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Exploring four year old Indigenous students' ability to pattern

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The gap between young Indigenous and non-Indigenous students' capability within mathematics is widely acknowledged. This gap is conjectured to exist at all levels of schooling, including pre-school, and widens as students mature. Most of these findings are based on research relating to students' understanding of number and space. Little is known about what knowledge Indigenous students bring to early years' settings with regard to patterning, an area that is widely acknowledged as fundamental to the development of concepts, process and knowledge of mathematics. One on one interviews were conducted with 35 Indigenous students (average age 4 years and 4 months) as they entered kindergarten. The results indicate that these students enter these settings with some intuitive understanding of repeating pattern, and that this knowledge is at odds with the hypothesised learning trajectory (Sarama and Clements, 2009) for repeating patterns. The results also begin to delineate the types of activities within these settings that have a significant impact on young Indigenous students' ability to pattern.

With regard to young Australian Indigenous students, research indicates that little is known about their numeracy capabilities as they enter kindergarten/Pre-prep settings. The results of a large Australian study, *Project Good Start* conducted with 506 students with an average age of 4.7 years, indicated that students from low SES backgrounds perform at a lower level than middle class students on school entry (Thomson, Rowe, Underwood, and Peck, 2005). This study also evidenced the wide gap between the numeracy achievement of non-Indigenous students and Australian Indigenous students as they entered pre-school settings. It is acknowledged in the study that when considering this finding it must be remembered there were only 48 Indigenous students in a sample of 1615 students, and these students were scattered across a range of sites, and the majority of these students were situated in multi-cultural centres in metropolitan areas. These findings were also based on the use of 'I Can do Maths Level A' (Doig and de Lemos, 2000), a test developed for 4 to 8 year olds focussing on the content areas of number, space and measurement. There were no questions directed to ascertaining young students' ability to pattern, an area of mathematics that is recognised as fundamental to the development of concepts, process and knowledge of mathematics (Cooper and Warren, 2008; Mulligan and Mitchelmore, 2009; Papic, 2007).

The power of mathematics lies in the relations and transformations which give rise to patterns and generalisations (Warren, 2005). The ability to discern a pattern is the precursor to the ability to generalise and abstract (Threlfall, 1999). Abstracting patterns is the basis of structural knowledge, the goal of mathematics learning in the research literature (Jonassen, Beissner and Yacci, 1993; Sfard, 1991). While patterning in its broadest terms impregnates a variety of areas including dance, design, music, and physical therapy, the common thread is the notion of repetition of a particular set of movements, sounds, or elements. It is the recognition of this repetition that lies at the heart of mathematics. An understanding of the repetition explicitly underpins the development of

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repeated addition, multiplicative thinking and concepts related to measurement. Multiplication requires the repetition of the same numerical unit (repeated addition) and measurement initially entails the iteration of the same nonstandard unit and later the same standard unit. Research has also evidenced that the exploration of repeating patterns with 4 year old students positively impacts on their understanding of mathematical concepts two years later (Papic, 2007). Thus, it is crucial to begin the exploration of repeating patterns within the early years setting as a foundation for the development of number and its operations. This paper explores young students' understanding of repeating patterns as they enter these settings and the types of activities that assist in its development.

Repeating patterns are patterns that have a discernable unit that repeats over and over again, a cyclical structure generated by the repeated application of a smaller proportion of the pattern. As such there is no beginning and no end. Students explore simple repeating patterns using shapes, colours, movement, feel and sound. Typically young students are asked to copy and continue these patterns, and find missing elements; a focus on single variational thinking where the variation occurs within the pattern itself. They are rarely challenged to identify the repeat; the fundamental activity that reveals the pattern's structure.

Research indicates that the pedagogy of teaching patterns in the early years of schooling is a concern (Liljedahl, 2004; Warren, and Cooper, 2007). The thinking that is commonly engendered in patterning activity tends to involve a focus on the patterns themselves with little consideration given to their structure or the mathematics that is embedded in the pattern (Liljedahl, 2004). In fact, many students have difficulty in identifying the repeating part with many teachers failing to understand the centrality of this component to mathematical learning. As a consequence pattern activities tend to become a cyclical chant (Watson, 2000).

