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Investigating Mathematics Teachers Candidates' Knowledge about Problem Solving Strategies through Problem Posing

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Abstract

The aim of the study was to determine mathematics teacher candidates' knowledge about problem solving strategies through problem posing. This qualitative research was conducted with 95 mathematics teacher candidates studying at education faculty of a public university during the first term of the 2015-2016 academic year in Turkey. Problem Posing Test (PPT) was used as a data collection tool which developed by researcher. It consists of 5 open ended questions. It was prepared according to the free problem posing situations. Teacher candidates were asked to pose problems that can be solved using problem solving strategies and problems were examined and categorized based on content analysis. Problems that were in accordance with the strategy were mostly related to daily life and solvable. Clinical interviews were conducted with 10 teacher candidates. Although some teacher candidates have knowledge about problem solving strategies, they did not pose a problem and in some cases they made mistakes in solving the problems they posed. Many teacher candidates stated that they posed similar problems they saw in textbooks, rather than act creatively.

Keywords: problem posing, problem solving strategies, mathematics teachers candidates

1. Introduction

Both problem solving and problem posing play an important role for mathematics education (Kwan & Leung, 2013) and have been investigated by many recent studies (Silver, 1994; NCTM, 2000; Abu-Elwan, 2002; Crespo, 2003; Stoyanova, 2003; Brown & Walter, 2005; Barlow & Cates, 2006; Korkmaz & Gür, 2006; Toluk-Uçar, 2009; Işık, 2011; Chapman, 2012; Tichá & Hošpesová, 2013; Silver, 2013; Kılıç, 2013; Işık & Kar, 2015; Kılıç, 2015). Considering that one of the most important aims of mathematics education is to improve these skills (Abu Elwan, 2002; Crespo, 2003; NCTM, 2000; Ministry of National Education of Turkey [MoNE], 2006, 2013; Kılıç, 2013), more importance should be given to problem solving and problem posing activities in schools.

Posamantier and Krulik (1998, p.1) defines problems "*is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known*". Problem is not an exercise or question that can be solved by applying previously learned method, it is a situation that can be solved by using knowledge and many abilities together (MoNE, 2006). Polya (1962) defines problem solving as trying to find a suitable action to reach a desired point but being unable to reach expected end. Solving problems is not only an aim of mathematics learning but also an important means of doing mathematics (NCTM, 2000, p. 52). Polya (1957) stated that problem solving consists of four stages: understanding the problem, planning for solution, implementation of the plan and testing the results. During the planning for solution step, person chooses an appropriate strategy; whereas during the implementation step, the chosen strategy is implemented on the relevant problem.

Teachers can be good problem solvers only by learning problem solving strategies, methods and how they are used (Posamantier & Krulik, 1998). Van de Walle (2004) defines the problem solving strategy as developing a method to solve a problem at hand. Posamantier and Krulik (1998) classifies problem solving strategies as working backwards, finding a pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing (visual representation), intelligent guessing and testing, accounting for all possibilities, organizing data and logical reasoning. Working backwards is a strategy that problem solver works back when there is a unique endpoint and variety of paths to get to the starting point. In finding a pattern strategy problem solver seeks a pattern and uses this pattern to find solution. Adopting a different point of view is looking at the problem from different perspective. Solving a simpler analogous problem strategy is to change the given problem into one that may be easier to solve. Considering extreme cases strategy is used where some variables are constant, while others are vary to extremes. In making a drawing strategy (visual representation) it is used diagrams or drawings to see relationships between situations and solve problems according to the drawings. In intelligent guessing and testing strategy guess is put and is tested to show it is indeed correct.

Accounting for all possibilities is a strategy that problem solver considers all options and chooses the most suitable one. Organizing data is reorganizing given data from the problem situation in a way different from the way it was presented. Logical reasoning is a thinking process such as if you say A, then it is expected that the response will B (Posamantier & Krulik, 1998). In order to learn the problem solving strategies and choose relevant strategy, students should be aware of what and why they are doing (Gök & Silay, 2009). It is believed that problem solving ability can be improved by learning problem solving strategies (MoNE, 2006; Abu Elwan, 2002) and using appropriate problem-solving steps and strategies will facilitate teachers' work in problem-solving instruction (Ersoy & Güner, 2014).

Gonzalez (1998) described as the fifth step of Polya's problem-solving step is problem posing that to create a new problem using a given situation and numbers (Silver, 1994; Stoyanova & Ellerton, 1996; English 1997; Ticha & Hospesova, 2009). Problem posing situation can be classified as free, semi-structured or structured. Free problem posing is a practice in which student is asked to pose a problem for a given situation without any limitation. Whereas structured problem posing is a practice in which students are given well-structured problems or situations and asked to pose an appropriate problem. Semi-structured problem posing is a practice in which student are given open ended situations or incomplete problems and asked to pose a problem (Stoyanova & Ellerton, 1996).

NCTM (2000, p.258) stated that students should learn to "*formulate interesting problems based on a wide variety of situations, both within and outside mathematics*". In Turkey Middle School Mathematics Education Program also emphasizes using different strategies and posing new problems rather than learning and using algorithms and rules for problem solving (MoNE, 2006; 2013). In addition it is necessary for students to pose problems using some cases and numbers rather than just solving given problems and reaching to the right answers (Ersoy, 2004; Baykul, 2009) and also mathematics teachers candidates should be given chance to pose problems for school mathematics settings (Ellerton, 2013). Nevertheless it is widely observed that schools can't provide the necessary environment to develop the problem posing skills of students (Crespo & Sinclair, 2008). In this regard since problem posing instruction could be done with knowledgeable teachers, it is essential to educate teacher candidates about problem posing that requires creative and logical thinking of individuals (Korkmaz & Gür, 2006) and developing pre-service teachers' mathematical knowledge structures during their education programme is very important for their professional development (Kılıç, 2015).

