academicJournals

Vol. 12(8), pp. 531-539, 23 April, 2017
DOI: 10.5897/ERR2017.3220
Article Number: A0BE00464006
ISSN 1990-3839
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Educational Research and Reviews

Full Length Research Paper

Identifying the concept "fraction" of primary school students: The investigation in Vietnam

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Received 17 March, 2017; Accepted 13 April, 2017

In Vietnam, primary school students explicitly learn the concept of fraction in Grade 4 and 5. Because this concept is introduced to them intuitionally, it is difficult for them to understand and apply it. Base on this point, we believe that the students will commit many errors when solving exercises related to this concept. The survey of 478 students showed that some remarkable errors were made by most students; when learning fractions, students still did not understand the nature of fractions as well as the equality of parts. Therefore, teachers should note the errors of students, and use the effective pedagogical measures to help them prevent, and correct the errors.

Key words: Fraction, teaching and learning fraction, mathematics in primary schools, mathematics education.

INTRODUCTION

The research papers of fractions were made by a lot of researchers around the world. Some authors showed that students find it difficult to understand fraction concepts. Stafylidou and Vosniadou (2004) tested 200 students by using a questionnaire to investigate the development of their understanding of the numerical value of fractions. The two researchers asked them to decide on the smallest/biggest fraction, to order a sequence of fractions, and to explain their answers. The results showed that students did not adopt the scientific concept of the fractions immediately. In particular, they gave their answers revealing their misconceptions. Prediger (2006) investigated students' difficulties with fractions in German schools. Most students wrongly found that multiplication makes it bigger.

Furthermore, they found it unacceptable to multiply a fraction by another. The reason for these misconceptions was that students were influenced by the knowledge of natural numbers. A survey of difficulties related to fractions in Primary School Leaving Examination was carried out by Ndalichako (2013).

The findings indicated that a large number of candidates were unable to do the operations with fractions because of their confusion of fractions with natural numbers. Indeed, they tended to treat numerators and denominators as separate entities. The reasons for these difficulties were lack of

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understanding of appropriate procedures to apply in solving a problem, the complexity of the tasks and over-generalization of procedures even in situations. In addition, the author proposed some relevant teaching methods to facilitate meaningful learning of fractions.

Besides, works related to the difficulties of students in learning fractions, many researchers studied the students' errors related to fractions. Lukhele et al. (1999) had a specific investigation of errors of students when they did the addition of fractions. The reasons for these errors were a weak understanding of the fraction concept, a very common limiting construction arising from learners' experience with whole numbers and the set algorithms which are taught for whole number arithmetic. The results of the study highlighted the fact that wrong strategies for adding fractions were applied by most students. Also, the researchers suggested some ways to deal successfully with the addition of fractions without grasping at rules and algorithms. Two Bruneian authors, Yusof and Malon (2003) carried out an interesting study of students' errors in the use of fractions. The main result was that five common kinds of errors were committed by 396 pupils in primary schools in Brunei. The researchers also pointed out the reasons for those errors.

For example, students' prior knowledge of learning natural numbers had a negative influence on their understanding of fractions and their operations. Moreover, they had lack of not only the basic knowledge but also the skills to do operations with fractions. Idris (2011) found out some kinds of errors made by 80 students in a secondary school in Malaysia when they did the operations of addition and subtraction of fractions.

In this study, three identified errors were careless errors, negligence errors and systematic random errors. More specifically, some students had problems such as: converting to the lowest common denominator, understanding fractions and dealing with improper fractions.

In order to increase the effectiveness of teaching fractions, several authors developed strategies for teaching fraction concepts. Amato (2005) proposed some ways to help students to understand fractions as an extension to the number system. As a result, it was very beneficial for students to use multiple representations for fractions equal to one unit and mixed numbers. Nicolaou and Pitta-Pantazi (2012) developed a new model for understanding fractions in primary schools including six factors: inductive reasoning, explanations, justifications, conception for the magnitude of fractions, representations and connections with other concepts. The results helped the researchers to verify the factors' effectiveness of understanding fractions very well.

