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TRANSITION FROM REALISTIC TO REAL WORLD PROBLEMS WITH THE USE OF TECHNOLOGY IN ELEMENTARY MATHEMATICAL EDUCATION

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Abstract: The availability of technology has a big impact on education, and that is the main reason for discussing the use of technologies in mathematical education in our paper. The availability of technology influences how mathematical contents could be presented to students. We present the benefits of learning mathematical concepts through real life situations in elementary mathematical education. Particularly, we will analyze classroom examples which will give students a clearer picture of the connection between the real world and learned mathematics. For that purpose we will use technology in the form of Geogebra.

Key words: mathematical education, Geogebra, technology

1. Introduction

The adoption of mathematical concepts in the earliest periods of education, affects not only the performance in mathematics, but also other aspects of education [6], [8]. Therefore, a lot of attention should be paid to the way of learning mathematics at an early age. Learning mathematics at the beginning of elementary school should encourage the development of mathematical thinking. According to Ginsburg [10], children should be able to interpret their real experiences in a mathematical form and understand the link between non-formal and mathematical expression. In order to achieve this, it is necessary to apply real-world situations to early mathematical education.

One of the approaches, called *realistic mathematics*, deals with the way of learning mathematics where students acquire mathematical concepts through a context that is understandable and close to them. That kind of context is referred to as realistic. At first, the students learn about the problem that they can visualize, and then understand and solve, which leads them from an informal to a formal mathematical language. The idea of realistic mathematics education has been set up by Hans Freudenthal [9]. He argued that mathematics is a humanistic achievement and must be connected to reality, close and understandable to students and important to society. Realistic mathematics is part of the educational program in some countries, such as the Netherlands, the United States and England and there are numerous studies that confirm the efficacy of realistic mathematical education [15], [7].

The impact of technology in mathematical education is constantly increasing. It is important to emphasize that technology can not only be a tool that helps the students learn to be more efficiently, but can also contribute to a deeper understanding of mathematical concepts [18]. According to Steinbring, mathematical knowledge is formed by linking experiences through interaction with ideas, processes or structures [20]. This is the reason why we should find ways of teaching mathematics with technology that can be applied in practice. This is certainly not an easy task. The rapid development of technology and its use in everyday life is not adequately appreciated in education [17]. There are many problems that accompany technology integration in the classroom, such as a lack of computer devices, integration into the curriculum or the teachers are not sufficiently trained. Despite this, the idea should not be abandoned: First, technology can bring mathematics closer to students regardless of their knowledge and age. Second, technology allows a real world context to be integrated into learning [18]. Third, many studies in education show that learning mathematics with technology can contribute to students' knowledge [19], [21]. Finally, technology enables visualization and mathematical modeling.

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2. Real world problems in the mathematical classroom

In this paper, we discuss the "reality" of classroom mathematics and provide examples how mathematics can be connected to primary school students' world. Also, we propose solutions to make mathematics a meaningful and purposeful tool with the help of technology at an early stage of education.

Usually, most elementary mathematical and real-world problems refer to realistic problems. Those problems are known as "experimentally real" to students and they can illustrate very well the importance and the use of mathematics in real life situations. These kinds of tasks always begin with a real-world context as a start for learning a mathematical context [14], but they are usually simplified and reduced to problems that develop only by solving routines. Even though students are engaged in these types of problems, which provide good grounds for mathematizing and developing critical and mathematical thinking, many consider these problems incomplete and distant from real life [2]. For example, if we pose the task of calculating the area of a garden that is needed to be tiled to 10-11 year old students, the shape of the garden will be simplified with a square or a rectangle, rather than a triangle or any other irregular polygon, even though it is more likely that the garden does not have a regular shape. Analyzing this example further, many other obstacles occur in constructing real life situations, which are often messy, complex, open problems without "nice" or single solutions. For instance, it is questionable if this task is interesting and real to the students of young ages, or would they receive benefits from it.

From the theoretic point of view, bringing reality to the classroom could be realized by following instructions in Figure 1 [3]. This picture shows that in the process of solving a real world problem with mathematical tools, the students should explore, explain, reflect and record. The starting point of the process would be investigation or exploration of the situation. The students should examine and check thoroughly the real world situation. It is crucial that the students understand the real world context, that they are familiar with the notions, the facts, or even the words that describe the situation. After examining, the students can start with explaining and solving the problem, they can write down, draw or simply explain with words what would be their idea of the path that will eventually lead them to the solution. In this part, they should discuss their observations, obstacles and the problems that they face in the process of solving. The reflecting part should be the part where the students together with their teacher connect the problem with mathematical knowledge. Finally, the students should write, draw or model theirs solutions.

