Using Video as a Stimulus to Reveal Elementary Teachers' Mathematical Knowledge for Teaching

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Introduction

Knowledge of mathematics is essential to effective teaching (Darling-Hammond, 2005; Hill, Sleep, Lewis, & Ball, 2007; Howard & Aleman, 2008). Given "that teachers may need to know subject matter differently than their students or non-teachers" (Hill et al., 2007, p. 122), however, knowledge of mathematics alone may be insufficient. This conjecture has resulted in the development of such constructs as pedagogical content knowledge (Shulman, 1986) and mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). Common to this set of constructs is the desire to ground teachers' knowledge of mathematics within the context of teaching.

This emphasis on mathematical knowledge has been coupled with increased attention to measuring teacher knowledge (Hill et al., 2007; Howard & Aleman, 2008). In light of current practices, Hill and col-

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leagues discussed the benefits and challenges of various approaches to measuring teachers' knowledge. In their conclusion, these authors encouraged the development of assessments focused on "measuring the mathematical knowledge used in teaching" (p. 150).

Heeding this call, our purpose was to explore the usefulness of a video-based tool for measuring teachers' mathematical knowledge for teaching. Unique to our tool was the use of video featuring a mathematical disagreement that occurred in an elementary classroom. We defined mathematical disagreements as instances in which students challenge each other's mathematical ideas (Barlow & McCrory, 2011). We anticipated that by having teachers view video of a mathematical disagreement and then respond to questions, we could effectively access their mathematical knowledge for teaching.

Given the need to measure teachers' mathematical knowledge within the teaching context (Hill et al., 2007), the significance of this work lies in its demonstration of using video of students' mathematical disagreements as a stimulus for revealing teachers' Mathematical Knowledge for Teaching (Ball et al., 2008). Our guiding research question was: What types of mathematical knowledge for teaching are revealed through a video-based tool featuring a mathematical disagreement?

Theoretical Framework

Our theoretical assumptions lie within two areas: teacher knowledge and geometric thinking. Literature in each area follows.

Teacher Knowledge

Pedagogical content knowledge. As emerging educational research on effective teaching focused on facets of the education process such as classroom management, cultural awareness, and individual differences, Shulman (1986) discussed "the missing paradigm" (p. 7) in educational research. According to Shulman, educational research in the 1980s was ignoring a key component: what teachers knew. In a departure from commonly held beliefs at the time, Shulman believed, for example, that subject matter knowledge itself was not enough to be able to answer confused students' questions. This teacher ability required specialized knowledge, and this type of knowledge was missing from the educational research.

Shulman (1986) called for the development of a theoretical framework that would describe this more specialized knowledge required of teachers. This framework would allow for multiple categories of content knowledge including subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. At that time, the notion of pedagogical content knowledge was new (Ball et al., 2008). In the 30 years that have passed since Shulman's introduction of pedagogical content knowledge, researchers have aimed to better understand the content knowledge unique to teaching (e.g., Ball et al., 2008; Campbell et al., 2014; Hill, Rowan, & Ball, 2005).

Although different research groups have, at times, defined content knowledge and pedagogical content knowledge differently (Depaepe, Verschaffel, & Kelchtermans, 2013), in a general sense, content knowledge "represents teachers' understanding of the subject matter taught" (Kleickmann et al, 2015, p. 116). In contrast, pedagogical content knowledge has been defined as "the knowledge needed to make that subject matter accessible to students" (Kleickmann et al., 2015, p. 116). Armed with these two definitions, researchers have sought to determine whether content knowledge and pedagogical content knowledge represent separate dimensions of the knowledge needed for effective teaching (e.g., Campbell et al. 2014; Krauss et al., 2008). In doing so, researchers have documented the influence of content knowledge and pedagogical content knowledge on instructional practices (Hill et al., 2005) and student achievement (Baumert et al., 2010; Campbell et al., 2014; Hill et al., 2005), at times noting pedagogical content knowledge to be more strongly related to student achievement (Baumert et al., 2010).

