of the word problem that is displayed on the slide, and provides a space where work is poured in. It provides a realistic situation that allows students to use informal ways to solve problems informal methods of student production play an important role in the rediscovery and construction of concepts, such as advice (Pape et al., 2003; Fuchs et al., 1996) that problem-solving with regard to everyday life should be the main focus of mathematics learning. Design tasks involving problem solving need to be considered so that students can develop their mathematical reasoning, as suggested (Francisco & Maher, 2005; Brodie, 2010 and Van de Wale, 2007).

The initial "worksheet" to each student aims to have each student practice pouring the work following RAT steps. This activity supports Fennema et al. (1999) that while teaching to emphasize mathematical reasoning, teachers should ensure that they establish class norms that enable students to engage in mental activity, provide mathematical tasks that facilitate much thought, incorporate equity that requires all students to have Opportunities to learn, and use assessment to constantly monitor the development of students' reasoning and understanding.

Beginning the work process students are given the opportunity to look at the information contained in the problem and write down the keywords that became the focus to help solve the problem. It aims to monitor that the student has been able to understand the terminology of the problem. Because understanding the problem is very crucial for students to continue their work. Furthermore, the teacher gives students the opportunity to pair up with a seatmate to discuss, exchange ideas related to solving problems they think. Working in pairs is one form of effective cooperative learning compared to working groups of more than two members. In accordance with the suggestion (Lie, 2004), the advantages of TPS type cooperative learning include: (a) increasing student participation, (b) more opportunities for the contribution of each group member, (c) easier interaction, and (d) more easily and quickly form it.

At the time students work in pairs the teacher provides scaffolding for couples to solve problems encountered by following the RAT steps in place that has been provided. Scaffolding in learning mathematics is the steps or the right action to make the process of learning mathematics or the process of solving a mathematical problem in the classroom. Such actions can be guidance, encouragement, and warning, breaking down problems into solving steps, giving examples, and other actions that allow the student to self-study. That kind of teacher action is called scaffolding. As Slavin (1997) argues that scaffolding is providing some assistance to students during the early stages of learning, then reducing aid and providing opportunities to take on greater responsibility once it can do so.

The RAT measures are as follows: (1) The process of mathematization by sketching figure, (2) Selecting and defining the solution strategy, (3) Determining the mathematical model and determining the completion with the chosen strategy, (4) Checking the answer, (5) Giving conclusions with reasonable reasons. Next, gives an explanation of each step given.

When the class has appeared to fill the worksheet, the teacher pointed at random a student representing the couple to deliver their work, and poured it on the board. Giving students opportunities to communicate the work is one of the activities to improve mathematical reasoning. Support the suggestion (Lincoln, NE & Marlene Grayer: 2009) that in order to build a class that focuses on reasoning, students need to present their ideas in class so that there is a class discussion involving all class members. The activity begins with the process of mathematization by sketching figure as a representation of word problems into the mathematics symbol.

The representation of the problem is translation, disclosure, reappraisal, mining or even modeling of ideas, concepts and relationships among those contained in a particular configuration, construction or situation presented in various forms in an effort to gain clarity of meaning, solution of a problem encountered (Mudzakir, 2006). Thus the process of representation can be divided into two stages, namely representation internally and externally. Internal representation is a process of thinking, while external representation is the result of embodiment in describing what is thought. The results of this embodiment may be expressed either orally, in writing in the form of words, symbols, expressions or notations, drawings, graphs, diagrams, tables, or other physical objects.

The next step in RAT lets the learner to choose and define a settlement strategy, followed by determining the mathematical model and determining the completion with the chosen strategy. When the student has sketched the drawing as a mathematical representation, the next step is to select and determine the completion strategy followed by determining the mathematical model and determining the completion with the chosen strategy. Eliciting activities play a role in motivating students to come up with ideas for strategies related to problem solving. When the eliciting activity has not been able to maximize the idea, the inserting activity plays a role to guide, add or insert information, new knowledge or explore the knowledge that has been possessed as necessary. So there is a link between inserting and scaffolding activities.

The step of checking the answers and concluding with reasonable reasons is an act of self-reinforcement. By checking the answers and giving an explanation of every step taken, will know for sure that the answer given is correct. More details are further discussed about Elip-Marc's activities by teachers to motivate students to improve their mathematical reasoning.

Eliciting. By Using classical questioning the teachers motivate the class to come up with ideas for strategies related to problem solving. Eliciting activity is an activity to motivate students to come up with new ideas, concepts, principles related to solving the problems discussed. This is support Fraiviling's (2001) finding that the early stage strategy of learning in Advancing Children's Thinking which takes into account the high-level taxonomy of Benyamin S. Bloom (analysis, synthesis, evaluation and creation), is the emergence of a variety of problem solving methods. It should be noted, however, that the activities provided as a learning experience at the beginning of the lesson should begin with activities that are relevant to the learning experiences that students have (Sridana: 2007). So, for the sake of supporting this eliciting activity in the learning process begins with giving the word problem in the form of problem solving.

