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The Common Core State Standards for Mathematics and College Readiness

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The Common Core State Standards were created with college and career readiness in mind to help prepare students to succeed upon graduation from high school. In this article, I examine college readiness as it has been described by both university mathematicians and educational researchers to precisely discern what will foster success in collegiate mathematics. This idea of college readiness is compared to the Common Core State Standards for Mathematics (CCSSM) to assess the degree to which they align with various mathematical aspects of college readiness. There is a strong alignment between what university mathematicians and educational researchers expect of college students and what the CCSSM expects of students. Faithful CCSSM-guided instruction has the potential to foster college readiness (and ultimately college success) in K-12 students.

"How can I prepare my students for success?" is the critical, driving question for many reflective K-12 educators around the country. Designed with college and career readiness in mind, the Common Core State Standards (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) has the potential to (at least partially) address this vital question, ensuring that high school graduates are prepared for whatever they chose to pursue next. Though the Common Core State Standards have been widely adopted across the United States, support for them has waned somewhat in recent years, with several states withdrawing from the initiative (Bidwell, 2014). Given the speed with which America can turn on its historical educational reforms (e.g., Dow, 1991), it remains topical to discuss whether or not the Common Core State Standards are beneficial for students, teachers, and the entire U.S. educational

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system, specifically in preparing our students for college.

In this paper, I examine college readiness with respect to the Common Core State Standards for Mathematics (CCSSM; NGA & CCSSO, 2010), to answer the question: to what extent do the CCSSM align with various mathematical aspects of college readiness? To answer this question, I begin with a brief overview of the CCSSM. I then explore what is meant by college readiness from the perspectives of both college mathematics instructors (mathematicians) and mathematics education researchers. Finally, I explore how the CCSSM addresses these aspects of college readiness.

The Standards for Mathematical Practice

The CCSSM are not curricula[1], prescriptive teaching methods, nor assessments. The CCSSM are a collection of mathematical standards that "define what students should understand and be able to do in their study of mathematics" (NGA & CCSSO, 2010, p. 4). They are divided into two parts: content standards and Standards for Mathematical Practice (SMPs). The content standards are the minimum requirements for what mathematically proficient students should know, understand, and do upon completion of each grade level. Students completing 8th grade, for example, should be able to "define, evaluate, and compare functions" as well as "use functions to model relationships between quantities" (NGA & CCSSO, 2010). The SMPs are eight mathematical practices, or habits of mind, that students should be engaged in across all grade levels. The two sets of standards function as a roadmap of mathematical learning for K-12. In this section, I focus first on the SMPs, then on college readiness, and finally on the relationship between the two.

[1] Here, by "curricula," I mean sequences of content paired with a set of suggested lessons and learning activities designed to facilitate learning that content, as is often found in textbook series. The CCSSM do not prescribe lessons or learning activities, and as such they are not curricula.

The SMPs outline the ways in which students (and mathematicians) engage with mathematical content at all levels. They represent the various areas of expertise of doing mathematics such as solving problems; creating, communicating, and critiquing reasoning, arguments, and proofs; modeling and representation; and drawing connections. Dispositions towards mathematics (as an area of study) and doing mathematics (as an activity) are also embedded in the SMPs; students are encouraged to see mathematics as a sensible, structured discipline, and to believe in their own ability to do mathematics.

There are eight SMPs presented in the CCSSM:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Making sense of problems (in SMP1) is often a matter of exploring meaning, both of mathematical operations and procedures, and of situations or phenomena being modeled with mathematics. Modeling with mathematics (SMP4), relatedly, refers to the use of mathematics to represent (and often predict) real-world situations and phenomena. Reasoning abstractly and quantitatively (SMP2) is closely related to SMP4 in that both standards involve decontextualizing situations (representing them with numbers, symbols, and/or equations) and contextualizing mathematical objects (reinterpreting numbers, symbols, and/or equations in the context of the original situation). Constructing viable arguments and critiquing the reasoning of others (SMP3), and attending to precision (SMP6) are matters of effective mathematical communication. Specifically, constructing viable arguments is a matter of using known definitions and logic to convince peers, and critiquing the reasoning of others requires making sense of and determining the validity or applicability of others' reasoning. "Tools" (in SMP5) is a broad term that can refer to anything from paper and pencil to computer programming languages. Looking for structure (SMP7), looking for repeated reasoning (SMP8), and perseverance (in SMP1) are not only good problem-solving strategies; they are also part of fostering an appreciation of mathematics as an interconnected discipline that is both sensible and useful. Altogether, the CCSSM practice standards provide a roadmap for engaging in mathematical material in ways that engender sense-making, conceptual understanding, and mathematical communication.