Prior to the application of the shown steps, the real world situation should be chosen. There are endless possibilities to choose the tasks from and the final decision depends on the mathematical concepts and practical skills that could be developed by the activities, the degree of improvement of critical thinking in the process of solving problems and the possibility to interact with other scientific or social disciplines [1], [5]. Some real world situations are more suitable than others, the choice should be shaped by the students' preoccupations. For example, the students from the rural parts would be more interested in the real-world problems related to agriculture, while the students form urban communities would be more curious about traffic or population problems. It is up to the teachers, textbook authors and software creator to find out what kind of real world problem would contribute to their students' knowledge.

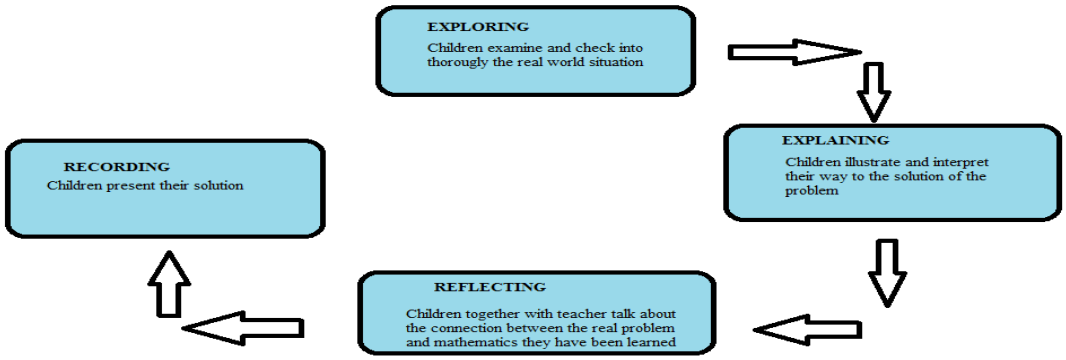


Figure 1. *Instructions for introducing students to the real world problems*

3. Use of technology in the process of learning mathematics through real world problems

Technology should be used in elementary schools in the process of teaching mathematics for several reasons: On the one hand, technology can be adjusted to the individual needs of the students; on the other hand, the students can collaborate with other students which allows them to learn in this process. The use of technology in elementary education prepares students for further education and for what is often called *lifelong learning* [4]. With personalized tasks with technology the students' discoveries and independent conclusions rely on their knowledge and experience. Also, solving the real world tasks with technology, linking mathematics and the world, as well as other objects, develops curiosity and problem-solving abilities. In addition to mathematics, there are technological solutions tailored, for elementary school, for learning reading and writing, drawing, research, creating presentations and conceptual maps [12], [22]. Technology today allows easy creation of applications for learning mathematics. Moreover, the combination of different types of technology opens new opportunities for learning.

In this paper we share the examples from the educational practice which describes how can "reality" be incorporated to primary education with the help of technology. We base our observations on applying mathematics in a real-world context with the help of technology in a primary school. The examples have been applied in elementary schools in Bijeljina, Bosnia and Herzegovina, and Ruski Krstur, Serbia with 10-11 year old students. In this part, we present the use of technology in the process of learning mathematics through real life situations. We discuss the use of educational software in the process of transition from the real world to the mathematical problem and we pay a significant attention to the fact that the use of technology gives a clearer picture of mathematical importance.

The first step in the process of learning mathematics through real-life mathematical problems was the selection of the real-world situation. Choosing a real-world problem that will be both, mathematically appropriate and close to the interests of 10-11 year old students is not an easy task. Their social and mathematical experience is narrow, and they are only familiar with the basics of both. While teaching mathematical concepts through real-world situations the cognitive competences of the students should be taken into consideration. Knowledge and skills that are gained during elementary mathematical education are: counting, cardinality, four basics operations and algebraic thinking, numbers and operations, fractions, expressions, equations and inequalities, geometric figures, metric units, etc. Elementary mathematical education as a starting point allows educators to build on the students' knowledge with an inquiry and a model-based approach, developing essential mathematical experiences in the classroom. The students' lack of social and mathematical experience can be used as an advantage, because the students in elementary schools can think about real world situations differently and unexpectedly. To do so, it is important to imply certain actions during mathematical modeling of the real-world problems. Actions, such as verbal, textual or illustrative demonstrations of the problem, identification of numerical variables and its graphical presentation, writing algebraic expressions and solving equations or interpretation of the expressions in the context can benefit the students' mathematical abilities. They can be developed through solving real-world problems during

the stages of examination, explanation, reflection and documentation. In our case, we have set the real-world situation in the library. Every school has a library and students at the elementary school are familiar it. The concept of a library can be used for explaining calculations in the real-world situations. For example, books could be borrowed or mathematically subtracted, or returned and mathematically added.

