

**PRESERVICE AND INSERVICE MATHEMATICS TEACHERS'  
PERSPECTIVES OF  
HIGH-QUALITY MATHEMATICS INSTRUCTION**

Sarah Clooney  
The College of New Jersey  
Ewing, New Jersey, USA  
cloones1@tcnj.edu

Robert F. Cunningham  
The College of New Jersey  
Ewing, New Jersey, USA  
bobc@tcnj.edu

***Abstract***

*This study examined the perspectives of what high-quality mathematics instruction looks like. Written responses from preservice (n=20) and inservice (n=16) mathematics teachers were collected and categorized according to the Ten Principles developed by Anthony & Walshaw (2009). The responses of preservice teachers more often than inservice teachers included the categories of Building on Student Thinking and Mathematical Communication, while the responses of inservice teachers more often than preservice teachers included the categories of Arrangements for Learning and Making Connections. Both preservice and inservice teachers least often included the category of Mathematical Language in their responses.*

**Introduction**

Both nationally and internationally teacher education is undergoing close scrutiny and over the last three decades there have been many moves to reform mathematics education. Reform documents such as *The Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), *Principles and Standards for School Mathematics* (NCTM, 2000) and the *Common Core State Standards* (NGA Center and CCSSO, 2011) describe the best practices of high-quality mathematics instruction. Many teacher education programs have integrated research-based reforms suggested by these documents into their curriculums. Preservice and inservice teachers may hold different perspectives of what high-quality mathematics instruction might look like, but teachers need to align their perspectives with research-based reforms so that they might become effective teachers able to enhance their students' mathematical proficiency (Anthony & Walshaw, 2009; Muntner, 2014).

Preservice teachers bring perspectives of, and beliefs about, mathematics teaching from their own schooling (Brown & Borko, 1992; Marks, 2007; Sherrf & Singer, 2012) and are presented with opportunities to refine their perspectives as they progress through teacher preparation programs that address reforms (Cooney, Shealy & Arvold, 1998). Many inservice teachers have completed similar programs and have had additional opportunities to refine their perspectives with classroom experience along with textbooks, curriculum documents, and professional development activities that address reforms. Examining the differences in perspectives held by preservice and inservice teachers might reveal which perspectives of high-quality mathematics instruction need further support. Of particular interest in this study is the perspectives held by preservice teachers near the end of their undergraduate program that promoted research-based reforms and those held by experienced inservice teachers who had previously completed the same program.

A set of ten research-based principles developed by Anthony & Walshaw (2009) will be used to categorize and examine the differences in perspectives of high-quality mathematics instruction held by preservice and inservice teachers. Those held by preservice teachers are important for teacher educators who are interested in preparing them to become effective instructors. Also, the perspectives held by inservice teachers might be used to tailor professional development activities designed to improve instruction. Both preservice and inservice teachers must understand what high-quality mathematics instruction looks like if they are to implement it.

### **Background**

In this study *Perspective* will refer to the mental process of recognizing or the way you understand something. This conforms to what participants in this study will be asked to do; recognize what high-quality mathematics instruction looks like. Because a *Perspective* is not a conviction such as a belief, it may be more susceptible to change over time (Hasan, 2016) and might provide a valuable prescription for support. Ultimately, teachers must recognize how high-quality mathematics instruction is defined by reform documents so that they can positively affect student proficiency.

Anthony & Walshaw (2009) developed a set of principles that overlap recommendations made by reform documents and are based on research syntheses and international studies that facilitate learning for diverse learners. The ten principles

“... underpin the kinds of pedagogical approaches found to develop mathematical capability and disposition within an effective learning community.”

The principles serve as the defining characteristics of high-quality instruction for this study. These principles will be used to categorize the perceptions held by preservice and inservice teachers so that they might be compared. The principles form a complex web of factors that affect student learning and are listed below in Figure 1.

