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Effects of Visual Working Memory Training and Direct Instruction on Geometry Problem Solving in Students with Geometry Difficulties

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We examined the effectiveness of (a) a working memory (WM) training program and (b) a combination program involving both WM training and direct instruction for students with geometry difficulties (GD). Four students with GD participated. A multiple-baseline design across participants was employed. During the Phase 1, students received six sessions of computerized WM training (Jaeggi et al., 2011); during Phase 2, they received six sessions of computerized WM training and six sessions of human-delivered direct instruction with Concrete - Representational - Abstract (CRA) representation sequence on triangle congruence. All four participants remarkably improved their WM, and enhanced their performance on the spatial rotation and general geometric problem-solving. However, the participants' performance on specific triangle-congruence tasks did not improve until direct instruction with CRA sequence was introduced. Practical recommendations were discussed. Practical recommendations of providing cognitive remediation combined with academic interventions were emphasized.

Geometry and spatial senses are fundamental components of mathematics learning (National Council of Teachers of Mathematics, 2004). Geometry interprets and reflects on the physical environment, and is fundamental for learning advanced topics in mathematics, science, geography, architecture, art, design, technology, and engineering in college or postgraduate studies. Learning geometry helps students develop the multiple skills including visual imaginary, conjecturing, deductive reasoning, logical argument and proof. Geometric representations also help students better learn other areas of mathematics, such as the linear representations of the number system, the relationships between the graphs of functions, and graphical representations of data in statistics (Jones, 2002).

Congruence is one of the most important topics that students have to learn in both basic and advanced geometry. Technically, two sets of points are called congruent when, and only when, one can be transformed into the other by an isometry, through a series of motions, such as a translation, a rotation, and/or a reflection (Clapham & Nicholson, 2009). Congruence is fundamental in Euclidean geometry and it is considered analogically similar to the concept of equality of numbers (Clapham & Nicholson). According to CCSSM (2011), in eighth grade students start to learn congruency and the similarity of geometric models. When students enter high school, they are taught skills with a greater level of complexity and depth, for example, students are expected to determine if two triangles are congruent using theorems and postulates.

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There is increased research interest in students with mathematics learning difficulties; however, most of the prior research has focused on numerical mathematics, such as arithmetic and algebra, whereas little research has addressed the specific difficulties in mathematic problem solving for students with geometry difficulties (GD). GD is a specific type of mathematics difficulty (Zhang, Wang, Ding & Liu, 2014). It is not uncommon to observe a percentage of students who demonstrate a discrepancy between their geometry achievement and their scores in other subjects, including other mathematics subjects such as numerical calculation, arithmetic and algebra.

The reasons why students encounter geometry difficulties may be multi-faceted. Individuals' spatial abilities (Clements, 1997) could be one of the most important factors. It has been well established (Clements, Battista, Sarama, & Swaminathan, 1997; Spelke, Lee, & Izard 2010) that spatial abilities highly correlate with students' geometric performance (Geary, 1996; Jeung, Chandler, & Sweller, 1997; Purcell & Gero, 1998; Verstijnen, van Leeuwen, Goldschmidt, Hamel, & Hennessey, 1998), and interestingly this correlation is even greater in students with poorer geometry performance (Battista, 1990). Deficits in visual and spatial working memory were documented to explain the difficulties of students with GD (Passolunghi & Mammarella, 2012). Visual working memory is a visual-spatial storage system to hold and represent information supporting visual-spatial related conceptual and procedural competencies (Baddeley, 2003), and is a significant predictor for student geometry problem solving performance (Geary, 1996; Jeung, Chandler, & Sweller, 1997; Purcell & Gero, 1998; Verstijnen, van Leeuwen, Goldschmidt, Hamel, & Hennessey, 1998). Unfortunately, working memory deficits are found in many children with mathematics learning disabilities (Geary, 2004; McLean, & Hitch, 1999).

However, there has been skepticism about the extent to which working memory capacity can predict students' geometry performance because many educators regard the subject of geometry as a holistic system. Students' experience and logical reasoning have been shown to play a critical role in developing geometric skills, and there is no doubt that inadequate geometry instruction during the elementary and middle school levels (Clements, 2004) is an important reason for students' failure in geometry learning. Battista (1990) reported that spatial visualization and logical reasoning were significantly related to both geometry achievement and geometric problem solving; Saad and Davis (1997) reported that language skills, in addition to spatial abilities, also predicted geometry performance; Lean & Clements (1981) found that not only spatial ability, but the knowledge of spatial conventions had an impact on students' geometric transformations and three- dimensional representations. Battista (1990) reported that high-achieving geometry students used an analytic approach rather than a visual approach to solve geometry problems more often than low-achieving students did, showing the greater influence of analytic skills over spatial abilities. Moreover, the Van Hiele geometric thinking theory, which breaks the geometry learning trajectory into five levels (i.e., visualization, analysis, informal deduction, formal deduction, and rigor), emphasized the importance of experience and logical reasoning in geometry learning. Unfortunately, although researchers recognize the importance of geometry knowledge and reasoning, a systematic review (Browning, Edson, Kimane, & Aslan-Tutak, 2014) suggested that mathematics teachers' overall conceptions in geometry and measurement were limited and weak, with many of them relying on memorized procedural processes.

In sum, as Battista (2007) commented, geometry is a holistic system, "a complex interconnected network of concepts, ways of reasoning, and representation systems that is used to conceptualize and analyze physical and imagined spatial environments" (p. 843), and growing cognitive abilities do not ensure a development of geometric understanding; children need to experience and engage in many varied activities that allow them to explore and construct geometric concepts (Battista, 2007).