Khairunnisak et al. (2011) applied theory of Realistic Mathematics Education to support fifth Grade students in Learning Multiplication of Fraction with Whole Number. The results showed that the learning process starts with the students discovering the contextual situation of fair division, where students extend their understanding of fraction concept associated with division and multiplication. Reimer and Moyer (2005) used virtual manipulatives to teach fractions to 19 third-grade students. They offered students opportunities to interact with several virtual manipulative applets in a computer lab. The results indicated that the virtual manipulatives:

1. Helped these students learn more about fractions by providing immediate and specific feedback
2. Were easier and faster to use than paper-and-pencil methods, and
3. Made students more interesting while learning mathematics.

Elisabet et al. (2012) showed that the early fraction learning is not just about shading some parts on any shape. Solving problems in contextual situations related to fractions helps students develop their understanding of the meaning of fractions.

In mathematical curriculum of primary schools in Vietnam, students study the topic "fraction" in Grade 4 and 5, in particular, the main contents of fraction are sufficiently mentioned in the textbook "Mathematics 4" (Toán 4) (Hoan, 2007a) , and reviewed in the textbook "Mathematics 5" (Toán 5) (Hoan, 2007b).

In the textbooks, the concept of fraction is defined as the number of equal parts of a whole. The students learn the concept through observing the picture of a circle divided into 6 equal parts in which 5 parts are colored in the same color, it is said that 5 over 6 of the circle is called a fraction (Figure 1). From this, a problem is posed: Is this approach beneficial for students to comprehend this concept? Moreover, before students learn fractions, they have to work with natural numbers for a long time.

Therefore, it is sure that they will be influenced by the previous knowledge when learning new knowledge. Also, operations with fractions are rather sophisticated. The second problem is: When doing operations with fractions, do they make errors? In Vietnam, they have been problems not to be done research before. From the aforementioned problems, five questions are formulated as follows:

Research question 1: In the fraction (1), the whole must be divided into **b** equal parts and take away a parts. However, in the case of the whole divided into **b** unequal parts and taken a unequal parts, do students still say that this is a fraction (2)?

Research question 2: In case, a whole is divided into **b** equal parts shaded in various colors, do the students identify the corresponding fractions?

Table 1. Grades in the survey.

Name of grades	Name of primary schools	City / Province
4A1, 4A4, 5A1, 5A2, and 5A3	Trung Hung 1	Co Do, Can Tho city
4A and 4C	Vo Truong Toan	Ninh Kieu ward, Can Tho city
4B and 5.7	Phan Boi Chau	Ninh Kieu ward, Can Tho city
5E	Mac Dinh Chi	Ninh Kieu ward, Can Tho city
5B	Cai Khe 1	Ninh Kieu ward, Can Tho city
5C and 5D	Vinh My	Vinh Chau district, Soc Trang province
5/3	Tan Long 2	Nga Nam county town, Soc Trang province

Research question 3: Do students meet difficulties when they add two fractions in the case of the two fractions represented by images?

Research question 4: In dividing two fractions, do students strongly believe that it is not correct to divide the numerator of the first one by the numerator of the second one, and the denominator of the first one by the denominator of the second one?

Research question 5: Because of the impact of adding natural numbers, are students wrong to add two fractions as follows: the numerator of the first one plus the numerator of the second one, and the denominator of the first one plus the denominator of the second one?

The study aim is to provide answers to the aforementioned questions. The results obtained will help us to recognize the limitations of how to introduce the concept of fraction in textbooks, and to know Vietnamese students 'difficulties and errors committed; this will be a useful lessons for us and primary school teachers of Vietnam to increase the quality of teaching mathematics in the primary school...

METHODOLOGY

Participants

Participants were students from 5 grades 4 and 10 grades 5 in primary schools of provinces in Mekong Delta – Vietnam (Table 1). The sum of students in 15 grades was 478. Also, they studied the contents of the fractions, the properties as well as calculations with fractions. This took place from January, 2016 to March, 2016.