**Figure 1.**  
Ten Principles

1. An Ethic of Care: Caring Classroom Communities that Are Focused on Mathematics Goals Help Develop Students' Mathematical Identities and Proficiencies
2. Arranging for Learning: Effective Teachers Provide Students with Opportunities to Make Sense of Ideas both Independently and Collaboratively
3. Building on Student Thinking
4. Mathematical Communication: Effective Teachers Facilitate Classroom Dialogue that Is Focused Towards Mathematical Argumentation
5. Mathematical Language: The Use of Mathematical Language Is Shaped When the Teacher Models Appropriate Terms and Communicates Their Meaning in a Way that Students Understand
6. Assessment for Learning
7. Worthwhile Tasks: Effective Teachers Understand that Selected Tasks and Examples Influence How Students Come to View, Develop, Use, and Make Sense of Mathematics

8. Making Connections: Effective Teachers Support Students to Create Connections, between Different Ways of Solving Problems, between Mathematical Topics, and between Mathematics and Everyday Experiences

9. Tools and Representations: Effective Teachers Carefully Select Tools and Representations to Provide Support for Students' Thinking.

10. Teacher Learning and Knowledge

Teacher knowledge: Effective Teachers Develop and Use Sound Knowledge to Initiate Learning and to Act Responsively towards the Mathematical Needs of All Their Students

The transition from preservice to inservice is not always a smooth. Studies have focused on the beliefs, conceptions, and perspectives of preservice mathematics teachers (Brown & Borko, 1992; Bush, 1986; Cooney, Shealy, & Arvold, 1998; Sandholtz, 2011). They have found that teachers have been impacted in significant ways by their experiences with mathematics and schooling long before they enter teacher preparation programs. Kilic (2014) found that preservice teachers had difficulty in both identifying the source of students' misconceptions, and errors and generating effective ways different than telling the rules or procedures to eliminate such misconceptions. Other research points to the difficulty preservice teachers often have integrating what they learned in their teacher preparation program with what they experience in schools (Cavanagh & Prescott, 2007; Hine, 2015). And, even though preservice teachers are regularly exposed to progressive pedagogical approaches, they nevertheless often shift to more traditional teaching practices as they move into the practicum and begin their teaching career (Marks, 2007)

Sandholtz (2011) studied documents generated by 290 elementary and secondary preservice teachers over a five year period to determine their conceptions of effective and ineffective teaching practices. Approximately 69% of the preservice teachers pointed to level of engagement as to the reason a lesson was successful or not, but failed to make a link to student understanding. Many mentioned that the students showed enthusiasm or listened attentively, but their reasoning overlooked the fact that classroom activities can be engaging without leading to student learning. Ineffective instruction was often characterized by too much direct instruction and not enough hands-on activities which again suggests the priority of engagement rather than understanding. Sandholtz (2011) also found that only 5% of ineffective instruction was attributed to the teachers lacking content knowledge or using the wrong terminology. These casual assumptions that engagement was equal to effective instruction, and that their lack of content knowledge or use of improper terminology were infrequently cited as the cause of ineffective instruction may also be true of inservice teachers.

Other studies have examined inservice teachers in transition and tried to understand the perspectives they hold while they struggle to participate effectively in research-based reforms in mathematics education (Marks, 2007; Muntner, 2014; Simon, Tzur, Heinz, Kinzel, and Smith 2000). In particular, Muntner (2014) studied the trajectories of inservice teachers and found that they are influenced over time by their settings and classroom experiences. Marks (2007) reported that due to experience, inservice teachers having a better knowledge of student understanding of concepts as opposed to preservice teachers who have limited experience.

Around the world there is a need to better understand this complex progression that **involves** transforming theory to practice and transforming identity from student to teacher (Da Ponte &

Chapman, 2006). Our intent is to improve understanding of this progression by highlight the perspectives of high-quality mathematics instruction held by preservice and inservice teachers.

**Research Question.** Given a sample of preservice teachers from a research-based mathematics teacher education program and a sample of inservice teachers, who completed the program in prior years at the same institution, how do their perspectives of high-quality mathematics instruction differ?

