Practice Perspective

Process-Driven Math: An Auditory Method of Mathematics Instruction and Assessment for Students Who Are Blind or Have Low Vision

Ann P. Gulley, Luke A. Smith, Jordan A. Price, Logan C. Prickett, and Matthew F. Ragland

Mathematics can be a formidable roadblock for students who are visually impaired (that is, those who are blind or have low vision). Grade equivalency comparisons between students who are visually impaired and those who are sighted are staggering. According to the U.S. Department of Education, 75% of all visually impaired students are more than a full grade level behind their sighted peers in mathematics, and 20% of them are five or more grade levels behind their sighted peers in the subject (Blackorby, Chorost, Garza, & Guzman, 2003). Overall low achievement in mathematics is directly related to the crisis in braille literacy that experts in the field have noted with alarm over the last 25 years (Amato, 2002; Mullen, 1990; National Federation of the Blind [NFB], 2009; Papadopoulos & Koutsoklenis, 2009; Schroeder, 1989). Over 80% of students who are visually impaired are educated in mainstream, state-run schools (American Printing House for the Blind [APH], 2014). These students are not receiving adequate instruction in the Nemeth Braille Code for Mathematics and Science Notation (hereafter, Nemeth code) and are not given adequate access to Nemeth code materials (Amato, 2002; Kapperman & Sticken, 2003; Rosenblum & Amato, 2004). As a result, visually impaired students often cannot persist in post-secondary education and are denied access to many career paths, especially those in science, technology, engineering, and mathematics.

Process-Driven Math is a fully audio method of mathematics instruction and assessment that was created at Auburn University at Montgomery, Alabama, to meet the needs of one particular student, Logan. He was blind, mobility impaired, and he could not speak above a whisper. Logan was not able to use traditional low vision tools like braille and Nemeth code because he lacked the sensitivity in his fingers required to read the raised dots. Logan's need for tools that would enable him to perform the rigorous algebraic manipulations that are common in mathematics courses led to the development of Process-Driven Math. With this method, he was able to succeed in both college algebra and pre-calculus with trigonometry. These tools may help other visually impaired students who are not succeeding in mathematics because they lack access to, or knowledge of, the Nemeth code.

Within many algebraic problems, there are processes (algorithms) that must be performed to simplify expressions and solve for variables. Process-Driven Math frees up working memory during the delivery of the mathematic content to the student, and during the student's subsequent manipulation of the equation while working toward a solution. As a result, the student who cannot use or access Nemeth code can better focus on the required algorithms because the cognitive load on working memory is greatly reduced. When using this method, a person who functions as the student's reader and scribe systematically reveals the algebraic expression in layers. The process is highly interactive and places control over the flow of information into the hands of the student. During the simplification processes, the student himself systematically transforms the equation in discrete, controlled steps that are carefully recorded so that every decision made is available to the

We thank Steve Noble for sharing his knowledge of synthetic voicing and speech rules for mathematics.

student for review. Take, for example, the following algebraic expression:

$$\frac{4x^2y + 4xy}{x^2 - x - 2} \div \frac{2xy + 8y}{x^2 + 2x - 8}$$

From the outset, the visually impaired student who either cannot use or access Nemeth code is at a severe disadvantage compared to her sighted peers. The sighted student can quickly assess the landscape of the expression and develop a plan of action based on its structure. If a student who is visually impaired is given an audio rendering of the same expression in which each number and symbol are read outright, he will have to recall all the details from working memory, analyze the relationships of the component parts, and then synthesize the landscape of the expression. The cognitive load created by this type of audio rendering will overload working memory (Cowan, 2010; Miller, 1956). When working memory is overloaded, that particular content within the subject becomes inaccessible to the student.