Unpacking College Readiness

Despite considerable and varied attention paid to the idea of college readiness, there is no consensus on what it actually means (Conley, 2008). Traditionally, college readiness was measured in terms of successful course completion, high school GPA, and standardized test scores. These metrics continue to have a great deal of influence on whether or not a student is accepted into any given college (Noble & Sawyer, 2004). Lately, researchers have begun to pay closer attention to what happens after the acceptance letter arrives (e.g., Chait &Venezia, 2009).

More recent conceptions of college readiness have less to do with being accepted to college and more to do with one's ability to thrive in college—successfully advancing through one's chosen field of study and graduating in a timely fashion. Despite higher attendance rates, college completion rates remain relatively unchanged. About half of college students complete their degrees (Chait & Venezia, 2009). Moreover, a substantial portion of first-year college students (between 25% and 33%) take remedial courses rather than engaging immediately in college-level material (Chait & Venezia, 2009).

These trends prompted researchers to take the struggles and experiences of college instructors and students into account, and thus expanded of the idea of college readiness.

Ultimately, college readiness is a broad, complex, and multifaceted construct that encompasses nearly everything that might affect one's ability to thrive and succeed in college (e.g., attitudes towards scholarship, behavioral characteristics surrounding work ethic, knowledge of available resources like financial aid and counselling services, etc.). Though researchers have used pre-college metrics (e.g., SAT scores and GPA) and post-college metrics (e.g., college drop-out rates and rates of remediation) to measure college readiness, the phenomena of being and becoming college ready is an ongoing process that spans one's entire academic career. It is not my goal in this article to refine definitions college readiness, nor is it my intention to consider college readiness beyond the extent to which it relates to mathematics content (see Conley, 2008, for a more expansive review of college readiness). Rather, I aim to explore various outlooks on mathematical aspects of college readiness as they relate to the CCSSM.

College Readiness According to Mathematicians

The expectations and perceptions of college mathematics instructors (i.e., mathematicians) offer insight regarding mathematical college readiness. Mathematicians have expressed a great deal of frustration regarding the college readiness of their students (Corbishley & Truxaw, 2010). For example, in Great Britain, university mathematics professors often expected their students to be much more capable than they perceived their students to be. This gap between university professors' expectations of their students' mathematical abilities and university professors' perceptions of their students' abilities was such a widely publicized issue that it became known as "the mathematics problem" (Howson et al., 1995). The problem is no less pertinent in the United States (e.g., Zucker, 1996).

University mathematicians have criticized several aspects of how students do mathematics. Some have criticized their students' lack of procedural fluency with basic skills (Conley, Drummond, Gonzales, Rosebloom, & Stout, 2011). Others lament that their students' learning focuses too much on basic skills (De Guzmán, Hodgson, Robert & Villani, 1998). In an extreme case, one mathematician asserted that high school graduates do not even know what learning is: "The fundamental problem is that most of our current high school graduates don't know how to learn or even what it means to learn" (Zucker, 1996, p. 863). In another case, several mathematicians stated "University teachers deplore the lack of prerequisite knowledge which makes the beginning at the tertiary level painful and difficult for many of their students" (De Guzmán et al., 1998, p. 751). While these sentiments are not representative of all the views of mathematicians nor are they supported with substantial evidence beyond personal experiences and perceptions, they represent a prevalent voice of mathematicians in the larger discourse on college readiness. Although much of the literature produced by mathematicians focuses more on deficiencies and college unpreparedness than college readiness (e.g., Howson et al., 1995), these accounts provide valuable insight into the mathematical expectations of college instructors. Mathematicians' and students' expectations sometimes conflict in ways that can hinder students' success. These discrepancies should inform our understanding of college readiness as a measure of students' ability to succeed in college.