Existing Geometry Interventions

One line of research has attempted to improve students' geometry performance via enhancing their spatial abilities. Bergstrom and Zhang (in press) reviewed 32 existing interventions in the U.S. published between 1980 and 2015 and reported 8 studies (Ben-Chaim, Lappan, & Houang, 1988; Boakes, 2009; Clements, Battista, Sarama, & Swaminathan, 1997; Jacobson & Lehrer, 2000; Tentomas, 2010; Zhang, Ding, & Mo, 2012; Zhang, Wang, Ding, & Liu, 2014) that focused on enhancing students' spatial skills. Two studies (Zhang et al., 2012; Zhang et al., 2014) explored the effects of a chunking strategy to help students improve their visual working memory and found this approach to be effective on measures of basic geometric tasks that required visual rotation and transformation. Clements et al. (1997) developed a curriculum called Flips, Turns, and Area, and reported that students improved their spatial abilities from the pretest to the posttest. Jacobson and Lehrer (2000) provided teacher training on understanding student reasoning on geometric designs and transformational geometry (rotations, reflections, and compositions), and reported that students of teachers who received the training showed greater learning and retention than other students on a measure of spatial tasks (i.e., eight problems involving flips, turns, and composition of motions). Ben-Chaim and colleagues (1988) developed a spatial visualization unit, and results reported significant effects on a spatial visualization test, whereas non-significant effects were found on a comprehensive mathematics achievement test. In addition to these successful results, two interventions were less successful. Boakes (2009) developed and evaluated a curriculum named Origami (the art of paper folding) mathematics lessons, involving extra lessons in varying paper folding activities; however, results did not find significant improvement on any measures between the experimental and the control groups. Tentomas (2010) also reported non-significant gains in his study where a 10-day curriculum, which aimed to help students improve spatial transformation skills through lectures and other activities, was employed.

The other line of research on geometry instruction focuses on the enhancement of student geometry content knowledge and reasoning skills. In the systematic review by Bergstrom and Zhang (in press), the majority of interventions (27 out of 32) targeted improving students' geometry knowledge and problem-solving strategies. Among the identified seven interventions for students with learning disabilities (Cass et al., 2003; Satsangi & Bouck, 2015; Strickland & Maccini, 2012; Worry, 2011; Xin & Hord, 2013; Zhang et al.,2012; Zhang et al.,2014), five (Cass et al.,2003; Satsangi & Bouck, 2015; Strickland & Maccini, 2012; Worry, 2011; Xin & Hord, 2013) focused on instructional content knowledge, strategies or technology and reported positive results. Three emphasized teaching area and perimeter (Cass et al., 2003; Satsangi & Bouck, 2015; Xin & Hord, 2013) using specific instructional strategies or technology and found students improved their performance. Worry (2011) compared project-based instruction with traditional lecture-based instruction in students who were at risk or were identified with disabilities, with a focus on trigonometric ratios, and the laws of sines, cosines, and vectors, and reported significant effects. Strickland and Maccini (2012) used the concrete-representational-abstract representation sequence to teach linear algebraic expressions to students with learning disabilities and reported satisfactory results. There are also a couple of studies that helped students to learn similarity and congruence through specific strategies: Seago et al. (2014) provided professional training to teachers about teaching geometric similarity, through presenting teachers with modules of videocassettes and encouraging them to use specific instructional strategies; and Pulley (2010) taught triangle congruence theorems by encouraging students to generate conjectures and emphasizing on exploration and discussion. Both studies reported positive results on students' scores of specific similarity/congruence tests.

In summary, there are two lines of research that have focused on helping students to learn geometry either through improving their cognitive spatial abilities, or through teaching specific geometric knowledge and problem-solving skills, and both lines of research seem to be effective. From the Bergstrom and Zhang (2016) review, it was noted that most interventions on improving cognitive spatial abilities were more likely to report effectiveness when basic spatial assessments were employed as outcome measures, while the interventions on improving geometry knowledge and strategies were more likely to use content-specific assessment as outcome measures.

Theoretical Framework

Considering the importance of development of spatial abilities, along with geometry content knowledge and reasoning skills, this study was aimed to provide interventions targeting enhancement of student skills on both domains. The framework for designing interventions was guided by emerging research on working memory training in cognitive psychology fields and on direct instruction in special education.

Working memory training. According to cognitive load theory (Sweller, 1994), many students experience learning difficulties because the problem-solving process demands more cognitive capacity than they can afford. Therefore, it is crucial to improve individuals' cognitive capacity and to ensure that instructional tasks and materials do not overload the cognitive capacity of students who have deficits in working memory. In recent years, cognitive psychologists have found participants' working memory scores can be improved through training (Jaeggi, Buschkuehl, Jonides, & Perrig, 2008; Jaeggi, Buschkuehl, Jonides, & Shah, 2011; Jausovec & Jausovec, 2012; Klingberg et al., 2005, Klingberg, Forssberg, & Westerberg, 2002; Olesen et al., 2003). Different types of training programs, such as Cogmed training (Bergman Nutley et al, 2011; Dahlin, 2011), Jungle memory (Alloway & Alloway, 2009), n-back training (Jaeggi et al., 2008; Jaeggi, Studer-Luethi, et al., 2010; Li et al., 2008; Seidler et al., 2010), and running span training (Dahlin, Neely, et al., 2008; Dahlin, Nyberg et al., 2008; Zhao, Wang, Zhou, Wang, & Tan, 2011), generally reported positive results in improving individuals' working memory scores.