Problem posing has many benefits both for students and teacher candidates. Problem posing exercises are effective tools for improving problem solving ability (Crespo & Sinclair, 2008; Cankoy & Darbaz, 2010). Besides problem posing helps students to contextualize mathematics in practical life (Stoyanova, 2003), improves mathematical reasoning (Silver, 1994), creativity (Kilpatrick, 1987; Silver, 1997; Yuan & Sriraman, 2010; Sheffield & Cruikshank, 2005; Van Harpen & Sriraman, 2013), logical reasoning (Cankoy & Darbaz, 2010), contributes to the conceptual learning (Akay, 2006; Lavy & Shriki, 2007; Işık, 2011; Korkmaz & Gür, 2006; Toluk-Uçar, 2009) and improves critical thinking (Nixon-Ponder, 1995). It can also be used to test and improves the pedagogical content knowledge of teacher candidates (Toluk-Uçar, 2009; Kılıç, 2013). In addition, problem posing is a powerful tool to assess the mathematical knowledge and ability both of the student and the teacher (Kar & Işık, 2014). Ellerton (2013) stated that pre-service teachers bring considerable mathematical and pedagogical insight into their involvement with problem posing in mathematics content classes. For that reasons problem posing can be used as a tool for investigating teacher candidates' knowledge about problem solving strategies. On the other hand problem posing is an important tool to connect everyday life with mathematics (Crespo & Sinclair, 2008; Yuan & Sriraman, 2010). In fact, real-life connections is extensively referred to within educational communities as a way of supporting students' learning and is highly recommended in mathematics education (Lee, 2012). Hence it is important for teacher candidates to pose problems in accordance with the daily life (Verschaffel, De Corte & Lasure, 1994). Lee (2012) stated that more work is needed to gain further insight into teacher candidates' perceptions of real-life connections.

Crespo (2003) emphasized that many researches focused on teacher candidates problem solving, but they passed over the problem posing issue. Whereas problem posing activities improve the problem solving skills of students positively (Brown & Walter, 2005; Silver, 1994; English, 1997) and there is a positive relation between problem posing and solving skills of students (Silver & Cai, 1993). According to the relevant literature researches on problem solving strategies generally investigated the strategies used by teachers and students to solve problems (Verschaffel, De Corte & Lasure, 1994; Silver & Cai, 1996; Yazgan & Bintaş, 2005; Elia, Heuvel-Panhuizen & Kolovou, 2009; Olkun, Şahin, Akkurt, Dikkartın, Gülbağcı, 2009; Çelebioğlu & Yazgan, 2009; Gür & Hangül, 2015; Aksoy, Bayazit & Kırnap-Dönmez, 2015). There are some researches on problem posing that investigated the problems posed by students and teacher candidates about fractions (Crespo, 2003; Işık, 2011; Kılıç, 2013;

Kılıç, 2015; Rizvi, 2004; Toluk-Uçar, 2009; Ünlü & Ertekin, 2012), algebraic equations (Akkan, Çakıroğlu & Güven, 2009; Işık & Kar, 2012; Aydoğdu- İskenderoğlu & Güneş, 2016), probability (Yıldız & Baltacı, 2015; Silber & Cai, 2017), absolute value (Güveli, 2015), ratio-proportion (Şengül & Katrancı, 2015), sets (Şengül & Katrancı, 2012) and geometry (Van Harpen & Sriraman; 2013; Lavy & Bershadsky, 2003) subjects. That researches revealed that students and teacher candidates had some difficulties regarding problem posing. This fact necessitated studies on further studies on problem posing. In short, in light of this researches, it was seen that studies related to problem posing generally examine the mathematics teacher candidates' problems about different mathematics subjects. However, when the literature is examined, there has been no study which directly investigated how mathematics teacher candidates pose problems about problem solving strategies. The problems posed about problem solving strategies will provide information both about competences of the teacher candidates for problem solving strategies and problem posing skills.

In addition little is known about the nature of the underlying thinking processes that form problem posing and plans through which students' mathematical problem posing can be examined and evaluated (Christou, Mousolides, Pittalis, Pitta-Pantazi & Shiraman, 2005). In order to understand problem in a deeper way, problem posing related to problem solving is essential (Korkmaz & Gür, 2006). Problem posing processes about problem solving strategies will provide information about mathematics teacher candidates' knowledge about problem solving and problem posing. Unlike other researches, this research investigates mathematics teacher candidates' knowledge about problem solving strategies in context of problem posing. Considering knowledge about problem solving strategies of mathematics teacher candidates is important for both mathematical learning and teaching, in this study it was aimed to investigate mathematics teacher candidates who are the ones first introduces children to the world, mathematics knowledge about problem solving strategies through problem posing. Besides by analyzing reasoning process during the problem posing, it could be obtained important evidence on the thinking processes of the teacher candidates. Results of the study will help teacher educators to develop their knowledge about problem solving strategies and problem posing.

2. Methodology

Qualitative case study was used to determine mathematics teacher candidates' knowledge about problem solving strategies through problem posing. "*Case studies can establish cause and effect, indeed one of their strengths is that they observe effects in real contexts, recognizing that context is a powerful determinant of both causes and effects.*" (Cohen, Manion, & Morrison, 2011, p.253).

2.1 Participants

The participants of the study consisted of mathematics teacher candidates who were in their second, third or fourth grade during the first term of the 2015-2016 academic year. Participants were studying at a public university faculty of education in Turkey. In the first stage the study was conducted with 95 mathematics teacher candidates. There were 76 female and 19 male students enrolled the study. Their ages ranged between 19 and 22 years old. Some of teacher candidates had enrolled in the Mathematics Teaching I method course in third year and some had enrolled elective Problem Solving Strategy course in their second year. Within the scope of Mathematics Teaching I and Problem Solving Strategy, problem, problem solving, Polya's problem solving stages, routine and non-routine problems, problem solving strategies and their applications were covered.

In the second stage 10 teacher candidates were selected from participants using purposeful sampling (Patton 1990) and performed clinical interviews with this sample. The sample contained both candidates who posed problems with the appropriate strategy and candidates who failed to do that. All of this teacher candidates had taken elective Problem solving strategies course. Teacher candidates were coded as Ö1, Ö2, Ö3, Ö4, Ö5, Ö6, Ö7, Ö8, Ö9, Ö10 and researcher was coded as R. Four of the teacher candidates were found to be at a low level, three of them were at a medium level and the remaining three were at a high level of success. Ö1 (male, 2nd grade, low level), Ö2 (female, 2nd grade, medium level), Ö3 (female, 3rd grade, high level), Ö4 (male, 3rd grade, medium level), Ö5 (female, 4th grade, low level), Ö6 (female, 4th grade, medium level), Ö7 (male, 2nd grade, high level), Ö8 (female, 3rd grade, high level), Ö9 (male, 4th grade, low level) and Ö10 (male, 3rd grade, low level).