The first broad point of discrepancy between mathematicians and their students is in the way each group perceives mathematics itself. Beginning university students mostly view mathematics as a set of un-related rules and formulae (Crawford, Gordon, Nicholas, & Prosser, 1994; Schoenfeld, 1992). They believe that doing mathematics means applying memorized rules to problems in order to get correct answers and recalling the rules at appropriate times in response to certain types of questions. Accordingly, to learn mathematics, students practice using rules and formulae on appropriate questions and examples until they are comfortable recognizing those questions, recalling the rules, and correctly applying them. This is an example of what Skemp (1976) called instrumental understanding or “rules without reasons” (p. 9). University mathematicians, on the other hand, tend to have what Skemp called “relational understanding”—not only knowing what to do, but also knowing why to do it. To illustrate the difference between instrumental and relational understanding, consider the quadratic formula and a hypothetical first-year university student who can recite the quadratic formula when asked and use it to find the roots of a quadratic equation. Indeed, this ability is crucial for success on standardized exams (e.g., the SAT) that impact one’s acceptance to a university. Mathematics professors are not only capable of reciting the quadratic formula, they can also demonstrate where it comes from and why it works, and explain the reasons and circumstances in which a particular quadratic equation might have zero, one, or two real roots but always two roots on the complex plane[2]. This kind of robust, connected understanding has several advantages over simply knowing a procedure for solving certain problems, including ease of recall (facilitated by knowledge of why and how the procedure works) and breadth of applicability (Skemp, 1976).

The second broad point of discrepancy between mathematicians and their students is in the way members of each group conceptualize what it means to learn, which closely aligns to the understandings that Skemp outlined. When students engage in mathematics at an instrumental level (Skemp, 1976) and believe that mathematics is a collection of rules and formulae, they also tend to believe that instructors should tell them the rules, show them the formulae, and provide examples and opportunities for practice (Crawford et al., 1994). Generating and justifying rules is typically not what first-year college students expect to do (Conley, 2008). Some have argued this discrepancy is not rooted in instructional expectations, but instead arises from differences of views

[2] Certainly some first-year college students can do the same, but our hypothetical first year student cannot. My claim here is not about the general abilities of most first-year college students, only about the relative merits of relational understanding over instrumental understanding.

regarding the nature of knowledge itself (Daempfle, 2003). Students who view knowledge as simple, absolute, certain, and clear-cut are significantly less likely to complete their degrees than those who view knowledge as difficult to construct and ultimately mutable (Daempfle, 2003).

Mathematicians often expect relational understanding and sense-making engagement from their students (Conley, Drummond, Gonzales, Rosebloom, & Stout, 2011; Howson et al., 1995). For mathematicians, mathematics is not a subject of rules and memorization, but the discipline of consequence, reasoning, and logical deduction—practices that students with instrumental understanding are not ready to learn or participate in (Zucker, 1996). With this in mind, one could argue that college-ready students are those who can

- make sense of the mathematical procedures;
- reason through the relationships between different mathematical topics to glean a sense of the interrelated structure of the subject;
- explain and justify why procedures and problem-solving techniques work;
- arrive at and defend sound conclusions via viable arguments; and
- take responsibility for their own learning and knowledge.

We see many of these same ideas mirrored in the SMPs (NGA & CCSSO, 2010). Making sense of mathematical procedures for relational understanding is part of making sense of mathematics, much like SMP1, make sense of problems and persevere in solving them. Reasoning through the relationships between different mathematical topics to understand the interconnectedness of mathematics as a discipline is an integral part of SMP2, reason abstractly and quantitatively, and SMP7, look for and make use of structure. Explaining and justifying why certain procedures and techniques work involves engaging in mathematical communication: constructing and critiquing arguments (SMP3) and attending to precision (SMP6). Constructing and defending arguments is also closely related to taking responsibility for one's own learning and knowledge. SMP7 and SMP8 involve looking for regularity, structure, patterns, and repeated reasoning. Students who are accustomed to looking for (and finding) these consistencies and using them in problem solving take responsibility for their own knowledge. There is a great deal of alignment between the expertise college mathematicians expect of their students and expertise detailed in the CCSSM practice standards.