Theoretical perspectives

Three perspectives underpinned this research, namely, Vygotskian theories of learning, the learning trajectory, and pedagogy for Indigenous student learning. Vygotsky suggested that students' learning is twofold, first it is an inter-psychological interaction where shared activities are co-constructed and second it has an intra-psychological aspect where the knowledge is internalised to enhance cognitive development (Vygotsky, 1978). This type of learning has also been viewed as an apprenticeship model where novices become experts through participation (Van Oers, 1996). In early childhood settings, Vygotsky's theory signifies the importance of play and the role of the adult in developing conceptual knowledge through interaction (Fleer, 2010). The child may initially imitate the concept, however through the internalisation of the concept the learner later engages in communication of justification and explanation which enhances their learning (Vygotsky, 1994). The dialectical nature of Vygotsky's theory suggests that when the child experiences the everyday concept this should then be linked to the scientific concept (Fleer, 2010). Through the engagement of play the students learn to communicate the mathematical language and then in turn transform their thinking of the mathematical concept (Steele, 2001). For instance, when students are involved in the play both physically and mentally they make connections from the ordinary language to the mathematical language and through this they construct meaning, which is crucial to mathematics (Pirie, 1998). This theory drove the style of intervention that occurred in the participating early childhood settings.

An ontology of students's learning that is currently dominating the early years' literature is the learning trajectory. From the perspective, learning consists of a series of 'natural' developmental progressions identified in empirically-based models of students' thinking and learning (Clements, 2007). In conjunction with viewing learning as a progression through development hierarchical levels, the learning trajectory sees teaching as the implementation of 'a set of instructional tasks

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designed to engender these mental processes' (Clements and Sarama, 2004, p.83). From this perspective, the act of teaching is secondary to the act of learning. The resultant curriculum consists of diagnostic tests, learning hierarchies and purposely-selected instructional tasks. Fundamental to this perspective is (a) the existence of a large repertoire of empirically-based research evidencing the development of particular concepts, such as number, number sense, and counting, and (b) conducting extensive field tests trialling various instructional tasks.

The learning trajectory is entrenched by the notion of hierarchic interactionalism (Sarama and Clements, 2009). This theory consists of three main tenets, namely, developmental progressions, domain specific progression, and hierarchic development. In summary these are: 'Most knowledge is acquired along developmental levels of thinking' (Sarama and Clements, 2009, p. 20). These developmental progressions seem to be more auspicious within particular mathematical domains or topics. Within this framework, while levels of thinking are often believed to be coherent and characterised by increased sophistication, the learning process is more often gradual and incremental. While the movement between levels can often range from slow to rapid, it is believed that a critical mass at one level must be constructed before movement to the subsequent level effectively occurs. The hypothesised learning trajectory for patterning in the early years is:

TABLE 1
HYPOTHESISED LEARNING TRAJECTORY FOR REPEATING PATTERNS

Age	Developmental progressions	Action with objects
3	**Pattern recogniser**	Has the capacity to recognise patterns, can operate on perceptual input and note regularities.
4	**Pattern fixer** – fills in the missing elements in an ababab pattern	Finds the missing element by continuing to produce the verbal sequence stored in the phonetic 'buffer' (or, visually through, the visuospatial sketch pad).
	Pattern duplicator AB – duplicates an ababab pattern	Can copy the pattern as long as the perceptual support is available for checking the duplication.
	Pattern extender AB – extends an ababab pattern	Can extend the pattern and has less need for constant perceptual support when doing so.
	Pattern duplicator – duplicates patterns without the need for model support	Duplicates longer patterns and patterns with more complex core units.