Despite this research, the theoretical framework that Shulman called for remains underdeveloped (Ball et al., 2008). To this end, Ball and her colleagues sought to clarify the knowledge necessary for effective mathematics teaching. Their work resulted in the introduction of the Mathematical Knowledge for Teaching construct, which is described in the following section.

Mathematical Knowledge for Teaching. Building on Shulman's call for the development of a theoretical framework, Ball et al. (2008) defined the construct of Mathematical Knowledge for Teaching (MKT) as, "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). They divided this knowledge into six domains—Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC)—and stated that collectively these domains represent the mathematical knowledge that teachers need. Research has indicated that a teacher's MKT contributes to the quality of mathematics instruction (Hill et al., 2007) and gains in student achievement (Hill et al., 2005).

The domains of KCS, KCT, and KCC are of particular interest in this study. According to Ball et al. (2008), KCS is the knowledge of how children come to understand mathematics. This domain involves understanding how students think about a particular mathematical concept, including common conceptions and misconceptions. Such knowledge serves to inform the teacher in selecting tasks for use in the classroom and in interpreting students' developing ideas.

Closely related, Ball and colleagues described KCT, which involves knowledge of how to develop mathematical understanding in students. KCT employs an interaction between teachers' mathematical understanding and their understanding of pedagogy that supports students' learning. While KCS supports the teacher in task selection, KCT supports the sequencing and effective implementation of those tasks. Additionally, KCC involves the teachers' knowledge of where particular mathematical topics fall within different grade levels. Teachers who possess KCC are aware of not only the content of the grade in which they teach, but also the content in previous and later grades as it relates to their instruction (Ball et al., 2008).

A teacher's mathematical knowledge varies depending on the mathematical topic of study (Hill et al., 2007). That is, a teacher who demonstrates strong MKT in the area of whole numbers, for example, does not necessarily possess the same level of knowledge in the area of rational numbers. Given that the mathematical disagreement featured in our research centered on geometric shapes, we utilized the van Hiele Levels of Geometric Thought as a means for identifying the related requisite knowledge for the MKT domains. In the following section, we will describe the van Hiele levels, linking this to MKT as appropriate.

Geometric Thought

Referencing the work of van Hiele and van Hiele-Geldof, Fuys, Geddes, and Tischler (1988) explained that, with appropriate instruction, students progress through five levels of geometric thought. Descriptions of these levels follow.

> Level 0. The student identifies, names, compares, and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.
>
> Level 1. The student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically (e.g., by folding, measuring, using a grid or diagram).
>
> Level 2. The student logically interrelates previously discovered properties/rules by giving or following informal arguments.

> Level 3. The student proves theorems deductively and establishes interrelationships among networks or theorems.
>
> Level 4. The student establishes theorems in different postulational systems and analyzes/compares these systems. (Fuys et al., 1988, p. 5)

Students must pass through all previous levels to achieve a certain level. Much of the research related to these levels has sought to determine their accuracy in describing the progression of students' thoughts (e.g., Burger & Shaughnessy, 1986; Clements & Battista, 1992; Fuys et al., 1988; Han, 1986). In the context of MKT, these levels represent the KCS a teacher should possess in order to design and/or select appropriate tasks for students.

Researchers have examined instructional implications of the van Hiele levels (Fuys et al. 1988; van Hiele-Geldof, 1984) and one particular implication is important for our study. When a teacher is providing instruction at a higher van Hiele level than that of a student and is using language and problem-solving processes at the higher level, then there will be confusion and a lack of understanding within the student (Burger & Shaughnessy, 1986; Fuys et al., 1988).

In addition, according to the work done by the van Hieles, the types of instructional experiences should match the level at which the students are operating. If instruction is provided at a higher level than that of the student, then the student will find ways to lower the requirements of the instruction to match the level in which they are operating. These instructional implications of the van Hiele Levels of Geometric Thought represent the KCT a teacher should possess in order to plan effective instruction for students.