College Readiness According to Educational Researchers

Not only do mathematicians' expectations for students align with the SMPs, educational researchers' definitions of college readiness have some overlap with the SMPs as well. Conley (2008) defined college readiness as a combination of four key components[3], one of which is *key cognitive strategies* such as "analysis, interpretation, precision and accuracy, problem solving, and reasoning" (p. 5). Conley described *analysis* as the evaluation of data and other sources or materials on the grounds of relevance and credibility, among other things, and *interpretation* as accurately describing events. These key cognitive strategies align well with SMP4, model with mathematics, and SMP2, reason abstractly and quantitatively, because they involve contextualizing data in situations and verifying that data are reasonable and make sense. Conley described *precision and accuracy* as consisting of three parts: (a) recognizing appropriate levels of precision for various tasks, (b) using precision to draw accurate conclusions, and (c) iteratively increasing the accuracy of

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[3] Conley identified four aspects of college readiness, including key cognitive strategies. The other three are (a) key content knowledge (e.g., algebra), which is discussed in a later section; (b) attitudes and behavioral attributes (e.g., time management, work ethic); and (c) contextual knowledge (e.g., how to apply for financial aid, awareness of available resources). The latter two are beyond the scope of the CCSSM.

approximations. Recognizing and applying appropriate levels of precision are crucial for the precise mathematical communication described in SMP6, attend to precision, which includes accurately using mathematical terminology, symbols, ideas, definitions, and arguments. Additionally, using precision to draw accurate conclusions is an element of SMP3, create viable arguments. Finally, iteratively increasing the accuracy of approximations is important in SMP2, reason quantitatively. Therefore, each part of Conley's description of precision and accuracy aligns with at least one of the SMPs. In addition, Conley's description of the key cognitive strategy *problem solving* includes using known strategies to solve routine problems and creating novel strategies to solve non-routine problems. This strategy relates to SMP1, make sense of problems; SMP7, look for and making use of structure; and SMP8, look for and make use of repeated reasoning: students learn to modify and adapt known problem-solving techniques to solve novel problems through structure and repeated reasoning. Finally, Conley described *reasoning* as constructing well-reasoned arguments, as well as accepting and providing logical critique. This strategy is nearly identical to SMP3, constructing viable arguments and critiquing the reasoning of others. Overall, there is a great deal of alignment between the key cognitive strategies aspect of college readiness described by educational researcher Conley and the SMPs.

My analysis is not the first to examine the relationship between the CCSSM and college readiness. Conley et al. (2011) used a survey to collect data from 1,897 post-secondary instructors regarding the applicability of the Common Core standards to the classes they taught. Of the participants, 302 were mathematics instructors; the others taught a variety of courses (e.g., science, English language arts, business management, etc.). The mathematics standards were rated as applicable by the majority of mathematics instructors, and the SMPs in particular were rated as applicable by all of the mathematics instructors. All instructors were asked, "Are the mathematics standards, taken as a whole, a coherent representation of the knowledge and skills necessary for success in your course?" (Conley et al., 2011, p. 82). From

1,706 responses to this question, 62% of instructors across all disciplines said yes. Among those reporting a mismatch between their courses and the CCSSM, many participants felt that certain parts of the CCSSM were simply not applicable to their course or went beyond the expectations they had for their students. However, the SMPs were rated as applicable and important by the majority of instructors across almost all content areas. There is a great deal of alignment between what college instructors perceive as important for success in their courses and what is described in the CCSSM.

Evidence from university mathematicians and educational researchers seems to agree that the SMPs are well-aligned with the mathematical aspects of college readiness.

The CCSSM Content Standards

The CCSSM content standards are divided into six conceptual categories: Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. The high school standards include an additional standard, Modeling[4]. Each of these conceptual categories is further divided into clusters of standards that fall under specific domains. For example, the conceptual category Functions includes domains such as Interpreting Functions and Building Functions. The cluster of standards within Interpreting Functions includes understand the concept of a function and use function notation, interpret functions that arise in applications in terms of the contest, and analyze functions using different representations.

College Readiness and Mathematical Content

With respect to mathematical content, elements of college readiness vary from person to person. For instance, an aspiring electrical engineer must be prepared to succeed in calculus and differential equations courses, whereas an aspiring biologist

[4] The modeling domain contains no individual standards, as it is best understood in relation to other standards rather than in isolation.

may only need to pass a single major-specific mathematics course focused on data analysis for their degree. College readiness for all students across prospective disciplines should entail preparing students to succeed in any mathematics course they choose to pursue. Specifically, for a student to be college ready, that student needs a broad foundation and thorough understanding of basic (algebra) skills that enables him or her to succeed in any number of a wide variety of college mathematics courses (Conley, 2008).