This hypothesised trajectory was based on previous research findings and pilot studies in four classrooms (Sarama and Clements, 2009). While the literature acknowledges the learning trajectory's limitations in terms of the types of patterning experiences it presents, little is known about its applicability to students from culturally diverse backgrounds such as young Australian Indigenous students. Little is also known about their capability as they begin to be exposed to patterning activities in this setting, particularly with regard to the support that assists them and the difficulties they encounter. This theory assisted in the sequencing of activities utilised in these settings.

Australian Indigenous students prefer a holistic approach to learning (Christie, 1994) and have little patience with an atomised curriculum (Barnes, 2000). In addition, it has been suggested that

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best practice for Australian Indigenous students to learn mathematics are teaching strategies that are kinaesthetic activity based to cater for individual needs, supported by direct instruction, modelled by peers and teachers, with a variety of representations, that has opportunities for discussion and listening moments for all members, and giving students the opportunity to reflect (Frigo, 1999; Warren and de Vries, 2009). Additionally, mathematical language can be a barrier for Australian Indigenous students and thus Indigenous teacher aides have an important role to play within these classroom settings (Warren, 2004). These theories were the basis for the selection and implementation of activities utilised in the settings.

This research project, undertaken by three researchers in two Indigenous kindergarten (Warren, deVries, and Thomas, 2009), looked at the mathematical experiences of Australian Indigenous kindergarten/Pre-prep students (average age 4 years and 4 months). The overarching object was to develop culturally appropriate best practice/research grounded teacher materials to support the transition of Australian Indigenous students from home to school. The specific aims of this paper are to (a) explore the understanding of repeating patterns that young Australian Indigenous students bring to the kindergarten/Pre-prep setting, (b) identify the types of learning activities that supports effective engagement with patterning activities, (c) gauge the appropriateness of Clement's and Sarama's purported learning trajectory for these students.

METHOD

Participants

The two Indigenous kindergartens were situated in a low socio-economic area of a large metropolitan city. The settings catered for young Australian Indigenous students, and were recognised as centres where teachers were willing to engage in professional dialogue with regard to student learning. The students came from a range of different ethnicities with the most predominant being Indigenous Australian (76.3%). The remaining students were predominantly Vietnamese. The first section of this paper reports on the Indigenous students from these centres (n=35). Thus the sample comprised 35 students, 18 male and 17 female, with an average age of 4 years and 4 months. One centre had two Pre-prep classes while the other had one. Prior to the administration of the test, the participants had not been exposed to exploring any types of patterns at these kindergarten centres. Not all of these students were present for the intervention or for the post test. The second section of this paper reports on the intervention sample, namely, 22 Australian Indigenous students (11 males and 11 females) from one centre. In Queensland, in the year prior to Pre-prep, students are either at home or in day care, or experiencing a mixture of both. While we cannot categorically proclaim that they had never experienced any type of patterning activities before the administration of the pre test, we can conjecture that it is highly unlikely. This conjecture was also supported by the parents who participated in the project, who shared in a post interview that patterning was an area that their students had had little experience prior to their participation in this project.

The intervention

The project began with a professional learning day. All teachers and assistants from one centre and the teacher from the other centre attended the day. The focus of the day was to share with the teachers and assistants successful learning activities from our previous work in Indigenous prep settings. These activities had been adapted to suit younger students. Two young students (aged 3 and 5) also attended the professional development day, grandchildren of one of the teachers. This added a 'real' dimension to the day as the ideas were trialled using these students as models. This then enabled the teachers and assistants to see the activities in action and how to implement these

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activities in their setting using a socio-cultural perspective on learning. Each centre was supplied with a kit of materials developed by the project. The conversations throughout the day were interspersed with dialogue with an early childhood expert, a mathematics practitioner, and a mathematics education academic. This allowed teachers to engage with the ideas through a variety of different perspectives. One dimension of the day involved the exploration of repeating patterns.