2.2 Data Collection Tools

Problem Posing Test (PPT) was used as a data collection tool which developed by researcher. It consists of 5 open ended questions. PPT was prepared in accordance with the free problem posing activities. Free problem-posing situations encourage students to reflect on their specific previous experience (Stoyanova & Ellerton, 1996). The result of the literature review the most common strategies such as working backwards, finding pattern, intelligent guessing and testing, accounting for all possibilities and making drawing (visual representation) were selected for this research (Altun & Arslan, 2006). Open ended questions were given below:

1. Pose a problem that can be solved using Working Backward Strategy.
2. Pose a problem that can be solved using Finding Pattern Strategy.
3. Pose a problem that can be solved using Intelligent Guessing and Testing Strategy.
4. Pose a problem that can be solved using Accounting for All Possibilities Strategy.
5. Pose a problem that can be solved using Making a Drawing (Visual Representation) Strategy.

It was consulted to three specialists to verify the data collection tool. Pilot study was performed with 10 teacher candidates and results of the pilot study proved that test was understandable and accurate. For PPT, clinical interviews were performed with 10 teacher candidates to determine their reasoning during problem solving and problem posing. Clinical interview is widely used to determine the thoughts of students in mathematics education (Merrifield & Pearn, 1999). It is a powerful tool to determine the mathematical reasoning underlying the questions and characterize the thoughts, strategies, knowledge and skills of students during mathematics education (Hunting, 1997; Karataş & Güven, 2003).

2.3 Data Collection and Analysis

Teacher candidates were asked to pose problems that can be solved using problem solving strategies in the Problem Posing Test. Data collected in this study consists of problems posed by teacher candidates and video recordings of interviews.

Candidates were given enough time to pose their problems and necessary measures were applied to prevent interaction. Problems were assessed individually by two academicians specialized on mathematics. Problems were examined and categorized based on content analysis. Then classifications were compared. When there was a discrepancy between categories, they discussed and reached to a consensus. Classification were “appropriate for strategy”, “inappropriate for strategy” or “Non-problem”. Afterwards problems were categorized based on whether they were associated with daily life or not. In the next phase, problems were categorized as solvable or unsolvable. The formula of Miles and Huberman (1994) was used to calculate inter-rater reliability and it was calculated as 92% for the classification. Examples of analyzing problems were given in Table 1:

Table 1. Examples of Analyzing Problems

Code	Problems
1. Appropriate for Working Backward Strategy, Related to Daily Life and Solvable	*1. A family is putting the apples they collected from the orchard into a basket. Father ate the half of the apples in the basket. Then the mother ate one third of the remaining apples. Then big brother ate the one fifth the remaining apples. Then the little brother ate one eight of the remaining apples. If seven apples are left in the basket; what was the total number of apples in the basket at the beginning?*
2. Appropriate for Making Drawing Strategy, Related to Daily Life and Unsolvable	*2. We want to plant trees with regular spaces between trees. If we plant one tree to every corner. How many trees can be planted to 190 orchards?*
3. Appropriate for Working Backward Strategy, Not Related to Daily Life and Solvable	*3. We add 5 to a number, then divide the result with two and multiply it with six, if we get 66 as the result what was the beginning number?*
4. Not Appropriate for Accounting for All Possibilities Strategy, Related to Daily Life and Solvable	*4. Ali, Mustafa, Mert, Ayşe, Fatma and Gül are all married couples. Ali is the big brother of Fatma; Mustafa is the ex-fiancée of Ayşe and brother of Fatma. Who is married to who?*

5. Not Appropriate for Working Backward Strategy, Related to Daily Life and Unsolvable	*5. A child ate 2/3 of his/her cake, his/her mother ate ¼ and his sister/bother ate 1/2. If cake has 24 slices, how many slices mother ate?*
6. Not Appropriate for Intelligent Guessing and Testing Strategy, Not Related to Daily Life and Solvable	*6. Estimate the sum of three times 25 and two times 12.*

Code1: Since the problem can be solved by working back, it was associated with real life, and it was used appropriate numbers and has a realistic answer, the problem was categorized as appropriate for working backwards strategy, related to daily life and solvable.

Code 2: The problem can be solved by using drawings and so this problem was appropriate to the making a drawing (visual representation) strategy and related to daily life. It was not solvable as it did not specify the gap we could use to plant trees so the problem was categorized as appropriate for making drawing strategy, related to daily life and unsolvable.

Code 3: This problem can be solved by working back and it has a solution but it was created by using mathematical number and it was not associated with real life so it was categorized as appropriate for working backwards strategy were not related to daily life and solvable.

Code 4: Problems that were not appropriate for accounting for all possibilities strategy were not related to daily life and solvable

Code 5: This problem could not be solved by working back so it was categorized as inappropriate for working backwards strategy. It associated with daily life but the sum of fractions is bigger than one so it was related to real life and unsolvable.

Code 6: This problem can be solved by estimation strategies, it was created by using number and problems have a solution so it was categorized as inappropriate for intelligent guessing and testing strategy, it was not related to daily life and solvable.

Categories determined after analysis was converted to percentage tables and frequencies. Moreover clinical interviews were recorded and analyzed by watching replays. Teacher candidates were informed about the interview and their informed consent was obtained. Interviews lasted about 25-30 minutes and were recorded, then they have been analyzed during replay. Content analysis method was used for data analysis (Miles & Huberman, 1994). Besides direct citations were given from clinical interviews in order to improve the internal reliability and validity of results.

3. Results

Teacher candidates were asked to pose a problem that can be solved by using problem solving strategy. Problems were classified as “appropriate for strategy”, “inappropriate for strategy” or “Non-problem”. Afterwards problems were categorized based on whether they were associated with daily life or not. In the next phase, problems were categorized as solvable or unsolvable. Frequencies (f) and percent (%) of problems were given in tables.