However, researchers have generally criticized the transfer effects of working memory training programs on improving students' academic achievement (Shipstead, Redick, & Engle, 2012). Some studies reported that participants were unable to transfer the improved working memory abilities to enhance their academic performance (Holmes et al., 2009; Horowitz-Kraus & Breznitz, 2009; Van der Molen et al., 2010). Nonetheless, among the varying training methods, the n-back training seemed to be more promising (Shipstead et al., 2012) regarding the transfer effects: Au et al. (2015) conducted a meta-analysis focusing on n-back working memory training studies for adults and reported small but significant transfer effects. Therefore, it is plausible to hypothesize and empirically examine that when using working memory training with an appropriate paradigm (i.e., n-back), there may be generalization effects on certain academic tasks, such as geometry problems, which are closely related to visual working memory.

Explicit instruction and CRA representation sequence. Special education literature has found explicit and direct instruction to be more effective for students with learning disabilities (Gersten et al., 2009). In explicit instruction, a teacher typically demonstrates step-by-step modeling of a strategy for solving a problem, and students are required to use the same procedures/steps demonstrated by the teacher to solve the problem. The concrete-representational-abstract (CRA) sequence, also called the concrete-semi concrete-abstract sequence, is commonly used together with explicit instruction. Teachers who use the CRA sequence first model the problem with actual objects such as toys, fruits, cubes, base-ten blocks, or fraction tiles to represent the problem scenarios. When students have demonstrated mastery using concrete materials, the concept is then represented with semi-concrete materials such as diagrams. At an abstract level, teachers use numbers and symbols to teach problem solving. The purpose of CRA is to ensure that students have a solid understanding of the mathematics concepts before progressing to abstract operations. It has been widely used for teaching a variety of mathematic topics, including arithmetic calculations (Harris, Miller, & Mercer, 1995), fractions (Joseph & Hunter, 2001), algebra (Maccini & Hughes, 2000; Strickland, & Maccini, 2012), area and perimeter (Cass, Cates, Smith, & Jackson, 2003).

Research Questions

This study aimed to examine the effects of (a) an n-back visual working memory training program, and (b) a combination program of n-back visual working memory training and direct instruction on the specific topic of triangle congruence on enhancing the geometry problem-solving performance of students with GD. The topic of triangle congruence was chosen because triangle congruence problems can be solved either with visual rotations which is cognitive-demanding, or with triangle congruence postulates which demand proficient understanding and deductive reasoning with specific geometric content knowledge. Therefore, if the n-back visual working memory training is able to result in fundamental spatial working memory enhancement, then there should be comprehensive improvements in performance on different types of spatial and geometry tasks, including specific triangle congruence tasks.

METHOD

Design

An adapted multiple probe design (Horner & Baer, 1978) across participants was employed to establish a functional relationship between the intervention and students' performance. With the multiple probe design, data are collected intermittently during the pre-intervention probe condition. Specifically, when a stable baseline on the n-back working memory program was observed for one participant, the intervention was introduced to this participant while other participants remained in the baseline condition.

When improvement on the n-back working memory program for Participant A was observed, Participant B was introduced to the intensive working memory training, and when improvement on the n-back program for Participant B was observed, Participant C was introduced to the intensive working memory training. In this design, replication of treatment effects from the second, third, and fourth participants was demonstrated if changes in performance occurred only when treatment was introduced.

Two phases were included in the intervention. To examine the first research question, the effects of working memory training alone, during Phase 1 participants received only the working memory training. To answer the second research question, the effects of the combination program, during the second phase the participants were involved in a combination program of working memory training and explicit instruction with a CRA sequence on triangle congruence postulates. In the first phase, participants received six sessions of intensive n-back working memory training, each lasting about 30 minutes; in the second phase, participants received six sessions of n-back working memory training and six sessions of explicit instruction on triangle congruence.

Participants

Six college students who reported difficulties in learning mathematics in high school initially signed up to participate in this study and the four who met our inclusionary criteria were selected to participate. All six participants were enrolled in a remediation program for undergraduate students who failed the college-screening test and thus had to take developmental mathematics courses before being eligible for regular college-level math courses. Participant recruitment was based on the recommendation of the coordinator of the remediation program. All participants reported that they had experienced geometry difficulties in middle school and high school.

Four out of the six students met the assessment criteria: (a) scoring under the 30th percentile rank in the KeyMath geometry subtest; and (b) scoring under the 30th percentile rank in the SAT math test. Student demographic information can be found in Table 1. Three out of the four participants reported that they received special education services to learn mathematics during high school. Among the four participants, their SAT scores in mathematics were from 17th to 26th percentile rank, and their Keymath Geometry Subtest percentile ranks were 1%, 9% and 25% and 25%.

Table 1. ***Demographic Information of Participants***

Variable	Tanya	Nicky	Monica	Megan
Gender	Female	Female	Male	Female
Ethnicity	African American	African American	Hispanic	African American
Age	19	19	19	19
University Screening Test In Math	Failed	Failed	Failed	Failed
SAT in Math	420 (20%)	NA	410 (17%)	440 (26%)
KeyMath Geometry Subtest	9%	1%	25%	25%

Measures

To examine the intervention effects on different measures with varying direct/indirect, close/far relations with the cognitive task that we trained, we adopted three different types of measures to examine the effectiveness of the intervention, and each category included two types of instruments as below.

Measures of students' visual working memory. The first category of measures directly assessed participants' visual working memory. First, the level of performance in the n-back program was recorded as a measure of student visual working memory capacity; for example, all students started at the 2-back level, and gradually increased to 3-back, 4-back, and so forth. Second, the accuracy of each session was recorded automatically by the computer program; for instance, a student performed at 30% during the first trial of the 2-back program, so his accuracy was 30% and his or her level was 2 for this trial.