Intervention began after the administration of the pre-test. Following the professional development day researchers visited the centres once a week for 18 weeks. With regards to patterning, the learning episodes focused on familiarising students with repeating patterns. The activities chosen for the intervention consisted of materials that allowed students to focus on the key ideas with regard to repeating patterns. For example, students worked in pairs using a set of colourful keys on a circular key ring to create patterns followed by verbally reading the pattern to their partner (e.g., red, blue, red, blue). The benefit of this activity was that threading keys on a key ring represented a pattern without a beginning or an end. The ring could also be opened to demonstrate the pattern in its linear form. Repeating patterns were also investigated by using grids. Students explored continuing repeating patterns on the grid by following a wrapping process (see figure 1a). In this instance when students reached the end of the row they placed the next element in the pattern at the beginning of the next row, thus continually continuing the pattern from left to right. They also explored continuing the pattern using a zig zag process, similar to the construction of a snakes and ladder board, where they continued the pattern from left to right and then right to left, placing the next element in the pattern directly below the last element placed in the preceding row. The use of grids enabled the students to identify different representations of patterns (e.g., left to right, top to bottom and diagonal) and established that patterns can be continued in both directions. Figure 1 presents three of the activities utilised in the intervention.

Figure 1: Activities used in the intervention

The students also focussed on identifying the repeating elements in the pattern. This was achieved through activities in which the students placed each repeating component on plates that could be physically separated, (e.g., a series of paper plates) or when reviewing the pattern, they covered each repeat with their hand or a cup. They then verbalised which part was repeating within the pattern (e.g., said ‘red, blue’ [pause] ‘red blue’ [pause] to the teacher). It is conjectured that identifying the repeating element assists students to understand that structure of the pattern and determine the missing elements in subsequent patterns.

Finally, it was important for the students to transfer these activities across different mediums to ensure that the understanding of patterning was being established. Transferring the pattern was achieved by exploring different mediums within a play context. For example, the students participated in kinaesthetic activities, such as, jump, clap, jump clap accompanied by replicating the pattern with a variety of concrete materials.

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During the project, University personnel visited the centres and worked with the teachers, trialling ideas. The aim of the trials was four fold. First, we were interested in ascertaining the appropriateness of the activities for these settings. Second, how young students engaged with these activities in these settings was important. Third, we were also interested in understanding what adaptations needed to be made to the activities and how they were delivered. Finally, we also wanted to explore the interplay between mathematics and play. This last dimension was particularly important, as it is our experience that many early years' teachers believe that mathematics is a very formal process, and thus unsuitable to be incorporated into a play context.

Data gathering techniques and procedures

At the commencement of the project all students participated in a one on one interview conducted by the researchers. The interview was designed by the researchers and focused on the concept of repeating patterns. The aim of the interview was to identify the preconceived knowledge about repeating patterns that students brought to the kindergarten/Pre-prep setting. The interview consisted of three tasks. Each pattern focused on ascertaining young students' ability to copy, continue, and complete repeating patterns. In Task 1 students were asked to copy, continue and complete an abababab pattern, and in Task 2 they were asked to copy, continue and complete an aabbaabbaabb pattern. While Task 3 they were given coloured paddle pop sticks and asked to create their own repeating pattern. Each part of the tasks was accompanied by a card with a picture of the pattern. Students were also supplied with the appropriate concrete materials needed for them to successfully complete the tasks. For example, for part 1 of Task 1 the students were instructed to copy the pattern using concrete lighthouses and ladybugs, and to place their pattern in the rectangle drawn below the picture of the pattern. Table 2 presents the cards used for each task together with the questions asked.

TABLE 2
CARDS USED FOR EACH TASK TOGETHER WITH THE INTERVIEW QUESTIONS

Task	Cards	Interview questions
Task 1 – part 1		Copy the pattern. Using your shapes, make the same pattern in the box.
Task 1 –part 2		Continue the pattern along the line. (Gesture along the line)
Task 1 – part 3		Complete the pattern. What is missing?
Task 2 – part 1		Copy the pattern. Using your shapes, make the same pattern in the box.
Task 2 – part 2		Continue the pattern along the line. (Gesture along the line)
Task 2 – part 3		Complete the pattern. What is missing?
Task 3		Using the paddle pop sticks create your own repeating pattern.