Technology allows for different opportunities of mathematical contents. It can be used for exploring various problems derived from the real-world situations in obvious, dynamic and interesting ways. For that purpose, two types of software could be used. Firstly, the educational software for learning mathematics through real-life situations with examples from the real-world which are solved mathematically in steps that should be followed by the students. This type of software was tailored to fit the educational purposes of elementary mathematics. It was designed to develop the students' mathematical modeling abilities through feedback and evaluations. Its designer was the second author of this paper and it was made as a part of her research about software usage in mathematical lessons and modeling at the University of East Sarajevo [15]. It is only available in Serbian language. The user interface that represents the introduction step is shown in Figure 2.

In Figure 2 we can see the short description of the software which refers to the modeling of real-world problems based on the visual and geometrical representations of the problems. The contents that are covered in this software are divided into eight parts, with special emphasis on the differentiated approach to the modeling of real world problems. There are different types of mathematical models, for example the real-world problem could be presented using a line segment (noted with 3.), tables (noted with 4.), rectangular (noted with 5.), graphs (noted with 6.), and blocks (noted with 7.). The introductory parts (noted with 1. and 2.) and tests are also included. They cover definitions and the basic stages of mathematical modeling, solving real world problems related to geometry, theoretical explanation model, the practical application of the model in solving real world problems and tasks with solutions. After every part, there are tests, which contain four real world problems with differentiated instructions and solutions. The program is designed in such a way that the students can ask for help three times, get feedback for each level, and choose the correct answer. It is appropriate for 10-11 year old students.

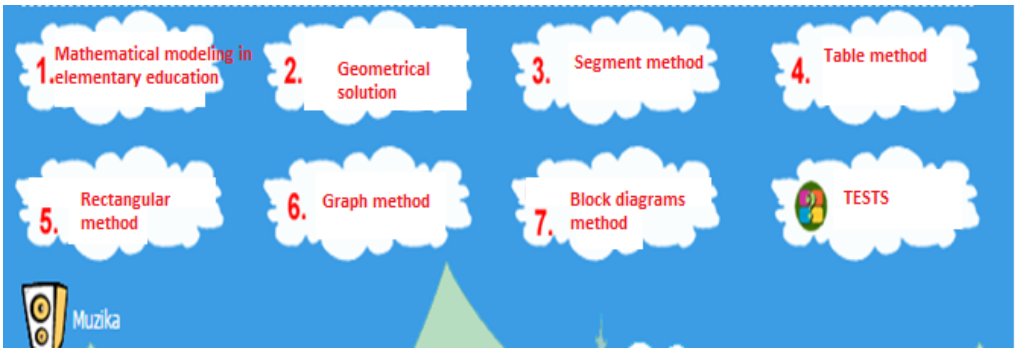
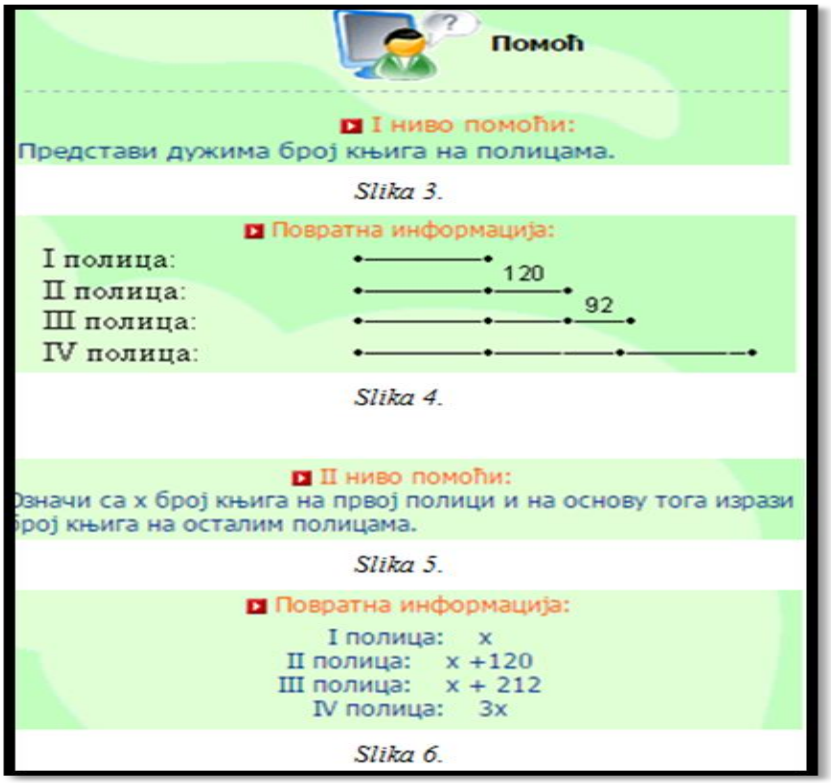