3.1 Problems about Working Backwards Strategy

Frequencies (f) and percent (%) of problems about working backwards strategy were summarized in Table 2:

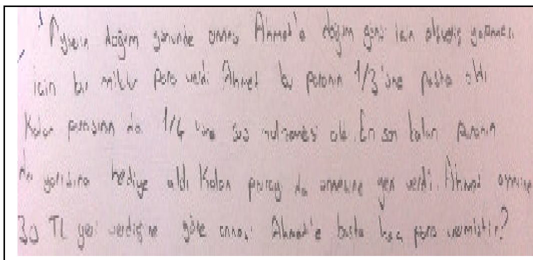
Table 2: Problem Types of Teacher Candidates

			f	%
Appropriate for Working Backwards Strategy	Real life Situation	Solvable	37	38.9
		Unsolvable	3	3.2
	Not Real life Situation	Solvable	25	26.3
		Unsolvable	-	-
Inappropriate for Working Backwards Strategy	Real life Situation	Solvable	16	16.8
		Unsolvable	3	3.2
	Not Real life Situation	Solvable	1	1.1
		Unsolvable	-	-
Non-Problem			10	10.5

When Table 2 was analyzed, it was seen that 68.4% of the candidates posed problems in accordance with the strategy. Of those appropriate problems, 42.1% was related to daily life and 38.9% was solvable. Whereas 26.3% was not related to daily life and unsolvable.

21.1% of the candidates failed to pose a problem in accordance with the strategy. Of those 20% was related to daily life, 16.8% was solvable and 1.1% was not related to daily life. 10.5% of the sample failed to pose a problem and things they wrote was not a problem.

According to the findings problems were mostly appropriate for working backwards strategy, related to daily life and solvable. Ö2 was one of the candidates that pose a problem in accordance with the working backwards strategy. Clinical interview results of that candidate below:



Ahmet's mother gave him money to spend for Ayşe's birthday celebration. Ahmet bought a birthday cake using 1/3 of the money. He spent the 1/4 of the remaining money to buy ornaments. Lastly he spent half of the remaining money to buy a present. He gave the remaining money back to his mother. If Ahmet gave 30 TL back to his mother, how much money was given to him at the beginning?

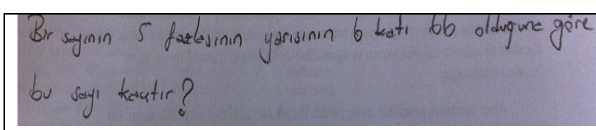
Figure 1. Ö2's Problem about Working Backwards Strategy

R: How did you pose this problem?

Ö2: First I imagined him to have 120 TL. If he was to spend three fourth of this money, there would be 80 TL left in his hand. If was to spend one fourth of this money, there would be 60 TL left in his hand. If was to spend half of this money, there would be 30 TL left in his hand. I posed the question based on this reasoning.

Analysis showed that candidate actually knew the working backwards strategy, she was able to pose an appropriate problem for strategy. In addition she applies the strategy to a real life situation and posed an solvable problem by using appropriate numbers.

It was seen that many problems posed as an exercise, but not related to daily life. Ö7 was one of the candidates that posed a problem in accordance with the strategy. Clinical interview results of that Ö7 candidate below:


Bir sayının 5 fazlasının yarısının 6 katı 66 olduğuna göre
bu sayı kaçtır?

We add 5 to a number, then divide the result with two and multiply it with six, if we get 66 as the result what was the beginning number?

Figure 2. Ö7's Problem about Working Backwards Strategy

R: How did you pose this problem?

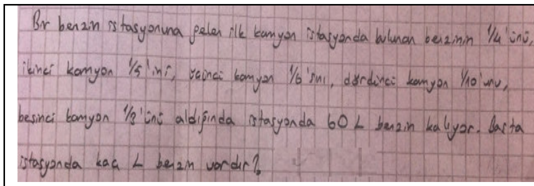
Ö7: Even a student who don't know the algebraic equations can easily solve this problem using working backwards strategy. He just have to work his way from the end condition toward the beginning condition.

R: Why did you pose this problem?

Ö7: I posed a short problem. When I was a student, we used to solve such problems. A long problem can intimidate children and they may quit.

Teacher candidate believed this problem can easily be read and understood by students as it was short. He also emphasized that he solved similar problems as a student.

The most common mistake done by candidates were using wrong numbers and providing insufficient information (such as forgetting the "remaining…." expression). The interview results of candidate Ö1, who failed to pose an appropriate question, was follows:



In a petrol station, first truck buys the 1/4 of the gasoline, second truck buys the 1/5, third truck buys 1/6, fourth truck buys 1/10, and fifth truck buys 1/3. There left 60 liters of gasoline in the station. How much gasoline there were at the beginning?"

Figure 3. Ö1's Problem about Working Backwards Strategy

R: How did you pose this problem?

Ö1: I wanted the student to find the initial gasoline starting from the remaining gasoline.

R: How do you solve this problem?

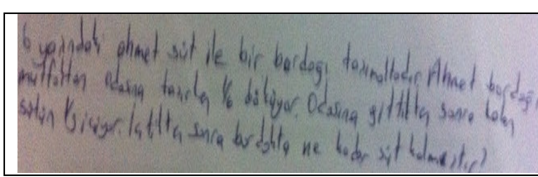
Ö1: I solved it by removing 1/3, 1/10, 1/6, 1/5, 1/4 from 60 liters. (Making calculations on his head) What is the number of which one of third is 60? 90… What is the number of which nine of tenth is 90? 100… What is the number of which five of sixth is 100? 120… What is the number of which four fifth is 120? 150… What is the number of which three of fourth is 150? 200… The answer is 200. (He is still not aware of his error).

R: How would you solve it otherwise?

Ö1: I would use fractions. If I equate the denominators and sum the fractions, I get sixty three divided by sixty…(He is thinking for a while). I think I made an error. Because sixty three divided by sixty is bigger than one. Trucks can't buy more gasoline than the existing. Where did I made mistake? (Candidate controls his calculations again). It is impossible that make mistake in the working backwards strategy. I made a mistake while summing the fractions (He is confident and can not find his mistake).

When problem was examined, it was seen that if it was phrased as "second truck buys 1/5 of the remaining gasoline" it could be solvable using working backwards strategy. But teacher candidate believed that the problem he posed was compatible with the working backwards strategy, so he tried to solve it using that strategy and failed to detect his mistake. This shows that teacher candidate knows the strategy, but he can't analyze the problem and tries to pose using his previous experiences. Although it was insisted on, the teacher candidate failed to realize his mistake.

Ö5 was one of the candidates that did not pose a problem in accordance with the strategy. Clinical interview results of that candidate were given below:



Ahmet is a 6 years old boy and he is cary a glass full of milk. He spilled 1/6 of the glass on the way from kitchen to his room. In his room, he drank 1/3 of the remaining milk. How much milk is left in the glass?