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The interview was approximately 15 minutes in length. The terms *copy*, *continue*, *complete*, and *create* were not explained to the students as it was necessary to identify if they understood the language used for describing patterns as well as ascertaining their ability to complete the tasks. Data was recorded on an answer sheet by the interviewer. Students' responses were marked as either correct or incorrect. Task 1 Part 2 and Task 2 Part 2 were allocated a possible score of 2, 1 for continuing the pattern in either direction or 2 for continuing the pattern in both directions. All other tasks were allocated a score of 1. Students used a variety of different strategies as they completed the tasks. The following section describes these strategies.

Copying the pattern

Three different strategies were used when copying the pattern. One group placed each shape on top of the pictures on the card and then pulled down each shape one by one so that they were in the box (Cover and move). Another group first copied all the shapes that were the same and then copied the remaining shapes (Copy part part). The third group copied the shapes in order from left to right (Copy left to right). Figure 2 illustrates each strategy.

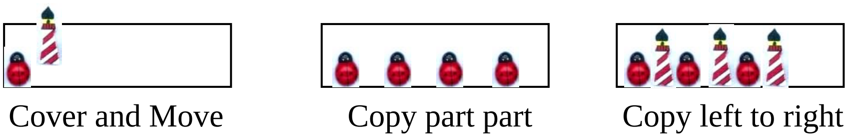

Figure 2: Copying strategies

Continuing the pattern

When continuing the pattern, there were also three different ways in which the students continued the patterns. They either continued the pattern to the right only, continued the pattern to the left only or continued the pattern both ways. When continuing to the left, some students placed a shape to the far left and then built the pattern to the existing pattern.

All of these strategies were recorded and coded. The data was entered into SPSS for analysis.

RESULTS OF PRE TEST

All 35 students completed the patterning interview. The maximum score for the patterning interview was nine. The mean score was 2.66 with a standard deviation of 1.8. The highest score obtained was 8 with the lowest being 0. Table 3 presents the frequency of students who successfully answered each task. It should be noted that for the continuing pattern parts of the tests responses were considered correct if the child continued the pattern in either direction or both directions.

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TABLE 3
FREQUENCY OF CHILDREN WHO SUCCESSFULLY COMPLETED EACH TASK (N=35)

Task	Pattern	Part	Frequency
Task 1	abababab	Copy	26
		Continue	8
		Complete	15
Task 2	aabbaabb	Copy	26
		Continue	4
		Complete	4
Task 3		Create	6

For Task 3, creating a repeating pattern using coloured paddle pop sticks, all 6 students created abababab patterns using two different strategies. The first strategy relied on colour to differentiate the elements of the pattern, for example, blue red blue red blue red. The second utilised the orientation of the sticks to differentiate the elements, for example, using the same coloured paddle pop sticks but placing one element on the vertical and another on the oblique.

Students were then categorised according to the strategy they used to copy the pattern. Table 4 summarises the frequency of students who were allocated to each category and who successfully copied the pattern.

TABLE 4
FREQUENCY OF CHILDREN WHO USED DIFFERENT COPY STRATEGIES AND WHO SUCCESSFULLY COPIED THE PATTERN (N=35)

Task	Pattern	Part	Strategy	Used the strategy	Successfully copied the pattern
Task 1	abababab	Copy	Cover and move	3	2
			Copy part part	11	4
			Copy left to right	21	20
			TOTAL	**35**	**26**
Task 2	aabbaabb	Copy	Copy and move	5	5
			Copy part part	17	8
			Copy left to right	13	13
			TOTAL	**35**	**26**

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As indicated in the above table students who used the copy part part strategy were less successful at copying the pattern than those who used the other two strategies. The next section discusses how students who used different strategies when copying the pattern performed on the different parts of each task. After copying the pattern, they were asked to continue the same pattern, and then complete the pattern (see Table 2). Table 5 presents the results of this section of the data analysis.