### Method

**Setting.** The study took place at a highly selective institution situated in Northeastern United States with an enrollment of approximately 6,500. There are approximately 320 mathematics majors of which about 100 are secondary mathematics education majors. These majors develop significant content knowledge while completing 13 content courses in mathematics. Additionally, they complete two research-based methods courses and are placed in practicums at local schools in each of their last 3 years of their 4-year program.

**Participants.** The preservice teachers (n=20) were all secondary mathematics education majors and had completed their junior year. In addition, they had completed two undergraduate methods courses that focus on research-based reform documents. Both courses meet for 15 weeks with two 80-minutes classes per week.

The first course introduces national and state standards for K-12 mathematics and how to teach according to these standards. Topics include the use of manipulatives, technology in teaching mathematics and the investigation of standards-based curricula. These topics are mostly situated in the context of middle school mathematics and the course is writing intensive. The second course also emphasizes national and state standards for K-12 but focuses more on secondary school mathematics. The course includes a research project along with a unit plan consisting of ten lesson plans, and field observations. The field placement consists of classroom observations and a two-week long teaching experience in which the student assumes a class of the cooperating teacher and teaches the unit. Both of these courses are taken prior to the full-time 15-week senior-student teaching practicum.

The inservice teachers (n=16) were all secondary mathematics education majors who had graduated previously from the same institution. Each teacher had been teaching mathematics at the secondary level for between 3 and 22 years. As undergraduates each had taken the two methods courses during their junior year as described. While the two undergraduate methods courses changed over the years since the publication of *The Curriculum and Evaluation Standards for School Mathematics* (NCTM,1989), they have consistently championed research-based reforms in mathematics education and prepared teachers with strong pedagogic knowledge. Over the time span of 22 years only three different instructors taught the two courses and 19 years were taught by two of the instructors, one of which is an author.

**Procedure.** The preservice students were asked to answer the question: “What would it look like if you walked into a classroom with a high-quality mathematics instructor?” They were instructed to describe using sentences or short paragraphs their perspectives of what high-quality mathematics instruction looks like. The inservice teachers were presented with the same question during an annual symposium designed to bring former secondary education mathematics majors

back to the institution to network, and to provide professional development. Each of the inservice teachers were former graduates of the program.

The written responses from the preservice and inservice participants were matched with the ten principles of effective instruction developed by Anthony & Walshaw (2009). Two mathematics educators and a mathematician independently served as reviewers and assigned responses to the appropriate principle. A total of 99 preservice responses and 63 inservice responses were categorized and used in the study. Eliminated from analysis were 11 responses that were either unclear, lacked detail, or resulted in contradictory categorizations by the reviewers.