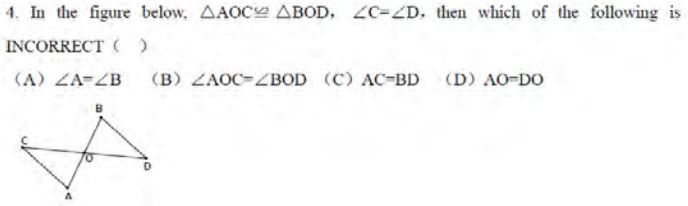
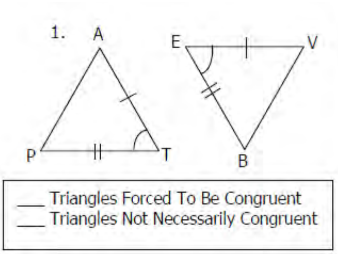
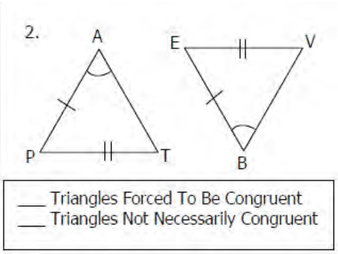
Measures of students' general visual spatial geometry problem-solving abilities. The second category of measures assessed participants' basic spatial abilities and general geometry problem-solving skills that are closely related to visual spatial abilities, and were used as near-generalization measures. Two standardized instruments were used: (a) Revised Purdue Spatial Visualization Tests: Visualization of Rotations (PSVT: R, Guay, 1976); (b) KeyMath-3 Geometry Subtest (Connolly, 2007). The PSVT: R is a well-validated prevalently used instrument especially in STEM research (Meada & Yoon, 2013) with sound properties (Yoon, 2011), for example, Cronbach's alpha was 0.84 (Maeda, Yoon, Kim-Kang, & Imbrie, 2013). The KeyMath Geometry Subtest (Connolly, 2007) is one of the Basic Concept subsets and includes 36 items. It assesses student early geometric awareness, two-dimensional and three-dimensional shapes, lines and angles, formulas, grids, and coordinate planes. The KeyMath basic concept subtests have been proved with a sound reliability (e.g., The alternate form reliability is 0.94) and validity (e.g., the intercorrelation between the total test and the basic concept subtests is 0.98; the correlation with Kaufman Test of Educational Achievement (II) is 0.85 for the basic concepts and mathematics).

Table 2. ***Results of Phase 1 and Phase 2 Interventions***

		Baseline	Probe after Phase 1	Probe after Phase 2	Maintenance Test
Tanya	Triangle Congruence Problem Solving	54%	41.67%	83.33%	N/A
	Triangle Congruence Identification	67%	75%	100%	N/A
	KeyMath Geometry	9%	75%	75%	N/A
	Visual Spatial (PSVT:R)	40%	73.35%	70%	N/A
Monica	Triangle Congruence Problem Solving	46.67%	46.67%	83.33%	100%
	Triangle Congruence Identification	41.67%	33.33%	93.33%	66.67%
	KeyMath Geometry	25%	50%	63%	50%
	Visual Spatial (PSVT:R)	23.33%	31.15%	36.67%	N/A
Nicky	Triangle Congruence Problem Solving	58%	53%	66.67%	80%
	Triangle Congruence Identification	33.33%	33.33%	66.67%	83.33%
	KeyMath Geometry	1%	16%	16%	25%
	Visual Spatial (PSVT:R)	46.67%	53.50%	43.33%	NA
Megan	Triangle Congruence Problem Solving	50%	33.33%	100%	100%
	Triangle Congruence Identification	58.33%	66.67%	66.67%	66.67%
	KeyMath Geometry	25%	37%	37%	63%
	Visual Spatial (PSVT:R)	56.56%	66.67%	83.33%	N/A

Measures of students' specific triangle congruence problem-solving abilities. Adapting items from mathematics texts and supplemental materials, the author developed two tests for assessing students' specific knowledge and problem-solving skills on triangle congruence: (a) the Triangle Congruence Identification Test, and (b) the Triangle Problem Solving Test. The Triangle Congruence Identification Test included 12 items that asked students to determine, according to given conditions, whether the two presented triangles were congruent. This test was designed to assess student basic knowledge and application of the five postulates of triangle congruence (i.e., AAS, SAS, SSS, ASA, and HL). The Triangle Problem Solving Test was designed to assess students' problem-solving skills of triangle problems that required knowledge of triangle congruence. Example items for these two tests can be found in Table 3. These two tests were administered during the baseline, posttest and maintenance test as generalization tests.

Table 3. ***Example Items from Triangle Problem Solving Test and Triangle Congruence Identification Test***

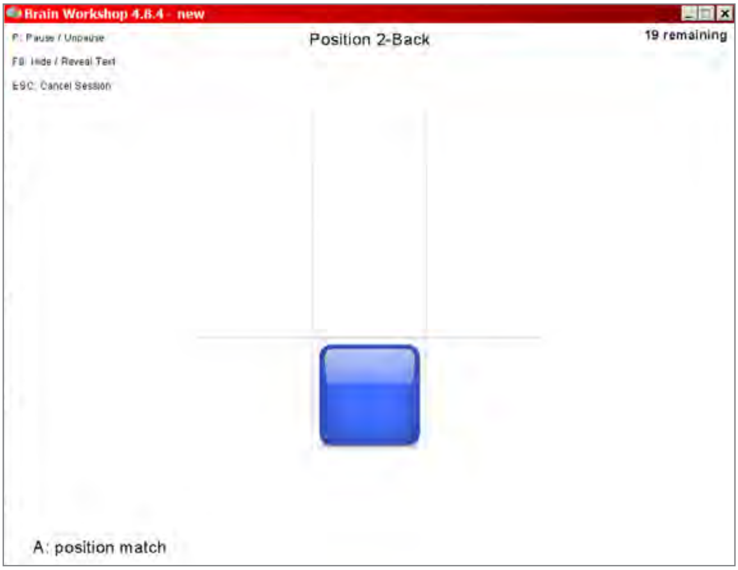
Assessment Instrument	Example Items	
Triangle Problem Solving Test	 4. In the figure below, $\triangle AOC \cong \triangle BOD$, $\angle C=\angle D$, then which of the following is INCORRECT ( ) (A) $\angle A=\angle B$ (B) $\angle AOC=\angle BOD$ (C) $AC=BD$ (D) $AO=DO$ 	
Triangle Congruence Identification Test	 ___ Triangles Forced To Be Congruent ___ Triangles Not Necessarily Congruent	 ___ Triangles Forced To Be Congruent ___ Triangles Not Necessarily Congruent