The Process-Driven Math method begins with an audio rendering of the algebraic expression that significantly reduces the spoken math syntax (that is, numbers, variables, symbols, and operators) found in the expression. It delivers the overall landscape of the expression without overwhelming the student's working memory. Numbers and symbols are temporarily hidden behind layers of appropriate mathematic vocabulary. For example, in the initial Process-Driven Math audio rendering of the expression above, the student will hear "rational divided by rational." (We chose the vocabulary term "rational" over the more common term "fraction" because it builds a strong foundation for the mathematic vocabulary that is often used in subsequent college mathematics courses.)

The student who hears "rational divided by rational" is now in control, ready to explore the pieces of the problem in a manner that is logical and consistent with the landscape presented. The student can often make decisions that advance the simplification process without having to hear the actual numbers and symbols in a particular part of the expression. For example, the second rational in the expression above must be inverted and the division sign changed to a multiplication sign. The student says, "Take the inverse of the second rational and change the operator that preceded it from division to multiplication." The student is able to focus on the mathematical process without having to hear the volume of syntax in the second rational. After the student directs the change, the reader-scribe then tells the student that the expression is "rational times rational" and rewrites the expression to reflect that change:

$$\frac{4x^2y + 4xy}{x^2 - x - 2} \times \frac{x^2 + 2x - 8}{2xy + 8y}$$

At some point in the process, the student will ask to hear the numerator of the second rational. Instead of saying "$x^2 + 2x - 8$," the reader-scribe says "term + term – constant." The use of proper vocabulary reinforces the student's understanding of the expression. *Terms* are defined as elements that are being added or subtracted. The word *constant i*mproves the student's understanding of this particular term because the student knows that a constant is a number that stands on its own without a variable. After hearing "term + term – constant" the student will ask to hear what is inside each of the terms, including the constant. The student will then have the information needed to identify the numerator of the second rational as a quadratic that should be factored.

With precision of language, the student will direct the reader-scribe to establish two sets of parentheses, each with space for two terms, and to place an operator of a plus sign in one parenthesis and an operator of a minus sign in the other.

$(+)(-)$

The student directs the scribe to put an *x* in the first slot of each parenthesis, a *4* in the second slot of the first parenthesis, and a *2* in the second slot of the second parenthesis.

$(x + 4)(x - 2)$

Without ambiguity, and without overloading working memory, the quadratic is simplified and the student moves on to work in another part of the problem.

The student continues directing the reader-scribe to read each section of the expression and to record the changes the student makes in each of those areas. After the student simplifies both of the numerators and both of the denominators properly, the expression becomes:

$$\frac{4xy(x + 1)}{(x - 2)(x + 1)} \times \frac{(x + 4)(x - 2)}{2y(x + 4)}$$

The reader-scribe tells the student that the expression is "rational times rational." The student directs the reader-scribe to multiply across the numerators and multiply across the denominators to create one rational. The reader-scribe follows the student's instructions and makes the following transformation:

$$\frac{4xy(x + 1)(x + 4)(x - 2)}{(x - 2)(x + 1)2y(x + 4)}$$

The reader-scribe tells the student that there is one rational. The student is then close to finishing the simplification of this expression and asks, "Are there any matching factors found both in the numerator and the denominator of this rational?" The reader-scribe responds in the affirmative, and the conversation for the remainder of the simplification sounds something like this:

Reader-scribe: "Yes, I see a factor of the quantity $x + 1$ in the numerator and the denominator."

$$\frac{4xy(x + 1)(x + 4)(x - 2)}{(x - 2)(x + 1)2y(x + 4)}$$

Student: "Cancel them. Are there any other matching factors found in both the numerator and the denominator?"

Reader-scribe: "Yes, I see a factor of the quantity $x + 4$ in the numerator and the denominator."

$$\frac{4xy(x + 4)(x - 2)}{(x - 2)2y(x + 4)}$$

Student: "Cancel them. Are there any other matching factors found in both the numerator and the denominator?"

Reader-scribe: "Yes, I see a factor of the quantity $x - 2$ in the numerator and the denominator."

$$\frac{4xy(x - 2)}{(x - 2)2y}$$

Student: "Cancel them. Are there any other matching factors found in both the numerator and the denominator?"