Figure 2. *Interface of the educational software for learning mathematics through real-life situations*

The interface of a task can be seen in Figure 3. There is a problem that is connected to the real world problem related to the school library. The problem is as follows: "There are four shelves in a school library with 1334 books. There are three times less books on the first shelf than on the fourth, 120 more books on the second shelf than on the first, and 92 less books than on the third shelf. How many books are on the shelves?" As it is shown in Figure 3, the hints of the solutions are offered in few steps. The hints are visual, represented with line segments, in order to help the students to get the picture of the real world situation. The steps are guiding students to the equation which represents a mathematical model. Manipulating with the equation the students receive the final solution of the real world problem. The help of the software is evident in the part of explaining and reflecting the real-world problem. Technology offers a visual explanation on the way to the solution and helps in connecting the real world situation with mathematics.

|There is a variety of educational software, which offers numerous possibilities for teaching mathematical lessons, and increases learning motivation. Besides that, technology developement provides the teachers with educational materials and simple software, that suits the needs of their students[13]. In regards to that, the educational computer software was created and experimentally tested. The software covered mathematical lessons related to equations with addition, substraction, multiplication and division with natural numbers and the zero. This software is suitable for 10-11 year old students. Mathematical content covered with this educational software is settled in a real-world context and its aim is to introduce the students with mathematical modeling. This educational software is created as a multimedia presentation with a goal to introduce the students with a mathematical context in an unusual and interesting way. It can be used for learning, exercising or testing, and it can be adjusted to the individual requirements of a student due to differentiated tasks. The application of the educational software was tested in a pedagogical experiment with an experimental (125 students) and a control group (125 students). After teaching with the use of the educational software, it was concluded that the experimental group achieved better results at the final test. The educational software provided interactions and information about progress, which made learning easier.

It was proven that the students who learn mathematics through real world situations with the help of technology benefited in many ways. Generally, the conclusion is that the students cannot stay passive while working with software, and visual and musical effects contribute to that. While working with software, the students are encouraged to improve and compete. This software offers possibilities of individual progress through received information about the achieved results.



Translation:

Help

1st level of help: Number of books represent with segment.

Picture 3.

Received information

1st shelve, 2nd shelve, 3rd shelve, 4th shelve.

Picture 4.

2nd level of help: Let x represents number of books at the first shelve and express number of books at other shelves.

Picture 5.

Received information.

1st shelve: x

2nd shelve: x+120

3rd shelve: x+212

4th shelve: 3x

Picture 6.

Figure 3. *The process of solving a real world problem with the help of technology*

In order to include more technology to the teaching process in 2016 we started to combine the mentioned software with the Geogebra software.

GeoGebra is a free educational software designed for learning algebra and geometry. Because of its properties it is widely used in high schools, where the application of technology is easier and more accepted. Geogebra also has certain possibilities for use in primary schools, and even in the lower grades. This dynamic software provides opportunities for application in primary schools (20) and the

developing of knowledge in algebra and geometry. There is also a large community of GeoGebra users, including teachers and students, who have produced plenty of material for various levels and themes that can be directly used in teaching, with or without adjustment. An example that we made refers to the specific problem of the students' lives and could be interesting for teachers in primary schools who want to teach mathematics through real situations.

To make the tasks more open and real, we used dynamic geometry properties of Geogebra. For example, we used the slider option to give the students the possibility to observe the number of the books in a library by changing values at the first shelf. The model was previously made by the teachers, and the students got instructions how to move the slider and to observe the changes. Using the Geogebra model, the students can propose a real world problem by themselves or they can check the results by calculating and practicing subtraction. The mathematical model of the problem made in Geogebra is shown in Figure 4. Geogebra is a well-known educational software. It is widely used for learning and teaching mathematics. This type of educational software is more suitable for high school education, but we propose its use in elementary school as well. It could be applied in the form of previously prepared materials suitable for elementary school students.