Procedures

The whole intervention program included two phases. During the Phase 1 intervention, students received six sessions of intensive n-back visual working memory training (Jaeggi et al., 2011); during Phase 2, they received both the computerized working memory training (six sessions) and human-delivered direct instruction with the CRA sequence on triangle congruence (six sessions) that explicitly taught triangle congruence postulates. The author and two trained research assistants implemented the trainings with the four participants.

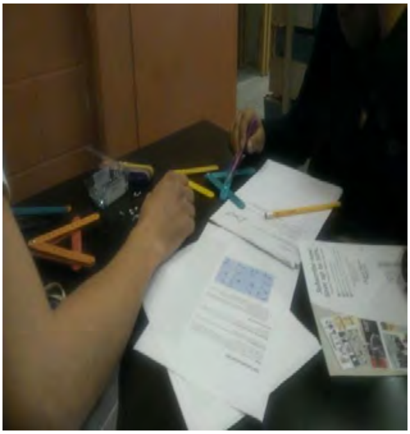
Phase 1. The aim of this phase of intervention was to examine the effects of an intensive working memory training program on improving participants' visual working memory capacity, and to determine whether the improvement of visual working memory could be generalized to improved geometry problem-solving performance. The four participants received a computerized n-back training program described by Jaeggi et al. (2011). A screen shot of the program can be found in Figure 1. Specifically, each participant was presented with a random sequence of visual stimuli (e.g., squares) in different cells that are located in different areas of the screen, and the participant decided whether the present stimulus was at the same location as the one *n* stimulus back. For example, if the first stimulation appears in the center cell of the screen, the second appears in the right top cell, the third appears in the center cell of the screen, the fourth appears in the left bottom cell…. So in a trial of 2-back level tasks, the participants have to decide if the third simulation appeared in the same cell where the first one did, if the fourth simulation appeared in the same cell where the second one did, if the fifth simulation appeared in the same cell where the third one did, and so forth. If it is a task at the 3-back level, then the participant have to decide if the fourth simulation appeared in the same cell as the first one did, if the fifth stimulation appeared in the same cell as the second one, and so forth. Every time when a stimulation appears in the screen (starting from the $(n+1)^{th}$ stimulation in an n-back task), participants were required to respond by pressing two different keys on the computer keyboard to indicate Yes or No. Each trial typically contained 20 -25 stimuli. Participants were allowed to take a break between trials. If a participant achieved over 80% correct in a trial for two out of three successive trials, then the participant moved to a more difficult level (e.g., from 2-back to 3-back). All students started from the 2-back level in the baseline assessment. The participants worked on this project for two or three sessions per week, 50 minutes per session, and six sessions in total.

***Figure 1.* A screen shot of the n-back visual working memory training. The program can be downloaded from http://brainworkshop.sourceforge.net**

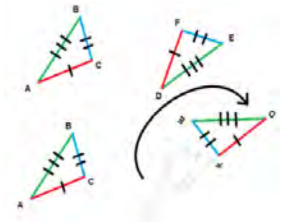


Phase 2. During this phase, each participant received a combination of the n-back working memory training and the direct instruction of postulates on triangle congruence with CRA sequence. First, the participants continued with working memory training for two sessions per week for three weeks, starting from the level where they ended during Phase 1. All procedures of the n-back training during Phase 2 were identical to those in Phase 1. Additionally, we added six sessions of direct instruction of geometry content with the CRA sequence specifically on the topic of triangle congruence. Five sessions were conducted on each of the five postulates of triangle congruence: AAS, SAS, SSS, ASA, and HL. We also provided a sixth session of comprehensive review on the five postulates, with the aim to help students choose the suitable postulate from the five to prove a congruence problem.

Figure 2. **Using the Concrete-Representational-Abstract sequence to teach the Side-Side-Side postulate.**



Concrete



Representational

Abstract: Side-Side-Side postulate states that if three sides of one triangle are congruent to three sides of another triangle, then these two triangles are congruent.

We integrated a CRA instructional sequence to teach each of the five postulates. The CRA instructional sequence of teaching the "Side-Side-Side (SSS)" postulate is illustrated in Figure 2. First, at the concrete level, the experimenters provided with parts (i.e., sticks) that can make triangles, then asked the participant to assemble a triangle in specific given conditions (e.g., a stick with a length of 4 units, one with a length of 3 units, and the other one with a length of 5 units on each side respectively). Then the experimenter instructed the participants to assemble a second triangle under the same conditions (i.e., using three sticks of the same lengths), and encouraged the participants to try as many approaches as possible to make different triangles under the given conditions. Afterwards, the instructor guided the participants to conclude that one cannot make two different triangles with the three given side lengths. Second, at the representational level, the instructor used visual diagrams to demonstrate with appropriate flipping and rotation procedures, two triangles with the congruent side lengths overlapping with each other. Lastly, at the abstract level, the instructor helped students to state the postulate verbally.