Literature Review

There are challenges involved in assessing teachers' knowledge via traditional methods such as multiple-choice tests (Hill et al., 2007; Kersting, Givvin, Sotelo, & Stigler, 2010; Schoenfeld, 2007). Teachers must not only possess knowledge, but the knowledge must be "organized and accessible in a flexible way" (Kersting et al, 2010, p. 179). Teachers may show that they possess knowledge through a pencil-and-paper examination yet "be unable to activate and apply that knowledge in a real teaching situation" (Kersting et al., 2010, p. 179). Alternatively, teachers may experience difficulty when attempting to answer "very general or decontextualized" (Jacobs & Morita, 2002, p. 155) test questions since they are accustomed to making instructional decisions which are based on professional judgments about classroom events (Jacobs & Morita, 2002). Thus, there is growing interest in studying teacher knowledge as it relates to the complex classroom environment (Hatch & Grossman, 2009; Jacobs & Morita, 2002; Kersting et al., 2010).

One route for accomplishing this is through the use of video clips. Video is a valuable tool in this regard because it allows teachers to review and analyze interactions that take place in the classroom (Sherin, Linsenmeier, & van Es, 2009). These analyses, according to Sherin and colleagues, can help teachers learn how to respond to students' mathematical thinking, and in turn, more can be learned about the mathematical knowledge that teachers possess for teaching.

In the research literature on teachers and teaching, researchers have primarily used video as a means for identifying instructional practices (e.g., Andrews, 2009) or for supporting the professional learning of preservice teachers (e.g., Alsawaie & Alghazo, 2010; Joon, Ginsburg, & Preston, 2009) and inservice teachers (Brantlinger, Sherin, & Linsenmeier, 2011; Nemirovsky & Galvis, 2004). Using video as a stimulus for measuring teachers' knowledge for teaching, however, was limited to two studies (Jacobs & Morita, 2002; Kersting et al., 2010). Although these researchers provided evidence of the usefulness of video for revealing teachers' knowledge (Jacobs & Morita, 2002; Kersting et al., 2010; Sherin et al., 2009), none utilized video of mathematical disagreements, which is the premise of the present study.

Methodology

To explore the usefulness of our video-based tool for measuring teachers' mathematical knowledge, we utilized qualitative inquiry as we asked teachers to view video featuring elementary students engaged in a mathematical disagreement and respond to a set of open-ended questions. The choice to use qualitative methods was appropriate given the exploratory nature of the work (Creswell, 2013). In this section, we describe the video-based tool and our methodology.

Video-based Tool

Background information. After logging into a secure website, participants responded to questions regarding basic background information (e.g., gender). At the end of these questions, participants clicked a submit button that linked them to Video Segment One.

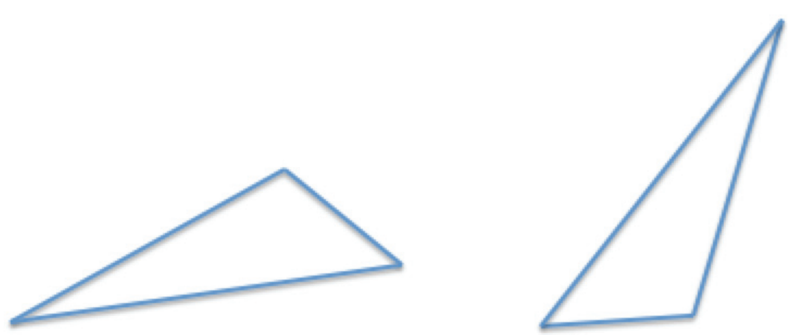
Video segment one. Next, participants viewed the first video. Video Segment One featured an embedded, eight-minute video followed by three questions. The video and questions were presented simultaneously, allowing the participants the opportunity to view the questions prior to viewing the video. In addition, participants had the option of watching the video multiple times prior to submitting their responses.

The video featured a third grade mathematics lesson involving two-dimensional shapes. The teacher indicated in the introduction to the video that her lesson goal was for students to be able to describe how two triangles were different based on the lengths of their sides and/or the size of their angles. During the lesson, the students indicated that a triangle and its rotated image were two "different" triangles. The video begins with the teacher's posing the following, "Here's the question. This is the original triangle. All right, watch. (Teacher rotates the triangle.) Is that the same triangle?" (See Figure 1).