Figure 4. Ö5's Problem about Working Backwards Strategy

R: How did you pose this problem?

Ö5: I constructed the problem by using examples we see in the classroom. A full glass is six out of six. (She wrote and at that moment teacher candidate realized her mistake). The problem is not compatible with the working backwards strategy.

R: Why?

Ö5: Because in this strategy, the last situation and changes from the beginning were given, then student was asked to work his/her way back to the original situation. But in this problem, original situation was given. If I have given the remaining milk (200 ml) and ask the original situation, it would be appropriate.

This problem was not solvable using the working backwards strategy as enough detail was not provided by the teacher candidate. But teacher candidate realized her mistake during the interview. She knew working backwards strategy but she failed to pose problem according to the working backwards strategy.

3.2 Problems about Finding Pattern Strategy

Frequencies (f) and percent (%) of problems about finding pattern strategy were summarized in Table 3:

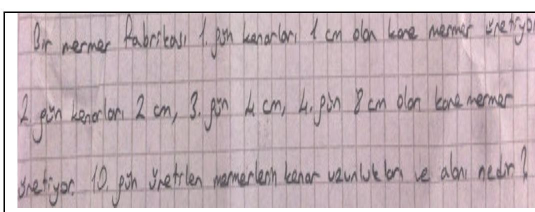
Table 3: Problem Types of Teacher Candidates

			f	%
Appropriate for Finding Pattern Strategy	Real life Situation	Solvable	13	13.7
		Unsolvable	7	7.4
	Not Real life Situation	Solvable	58	61.1
		Unsolvable	-	-
Inappropriate for Finding Pattern Strategy	Real life Situation	Solvable	2	2.1
		Unsolvable	1	1.1
	Not Real life Situation	Solvable	3	3.2
		Unsolvable	-	-
Non Problem			11	11.6

When Table 3 was analyzed it was seen that 82.2% of the candidates posed problems in accordance with the strategy. Of those appropriate problems, 21.1% was related to daily life and 13.7% was solvable, 7.4% was unsolvable. Whereas 61.1% was not related to daily life and of those 61.1% was unsolvable.

6.4% of the candidates failed to pose a problem in accordance with the strategy. Of those 3.2% was related to daily life, 2.1% was solvable and 1.1 unsolvable. Whereas 3.2% was not related to daily life and solvable. 11.6% of the sample failed to pose a problem and things they wrote was not a problem.

Ö1 was one of the candidates that pose a problem in accordance with the strategy. The clinical interview results of that candidate were given below:

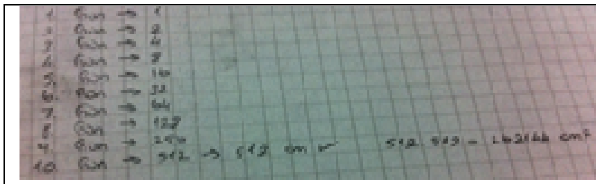

Bir mermer fabrikası 1. gün kenarları 1 cm olan kare mermer üretiyor 2. gün kenarları 2 cm, 3. gün 4 cm, 4. gün 8 cm olan kare mermer üretiyor. 10. gün üretilen mermerlerin kenar uzunlukları ve alanı nedir?

A marble factory is producing square marbles with 1 cm edges in the first day. It is producing marbles with 2 cm, 4 cm, and 8 cm respectively in the second, third and fourth days. What will be the edge size and area of the marbles produced in the tenth day?

Figure 5. Ö1's Problem about Finding Pattern Strategy

R: How did you pose this problem?

Ö1: First I imagined a pattern developing as the powers of two; such as second power of two, third power of two; fourth power of two, fifth power of two, sixth power of two (He wrote the answer on paper)… Then I asked the 10th day. If it is 8 cm in the fourth day. It is 8.2=16, in the fifth day … it will be 512 cm in the tenth day. When you find the edge size, you can calculate the area.

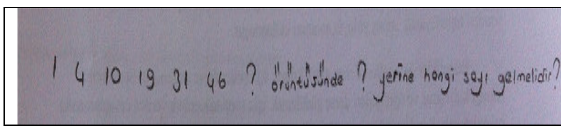


1.day=1	6.day=32
2.day=2	7.day=64
3.day=4	8.day=128
4.day=8	9.day=256
5.day=16	10.day=512

Figure 6. Ö1's Problem Solving about Finding Pattern Strategy

The teacher candidate stated that he first imagined a pattern and posed the problem on this pattern. He knew the finding pattern strategy and he could pose problem about this strategy.

Ö8 was one of the candidates that posed a problem in accordance with the strategy, was not related to daily life and solvable. After clinical interviews it was investigated that he had a knowledge about finding pattern strategy and he could pose problem. Clinical interview results of that candidate below:



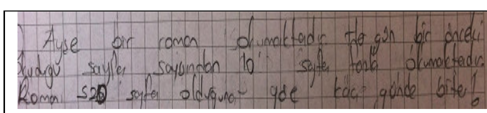
In the 1 4 10 19 31 46 ? pattern, what number should replace the symbol?

Figure 7. Ö8's Problem about Finding Pattern Strategy

R: How did you pose this problem?

Ö8: First I imagined a pattern. My first number was one, second number was 1+3.1=4; third number was 4+3.2=10, fourth number was 10+3.3=19, and fifth number was 19+3.4=31.

Ö4 was one of the candidates that posed a problem in accordance with the strategy, related to daily life and was not solvable. Clinical interview results of that candidate below:



Ayşe is reading a novel. She reads 10 pages more than the previous day. How many days does it take to finish a novel with 520 pages?

Figure 8. Ö4's Problem about Finding Pattern Strategy

R: How did you pose this problem?

Ö4: I thought increasing by ten creates a pattern. 10, 20, 30 and so on. Because it is increasing with a rate, sequentially at any rate…

R: Could you solve this problem?

Ö4: If the number of pages read in the first day is x, second day can be calculated as x+10, third day x+20, fourth day x+30, ,…. (Candidate thought and started to write). I can't solve this problem. I should have given the number of pages she read in the first day. I didn't.

Here, it was seen that the candidate did not know the finding pattern strategy and has inadequate in problem posing.

According to the findings problems posed by teacher candidates about finding pattern strategy was generally not related with the daily life. Besides, although they know the strategy and how to use it, they posed problems lacked data to be solvable.