After introducing the postulate with the CRA sequence, the instructor adopted an explicit modeling - guided practice - independent practice sequence to help the participants to solve geometry problems using the postulate that was just taught. First, the instructor explicitly modeled three problems, including both triangle congruence proof problems, or explorative triangle problems that required a step of triangle congruence proving (e.g., to find out the degree of an angle or the length of a side of a triangle, students had to first prove this triangle was congruent with another triangle of which the angle/side information was given or could be calculated). Next, the instructor asked the participants to solve another three problems, and provided guidance or cues when they needed help. For example, the instructor may ask "how can we decide if two triangles are congruence" "Can you find any pairs of sides/angles that are the equivalent?" Finally, we asked the participants to independently work on three problems.

Two trained research assistants implemented the instruction to the four participants. One research assistant was a master student in a mathematics education program, with a bachelor degree in mathematics, whereas the other research assistant was a psychology major. Before implementing the interventions, both research assistants thoroughly reviewed the middle school geometry curriculum on triangle congruence and related properties, and worked on a variety of triangle problems on their own to make sure they had adequate content knowledge and problem solving skills to teach the lessons. The two instructors also practiced teaching the lessons with the CRA sequence before teaching it to the participants with GD.

Treatment Fidelity

The author and two research assistants developed a teaching script and a lesson plan for each of the six tutoring sessions on triangle congruence. The teaching script and the lesson plan were used to guide explicit instruction with a CRA instructional sequence during the intervention on triangle congruence postulates as shown in the appendix. The author observed 30% of treatment sessions to assess fidelity or quality of implementation of specific performance indicators. In addition, a checklist of the instructional steps was developed to assess the instructor's adherence to the instructional components, which was judged on the presence or absence of the features listed on the fidelity checklist. These components were included if the instructor followed the CRA sequence, if the instructor explicitly modeled three problem-solving processes, if the instructor guided students to solve problems, if the instructor provided feedback appropriately, and so forth. Treatment fidelity was calculated as the percentage of steps correctly completed and was 100%.

Inter-rater Reliability

Inter-rater reliability was checked by an undergraduate research assistant who was unaware of the purpose of this study. Participants' working memory level and accuracy in the n-back program was automatically recorded in the computer, so we only checked the reliability of data of the four generalization tests (i.e., the KeyMath Geometry Subtest, the PSVT: R, the Triangle Problem Solving Test, and the Triangle Congruence Identification Test). The research assistant randomly chose 30% of the tests, and re-scored all the items of the selected tests. The inter-rater reliability was 100%.

RESULTS

Results with single-subject design graphing and visual analyses were presented to evaluate the effectiveness of Phase 1 and Phase 2 interventions. We found that (a) with the completion of the n-back visual working memory training, the participants enhanced their performance on the measures of visual working memory span and on the measures of overall geometry abilities (visual-spatial cognitive test [PSVT:R] and the KeyMath Geometry Subtest; and (b) the participants' improvement on measures of specific triangle-congruence knowledge and problem-solving skills did not occur until the Phase 2 direct instruction with CRA sequence was introduced.

Baseline

All participants started from the 2-back level in the working memory training program. They scored poorly on both the two measures of general geometry abilities. Specifically, none of the four participants exceeded the 25th percentile on the Geometry Subtest of KeyMath. Their performance on the PSVT: R was low to average (i.e., 40%, 23%, 46%, and 56%). They also scored low on the two specific triangle congruence tests: Their problem solving accuracy ranged between 47% and 58% on the Triangle Problem Solving Test, and between 33% and 67% on the Triangle Congruence Identification Test.

Phase 1 Effects of the Working Memory Training Alone

Improvement in visual working memory. All four participants steadily improved their performance on the working memory training program. All participants started from the 2-back level, and when students achieved over 80% correct for two out of three consecutive trials, they moved to the next level. By the end of the Phase 1 with the completion of six sessions of working memory training, one participant (Nicky) improved to the level of 6-back, and three participants (Tanya, Monica, and Megan) improved to the level of 5-back.

Improvement in spatial ability and general geometric problem solving. All four participants demonstrated some improvement on both the general geometry problem solving test (i.e., KeyMath Geometry Subtest) and on the spatial ability test (i.e. PSVT: R). Specifically, on the KeyMath Geometry Subtest, Tanya improved from 9% during the baseline to 75% after Phase 1 intervention, Monica improved from 25% to 50%, Nicky improved from 1% to 16%, and Megan improved from 25% to 37%. On the visual rotation test PSVT: R, Tanya improved from 40% to 73%, whereas the other three students maintained at approximately the same level: Monica from 23% to 31%, Nicky from 47% to 53%, and Megan from 57% to 67%.

Improvement on specific triangle congruence tests. Results suggested that the four participants did not make remarkable improvement on either of the two tests assessing students' specific triangle congruence knowledge or problem-solving skills. On the Triangle Congruence Identification Test, Tanya (67% to 75%) and Megan (58% to 67%) somewhat improved in accuracy, Nicky stayed at the same level of accuracy with 33% correct, whereas Monica somewhat decreased her accuracy from 42% to 33%. On the Triangle Problem Solving Test, none of the four participants showed improvement. In sum, it seemed that the visual working memory training during Phase 1 did not make a notable difference in students' performance on specific triangle congruence knowledge or problem-solving performance.

Phase 2 Effects of the Working Memory Training Combined with Direct Instruction and CRA Sequence

Improvement in visual working memory training. All participants showed an improving trend during the second phase of working memory training. On the posttest by the end of the Phase 2 intervention, Nicky improved from 6-back to 9-back, Monica and Megan improved from 5-back to 6-back, and Tanya improved from 5-back to 7-back.