Reader-scribe: "Yes. I see a *y* in the numerator and the denominator."

$$\frac{4xy}{2y}$$

Student: "Cancel them. Are there any other matching factors found in both the numerator and the denominator?"

Reader-scribe: "No."

$$\frac{4x}{2}$$

Student: "Read what is there."

Reader-scribe: "A rational with a numerator of $4x$ and a denominator of 2."

Student: "Reduce the 4 and the 2. Make the 2 in the denominator into a 1 and make the 4 in the numerator into a 2. Read back what is left."

Reader-scribe: "A rational with a $2x$ in the numerator and a 1 in the denominator."

$$\frac{2x}{1}$$

Student: "My answer is $2x$."

The role of the reader-scribe must be filled by an individual who has mastery over the course content, and training is critically important. A reader-scribe who mistakenly says "the square root of $x + 1$ divided by 2" will put the student at a disadvantage because that statement can be interpreted five different ways:

$$\sqrt{x} + \frac{1}{2} \text{ or } \frac{\sqrt{x} + 1}{2} \text{ or } \frac{\sqrt{x + 1}}{2}$$

$$\text{or } \sqrt{x + \frac{1}{2}} \text{ or } \sqrt{\frac{x + 1}{2}}$$

The Process-Driven Math reader-scribe is trained to hide mathematic syntax by breaking the expression into pieces, a method that ensures a consistent and unambiguous rendering of the equation. Likewise, the student must also be trained to give an accurate audio rendering of the mathematic manipulations being executed.

The interactive, bidirectional communication also allows the student to render mathematical formulas without ambiguity. We will take, for example, the rendering of the quadratic formula. In written form, the quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The student demonstrates proficiency with the formula by making the following statements to a reader-scribe:

Student: "The quadratic formula is x equals a rational."

$$x = \frac{numerator}{denominator}$$

Student: "The numerator of the rational is term plus or minus term."

$$x = \frac{term \pm term}{denominator}$$

Student: "The first term is negative b, and the second term is a square root."

$$x = \frac{-b \pm \sqrt{\square}}{denominator}$$

Student: "The inside of the square root is term minus term."

$$x = \frac{-b \pm \sqrt{term - term}}{denominator}$$

Student: "The first term in the square root is b squared, and the second term in the square root is $4ac$."

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{denominator}$$

Student: "The denominator of the rational is the factor 2*a*."

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Without using Process-Driven Math, a person speaking the quadratic formula might say "*x* equals negative *b* plus or minus the square root of *b* squared minus 4*ac* divided by 2*a*," a statement that could be transcribed in many different ways. The student using the Process-Driven Math approach to verbally render mathematical formulas can be graded using the same standard for accuracy that is used for sighted students. Thus, that student is competing on a more equal footing with sighted peers.

Logan successfully solved well over one thousand mathematics problem using Process-Driven Math. He fully drove all of the intellectual processes and proved his competency in the mathematics classes required by the university. Logan always had the ability, but what he lacked, and what threatened his persistence in college, was access to appropriate tools. Logan's role has been far more than that of a student using a method created for him. He is one of the developers of the Process-Driven Math method. His insights into the experiences of a person doing mathematics solely by listening and speaking provided crucial guidance in the creation of the method. His success led to the establishment of a university-funded research program called the Logan Project. Researchers sought and received IRB approval from Auburn University at Montgomery for the preliminary collection of data, and participants signed informed consent forms for their participation in research activities.

Braille is an essential literacy tool for people who are visually impaired, and its use is directly related to educational attainment, employment opportunities, and independent living. Braille and the Nemeth code will continue to be important and necessary tools for individuals who are visually impaired, but additional tools are needed for people who have tactile limitations and for others who have not been taught to read braille or Nemeth code. Process-Driven Math was developed to help these students so they, too, would have the opportunity to succeed in mathematics and persist in college.