3.3 Problems about Intelligent Guessing and Testing Strategy

Frequencies (f) and percent (%) of problems that can be solved by using intelligent guessing and testing strategy were summarized in Table 4:

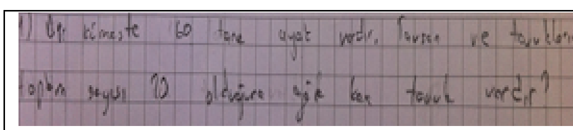
Table 4: Problem Types of Teacher Candidates

			f	%
Appropriate for Intelligent Guessing and Testing Strategy	Real life Situation	Solvable	38	40
		Unsolvable	2	2.1
	Not Real life Situation	Solvable	14	14.7
		Unsolvable	-	-
Inappropriate for Intelligent Guessing and Testing Strategy	Real life Situation	Solvable	14	14.7
		Unsolvable	4	4.2
	Not Real life Situation	Solvable	11	11.6
		Unsolvable	-	-
Non-Problem			12	12.6

When table was analyzed, it was seen that 56.8% of the candidates posed problems in accordance with the strategy. Of those appropriate problems, 42.1% was related to daily life. Of those problem 40% was solvable, 2.1% unsolvable. Whereas 14.7% was not related to daily life and solvable.

30.5% of the candidates failed to pose a problem in accordance with the strategy. Of those 18.9% was related to daily life, 14.7% was solvable and 4.2% was unsolvable. 11.6% was not related to daily life and solvable. 12.6% of the sample failed to pose a problem and things they wrote was not a problem.

Problems that were in accordance with the strategy were mostly related to daily life and solvable. Ö3 was one of the candidates that pose a problem in accordance with the intelligent guessing and testing strategy. Clinical interview results of that candidate were given below:



There is 60 feet in a hen. If there is 20 chicken and rabbit in total, how many chicken there is in the hen?

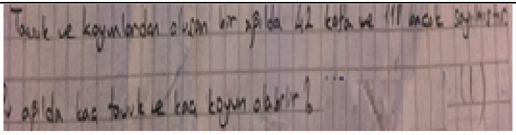
Figure 9. Ö3's Problem about Intelligent Guessing and Testing Strategy

R: How did you pose this problem?

Ö3: We can determine random numbers, because we know the number of feet and animals. For example, we can choose 11 chicken and 9 rabbits. 11 chicken has 22 feet and 9 rabbit has 36 feet. It is 58 in total. Our estimate is wrong. I have to increase the number of rabbits to reach to 60. 10 rabbits and 10 chicken. 10 chicken has 20 feet and 10 rabbit has 40 feet. This makes 60 feet. I constructed the problem on this reasoning.

Teacher candidate had a knowledge about intelligent guessing and testing strategy but the problem posed by Ö3 is unsolvable as it does not specify there only chickens and rabbits in this hen. Ö1 posed a similar problem with Ö3 problem. He stated that "there are only chickens and sheeps.

Problem and clinical interviews were given below:



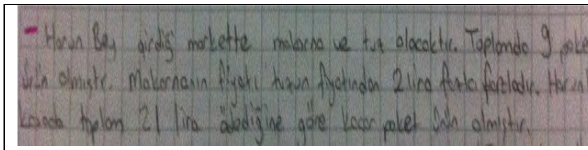
There are only chickens and sheeps in a fold. If there are 42 head and 118 feet in this fold, how many chicken and sheep there is in this fold?

Figure 10. Ö1's Problem about Intelligent Guessing and Testing Strategy

R: How did you pose this problem?

Ö1: Both chickens and sheep have only one head, but chicken have two whereas sheep have four feet. Thus one can estimate the number of sheep and chicken.

Ö10 was one of the candidates that pose a problem in accordance with the strategy, related to daily life but this problem is not solvable using the intelligent guessing and testing strategy as enough detail is not provided by the teacher candidate. Clinical interview results of that candidate below:



Harun bought salt and pasta from a market. He bought 9 packages in total. Pasta costs 2 liras more than salt. If Harun paid 21 TL, how many packages he bought from each product?

Figure 11. Ö10's Problem about Intelligent Guessing and Testing Strategy

R: How did you pose this problem?

Ö10: I tried to pose a problem similar to problems we see in the textbooks. A daily problem, we all go to market and buy stuff.

R: Could you solve this problem?

Ö10: If I call the price of pasta x, and price of salt x+2… He is buying 9 packages. 1 package of pasta and 8 packages of salt. 1.x + 8.(x+2)=21, x=5/9 Is that wrong? (He thinks for a while). It is not solvable, because it changes as the x changes. I should have given the price of pasta.

Teacher candidate did not pose appropriate problem for intelligent guessing and testing strategy. He realized his mistake during the interview but he could not pose new problem about the strategy during the interview.

3.4 Problems about Accounting for All Possibilities Strategy

Frequencies (f) and percent (%) of problems that can be solved by using accounting for all possibilities strategy were summarized in Table 5:

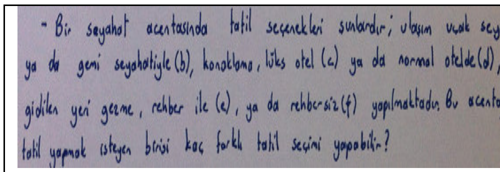
Table 5: Problem Types of Teacher Candidates

			f	%
Appropriate for Accounting for All Possibilities Strategy	Real life Situation	Solvable	46	48.4
		Unsolvable	-	-
	Not Real life Situation	Solvable	21	22.1
		Unsolvable	-	-
Inappropriate for Accounting for All Possibilities Strategy	Real life Situation	Solvable	9	9.5
		Unsolvable	-	-
	Not Real life Situation	Solvable	1	1.1
		Unsolvable	-	-
Non-Problem			18	18.9

When table was analyzed, it was seen that 70.5% of the candidates posed problems in accordance with the strategy. Of those appropriate problems, 48.4% was related to daily life and solvable. Whereas 22.1% was not related to daily life and solvable.

10.6% of the candidates failed to pose a problem in accordance with the strategy. Of those 9.5% was related to daily life and solvable and 1.1% was not related to daily life. 18.9% of the sample failed to pose a problem and things they wrote was not a problem.