The need for first-year college students to have a broad foundation of basic skills often runs counter to the desire for students to have exposure to as much advanced content (e.g., calculus) as possible. On one hand, enrollment in high school calculus courses has been steadily increasing over the last 30 years, with mounting pressure on both high school students to take calculus and on high schools to offer it (Bressoud, 2010). On the other hand, the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) are in agreement that calculus should be left for college (MAA & NCTM, 2012). Mathematicians place greater value on a rigorous and thorough understanding of algebraic fundamentals than on brief exposure to advanced content, such as calculus (Chait & Venezia, 2009). In fact, at my university, the head of the engineering program prefers students arrive ready to learn calculus rather than having already taken it (R. Peck & R. Balasubramanian, personal communication, 2014).

In addition to clashing with the preferences of college mathematicians, taking calculus in high school is not always beneficial for students. For example, among a representative sample of high school classes of 2004, one in six students who had completed a high-school calculus course enrolled in remedial mathematics courses at the college level (National Center for Education Statistics, 2010). Of the students who took the Calculus AB advanced placement (AP) exam in 2012, 43% failed (National Science Board [NSB], 2014). Furthermore, it is common for students who passed the exam to choose to re-take calculus in college (Sadler, 2014)[5]. The high rates of remediation and AP exam failure suggests that high school calculus courses do not always prepare students for college mathematics, and the drive to get students into high school calculus courses may have adverse effects on their learning of more foundational mathematics (Bressoud, 2010). The MAA and the NCTM prefer that K-12 education focus on fundamentals, as described in their joint position on Calculus:

> The ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. (MAA & NCTM, 2012, p. 1)

Moreover, Conley (2008) suggested that mathematical college readiness is a matter of understanding algebra: "Most important for success in college-level math is a thorough understanding of the basic concepts, principles, and techniques of algebra, since a great deal of mathematics that students will encounter later on will draw upon or utilize these principles" (p. 15).

For many students, mathematical college readiness depends on success in introductory calculus during their first year of college; for others, mathematical college readiness depends on preparation for courses in statistics, data analysis, discrete mathematics, probability, or other non-calculus courses. To account for this wide variety of needs, Conley's (2008) and MAA & NCTM's (2012) suggestions focused on mathematical foundations and basic principles that are

[5] One possible explanation for the high percentage of students taking calculus at the high school level and again in college is that calculus is often taught differently across these levels. Compared to high school teachers, college instructors tend to have different goals for calculus instruction, moving more quickly through topics while focusing on different aspects of the material (Conley, 2008).

applicable across a variety of mathematical contexts. A solid foundation in the fundamentals of algebra, functions, and modeling can help students succeed in calculus, as well as most other introductory college mathematics courses (Conley, 2008).

The CCSSM Content Standards and College Readiness

There is a wide variety of different introductory college mathematics courses for first-year students including calculus, statistics, data analysis, business mathematics, and many other major-specific courses. Evaluating the relevance and applicability of the CCSSM content standards to each of these introductory mathematics courses is beyond the scope of this paper[6]. Instead, I focus specifically on calculus, "the lodestar of the K-12 curriculum and the bedrock of post-secondary preparation for science and engineering" (Bressoud, Mesa, & Rasmussen, 2015, p. vi).

Calculus 1 includes four major content areas: limits and continuity, derivatives, integrals, and sequences and series (Burn & Mesa, 2015). In the following section, I list some of the CCSSM content standards required for a thorough understanding of each of these major content areas. The foundations of understanding limits and continuity are embedded in the following CCSSM conceptual categories:

- Number and Quantity: In the case of limits of functions, understanding what is meant by "*x* approaches *a*" involves an appreciation of the density of the real number system.
- Functions: Making sense of continuity requires students to have a firm grasp of the behavior of functions. Evaluating functions both numerically (given certain coordinates) and symbolically (in the

[6] Conley et al. (2011) provided a corpus of empirical data regarding the applicability of each of the CCSSM standards to courses taught by 1,897 collegiate instructors across a variety of disciplines.

case of function composition) is key to understanding limits.