It is noticeable that the first type of software is suitable for realistic problems. The given example is concerned with one particular, maybe imaginary, school library with 1334 books. The problem is adjusted to the integer solution. Geogebra through its dynamic geometry properties enables the library problem to become real. With the slider the students can change the actual number of books in their library and see what happens on the shelves. The example also refers to the number of books in the library. The amount on the first shelf is defined by the slider. The second shelf has 120 books more, the third shelf three times more and the fourth has 212 books more than the first shelf. The task that the students should solve could refer to the problem of the total amount of books in the library while the number of the books on the first shelf is changeable.

For example, the slider is represented with b and it represents the number of books on the first shelf. The first shelf is colored with red and named "prva". The second shelf is presented with purple and named "druga". The number of books on the second shelf is equal to the sum of the books number on the first shelf and another 120 books extra. The third shelf is named "treca" and it is in blue color. The amount of books is equal to the sum of the first shelf and the number of 212 extra books.

The fourth shelf is named "cetvrta", it is in green color, and its amount of books is three times bigger than on the first shelf. The dashed line represents the overall number of books in the library and it is named "ukupno". During the process of changing the slide values, the students can calculate and check their solutions and compare them with Geogebra results (www.geogebra.org/m/BsvhEGUN).

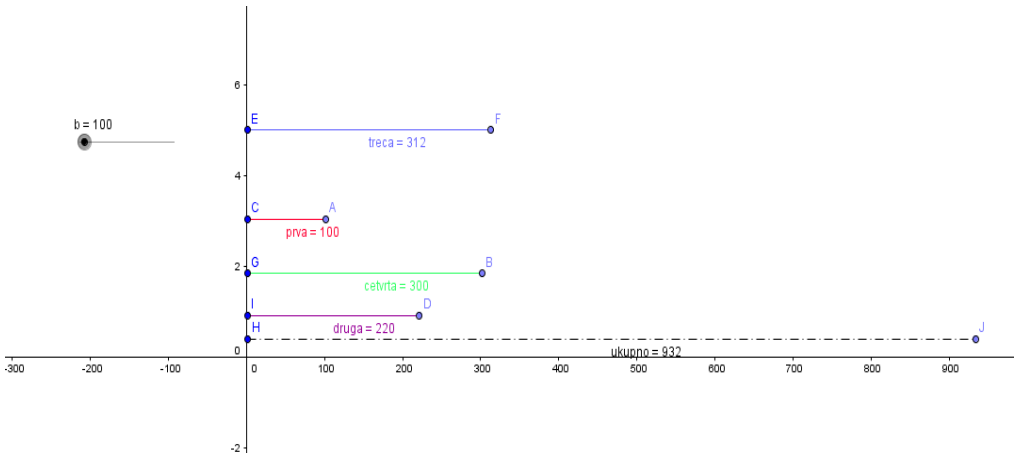


Figure 4. *Exploring a real world problem with the help of Geogebra*

3. Conclusion

Technology and children interests are important variables in the educational process. Technology is unavoidable in young students' everyday lives, since it is widely used. The students are also interested in the purpose of the learned content, even in elementary school. Many times they question teachers about the connection of mathematics to reality. The application of technology in the education process and finding appropriate real world situation to illustrate the use of mathematics, are not straightforward. Both require careful preparation and research in order to provide the students with knowledge and understanding of mathematical concepts and their application in real life. Solving the realistic and real life tasks with the help of technology can illustrate usefulness of mathematics to the students and fulfills the expectation of the educational process. The availability of technology has a big effect on education in general and its usage can provide profound exploration of the problems, even at an early stage.

In this paper, we have analyzed ways of applying differently a software. The software implemented in Bosnia and Herzegovina differs from GeoGebra, but it was proven beneficial in 2006, and it can be used today in teaching. The application of GeoGebra is at an early stage but qualitative results show the benefit of this kind of learning. Technology has not been introduced in the curriculum of primary schools of Bosnia and Herzegovina, or in Serbia. Technology supported teaching is also affected by equipment problems. Regardless of the obstacles, educational professionals are obliged to point out the importance and the benefits of technology application in the classroom from the earliest possible age, even though sometimes the conditions are discouraging. There is no ideal educational software that will completely teach the students, without any obstacles. But continuous searching and combining different approaches and software we might come close to the desired results of better mathematical understanding.

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