Inserting. When eliciting activities are not maximized, teachers use digging and / or guiding questions to add or insert information, new knowledge or explore the knowledge that students already possess necessary to solve the problem. This support opinion (Henningsen & Stein, 1997; Fraivilling et al., 1999) that higher level questions are focused on creating a conducive environment for students to think critically. While the question of directing or guiding (Prompting Question), and question digging (Probing Question), is a follow-up question to encourage students to further explore the answer itself. These questions are an inserting activity. This supports the research reported by Emerson (2010) that, although students are hampered by high level questions, but if persistent teachers regularly use them in the classroom learning process, and help them with digging questions and lead questions, Improves students' understanding and can make connections between mathematical concepts he has learned. This is also support the suggestion (Davis, 1996; Noddings, 1990), a constructivist view that mathematics knowledge is built into the mind through assimilation and / or accommodation processes. New information should be linked to its experience of the world through a logical framework that transforms, organizes, and interprets its experience.

Pressing. Pressing Activity is an activity that motivates students to emphasize, clarify, justify or explain more clearly from the ideas that arise, teachers can use the questions "why...", "how..., mention other examples, and so on. Through the contribution of students' answers, the teacher gives students the opportunity to explain the ideas that come up. This supports the opinion (Brodie, 2010: 76) which says that "When students give mathematical reasons, by explaining, or justifying, train students to communicate mathematically. Similarly, in the opinion of Kilpatrick et al. (2001) students should be able to explain their ideas to clarify their reasoning, by explaining the emerging ideas can hone the ability of reasoning and improve conceptual understanding. Confirmed by Kidron & Dreyfus (2010) that: Justification of the major and important components of mathematical reasoning. In the process of justifying mathematical phenomena, students often need to expand their knowledge and to build new knowledge.

Maintaining. Maintaining activity is an activity that motivates and encourages students to affirm the idea by repeating it, or asking other students to explain in another way according to their own language. This activity, as an emphasis of pressing activity. But in the case of mathematics problem solving tends to be an activity that motivates students to make other strategies or other ways to find the same answer. This is support Ginnis's (2008) suggestion that teachers at all times give students the opportunity to view a common concept from different ways. Because such things can train students to think creatively.

Reflecting. Reflecting activity is an activity that motivates students to make essays about: important things from the material that has been learned, the benefits of learning in the short and long term and the consequences if not mastered the material he studied. This is support the suggestion (Schon, 1987; Boody, 2008; Rudd, 2007; Artzt, 1999; Van Es & Sherin, 2008) reflecting at the end of each learning activity is seen as a center for improving mathematics learning. By reflecting at the end of each learning process students will be aware and control their learning actively, assess what they know, what they need to know, and what the consequences are if they do not know it. It also supports Sezer (2008) has said that reflection bridges the gap in learning situations.

Confirming. Confirming Activity is an activity at the end of the lesson to emphasize the right ideas, provide evidence, and make conclusions and affirmations. This activity supports McMillan's (2004) opinion that the purpose of asking questions at the end of the learning process is to (a) assess student progress; (B) to control students; and (c) to review the important content of the lesson.

5. Conclusion

The purpose of this study was to develop a model of interaction through Elip-Marc activities and *TPS* cooperative learning oriented to improve students' mathematical reasoning that have valid, practical and effective criteria. All products are declared valid as constructs and content by expert validator. By Analysis of the teacher's Elip-Marc activity and the respon of student's in the learning process obtained conclusions that interaction model is practical and effective, so can improve student's mathematical reasoning as the intervention results in desired outcomes

The emphasis at this event how Elip - Marc activity in the learning process. It turned out that the results of this study support the previous research such that: If (1) the teacher is pleased to be patient and persistent engage students in the learning process through Elip Marc activities (Brodie, 2010), (2) is supported by the task of "word problems" challenging (Pape & Yetkin, 2003), (3) is motivated by the question that digging (Prompting Questions) and guided (Probing Question), (Jacobsen et al., 2009; McMillan, 2004), (4) and provide students the necessary scaffolding (Anghileri, 2006). (5) provide space whorksheet adequate workplace (Dragonosky, 2012),

(5) following the *RAT* steps (Alajmi, 2010) students were motivated to improve mathematical reasoning. Mathematical reasoning is measured by how students respond *Elip-Marc* activity in worksheet outlined with *RAT* format by steps (a) sketching picture, (b) determining a preferred strategy, (c) translating problems into mathematical models and using options sterategy for resolving the problem, (d) checking the work, (e) making conclusion of the work to provide a reasonable answer (Wisulah, 2013). The material in this study is "Three dimension geometry problem solving ". However, the authors suggest that the results of this study can be adapted to the other materials in mathematics learning.

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