The video featured students presenting their views regarding whether the two triangles were different. We categorized the ensuing discussion as a mathematical disagreement, given the contrasting views presented by the students. Initially, two students argued that the two triangles were not different, stating that nothing had changed. One might assume that these two students were operating at Level 1 of the van Hiele Levels of Geometric Thought. These students changed their minds, however, after listening to their classmates describe how the two triangles looked different. By focusing on the appearance of the triangles rather than their properties, the students in the class were operating at Level 0 of the van Hiele Levels of Geometric Thought. The video segment ended without resolution of the mathematical disagreement.

Participants responded to three questions after viewing the video. Specifically, we asked participants to describe the mathematical disagreement, students' mathematical understandings and misunderstandings, and

Figure 1.



a strategy for resolving the disagreement. After responding, participants clicked the submit button that linked them to Video Segment Two.

Video segment two. The second video segment featured an embedded, seven-minute video followed by questions. Like Video Segment One, the video and questions were presented simultaneously, allowing for multiple viewings.

The video segment began where the previous video had ended. The teacher began by cutting out four congruent triangles. Students confirmed that when the triangles were placed on top of one another they were the "same." Once the teacher rotated one of the triangles, however, students indicated it was "different." The teacher placed a pencil mark on the triangle noted by students as "different" and placed all four triangles in a brown paper bag. After shaking the bag, the teacher asked students to tell how she might find the "different" triangle in the bag without looking. Initially, some students argued that the teacher should be able to feel the pencil mark thus indicating the different triangle. In this way, they were focusing on irrelevant features of the triangle. The teacher noted, however, that the students felt the triangle was different before the pencil mark was placed on it. As the discussion advanced, the students determined that the triangles were not different. It is worth noting that the teacher in this video segment removed the irrelevant features on which students had previously focused, thus the instruction should have supported students in transitioning to Level 1 of the van Hiele Levels of Geometric Thought.

After watching the video, participants responded to another set of questions. Our intent was to gain insight into participants' perceptions of the featured instruction. In particular, we were curious as to whether the participants would find value in the teachers' approach of removing the irrelevant features. After typing the responses, participants clicked the submit button.

Participants

Given the qualitative nature of our study and its exploratory purpose, we aimed to recruit a sample of five to six participants. To do so, our colleagues provided e-mail addresses of elementary teachers they had known previously. We sent an initial e-mail to these teachers inviting them to participate in the research. No incentives were offered. When a response was received, we sent a second email, providing the website address, username, and password. Usernames were distributed randomly and no attempt was made to track which participant used a particular username. A total of 88 invitations were sent with 13 teachers indicating their willingness to participate. Of these, seven teachers either did not complete the survey or only partially completed the survey. Therefore, our data were drawn from the six completed surveys. Given the exploratory nature of our work, this small sample served our needs. Table 1 presents background information on the participants.

Data Analysis

In our analysis, we utilized Patton's (2002) recommendations for content analysis. Three researchers separately and then collectively identified "core consistencies and meanings" (Patton, 2002, p. 453) represented in the data. From these, we then developed three questions to guide our continuing analysis. First, what did participants perceive as the mathematical misunderstanding(s) that formed the basis of this disagreement? Second, what instructional strategies would participants use to resolve the disagreement? Third, what were participants' ideas related to the teacher's resolution of the mathematical disagreement in the video? With these questions identified, we returned to the data and independently coded the responses in relation to each question. Afterwards, we met as a group and collectively agreed upon the coding of statements within the data. Finally, we linked the coding categories to the domains of MKT as appropriate.