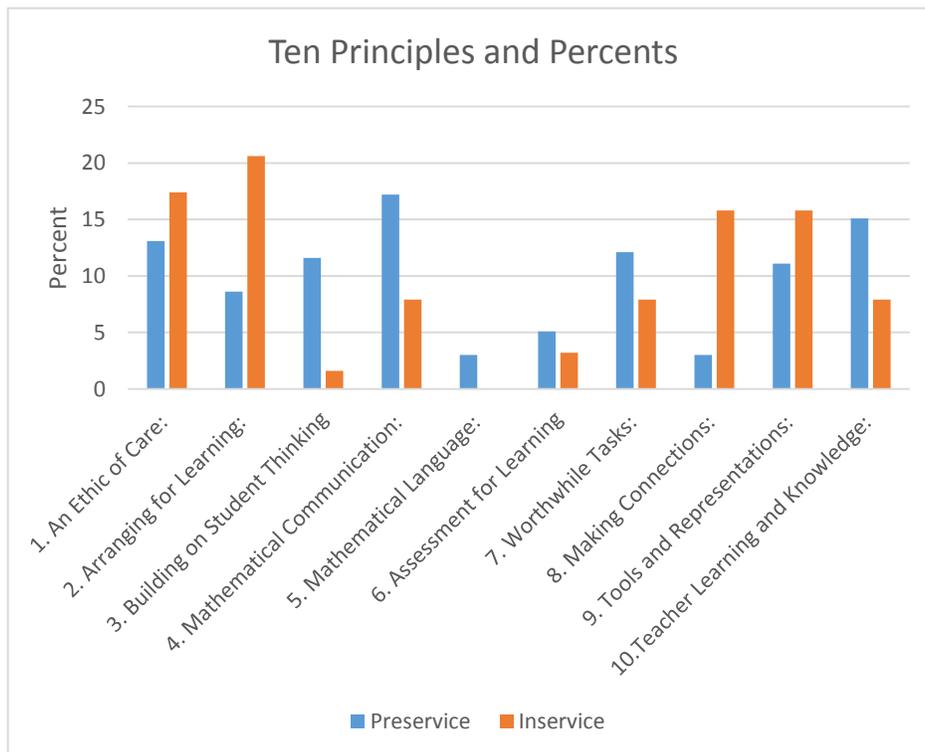
### Results

Preservice teachers' responses appeared in two categories at a higher percentage than those of inservice teachers. The two largest percentage differences were in the categories of *Building on Student Thinking* (11.6% vs. 1.6%) and *Mathematical Communication* (17.2% vs. 7.9%).

Inservice teachers' responses appeared in two categories at a higher percentage than those of preservice teachers. The two largest percentage differences were in the categories of *Arrangements for Learning* (20.6% vs. 8.6%) and *Making Connections* (15.8% vs. 3%).

In addition, few preservice teachers (3%) and no inservice teachers' responses appeared in the category of *Mathematical Language*. The results for each Principle appear below in Figure 2.

**Figure 2.**  
Percent of Preservice and Inservice Teacher Responses Corresponding to Principle



### Discussion

Preservice teachers' responses, more often than those of inservice teachers, corresponded to the categories of *Building on Student Thinking* and *Mathematical Communication*. The category

of *Building on Student Thinking* is one of the aspects of Pedagogic Content Knowledge (PCK) identified by An, Kulm, and Wu (2004). They noted that teachers need to relate students' prior knowledge with new knowledge through various representations, examples, and manipulatives. A higher number of responses in this category given by these preservice teachers might have been prompted by the use of a lesson plan template in their methods courses that includes a component on student prior knowledge. Bush (1986) points to the strong influence on preservice teachers exerted by undergraduate faculty and their consideration of lesson plans. In theory it is important for teachers to have an accurate knowledge of student proficiencies. But unlike inservice teachers, preservice teachers may have yet to realize the significant challenge it presents. Faculty who teach preservice teachers might caution their students about the difficulty associated with determining student proficiencies. A good example of such difficulties is presented by Schoenfeld, Smith & Arcavi (1993) where a single student is videotaped for several hours while learning about graphs. A set of evaluators are tasked with determining when the student actually learned a concept only to be met with many false assumptions and limited success. But, preservice teachers must realize that even for an experienced instructor determining student proficiencies for an entire class of students is at best an approximation. Inservice teachers may have mentioned this category less often because they consider anything other than an approximation out of reach even for the best teachers.

*Mathematical Communication* involves orchestrating classroom dialogue focused toward mathematical argumentation. The demands required by such orchestration goes well beyond traditional classroom discourse. The classroom dialogue used by most teachers is primarily in the form of Initiate-Respond-Evaluate (I-R-E), (Cirillo, Steele, & Herbel-Eisenmann, 2013). A high level of planning and skill is required to engage students in mathematical explanations and justification. This type of dialogue is referred to by Cirillo, Steele, & Herbel-Eisenmann (2013) as Teacher Discourse Moves (TDM). It can increase student learning, and support teachers in understanding and assessing student thinking. While TDM is again worthwhile in theory, it requires great skill and experience to implement. Teachers must accommodate multiple explanations and relate them to the underlying mathematics which magnifies the complexities of the teacher's decision making (Choppin, 2007). Cirillo, Steele, & Herbel-Eisenmann found that inservice teachers were often reluctant to adopt TDM (2013). There were many instances in which participants would say things such as, "My low-level kids could never do this." In addition many of the participants continued to express concern about the amount of time it took to have these kinds of discussions. Such illustrations may reveal why the inservice teachers mentioned *Mathematical Communication* less often in their responses.