Questions constructed for testing students

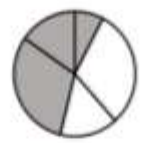
In order to find out students' answers to the aforementioned research question, we constructed five questions (as below) for students to complete them within 15 min. Table 2 indicated the objectives of each question.

Questions for testing students

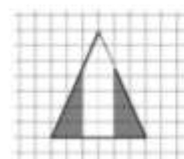
Question 1. (for the research question 1)

Fill in the blank with T (True) or F (False)

a. Fraction indicating the colored parts in the circle is $\frac{2}{5}$ ☐



b. Fraction indicating the colored parts in the circle is $\frac{3}{5}$ ☐

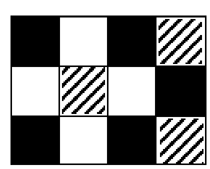


c. Fraction indicating the crossed parts in the triangle is $\frac{2}{3}$ ☐

d. Fraction indicating the crossed parts in the rectangle is $\frac{3}{4}$ ☐

Question 2. (for the research question 2)

Fill out the appropriate fractions to place dots.

a. 

The rectangle includes all 12 squares alike.

- Fraction which indicates the black parts is……
- Fraction which indicates the white parts is……

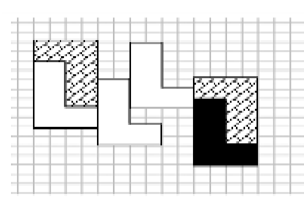
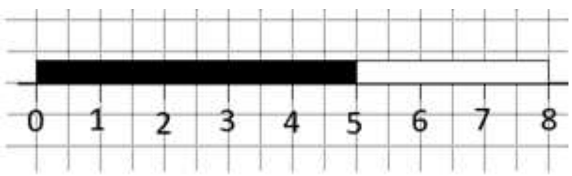
b. 

Table 2. The objectives of questions used to test students.

Objective of question	Students' ability to identify "fraction" given by images	Students' ability to identify "fraction" and do "+" operation of "fraction" given by images	Students' ability to identify correction of "x" and ":" operations
Question	Question 1; question 2	Question 3	Question 4; question 5

The earlier shown shape includes all 6 L- shaped blocks alike.

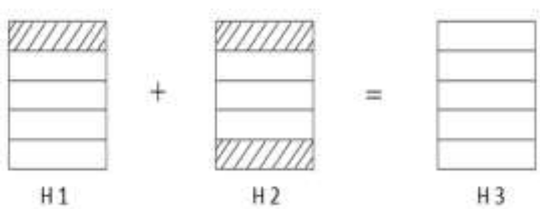
Fraction which indicates the crossed parts is……

c. 

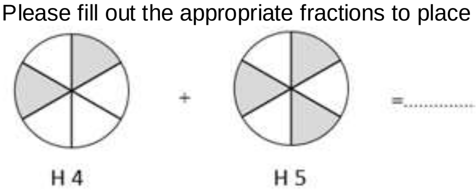
Fraction which indicates the black parts in the above shape is……

Question 3. (for the research question 3)

a. The crossed parts of the pictures H1 and H2 represent fractions. Let's represent the sum of the two fractions on the picture H3 by crossing out.



b. The colored parts of the pictures H4 and H5 represent fractions. Please fill out the appropriate fractions to place dots.