Limitations

As with any study, there are limitations to this work that must be considered prior to presenting the results of the study. The first limitation involves the analysis of a single source of data. Given our desire to explore the usefulness of our video-based tool for measuring teachers' mathematical knowledge, we felt that a single source of data was appropriate. While this may be viewed as limiting in nature and clearly triangulation of the data would strengthen the results, we feel that our

Table 1
Participants

Pseudonym	*Gender*	*Race/ Ethnicity*	*Years Teaching*	*Current Grade Level*	*Currently Teaching Math*
Ann	Female	White	4	4th	Yes
Beth	Female	White	10	4th	Yes
Cathy	Female	White	7	4th	Yes
Delia	Female	White	39	3rd	Yes
Emma	Female	White	9	3rd	Yes
Frank	Male	White	21	6th	Yes

use of anonymous elicited texts, as defined by Charmaz (2006), in some ways offsets this concern. Specifically, Charmaz stated,

> Elicited texts involve research participants in writing the data. . . . Internet surveys containing open-ended questions are common sources of these texts. . . . Anonymous elicited texts can foster frank disclosures . . . [and] work best when participants have a stake in the addressed topics, experience in the relevant areas, and view the questions as significant. (pp. 36-37)

Although Charmaz acknowledged the desire to have multiple forms of data, she stated that it is not uncommon for qualitative researchers to utilize a single source, such as elicited texts, without the opportunity to collect additional data. In the present study, the use of anonymous elicited texts enhanced the honesty represented within the data. However, it also prevented the collection of additional data.

The anonymity of the participants additionally resulted in the second limitation of the study: an inability to perform member checking as a validation strategy (Creswell, 2013). We have utilized other validation strategies, however, including multiple researchers and independent coding, as described by Creswell (2013).

Results and Discussion

Question 1: Mathematical Misunderstandings

The six participants were able to describe with reasonable accuracy the featured disagreement, but they gave varied responses regarding students' understandings and misunderstandings. Specifically, the analysis revealed two codes. Each of these codes is described below.

Prerequisite knowledge / experiences. Three participants hypothesized that students lacked prerequisite knowledge or experiences needed for thinking about the rotated figure. When asked about students' mathematical understandings and misunderstandings, Frank wrote, "Students that understand rotation and those that don't." This response indicated that knowledge of rotations could be considered prerequisite knowledge for this work.

Alternatively, Emma felt that students' prior experiences had been limited to "typical" triangles and noted the need to have worked with "atypical" triangles. She said:

> The students seem to have an idea of what a typical triangle should look like, and they believe when turned it is a completely different triangle. . . . Taking away that typical triangle would probably help them to see that it was still the same triangle.

In addition, Emma indicated that students needed to have had "earlier discussions on congruent figures, which would help them to understand that if objects are the same shape and size they are congruent no matter which way they are turned."

Finally, Ann stated that the students lacked general vocabulary necessary for expressing their ideas. She wrote:

> The few students that believe [the triangles] are the same are saying that nothing changed, however they do not have the vocabulary to express the difference is position. The one girl used the most vocabulary that describes triangles, but she still does not know how to explain her reasoning.

From this quote, it appeared that Ann was concerned about students' lack of prerequisite vocabulary.

Although their ideas were quite different, Frank, Emma, and Ann each described an experience or knowledge piece that students should have gained prior to participating in the mathematical disagreement.

Orientation. Four of the six participants suggested that changing the orientation of the triangle was the undergirding mathematical misunderstanding. Three of these participants indicated that the students likely believed that the change in orientation caused a change in the triangle's attributes. For example, Beth wrote:

> Students are debating if rotating or translating a triangle changes its shape. Students didn't understand that a shape's change of orientation does not change its shape. Maybe have students measure the triangle's angles and/or sides . . . students should conclude the triangle did not change because its measurements didn't change.

In contrast, Ann indicated that the change in the orientation caused the triangles to look different. She stated:

> For the most part, the students say the two triangles are different because they look different. The few students that believe they are the same are saying that nothing changed, however they do not have the vocabulary to express the difference is position.

Although both Beth and Ann specified that the triangle's change in orientation was the stimulus for the students' disagreement, Ann did not indicate that the students believed that the triangle's angles and/or side lengths had changed as Beth did. Rather, Ann stated that the students appeared to be focusing on the triangle's change in appearance.