While preservice teachers mention *Building on Student Thinking* and *Mathematical Communication* categories more often in their perspectives of high-quality instruction, they have yet to realize the level of experience and skill necessary to orchestrate them. In theory they are important components of high-quality instruction and student learning, yet they are difficult to practice even for experienced teachers.

Inservice teachers' responses, more often than those of preservice teachers, corresponded to the categories of *Arrangements for Learning* and *Making Connections*. *Arrangements for Learning* involves providing students with arrangements that are responsive to their needs. This category included employing opportunities for individual and small group work as well as whole class discussions. Cooperative learning was mentioned most often in this category with the reference to its usefulness for engaging students. Sandholtz (2011) notes that a majority of

teachers studied highlighted the importance of student engagement as the reason a lesson was successful but failed to link it with student understanding. Teachers may need to be reminded that care must be taken in equating engagement with learning.

The category of *Making Connections* includes connecting topics within mathematics, connecting topics to other subject areas, and connecting mathematics to real life. This last connection to real life was most often listed by inservice teachers with the intention of making mathematics more interesting and relevant to students (Anthony & Walshaw, 2009). Inservice teachers have a strong inclination to generate student enthusiasm in their classrooms. The teaching of disinterested secondary students devalues efforts made by teachers and can be extremely disheartening, so inservice teachers value strategies which might increase student interest. Many preservice teachers have yet to fully realize the challenge of teaching such students which may not mirror their own strong interest in mathematics as students.

Inservice teachers identified *Arrangements for Learning* and *Making Connections* more often in their perspectives of high-quality instruction because they offer opportunities to enhance student engagement and interest. Inservice teachers may have cited them as part of high-quality instruction as they fully appreciate the challenge of trying to engage all students. Nevertheless, teachers should be cautious not to settle for engagement at the expense of learning or even equate the two.

The category that received the fewest responses for both preservice and inservice teachers was the category of *Mathematical Language*. This category involves the teacher's use of appropriate terms and communicates their meaning in a way that students understand. This result may reflect the casual perspective held by both preservice and inservice teachers that they are using the correct vocabulary and terms as well as communicating effectively. This may also result from the false assumption that since they understand the meanings associated with vocabulary and terms that their students also understand. While preservice teachers usually receive some feedback from an experienced mathematics educator during their student-teaching practicum on their use of *Mathematical Language*, many times inservice teachers are evaluated infrequently and by administrators who do not have the expertise to evaluate their use of *Mathematics Language*. Marks (2007) points out once preservice teachers graduate and enter their own classroom they believe themselves to be successful. Similar to Sandholtz (2011) findings, *Mathematical Language* seems to be an under-evaluated and overlooked category, yet imperative for high-quality mathematics instruction

### **Conclusion**

Mathematics teacher education programs have integrated research-based reforms into their curriculums in the hope that their students would not only recognize high-quality instruction but also implement it. Teacher educators have integrated these reforms into their methods classes but they must realize that their students might value them only temporarily (Marks, 2007), and as they become classroom teachers their perspectives of high-quality instruction might change. The principles used in categorizing the participants' perspectives for this study are certainly not held in isolation but rather form a complex web of factors that represent best practices. Despite the complexity, there were differences in perspective of high-quality mathematics instruction cited by preservice and inservice teachers. These differences might suggest a starting point for further research. However, the findings do indicate that some perspectives need more support than others since both preservice and inservice teachers must align their perspective with research-based reforms so that they can effectively enhance their students' mathematical proficiency.

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