Problems that were in accordance with the strategy were mostly related to daily life and solvable. Ö6 was one of the candidates that pose a problem in accordance with the strategy. Clinical interview results of that candidate were given below:


- Bir seyahat acentasında tatil seçenekleri şunlardır; ulaşım uçak sey
ya da gemi seyahatiyle (b), konaklama, lüks otel (c) ya da normal otelde (d),
gidilen yeri gezme, rehber ile (e), ya da rehbersiz (f) yapılmaktadır. Bu acenta
tatil yapmak isteyen birisi kaç farklı tatil seçimi yapabilir?

A travel agency have these options: (a) travel by airplane or (b) travel by ship; (c) accommodation at a luxury hotel or (d) accommodation at a regular hotel; (e) guided walk or (f) unguided walk. How many alternatives can a customer choose?

Figure 12. Ö6's Problem about Accounting for All Possibilities Strategy

R: How did you use the accounting for all possibilities strategy in this problem?

Ö6: First we chose our transportation, accommodation and guidance. By listing we can see all options. (She writes all options). Airplane, luxury hotel, guided (a, c, e); airplane, luxury hotel, unguided (a, c, f)...

Problem solving of teacher candidate was given below:

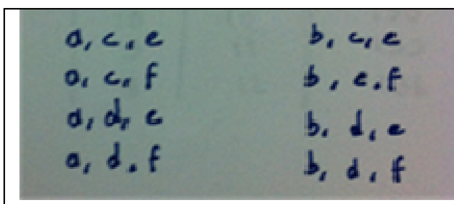
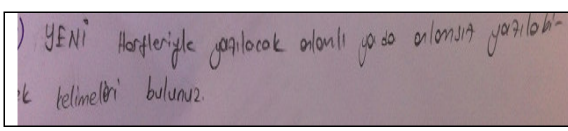

a, c, e	b, c, e
a, c, f	b, c, f
a, d, e	b, d, e
a, d, f	b, d, f

Figure 13. Ö6's Problem Solving about Accounting for All Possibilities Strategy

Ö8 was one of the candidates that pose a problem in accordance with the strategy but did not related to daily life and clinical interview results of that candidate below:


) YENİ Harfleriyle yazılacak anlamlı ya da anlamsız yazılabi-
k kelimeleri bulunuz.

By using the letter in the word "YENİ", how many word can we create irrespective of meaningfulness?

Figure 14. Ö8's Problem about Accounting for All Possibilities Strategy

R: How did you pose this problem?

Ö8: In order to determine how many words we can create, first we need to reorder them systematically. I based my problem on this reasoning.

R: Could you solve this problem?

Ö8: First I wrote the Y letter, then by keeping the second letter same I switched the third and *fourth letters. This created 6 alternatives. Then I repeated this process using every letter.*

Problem solving of teacher candidates was given below:

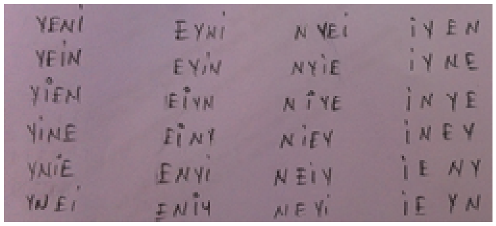


Figure 15. Ö8's Problem Solving about Accounting for All Possibilities Strategy

3.5 Problems about Making a Drawing (Visual Representation) Strategy

Frequencies (f) and percent (%) of problems that can be solved by using making drawing (visual representation) strategy were summarized in Table 6:

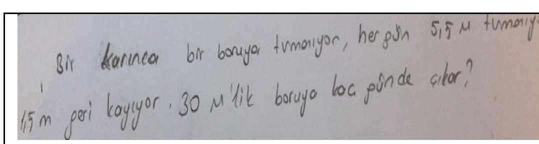
Table 6: Problem Types of Teacher Candidates

			f	%
Appropriate for Making a Drawing Strategy	Real life Situation	Solvable	70	73.7
		Unsolvable	3	3.2
	Not Real life Situation	Solvable	8	8.4
		Unsolvable	-	-
Inappropriate for Making a Drawing Strategy	Real life Situation	Solvable	-	-
		Unsolvable	-	-
	Not Real life Situation	Solvable	4	4.2
		Unsolvable	-	-
Non-Problem			10	10.5

When table was analyzed, it was seen that 85.3% of the candidates posed problems in accordance with the strategy. Of those appropriate problems, 76.9% was related to daily life. Of those problem 73.7% was solvable, 3.2% non-solvable solvable. Whereas 8.4% was not related to daily life and solvable.

4.2% of the candidates failed to pose a problem in accordance with the strategy. Of those 4.2% was not related to daily life and solvable. 10.5% of the sample failed to pose a problem and things they wrote was not a problem.

Problems that were in accordance with the strategy were mostly related to daily life and solvable. Ö7 was one of the candidates that pose a problem in accordance with the strategy, related to daily life and solvable. Clinical interview results of that candidate below:



An ant is climbing to a pipe. Every day it climbs up 55 meters, but slips 1, 5 meters back. How many days does it take to the top of a 30 meters long pipe?

Figure 16. Ö7's Problem about Making a Drawing Strategy

R: How did you pose this problem?

Ö7: Child will subtract 1.5 from 5.5 and find 4. Then he will try to divide 30 by 4, but it is not an aliquot.

Child will be forced to draw the situation.

R: Could you solve this problem now?

Ö7: If ant climb 4 meters a day, it reaches to the 28th meter at seventh day. So it climbs the last two meters in the eight day.

Visual Representation of teacher candidates was given below:

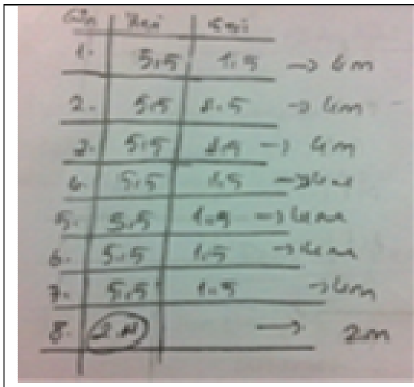
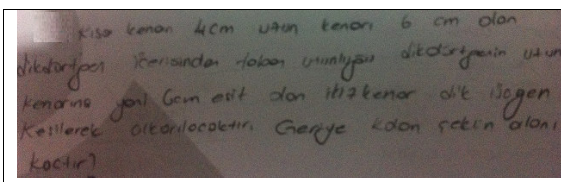


Figure 17. Ö7's Problem Solving about Making a Drawing Strategy

Ö9 was one of the candidates that pose a problem in accordance with the strategy but did not related to daily life and non solvable. Clinical interview results of that candidate were given below:



We want to extract an isosceles right angled triangle with the edge length 6 cm out of a rectangle with edges 4 and 6 cm long. What is the area of remaining figure?