Discussion. Students in the featured video were operating at Level 0 of the van Hiele Framework for Geometric Thinking. Rather than focusing on the triangle's defining characteristics (i.e., angle measures and side lengths), the students focused on the overall appearance of the triangle, noting whether it was pointed upward or sideways. A teacher's knowledge in this area falls within the KCS domain of the MKT Framework (Ball et al., 2008). Only one participant, Ann, correctly identified the students' misunderstanding as centered on how the triangle looked, thus indicating that she held the requisite KCS to effectively design instruction that could potentially resolve the mathematical disagreement.

Other participants incorrectly believed that students thought the size of the triangles' attributes changed as a result of the rotation. Furthermore, participants provided additional ideas regarding experiences or knowledge pieces that students should have had prior to this lesson. Because participants described these in the context of prerequisite knowledge/experiences, there seemed to be a general belief that had the students been privy to this knowledge or these experiences, the disagreement would not have occurred. Collectively, these participants indicated an overall lack of knowledge regarding the students' misunderstandings, or lack of KCS, related to this mathematical disagreement.

Question 2: Instructional Strategies

Our analysis revealed four codes related to instructional strategies for resolving the disagreement. Participants often provided responses that were assigned with multiple codes. Each code is described in the following paragraphs.

Different triangles/figures. Three participants desired to have students work with triangles or figures that were different from the triangle the teacher had rotated. Ann and Emma both indicated that students should work with different types of triangles next. Ann explained that having multiple types of triangles may provide for a visual comparison that could be helpful for the students in examining what happens when the triangles are rotated. Emma added that she would be sure to use acute and obtuse triangles.

Like Ann and Emma, Cathy described having the students explore the rotation of figures. Rather than working with other triangles, however, Cathy stated that she would use a student to demonstrate that changing the orientation of a figure did not change the figure. She wrote:

> I would have a student lay [*sic*] on the floor and then rotate them and ask the class if it is still the same student. I would discuss how we did not take anything away or add anything, but that we still have the same student turned in a new direction. Then bring it back to the polygons. Show how even though the shape is rotated, it is still the same shape.

Measuring sides and/or angles. Two participants, Beth and Emma, indicated that students needed to measure the sides and/or angles of the triangle and its rotated image. For example, Beth wrote, "Maybe have students measure the triangle's angles and/or sides on the overhead and then move the shape through transformations. The students should conclude the triangle did not change because its measurements didn't change."

Teaching a different lesson. One participant, Emma, suggested the need to teach a different lesson. In addition to having students measure the sides of the triangle, Emma stated, "I would also teach a lesson on congruent figures, using other polygons. This would help them to understand that the triangle was the same." Emma did not provide information regarding the structure of this proposed lesson on congruent figures.

Individual or small group exploration. The participants not only offered various foci in resolving the disagreement, but they also offered ideas about how they would structure the proceeding instruction. Five participants suggested using either individual or small group exploration. Ann and Emma indicated that students should explore in small groups the position of the triangles. Alternatively, Beth, Delia, and Frank all offered the idea that allowing students to individually explore the rotation of triangles would be their route to resolving the disagreement.

Discussion. Recognizing that the students' were operating at Level 0, instruction designed to begin students' progression to Level 1 should support students in focusing on the relevant features of the triangles, such as the angles and sides, as opposed to the irrelevant features, such as the direction the triangle is pointed. Such knowledge falls within the domain of KCT within the MKT Framework (Ball et al., 2008).

Participants' descriptions of how they would proceed in resolving the disagreement failed to provide evidence of possessing the required KCT with regard to this particular disagreement. Accounts of working with alternative figures through either individual or small group exploration did not address how such activities would draw students' attention away from the irrelevant features of the figures and toward the relevant features. Similarly, several participants suggested having students measure the side lengths and angle measures. Although these actions may draw students' attention to the relevant features, it was not clear how these actions would draw students' attention away from the irrelevant features. In addition, skills associated with measuring angles would likely appear in a later grade level, indicating a lack of KCC. The same is true of introducing the terminology of congruent figures.