Question 4. (For the research question 4)

Fill in the blank with T (True) or F (False)

a. $\frac{35}{12}:\frac{5}{6}=\frac{7}{2}$ ☐

b. $\frac{22}{9}:\frac{2}{3}=\frac{11}{3}$ ☐

c. $\frac{7}{3}:\frac{4}{9}=\frac{21}{4}$ ☐

d. $\frac{15}{8}:\frac{5}{4}=\frac{3}{2}$ ☐

e. $\frac{5}{6}:\frac{2}{5}=\frac{25}{12}$ ☐

Question 5. (For the research question 5)

Fill in the blank with T (True) or F (False)

a. $\frac{9}{8}+\frac{2}{7}=\frac{11}{15}$ ☐

b. $\frac{3}{2}+\frac{5}{2}=4$ ☐

c. $\frac{6}{7}+\frac{5}{6}=\frac{11}{13}$ ☐

d. $\frac{5}{2}+\frac{5}{7}=\frac{5}{9}$ ☐

e. $\frac{3}{7}+\frac{5}{7}=\frac{8}{7}$ ☐

Initial comments of questions

This section includes our intentions of each question posed to the student, the basis of the questions raised, the correct answers to the questions as well as our predictions of the answers of students

For question 1

We want to know the answer of the question: Do students consider the parts of a whole as fractions, if they are divided in the whole unequally? In the process of teaching, if teachers do not inadvertently give attention to students about the "equality of the chosen parts", it is likely that students will not realize this. Therefore, we think that students are very easy to make mistakes in identifying the unequal parts. Here are purposes of items and the correct answers to the question (Table 3):

(1) Item a: we propose a correct item, and it is very familiar to students as background for the following items.

(2) Item b: we divided the circle with the unequal parts. Clearly, we found that the fraction of the colored parts is smaller than $\frac{3}{5}$, (3) and that of the remaining parts is smaller than $\frac{0.5}{5}$ (4).

(3) Item c: we can easily say that the fraction of the crossed parts is smaller than $\frac{2}{3}$ (5) because that of the white parts is larger than $\frac{1}{3}$ (6). By counting the numbers of squares, students find that the white parts are not equal to the crossed parts.

(4) Item d: It is very clear and intuitive for students to know that the white triangular part is equal to the crossed part (Table 3).

For question 2

We want to check if the students have to find the essence of

Table 3. The correct answers to the question 1.

Items	a	b	c	d
Correct answer	T	F	F	T

the fractions or not when they are presented in an unfamiliar shape. For the concept of fractions in mathematical textbooks, they are often introduced by the whole divided into equal parts, and the colored or taken parts. In reality, the taken parts are usually colored in the same color. In our shape, the parts are black, white and cross. In this case, it can cause more obstacles for students greatly in identifying the fractions correctly. The reason for this is that students will find it confused to determine denominators.

Item a:

Possible strategies:

Strategy 1 (S1): (Correct answer)

There are five black cells of 12 cells, the fraction which indicates the black cells is $\frac{5}{12}$ (7). Meanwhile, there are five white cells of 12 cells, the fraction which indicates the white cells is $\frac{7}{12}$ (8)

Strategy 2 (S2): (Incorrect answer)

Students take five black cells as the numerator, then take the total number of white cells as the denominator, and they write $\frac{5}{7}$ (9)

Strategy 3 (S3): (Incorrect answer)

Wrong answers may be $\frac{5}{12}; \frac{4}{7}$ (10). The reason for this answer is that students ignore five black cells.

Strategy 4 (S4):

Students can raise some fractions to complete assignments. For example, they are $\frac{4}{12}, \frac{3}{12}, \frac{3}{10}, \frac{4}{12}$ (11)

Item b:

Possible strategies:

Strategy 1 (S1): (Incorrect answer)

The students' answer may be $\frac{2}{3}$ (12). They may consider the numbers of crossed L-shapes as the numerator and the numbers of white L-shapes as the denominator.

Strategy 2 (S2): (Correct answer)

The right answer is $\frac{2}{6}$ (13) due to correctly identifying the chosen parts of a whole.

Strategy 3 (S3): (Incorrect answer)

The wrong answer may be $\frac{2}{4}$ (14). The explanation of students is: The numerator is the number of slashed L-shapes, the denominator is the number of remaining L-shapes.

Strategy 4 (S4):

Students can raise some fractions to complete assignments.