TABLE 5
FREQUENCY OF SUCCESSFUL RESPONSES FOR THE CONTINUE AND COMPLETE PARTS OF THE TASKS (N=35)

ababab Pattern		
Strategy	Continue	Complete
Copy and move (3)	1	0
Copy part part (11)	1	2
Copy left to right (21)	5	13
aabbaabbaabb Pattern		
Strategy	Continue	Complete
Copy and move (5)	0	0
Copy part part (17)	1	1
Copy left to right (13)	2	3

The results indicate that students who used the strategy of copying from left to right were more successful at both continuing the pattern and completing the pattern. In addition, the 6 students who successfully created a pattern (see Table 2), 4 of these used the strategy copy left to right when copying the patterns.

In summary, students found it easier to copy patterns than they did to continue and complete patterns. They also found it easier to complete an ababab pattern than they did to continue the pattern. Duplicating a more complex pattern (aabbaabb) proved less difficult than continuing and completing a less complex pattern. Finally, students who used the strategy of copying a pattern from left to right were more successful than other students on all parts of the test, including their ability to create their own repeating pattern.

RESULTS OF THE POST TEST

Post testing occurred five months after the pre test. Only 22 students were present. Thus the sample reported in the next section focuses on the results of these 22 students. The maximum possible score for the pre and post test was 9. For the pre test the mean score was 2.73 with a standard deviation of 1.6. The range of the pre test score was 0–7. For the post test the mean score was 4.59 with a standard deviation of 2.3. The range was 1–8. The results of paired t tests indicated that there was significant improvement in the students’ ability to pattern after the intervention

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(t_{21}= 4.2, p=0.0). Table 6 presents the frequency of students who successfully answered each task. It should be noted that for the continuing pattern parts of the test students were considered correct if they continued the pattern in either direction or both directions.

TABLE 6
FREQUENCY OF INDIGENOUS CHILDREN (N=22) WHO SUCCESSFULLY COMPLETED EACH TASK IN THE PRE TEST AND POST TEST

Task	Pattern	Part	Correct pre test	Correct post test
Task 1	abababab	Copy	18	22
		Continue	4	15
		Complete	10	10
Task 2	aabbaabb	Copy	18	21
		Continue	2	8
		Complete	3	4
Task 3		Create	4	8

The only component in which students did not make a significant improvement was completing the pattern. Table 7 summarises the frequency of students who used different strategies when copying the pattern and successfully copied it.

TABLE 7
FREQUENCY OF CHILDREN WHO USED DIFFERENT COPYING STRATEGIES AND WHO SUCCESSFULLY COPIED THE PATTERN (N=22)

				Pre test		Post test	
Task	Pattern	Part	Strategy	Used the strategy	Successfully copied the pattern	Used the strategy	Successfully copied the pattern
1	ababab	Copy	Cover and move	0	0	7	7
			Copy part part	6	3	3	3
			Copy left to right	16	14	12	12
			Total	**22**	**17**	**22**	**22**
2	aabbaabb	Copy	Copy and move	4	4	9	8
			Copy part part	11	6	1	1
			Copy left to right	7	7	12	12
			Total	**22**	**17**	**22**	**21**

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Interestingly, for the abababa pattern the majority of students used the strategy of copying from left to right but for the aabbaabbaabb the frequency of students utilising the copy part part strategy increased. Table 8 summarises the frequency of students who utilised the three different copying strategies and their success rate for the other parts of the test.