Overall, participants did not provide instructional suggestions that would likely support students' movement towards Level 1 thinking. That is to say, they failed to provide evidence of possessing the KCT required for designing instruction with a goal of resolving the mathematical disagreement. In addition, in some instances participants revealed a lack of KCC with regard to this mathematical content.

Question 3: The Teacher's Resolution

Regarding the teacher's process for resolving the disagreement in the video, three participants felt the teacher's instructional strategy was appropriate. Specifically, Cathy and Emma felt the instruction was appropriate, noting that the teacher had removed the visual aspects of the figures. In contrast, Delia also stated that the instruction was appropriate but she did not justify her thoughts in terms of removing the visual aspects of the figures. Rather, she focused on the opportunities students had for developing a deeper understanding through the defense of their ideas and self-realization of mistakes.

The remaining three participants either expressed a neutral view of the instruction or dissatisfaction. Ann's neutral response follows.

> The teacher's discussion is a little confusing because of the term different because the four triangles are different ones, but they are congruent. It would have helped to refer back to the vocabulary and characteristics of a triangle that were on the board.

Ann did not clearly state that she felt the instruction was appropriate or inappropriate, although one might construe her criticism of the teacher's terminology combined with her instructional suggestions as disapproval of the overall instructional process.

Unlike Ann, Beth and Frank were more forthcoming with their opinions of the lesson. Beth stated, "It was an adequate way to handle the disagreement, but I'm not sure that all the students were able to relate to it." Similarly, Frank wrote, "It is definitely appropriate to have the mathematical disagreement, but I am not sure the approach used in the video was the most effective way." Frank continued by sharing an alternative approach to resolving the disagreement.

Discussion. In the video, the teacher attempted to have students describe the triangle's relevant features as a means for identifying the unseen triangle. In doing so, this instruction likely supported students in beginning to transition towards Level 1 thinking as it removed the triangle's irrelevant features. Two participants noticed this key aspect of the instruction, which seemed to indicate the possession of KCT at least to some degree. Other participants failed to identify this key feature of the instruction.

Conclusion

Shulman (1986) noted that earlier researchers had omitted a component in their quests to understand effective teaching and learning. Recognizing the need for knowledge beyond subject matter knowledge, he believed researchers could begin to fill the "missing paradigm" (Shulman, 1986, p. 6). With increased attention given to the mathematical knowledge that teachers should possess, the need for alternative means for assessing teachers' mathematical knowledge has been expressed (Hill et al., 2007). To this end, researchers have demonstrated the potential of using video as a stimulus to reveal teachers' knowledge (Jacobs & Morita, 2002; Kersting et al., 2010; Sherin et al., 2009). Unlike this previous research, however, the present study aimed to examine the use of video featuring a mathematical disagreement as a stimulus for accessing teacher knowledge.

This use of video featuring a mathematical disagreement proved to be an effective way to access the participants' knowledge. As indicated, we were able to conclude that our participants, for the most part, did not provide evidence of possessing the requisite pedagogical content knowledge for effectively supporting students' resolution of the mathematical disagreement. The analyses of descriptions of students' misunderstandings and proposed instructional strategies revealed a failure by participants to provide evidence of possessing the KCS and KCT in relation to the featured mathematical topic. Furthermore, in some instances participants provided evidence of a lack of KCC, as suggestions emphasized concepts or skills that would occur in later grade levels. Such instruction would likely result in confusion and a lack of understanding (Burger & Shaughnessy, 1986; Fuys et al., 1988).

These findings highlight the usefulness of using a video-based tool that includes a mathematical disagreement in accessing the types of knowledge that teachers possess, which not only contributes to the available research methodologies involving video but also continues the mathematics education community's response to Shulman's (1986) desire to understand more about what teachers know. By accessing teachers' MKT in this way, more can be known so that a potential lack of knowledge can be addressed. One should note, however, that different results might have been found had the mathematical disagreement involved a different focus. Future work should examine the use of the video-based tool featuring a mathematical disagreement centered on dissimilar content.

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