- Algebra: Evaluating limits in calculus courses often involves algebraic and arithmetic competency with polynomial and rational expressions.

The foundations of understanding both derivatives and integrals are embedded in the following CCSSM conceptual categories:

- Functions: Both the derivative and integral involve the behavior of functions in terms of rate of change and accumulation, respectively. Interpretation of the behavior of various families of functions is fundamental.

- Algebra: Differentiation and integration at this level are often accomplished using 'rules' (e.g., the power rule, the chain rule, etc.). Students need a solid grounding in algebraically manipulating expressions to understand these rules.

- Geometry: Beyond the rules for analytically finding equations for the derivatives and integrals of functions, students should have a firm grasp of geometry of such ideas. Graphically, derivatives measure slopes of lines tangent to a curve and integrals measure the area between the axis and a curve.

- Modeling: Applying the ideas of derivative and integral to problems is a key element of calculus, and is fundamental to certain problem-solving activities such as optimization, related rates, and volume calculations.

The foundations of understanding sequences and series are embedded in the following CCSSM content conceptual categories:

- Functions: Sequences and series can be understood as functions on whole-number domains. Understanding the behavior of sequences and series is often a matter of understanding the underlying function.
- Algebra: Working with sequences and series requires doing arithmetic with polynomial and rational expressions, as well as reasoning with equations and inequalities.
- Modeling: Modeling is required to understand tangent lines as the limit of a series of secant lines, and to understand the integral as the limit of a series of Riemann sums.

In my evaluation, each of the four major content areas of Calculus 1 is thoroughly grounded in the CCSSM high school content domains.

Although calculus is not among the six domains of the CCSSM content standards, the decision to not require high schools to teach calculus material aligns with the above discussion of college readiness and mathematical content. As suggested by MAA & NCTM (2012) and Conley (2008), the CCSSM are focused on fundamentals and basics, so that high school graduates will be well prepared for whatever mathematics they choose to pursue in college. Also note that the CCSSM are *minimum* standards—calculus is not prohibited, and it is certainly possible for high schools to offer calculus while still adhering to the CCSSM. However, the content standards are focused primarily on fostering a solid mathematical foundation for students to take calculus at university by emphasizing algebra, equations, functions, modeling, and interpretation, which is precisely what Conley (2008) described as "most important for success in college-level math" (p. 15).

Conclusion

The concept of college readiness is broad, complex, and multifaceted. It encompasses a wide variety of characteristics and abilities that must be applied in a wide variety of situations and circumstances in order for an individual to succeed and thrive in college. I examined some of the characteristics and abilities that enable a student to successfully complete their university mathematics courses from various perspectives. College mathematicians (e.g., De Guzmán et al., 1998) expect their students to engage in sense-making, critical thinking, and the construction of arguments, just as the SMPs suggest. Education researchers (e.g., Conley 2008) have identified key cognitive strategies for college readiness such as analyzing and interpreting, the core ideas of which are mirrored in the SMPs. There is a general consensus (Conley, 2008; MAA & NCTM, 2012) that rather than take calculus in high school, students should arrive at college with a rigorous and thorough understanding of fundamentals so that they can succeed in whatever mathematics courses they choose to pursue. This, too, is reflected in the CCSSM content standards. The CCSSM are, ultimately, well aligned with each of these various perspectives on college readiness.

References

Bidwell, A. (2014, August 20). Common core support in free fall. *U.S. News & World Report*. Retrieved from http://www.usnews.com/news/articles/2014/08/20/common-core-support-waning-most-now-oppose-standards-national-surveys-show

Bressoud, D. M. (2004, August/December). The Changing Face of Calculus: First-Semester Calculus as High School Course, *MAA Focus, 24*(6), 6–8.