Item c:

Possible answers:

The aim of item c is to answer the question: "Do students identify fractions as they are expressed in linear approach?" Some answers of the students:

$\frac{5}{8}$ (15). Students identify the fraction correctly.

The answers are different from $\frac{5}{8}$ (16), so the linear approach to fractions is difficult for students.

For question 3

We want to test students on adding two fractions. Particularly in the item a, we also require them to express the sum of two fractions in the "shape" form. Typically, the exercises on adding fractions are presented and designed by two fractions written in symbols explicitly. Adding fractions illustrated by shapes does not appear in the mathematical textbooks.

However, if students understand the nature of the fractions and the rule of adding two fractions, the calculation of adding two fractions (illustrated by shapes) is not much different than that of the two fractions denoted and illustrated by fractional symbols. Besides, we think that this question can form creative thinking for students because it has multiple right solutions. Some possible answers:

(1) Students calculate the sum of two fractions by crossing out 3 parts in 5 parts.

(2) Because students do not know how to calculate the sum of two fractions, they have a vacant or wrong answer.

For question 4

Normally, teachers present the rule of dividing two fractions very carefully. For example, "To divide two fractions, we multiply the first fraction by the reciprocal of the second fraction". Because

Table 4. The correct answers to the question 4.

Items	a	b	c	d	e
Correct answer	T	T	T	T	T

Table 5. The correct answers to the question 5.

Items	a	b	c	d	e
Correct answer	F	T	F	F	T

influenced by this rule, students will say that it is incorrect to divide two fractions by dividing the two numerators together, and the denominators together. Table 4 presented the correct answers to the question 4. Two possibilities may occur:

Possibility 1: Students give five correct answers. Possibility 2: Students give five wrong answers.

Question 5

We want to test whether students add fractions as natural number or not. In fact, in the item d, we want to know if they add the two denominators and keep the numerators unchanged or not. There are two correct rules of adding fractions in the mathematical textbooks as below:

Rule 1: In order to add two fractions with a common denominator, we add the two numerators and keep the denominators unchanged.
Rule 2: In order to add two fractions with different denominators, we make them have a common denominator, then do the addition.

For teachers, there is nothing difficult, but for students in primary schools, because they are thoroughly conversant with adding two natural numbers, their knowledge of adding two fractions may be similar to that of adding two natural numbers. In an inertial way, that the students add two fractions as natural numbers is entirely probable. Table 5 presented the correct answers to the question 5.

RESULTS AND DISCUSSION

The salient result in Table 6 was that 98% of students had correct answers in the item a because of their familiarity with the equal parts of fractions. In addition, Table 1 also showed that many students did not pay attention to the condition of "equal parts" to identify the concept of fraction, namely a lot of students came up with the wrong answers (40% in the item b and 67% in the item c). Besides, in the item d, many students were not concerned about the equality of parts, in particular, the white triangle parts were not equal with rectangular crossed parts (accounted for 28.5%).

Specially, a student did have no right answers. This made us very surprised. Moreover, there were 2 students with 0.4% respectively having no answers in

Table 6. Students' answers to question 1.

Items	Answers	% of students
a	Correct	469/478 (98)
	Incorrect	9/478 (2)
b	Correct	289/478 (60)
	Incorrect	189/478 (40)
c	Correct	156/478 (32.6)
	Incorrect	320/478 (67)
	No answer	2/478 (0.4)
d	Correct	340/478 (71.1)
	Incorrect	136/478 (28.5)
	No answer	2/478 (0.4)

both items c and d. Those results allowed us to assert that in the case of the whole divided into **b** unequal parts and taken **a** unequal parts, students still say that this is a fraction $\frac{a}{b}$. (17).

According to Table 7, the right strategies for item a (S1) and b (S2) were chosen by a lot of students, respectively as follows:

292/478 (61%) and 339/478 (71%).