TABLE 8
FREQUENCY OF CHILDREN WHO USED THE THREE DIFFERENT COPYING STRATEGIES AND THEIR SUCCESS RATE ON THE PARTS OF PRE AND POST TEST (N=22)

Pattern	Pretest			Post test		
abababab	Strategy	Continue	Complete	Strategy	Continue	Complete
	Copy and move (0)	0	0	Copy and move (7)	4	2
	Copy part part (6)	0	0	Copy part part (3)	1	1
	Copy left to right (16)	4	10	Copy left to right (12)	8	7
aabbaabb	Strategy	Continue	Complete	Strategy	Continue	Complete
	Copy and move (4)	0	0	Copy and move (9)	0	1
	Copy part part (11)	1	1	Copy part part (1)	1	1
	Copy left to right (7)	1	2	Copy left to right (12)	7	2

As can be seen from the above table, the most successful students used the strategies copy and move or copy left to right. Those who utilised the copy part part strategy were generally unsuccessful on all parts of the test. It should also be noted that there was also a significant increase in the number of students who could continue the patterns to the left and the right. The increase for the abababa pattern was from one child to seven students in the post test, and for the aabbaabbaabb pattern the increase was from zero to six students. All of these students used the third strategy, namely, copy left to right. The same trend occurred when students were asked to create their own pattern. For the pre test three out of four of the students who created a repeating pattern used the copy left to right strategy when copying the pattern. For the post test there were six out of eight students who used this strategy in the post test.

DISCUSSION AND CONCLUSIONS

This paper begins to document the thinking about repeating patterns that these young Indigenous students brought to the kindergarten/Pre-prep setting. Given the relatively small sample of the Indigenous students it is recognised that the generalisability of these results is tentative. Consequently, the researchers recognise that these results might be different if the study was conducted in a rural or remote Indigenous kindergarten settings. Despite this limitation, the results in this study provide further insights into the conjectured learning trajectory (Clements, 2007). Table 9 represents a conjectured learning trajectory for young Australian Indigenous students who participated in this research.

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TABLE 9
CONJECTURED LEARNING TRAJECTORY FOR REPEATING PATTERNS FOR YOUNG INDIGENOUS CHILDREN

Age	Developmental progressions	Action with objects
3	**Pattern Duplicator AB**	Can copy the pattern with visual support
3	**Pattern Duplicator AABB**	Duplicates longer patterns with more complex units
4	**Pattern continue AB**	Extends the pattern and moves away from visual support to holding the pattern in the mind
4	**Pattern Fixer AB**	Completes missing elements with visual support
4	**Pattern continue AABB**	Extends the pattern moving away from the visual support and can recognise the repeating element
4	**Create ABAB Pattern**	Creates a pattern
4	**Pattern fixer AABB**	Completes missing element and can continues to read the pattern to ensure that the fixer is correct

This learning trajectory differs from the learning trajectory (see Table 1) presented earlier in the paper. The main difference is that students appear to experience greater difficulty with completing patterns than continuing patterns. The strategy used for copying patterns was an indicator of their ability to understand the structure of the pattern and a predictor for their success in continuing and creating repeating patterns. The use of the copying left to right strategy also seemed to diminish as the pattern became more complex. We conjectured that this was a result of having difficulty in identifying the pattern structure of these more complex patterns. There were four main conclusions drawn from this study.

First, as evidenced by the results (see Table 1) most of the students had already begun their 'patterning journey' as they entered kindergarten. The results also indicated that the exact order presented for the learning trajectory is somewhat at odds with what these Australian Indigenous students could do. For example, many of these students found duplicating a pattern easier than 'fixing' the pattern. They also found duplicating a more complex pattern easier than 'fixing' and 'extending' easier patterns. Does this mean that the learning trajectory for patterning is different for Australian Indigenous students or should we as researchers be more flexible in the 'hierarchical steps' that we propose students pass through as they begin to learn new concepts?