Bressoud, D., Mesa, V., & Rasmussen, C. (Eds.) (2015). *Insights and recommendations from the MAA national study of college calculus*. Retrieved from http://www.maa.org/sites/default/files/pdf/cspcc/InsightsandRecommendations.pdf

Burn, H., & Mesa, V. (2015). The calculus I curriculum. In D. Bressoud, V. Mesa, & C Rasmussen (Eds.), *Insights and recommendations from the MAA national study of college calculus* (pp. 45–57). Retrieved from http://www.maa.org/sites/default/files/pdf/cspcc/InsightsandRecommendations.pdf

Burton, N.W., & Ramist, L. (2001). *Predicting success in college: SAT studies of classes graduating since 1980* (Research Report No. 2001-2). New York: College entrance Examination Board.

Chait, R., & Venezia, A. (2009). *Improving academic preparation for college: What we know and how state and federal policy can help*. Retrieved from http://www.americanprogress.org/issues/labor/report/2009/01/27/5411/improving-academic-preparation-for-college/

Conley, D. T. (2008). *Redefining college readiness*. Retrieved from Educational Policy Improvement Center website: http://epiconline.org/publications/redefining-college-readiness

Conley, D. T., Drummond, K. V., de Gonzalez, A., Rooseboom, J., & Stout, O. (2011). *Reaching the goal: The applicability and importance of the common core state standards to college and career readiness.* Retrieved from http://www.epiconline.org/publications/document-detail.dot?id=09535d00-466a-498f-ac44-98f4c372d9c1

Corbishley, J. B., & Truxaw, M. P. (2010). Mathematical readiness of entering college freshmen: an exploration of perceptions of mathematics faculty. *School Science and Mathematics, 110*(2), 71–85.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1994). Conceptions of mathematics and how it is learned: The perspectives of students entering university. *Learning and Instruction*, *4*(4), 331–345.

Daempfle, P. A. (2003). An analysis of the high attrition rates among first year college science, math, and engineering majors. *Journal of College Student Retention: Research, Theory and Practice*, *5*(1), 37–52.

De Guzmán, M., Hodgson, B. R., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education. In G. Fischer, & U. Rehmann (Eds.), *Proceedings of the international Congress of Mathematicians* (Vol. 3, pp. 747–762). Berlin: Documenta Mathematica.

Dow, P. B. (1991). *Schoolhouse politics: Lessons from the Sputnik era.* Cambridge, Massachusetts: Harvard University Press.

Hong, Y. Y., Kerr, S., Klymchuk, S., McHardy, J., Murphy, P., Spencer, S., … & Watson, P. (2009). Teachers perspectives on the transition from secondary to tertiary mathematics education. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 241–248). Wellington, New Zealand: MERGA.

Howson, A. G., Barnard, A. D., Crighton, D. G., Davies, N., Gardiner, A. D., Jagger, J. M., ... & Steele, N. C. (1995). *Tackling the mathematics problem*. London: The London Mathematical Society.

Mathematical Association of American and the National Council of Teachers of Mathematics. (2012, April 3). *Calculus: A joint position statement of the Mathematical Association of America and the National Council of Teachers of Mathematics*. Retrieved from http://www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/Calculus%20MAA%20NCTM%20Joint%20Statement.pdf

National Center for Educational Statistics, U.S. Department of Education (2010). *Science and Engineering Indicators: 2010* (NSB Report No. 10-01). Retrieved from https://wayback.archive-it.org/5902/20150818151020/http://www.nsf.gov/statistics/seind10/append/c1/at01-20.pdf

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington DC: Authors.

National Science Board (2014). *Science and Engineering Indicators 2014* (NSB Report No. 14-01). Arlington VA: National Science Foundation.

Noble, J. P., & Sawyer, R. L. (2004). Is high school GPA better than admission test scores for predicting academic success in college? *College and University, 79*(4), 17–22.

Sadler, P. M. (2014, October). Factors influencing college success in mathematics: How do decisions made by high school students and teachers play out in college calculus? [Video file]. Retrieved from http://www.kaputcenter.umassd.edu/events/cs/14-15/index.php?event=1_sadler_100814&vid

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy*, *18*(1), 253–286.

Seymour, E., & Hewitt, N. M. (1994). *Talking about leaving: Factors contributing to high attrition rates among science, mathematics and engineering undergraduate majors*. Boulder, CO: University of Colorado.

Skemp, R.R. (1976). Relational and instrumental understanding. *Mathematics Teaching*, *77*, 20–26.

Zucker, S. (1996). Teaching at the university level. *Notices of the American Mathematical Society*, *43*(8), 863–865.