Auburn University at Montgomery is now employing Logan to train tutors to become reader-scribes for Process-Driven Math. Members of the Logan Project have since worked with a second student who was severely visually impaired. This student had never been taught braille or Nemeth code, but instead used enlarged text, screen readers, speech to text, and typing to access her curricula. These tools had been sufficient for all of her courses except mathematics. She struggled to properly discern exponents, even when materials were enlarged. More significantly, she did not find that typing adequately supported her efforts to demonstrate a series of complex transformations when simplifying algebra expressions. She could write by hand, but the process caused significant fatigue as she spent long periods of time bent over the desk to keep her eyes extremely close to the paper while writing. She was given the opportunity to use any of the tools available to her, including Process-Driven Math, throughout her college algebra course. She consistently chose to use the Process-Driven Math approach because it significantly reduced the fatigue she experienced when writing by hand. She successfully completed her college algebra course and is continuing to move forward in her education.

One limitation of the Process-Driven Math method is the subjectivity that exists between the student and the reader-scribe during the communication of the mathematics. In addition, Process-Driven Math is a highly labor-intensive process in which the student is dependent on the interaction with a trained reader-scribe to work through lengthy problems. In response to these limitations, members of the Logan Project are seeking to develop Process-Driven Math into a software application. Software would eliminate the person who is in the role of the reader-scribe and improve the autonomy of students using the method. Currently, the role of the reader can be met with software tools that produce unambiguous synthetic mathematic speech. One option is the use of software that implements MathSpeak, a set of speech rules created by Abraham Nemeth that map directly to Nemeth code symbols (gh, 2004–2006; Nemeth, 2013). Another option is software that integrates ClearSpeak, a set of speech rules and preferences that allows synthetic speech to closely approximate the way mathematics are spoken in the classroom (Frankel, Brownstein, Soiffer, & Hansen, 2016). A Process-Driven Math software tool would combine existing synthetic mathematic speech capabilities with a component to automate the role of the scribe. Thus, students could independently make real-time transformations to the mathematics in discrete, user-controlled steps. The goal of the project is to help many more people who might be facing hurdles similar to those that Logan and other students have faced so that they can be successful in college mathematics.

References

Amato, S. (2002). Standards for competence in braille literacy skills in teacher preparation programs. *Journal of Visual Impairment & Blindness*, *96*(3), 143–153.

American Printing House for the Blind. (2014). *Annual report 2014: Distribution of eligible students based on the federal quota census of January 7, 2013 (fiscal year 2014).* Retrieved from http://www.aph.org/federal-quota/distribution-2014

Blackorby, J., Chorost, M., Garza, N., & Guzman, A. (2003). *The academic performance of secondary school students with disabilities.* (NLTS2 Report). Washington, DC: U.S. Department of Education. Retrieved from http://nlts2.sri.com/reports/2003_11/nlts2_report_2003_11_ch4.pdf

Cowan, N. (2010). The magical mystery four: How is working memory capacity limited, and why? *Current Directions in Psychological Science*, *19*(1), 51–57.

Frankel, L., Brownstein, B., Soiffer, N., & Hansen, E. (2016). Development and initial evaluation of the ClearSpeak style for automated speaking of algebra. *ETS Research Report Series*, *2016*(2), 1–43.

gh. (2004–2006). *MathSpeak core specification grammar rules*. Retrieved from http://www.gh-mathspeak.com/examples/grammar-rules

Kapperman, G., & Sticken, J. (2003). A case for increased training in the Nemeth code of braille mathematics for teachers of students who are visually impaired. *Journal of Visual Impairment & Blindness*, *97*, 110–112.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, *63*, 81–97.

Mullen, E. A. (1990). Decreased braille literacy: A symptom of a system in need of reassessment. *RE:view, 22*(3), 164–169.