This showed that many students recognized fractions expressed by many factors. However, there were also quite a lot of students who did not know how to identify fractions. The students' answers were given as follows:

(1) $\frac{3}{5};\frac{4}{5}$ (18). Students found that the slashed part was the numerator and the denominator was the black part, while the numerator was the crossed part, the denominator was the white part.

(2) $-\frac{5}{3};\frac{5}{4}$ (19). Students acted in a similar way to the above case.
When there were many factors in the whole, they faced embarrassment in determining the numerator as well as the denominator, then led to the misidentification of fractions.
(3) 5, 4: The incorrect answers of students were two natural numbers, while the request was writing fractions. In this case, because the students could not determine denominators, they only wrote the numerators. When students did not choose the right strategy, they considered S4 as an answer. The most obvious result in the survey was 185/478 (40%) of students who chose this strategy.

In general, 70% of students gave the right strategy. Therefore, we concluded that in the case of whole

Table 7. Students' answers to question 2.

Items	Answers		% of students
a	Strategies	S1	292/478 (61)
		S2	1/478 (0.2)
		S3	0/478 (0)
		S4	185/478 (38.8)
	Correct		292/478 (61.1)
	Incorrect		182/478 (38.1)
	No answer		4/478 (0.8)
b	Strategies	S1	0/478 (0)
		S2	339/478 (71)
		S3	18/478 (4)
		S4	121/478 (25)
	Correct		339/478 (71)
	Incorrect		133/478 (27.8)
	No answer		6/478 (1.2)
c	Correct		367/478 (76.8)
	Incorrect		103/478 (21.5)
	No answer		8/478 (1.7)

Table 8. Students' answers to question 3.

Items	Answers	% of students
a	Correct	436/478 (91)
	Incorrect	16/478 (4)
	No answer	26/478 (5)
b	Correct	213/478 (44.6)
	Incorrect	218/478 (45.6)
	No answer	47/478 (9.8)

divided into **b** equal parts shaded in various colors, the students could identify the corresponding fractions.

According to Table 8, we found that there was a big difference in the percentages of students between items a and b. Indeed, the percentages of students giving the correct answers in both item a and b were 91 and 44.6%. The reason for this was easily understood as follows:

When asked to present a total of two fractions illustrated by shapes, many students came up with the right solutions.

Sometimes, students looked visually through the shapes, then represented a fraction of the total shape, but they still did not understand. Specifically, students could choose the strategy represented: Because the shape 1 has one part crossed out, and the shape 2 has two parts crossed out, they crossed out three parts in the shape 3 in an inertial way.

This could be clearly explained in the item b, and many students made a mistake because in the item b, this could not happen. In short, students met difficulties when they add two fractions in the case of the two fractions represented by images.

According to Table 9, the percentage of students giving the wrong answers to item a, b and d was nearly 50% (54%, 53% and 51%) and item d was 74%. When the students observed fractional division which took the numerator (the first fraction) divided by the numerator (2nd fraction) and the denominator (the first fraction) divided by the denominator (2nd fraction), they immediately said that it was wrong. Hence, there were some very careful students, so they checked again the

Table 9. Students' answers to question 4.

Items	Answers	% of students
a	Correct	222/478 (46)
	Incorrect	255/478 (54)
	No answer	6/478 (5)
b	Correct	222/478 (46)
	Incorrect	251/478 (53)
	No answer	5/478 (1)
c	Correct	117/478 (25)
	Incorrect	355/478 (74)
	No answer	6/478 (1)
d	Correct	229/478 (48)
	Incorrect	242/478 (51)
	No answer	7/478 (1)
e	Correct	359/478 (74)
	Incorrect	119/478 (25)
	No answer	7/478 (1)

Table 10. Students' answers to question 5.

Items	Answers	% of students
a	Correct	309/478 (65)
	Incorrect	169478 (35)
	No answer	6/478 (1)
b	Correct	186/478 (39)
	Incorrect	283/478 (59)
	No answer	9/478 (2)
c	Correct	312/478 (65)
	Incorrect	157/478 (33)
	No answer	9/478 (2)
d	Correct	394/478 (82)
	Incorrect	74/478 (16)
	No answer	10/478 (2)
e	Correct	385/478 (81)
	Incorrect	84/478 (17)
	No answer	9/478 (2)

results by doing fractional division based on the rule in the mathematical textbooks.