The literature presents a second perspective with regard to the ontology of student learning: the learning-teaching trajectory (e.g., Van den Heuvel-Panhuizen, 2001). While both perspectives have many commonalities, the main differences lie in their emphasis on the act of teaching in the learning process, and the prescriptiveness of the resultant curriculum. In contrast to the learning trajectory, the learning-teaching trajectory has three interwoven meanings, each of equal importance. These are; (a) a learning trajectory that gives an overview of the learning process of students; (b) a teaching trajectory that describes how teaching can most effectively link up with and stimulate the learning process; and finally, (c) a subject matter outline, indicating which core elements of the mathematical curriculum should be taught (Van den Heuvel-Panhuizen, 2001). It provides a 'mental education map' which can help teachers make didactical decisions as they interact with students' learning and instructional tasks. It allows for a degree of flexibility in the learning sequence, and acknowledges that quality teaching is a key dimension of effective learning. This ontology may provide a theoretical perspective that better aligns with our project.

Second, the findings indicate that how a student copies a pattern provides insights into their ability to see the structure of the pattern as a whole, rather than seeing the pattern consisting of two parts, the lighthouses and the ladybirds. The most successful copying strategy, copying the

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elements in succession from left to right was also accompanied by a verbal or nonverbal 'chant', ladybird lighthouse ladybird lighthouse ladybird lighthouse (see Table 2), reading the grain of the pattern (Warren, 2005). We are suggesting that these actions in unison assisted students to continue the pattern. They were beginning to 'see' the structure of the pattern. But seeing structure entails more than this, as many of these students had difficulties continuing the pattern to the left. Continuing to the left required them to start with a lighthouse instead of a ladybird. A common mistake that many students made was to duplicate the ladybirds as they continued the pattern to the left, a strategy we termed as mirroring the pattern. We hypothesise that 'seeing structure' of repeating patterns requires the identification of two components, identifying the rhythm of the pattern, and breaking this rhythm into the repeating component (Warren, 2005).

Third, young Australian Indigenous students do enter kindergarten/Pre-prep with some understanding of patterning. In fact our past research with prep aged students indicates that there is no significant difference between these students' ability to pattern as compared with non-Indigenous students (Warren and deVries, 2009). The results of this pilot study with three groups of students, namely, Indigenous students (n = 14, average age = 4 years 11 months), Other Culture students (n = 11, average age = 4 years 11 months) and Caucasian students (n = 23, average age = 5 years) also indicated that the main significant difference between these three groups of students as they began school was their understanding of number, and not patterning nor mathematical language, and with appropriate teaching actions it was possible to close this gap. It must be remembered that statements such as 'low socio-economic status students show less proficient mathematical performance than do their middle-SES peers, particularly when meta-cognition is required, but do not lack basic concepts and skills' (Ginsburg, Lee and Boyd, 2008, p5.) are based on the tests results after these students have participated in school for some years. Ginsburg, Lee and Boyd (2008) point out that these low SES students are also exposed to a pervasive risk factor as they proceed through school, namely, low school quality. Many teachers in these schools fail to provide opportunities for mathematical learning (Ginsburg, Lee and Boyd, 2008). Thus an important finding of this research project is that this is a false assumption. These young Australian Indigenous students did begin school with an intuitive understanding of patterning, and this understanding substantively 'grew' with exposure to engaging and mathematically sound learning experiences.

Fourth, the types of activities students experience in the kindergarten setting impacts on their ability to pattern. This supports the findings of Papic (2007). The materials chosen and the activities presented focused on the core learning associated with patterning, namely, repeating patterns continue in both directions and are never ending (e.g., the use of the keys and key rings). The structure of repeating patterns is that they consist of repeating components (e.g., the use of plates to place repeats on). Transferring patterns to other mediums increases awareness of pattern structure (e.g., kinaesthetic movement and singing pattern songs).

This paper begins to share some of our results from our project in two Indigenous kindergarten settings. Briefly, the students' ability to pattern significantly improved over the year and the early childhood teachers' understanding of how they construct themselves as educators engaged in both a play-based pedagogy and mathematics as a curriculum discipline also changed (Thomas, Warren and deVries, 2010). This change was not only assisted by the types of activities utilised in this project but also by an ongoing discussion about theories of learning in these settings, and particularly with regard to a Piagetian approach compared to a Vygotskian (Fleer, 2010).

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