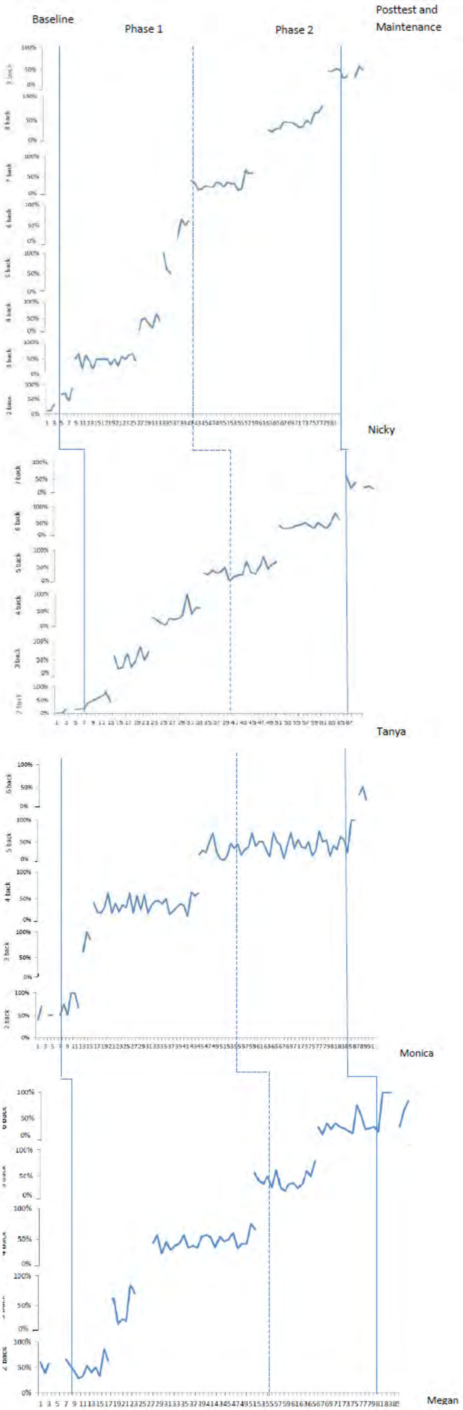
Improvement in spatial ability and general geometric problem solving. All four participants maintained a similar percentile rank on the KeyMath Geometry Subtest in Phase 2. Tanya maintained in the 70th percentile. Nicky maintained in the 16th percentile and Megan maintained in the 37th percentile. Only Monica improved from the 50th percentile rank to the 63rd percentile rank. Similar results were also found on the PSVT: R. Tanya and Monica maintained their percent correct at 70% and 37%, respectively. Megan somewhat improved from 67% to 83%, whereas Nicky decreased her accuracy from 53% to 43%.

Improvement on specific triangle congruence tests. Remarkable improvement in problem-solving accuracy was found after the Phase 2 intervention across all four participants on both the specific Triangle Congruence Identification and Triangle Problem Solving Tests. On the Triangle Congruence Identification Test, Tanya improved from 75% accuracy correct to 100%, Monica from 33% to 93%, Nicky from 33% to 67%, and Megan maintained at 67% from the end of Phase 1 to the end of Phase 2 intervention. On the Triangle Problem Solving Test, Tanya and Monica both improved their accuracy from 42% to 83% and 47% to 83% correct, respectively. Megan improved from 33% to 100% and Nicky improved from 53% to 67%.

Maintenance Effects

Researchers administered a maintenance assessment with all three categories of instruments two weeks after the termination of the Phase 2 intervention. All four participants maintained a high level of performance on the n-back working memory training program during the maintenance test (e.g., Nicky maintained at the 9-back level, Tanya maintained at the 7-back level, and Megan and Monica maintained at the 6-back level). Unfortunately, due to scheduling issues (i.e., summer started and students left the university after finals), one participant (Tanya) was unable to take any of the four generalization tests, Megan only completed the two specific triangle congruence tests, and the other two participants (Nicky and Monica) completed three of the four generalization tests except the PSVT:R. Monica, Nicky, and Megan maintained a satisfactory level on both the two specific triangle congruence tests (100%, 80%, and 100% correct on the Triangle Problem Solving Test, and 66.67%, 83.33%, and 66.67% on the Triangle Congruence Identification Test, respectively) and the KeyMath Geometry Subtest (Monica maintained at the 50th percentile rank, and Nicky showed some improvement to the 25th percentile).

Figure 3. **Participants' progress on the n-back visual working memory training**



Social Efficacy

To examine the social validity of the working memory training program during Phase 1 and the combination program during Phase 2, the researchers developed a questionnaire to assess students' evaluation of the program. Participants were asked to respond to statements about the helpfulness of the two phases of interventions. The following three statements evaluated the n-back working memory program: "I think the working memory training program is helpful," "I think the working memory program is interesting," and "I will recommend this working memory training program to my friends who also have difficulties with working memory problems." A 5-point scale was used, in which 1 = *strongly disagree*, 2 = *disagree*, 3 = *neutral*, 4 = *agree*, and 5 = *strongly agree*. Three participants (Monica, Nicky, and Tanya) scored a median of 4 on these three items, whereas the other participant (Megan) scored a median of 3 points.

Participants' evaluation of the direct instruction program included the following five statements: "I like the tutoring program," "I think it is helpful that we reviewed the related theorems and properties during sessions," "I like the manipulatives which helped me better understand the triangle postulates," "I like the direct instruction in which the tutor explicitly taught me the steps of problem solving," and "I feel the guided practice is helpful." All four participants reported a median score of 4.5.