At this point, it was reasonable to say that in dividing two fractions, students strongly believe that it is not correct to divide the numerator of the first one by the numerator of the second one, and the denominator of the first one by the denominator of the second one.

According to Table 10, 65% of students had incorrect answers in the item b. The difference of the item b from the others was that we simplified the result fraction $(\frac{3}{2}+\frac{5}{2}=4)$. Maybe, that was the main reason for the errors of students in the item b. In the item d, 74 students (16%) wrongly did the addition as below: keep the numerator unchanged and add the two denominators. However, the number of students having right answers was rather large. The percentages in the items were 65% (a and c), 82% (d) and 81% (e).

Therefore, we could assert that because of the impact of adding natural numbers, students will be wrong to add two fractions as follows: the numerator of the first one plus the numerator of the second one, and the denominator of the first one plus the denominator of the second one.

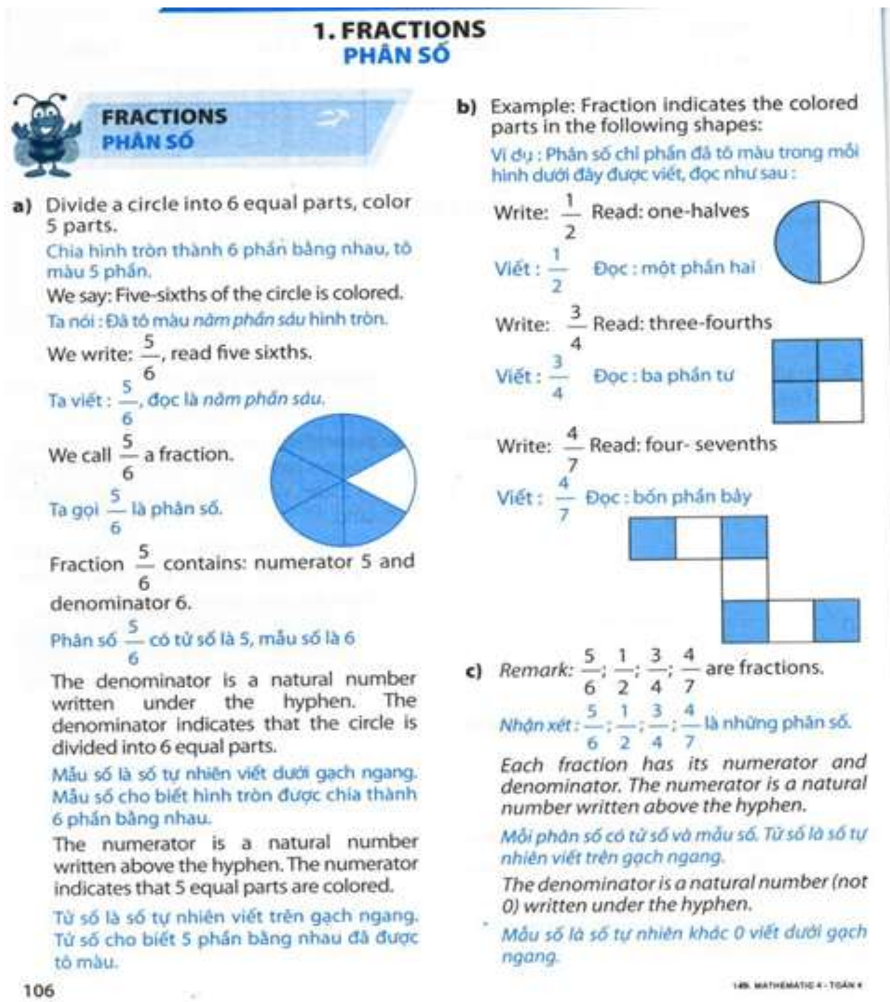
CONCLUSION

The results of the investigation showed that students made mistakes in identifying fraction. In our opinion of this study, students' mistakes occurred due to the following reasons:

(1) Primary school students learned the concept of fraction intuitionally and in an informal way. They studied this concept by observing visual figures, not by formal definition of fraction (Figure 1). Furthermore, in system of exercises to identify "fraction" in the textbook, there were not any exercises considered as non – examples of a whole divided into unequal parts like the question1 (for verify the research question 1). It is noted that in teaching a mathematics concept, the use of examples and no-examples is very helpful for students to understand the concept.
(2) The notion of a fraction itself is a difficult concept because it is related to the concept of rational numbers. Therefore, in order to understand the fraction concept, students need to do several exercises related.
(3) Due to the habit of doing mathematics in environment of natural number before, students also easily make the mistakes because of their confusion of fractions with natural numbers (Ndalichako, 2013).

In order increase the effectiveness of teaching the topic of fraction, together with the suggestions of Amato (2005), Nicolaou and Pitta-Pantazi (2012) and Khairunnisak et al. (2011) which were mentioned earlier, From of this study, two measures should be used:

(1) Using examples and non – examples to help students understand fraction in a correct way
(2) Using different representations during process of teaching topic "Fraction" such as: number lines, area



1. FRACTIONS
PHÂN SỐ

FRACTIONS
PHÂN SỐ

a) Divide a circle into 6 equal parts, color 5 parts.
Chia hình tròn thành 6 phần bằng nhau, tô màu 5 phần.
We say: Five-sixths of the circle is colored.
Ta nói : Đã tô màu năm phần sáu hình tròn.
We write: $\frac{5}{6}$, read five sixths.
Ta viết : $\frac{5}{6}$, đọc là năm phần sáu.
We call $\frac{5}{6}$ a fraction.
Ta gọi $\frac{5}{6}$ là phân số.
Fraction $\frac{5}{6}$ contains: numerator 5 and denominator 6.
Phân số $\frac{5}{6}$ có tử số là 5, mẫu số là 6
The denominator is a natural number written under the hyphen. The denominator indicates that the circle is divided into 6 equal parts.
Mẫu số là số tự nhiên viết dưới gạch ngang. Mẫu số cho biết hình tròn được chia thành 6 phần bằng nhau.
The numerator is a natural number written above the hyphen. The numerator indicates that 5 equal parts are colored.
Tử số là số tự nhiên viết trên gạch ngang. Tử số cho biết 5 phần bằng nhau đã được tô màu.

b) Example: Fraction indicates the colored parts in the following shapes:
Ví dụ : Phân số chỉ phần đã tô màu trong mỗi hình dưới đây được viết, đọc như sau :
Write: $\frac{1}{2}$ Read: one-halves
Viết : $\frac{1}{2}$ Đọc : một phần hai
Write: $\frac{3}{4}$ Read: three-fourths
Viết : $\frac{3}{4}$ Đọc : ba phần tư
Write: $\frac{4}{7}$ Read: four- sevenths
Viết : $\frac{4}{7}$ Đọc : bốn phần bảy

c) Remark: $\frac{5}{6}; \frac{1}{2}; \frac{3}{4}; \frac{4}{7}$ are fractions.
Nhận xét : $\frac{5}{6}; \frac{1}{2}; \frac{3}{4}; \frac{4}{7}$ là những phân số.
Each fraction has its numerator and denominator. The numerator is a natural number written above the hyphen.
Mỗi phân số có tử số và mẫu số. Tử số là số tự nhiên viết trên gạch ngang.
The denominator is a natural number (not 0) written under the hyphen.
Mẫu số là số tự nhiên khác 0 viết dưới gạch ngang.

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Figure 1. How to present the concept “fraction” in Mathematics 4 – Vietnam.

models, volume models.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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