National Federation of the Blind Jernigan Institute. (2009). The *braille literacy crisis in America: Facing the truth, reversing the trend, empowering the blind.* Retrieved from https://nfb.org/images/nfb/documents/pdf/braille_literacy_report_web.pdf

Nemeth, A. (2013). *MathSpeak* (a talk on verbalizing math by Dr. Abraham Nemeth, creator of the Nemeth math braille code). Retrieved from http://accessinghigherground.org/handouts2013/HTCTU%20Alt%20Format%20Manuals/Math%20Accommodations/07%20MATHSPEAK.pdf

Papadopoulos, K., & Koutsoklenis, A. (2009). Reading media used by higher-education students and graduates with visual impairments in Greece. *Journal of Visual Impairment & Blindness*, *103*(11), 772–777.

Rosenblum, L. P., & Amato, S. (2004). Preparation in and use of the Nemeth braille code for mathematics by teachers of students with visual impairments. *Journal of Visual Impairment & Blindness, 98*(8), 484–498.
Schroeder, F. (1989). Literacy: The key to opportunity. *Journal of Visual Impairment & Blindness*, *83*(6), 290–293.

Ann P. Gulley, B.S., *math and sciences student services coordinator, doctoral student, Learning Center, Auburn University at Montgomery, P.O. Box 244023, Montgomery, AL 36124; e-mail: agulley@aum.edu.* ***Luke A. Smith, Ph.D.,*** *assistant professor, College of Education, Auburn University at Montgomery, 7400 East Drive, Montgomery, AL 36117; e-mail: lsmith4@aum.edu.* ***Jordan A. Price, B.S.,*** *graduate student, Samuel Ginn College of Engineering, Auburn University, 1301 Shelby Center, Auburn, AL 36849; e-mail: jprice15@aum.edu.* ***Logan C. Prickett,*** *undergraduate student, Learning Center, Auburn University at Montgomery; e-mail: lprickett@aum.edu.* ***Matthew F. Ragland, Ph.D.,*** *associate provost and professor, Auburn University at Montgomery; e-mail: mragland@aum.edu.*

Practice Report

A Comparative Analysis of Contracted Versus Alphabetical English Braille and Attitudes of English as a Foreign Language Learners: A Case Study of a Farsi-Speaking Visually Impaired Student

Mohsen Mobaraki, Saber Atash Nazarloo, and Elaheh Toosheh

Education is an absolute right of all human beings, especially in today's world, and there is a huge responsibility on those teachers who work with learners with disabilities. A very special kind of education is needed for people who are visually impaired, and braille is one method that is utilized by individuals to access educational materials.

There are two kinds of braille, alphabetic and contracted. In alphabetic braille, there are graphemes for each character of the English alphabet, which consumes a lot of space, resulting in books that are very long. Contracted braille, on the other hand, uses 189 contractions, many of which represent whole words with one grapheme (*the, can,* and *people,* for example), which has led to shorter publications. The creation and utilization of contracted braille has had an effect on the education of visually impaired students. As a result, there has been extensive research in this area.

Lee and Hock (2014) investigated the effect of using contracted braille on the spelling proficiency of visually impaired students in a bilingual setting. Through doing quasi-experimental research, they concluded that the most frequent errors were grapheme substitution and direct translation of syllables of the first language. Troughton (1992) stated that the students who learn contracted braille later in school show a better performance in reading skills than those who learned contractions earlier. Wall Emerson, Holbrook, and D'Andrea (2009), on the other hand, concluded that if visually impaired students learn contractions at an earlier time, they will show better performance in their reading skills.

In the educational system of Iran, in which English is a foreign language, the duration of primary school is six years. After that, these English as a foreign language students enter high school and start to learn English during an additional six-year period. In years seven and eight, English textbooks are embossed in alphabetic English braille; but, for the next four years, these books are prepared in the contracted form. Visually impaired students in Iran use alphabetic braille to read and write in their own mother tongue (Farsi). Contracted braille is, therefore, a foreign concept to these students.

In Iran, the exposure of students who are learning English to contracted braille in high school has affected the reading skills of these students. The present researchers interviewed four teachers regarding the teaching of