Discussion

In this study we examined the effects of visual working memory training on students' working memory skills and explored if working memory improvement could be generalized to students' general visual spatial abilities, general geometry problem- solving performance, and specific triangle congruence problem-solving skills. We also investigated the effects of a combined program of visual working memory training and direct instruction with CRA sequence on improving the geometry performance in students with GD. As we hypothesized, all four participants demonstrated remarkable increased scores on the *n*-back working memory training. Some enhancement of performance was also found on the basic visual spatial rotation task (PSVT:R) and on general geometric test (i.e., the KeyMath Geometry Subtest). However, students did not demonstrate gains in specific geometry tasks until direct instruction with CRA sequence was introduced, suggesting that the increased scores in the n-back working memory program did not automatically transfer to solving specific geometric problems.

Participants' improvement during the working memory training was consistent with the literature (Jaeggi et al., 2011; Klingberg, 2010) showing that working memory can be improved through training. More importantly, our study contributes to the literature by providing evidence for the differential generalization effects of the working memory training on improving different types of geometry tasks. In the literature, the improvement of working memory has been questioned regarding whether an improved score on the n-back working memory program itself is a real indicator of an enhancement of working memory capacity; rather, the increased scores on the working memory program may be the result of participants discovering certain test-taking strategies or merely because of practice effects (Owen et al., 2010). The critics claim that if students really have improved their cognitive capacity, which is the foundation for many academic problem- solving activities, then they should be able to naturally demonstrate an improvement in a very broad range of skills and abilities. In this study, we found very different levels of enhancement in different types of generalization tasks: some improvement on basic and general geometric tasks that required less geometry content knowledge (i.e., the PSVT:R and KeyMath Geometry Subtest), but very little improvement on the measures that required specific geometric content knowledge (i.e., Triangle Congruence Identification Test or the Triangle Problem Solving Test). In conclusion, results shows that when discussing the generalization effects of the working memory tasks, one should be cautious and clarify what kind of generalization tasks were employed and to what extent the generalization tasks were correlated with the trained cognitive ability such as working memory.

The improvement on the basic spatial skills and the general geometry test on the Phase 1 posttest indicated the optimism that the n-back working memory training was very likely to have transfer effects on these geometric or visual spatial tasks that are highly associated with visual working memory skills. Like a number of previous studies (Brinkmann, 1966; Onyancha, Derov, & Kinsey, 2009; Sorby, Casey, Veurink, & Dulaney, 2013) suggested, spatial ability is a skill that can be developed through practice, and this study shed light on the possibility of improving students' basic spatial abilities through cognitive training programs. Although in this study we were unable to identify the underlying mechanism of working memory improvement during the Phase 1 training, it is plausible to assume that the intensive practice on holding visual information during the training program may have helped the participants biologically improve their brain function (e.g., Dahlin, Nyberg, Bäckman & Neely, 2008; Hempel et al., 2004), or the participants successfully developed certain strategies (Morrison & Chein, 2011), such as rehearsal (Ford, Pelham, & Ross,1984; Turley-Ames & Whitfield, 2003) or elaborative encoding (Carretti, Borella, & De Beni, 2007; Cavallini, Pagnin, & Vecchi, 2003), to help them with encoding, maintenance, and/or retrieval from working memory. In turn, they may have transferred these strategies to items in the basic spatial and geometry tasks where working memory was highly required for problem solving. Future research is warranted to further explore the mechanism of the training effects and accordingly design working memory training programs with greater effectiveness.

The unsuccessful generalization effects on the two specific tests on triangle congruence during Phase 1 may imply that for specific geometry tasks that highly require content knowledge, pure cognitive training is inadequate for making significant improvement on student performance. In contrast, a notable finding after Phase 2 was that once the explicit tutoring of the triangle congruence postulates was introduced, all four participants showed remarkable improvement on the two specific triangle congruence tests. Results suggest that teaching geometry content knowledge and deductive reasoning skills is even more directly helpful than basic cognitive visual working memory training for affecting students' performance on a specific geometric subject. As previous research (Battista, 1990) suggested, the balance between visual-spatial and verbal-logical thinking is critical for geometry learning. Battista (1990) found students with GD relied more on visual spatial skills to solve geometry problems than their normal-achieving peers, whereas the high-achieving students show a higher correlation between verbal logical reasoning and geometry achievement than students with GD. In this study, we found merely improving visual spatial skills might be inadequate for improving student performance on specific geometry subjects. Therefore, teaching content knowledge and logical reasoning is especially important for students with GD. In addition, this study contributed to the literature through an innovative extension of the CRA instructional sequence and explicit instruction from teaching numerical or algebra subjects to teaching advanced geometry topics. In future research, we will investigate if working memory training influences students' improvement by comparing the effectiveness of a combination program (i.e., providing both working memory training and explicit instruction) and the explicit instruction-only program.

Limitations and Future Research

This study has a few limitations. First, the single-subject design of this study limited the generalizability of the conclusions to a larger population of students with GD. Results need further validation by randomized controlled trial research. Second, it remains unclear whether working memory training played a role in boosting the effects of explicit instruction on triangle congruence problem solving, and future research is warranted to examine whether an integration program that provides both working memory training and explicit content instruction yields greater effect sizes than a program that provides only explicit content instruction. Third, given that working memory is a task-specific cognitive ability involved in specific mathematics problem-solving procedures (Peng & Fuchs, 2014), it is plausible that specific geometry working memory training, rather than general working memory training such as the n-back paradigm, could be more effective in enhancing student geometry problem solving. We are currently designing procedure-specific working memory training activities that imitate the problem-solving procedures in which working memory is involved in specific mathematics tasks; for example, for improving arithmetic skills, we train students to visually imagine the numerical operations (i.e., imagining vertical or columnar addition, etc.); for geometric problems, we train students to practice on visualizing and mentally manipulating the geometric shapes and relations, especially in three-dimensional geometry problems. Results of effects of these problem-solving-procedure-specific working memory training programs are forthcoming.

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