Figure 18. Ö9's Problem about Making a Drawing Strategy

R: How did you pose this problem?

Ö9: If we draw a triangle inside a rectangle, we can solve this problem.

R: Could you solve it now?

Ö9: He draw a rectangle and he tried to draw isosceles right angled triangle with the edge. (He thinks for a while). It is impossible. I wrote wrong problems I think.

Teacher candidates knew the strategy but he could not pose a solvable problem. Through clinical interviews he could not solve this problem because when he draw a rectangle and tried to draw triangles, he saw the problem was non-solvable.

4. Discussion

The aim of the study was to determine mathematics teacher candidates' knowledge about problem solving strategies through problem posing. Teacher candidates were asked to pose problems that can be solved using problem solving strategies in the Problem Posing Test.

Results show that teacher candidates are skilled in problem posing in accordance with problem solving strategies. However previous studies concluded that teachers and teacher candidates had some difficulties with problem posing (Crespo, 2003; Rizvi, 2004; Akkan, Çakıroğlu & Güven, 2009; Toluk-Uçar, 2009; Işık, 2011; Ünlü & Ertekin, 2012; Işık & Kar, 2012; Kılıç, 2013). This can be explained with the addition of problem posing and problem solving strategies to the new Middle School Mathematics Education Program. Besides it was believed that Problem Solving Strategies and Mathematics Teaching lessons taken by teacher candidates also contributed to this result. Ellerton (2013) emphasized the importance of preparing learning environments in which teacher candidates can pose their own problems instead of teacher education, which focuses on finding and solving difficult problems from the internet and books. In order to teach individuals how to pose creative and rational problems, teachers that have the required knowledge and skills were needed (Korkmaz & Gür, 2006). Teachers should pose different problems which are not solvable by using simple operations (Altun & Arslan, 2006). In addition learning to pose mathematical tasks is important learning to teach mathematics (Crespo, 2003). Thus it is essential to educate teacher candidates on the problem posing. A teacher equipped with necessary skills to pose problems in accordance with the problem solving strategies can easily be successful at teaching mathematics (Altun & Arslan, 2006). Besides, even though few in number, there were teacher candidates who can not pose appropriate problems or any problem at all. This result was compatible with the findings of Chen, Van Dooren, Chen & Verschaffel (2011) and Kılıç (2015).

The types of problems that mathematics teacher candidates posed were analyzed. It is important for mathematical problems to be related to daily life (Işık & Kar, 2012). Research revealed that problems posed by teacher candidates were related with real life. Kılıç (2015) also found pre-service teachers were able to pose non-standard problems, which can be solved by using arithmetical operations and taking into account real life situations. Teachers should learn how to apply problem solving strategies not only to mathematical situations but also to daily life situations (Posamantier & Krulik, 1998). Otherwise, students will be disappointed that they will not see the benefits of mathematics and their relations of it to daily life (Steen & Forman, 1995). The association of problems with everyday life will also provide meaningful learning of students in mathematics (Verschaffel, De Corte & Lasure, 1994). Therefore, it is very important for teacher candidates to pose problems that related mathematics to everyday life.

When posing a problem, its solution should always be kept under mind (Cai & Hwang, 2002). When the problems were examined in terms of solvability, problems posed by mathematics teacher candidates were generally solvable. Korkmaz & Gür (2006) also stated that problems posed by teachers in their research are generally solvable and appropriately expressed. There were also teacher candidates who posed unsolvable problems. Teacher candidates did not check whether there was a solution or not. This result was compatible with the findings of Crespo (2003) and Kılıç (2013). Main reasons behind this problem were using wrong numbers and inadequate data.

During clinical interviews, it was analyzed how teacher candidates posed problems and whether they can solve their own problems. Many teacher candidates stated that they posed problems similar to the problems they saw in textbooks and lessons, rather than act creatively. This is compatible with the findings of other studies (Barlow & Cates, 2006; Korkmaz & Gür, 2006; Crespo & Sinclair, 2008; Işık, Işık & Kar, 2011; Silver & Cai, 1996; Stickles 2006).

The most common mistake done by teacher candidates were using wrong numbers and providing insufficient information (forgetting the "remaining…." expression) such as *"In a petrol station, first truck buys the ¼ of the gasoline, second truck buys the 1/5, third truck buys 1/6, fourth truck buys 1/10, and fifth truck buys 1/3. There left 60 liters of gasoline in the station. How much gasoline there were at the beginning?"* in working backwards problems. Sometimes teacher candidate knows the strategy, but they can not analyze the problem and they try to pose using their previous experiences. Knowing problem solving strategies is not sufficient to pose appropriate problems. In addition many problems posed as an exercise, but not related to daily life such as *"We add 5 to a number, then divide the result with two and multiply it with six, if we get 66 as the result what was the beginning number?"*

Teacher candidates were more successful at posing problems in accordance with pattern finding strategy. Problems posed with this strategy were not related to daily life, but solvable. There were many problems like "Find the next number in… pattern?" This shows that teacher candidates were not successful at relating patterns in daily life. Therefore, examples which the patterns are related to daily life should be given in the lessons.

It was seen that problems about intelligent guessing and testing strategy both not related to daily life and unsolvable. Findings of the research revealed that teacher candidates often confuse between computational estimation and intelligent guessing and testing strategy. There were also many unsolvable problems because of inadequate information. Besides there were many problems posed according to according all possibilities strategy, but not related to daily life, such as *"How many numbers can we write using numbers 1, 2, 3, and 4?"* and unsolvable such as *"Harun bought salt and pasta from a market. He bought 9 packages in total. Pasta costs 2 TL more than salt. If Harun paid 21 TL, how many packages he bought from each product?"*

Teacher candidates was most successful at posing problems according to making drawing strategy. Most of the problems on this category was related to daily life and solvable. Larkin and Simon (1987) stated using diagrams instead of symbols enhance the intelligibility of problems. Besides visualized problems that these candidates see since their childhood can be a contributing factor for this success.

Considering the importance of problem posing and problem solving strategies it is important to educate teacher candidates during their training to become a teacher (Kılıç, 2015). Understanding teacher candidates' problem posing performance in the context of problem solving strategies have an important contribution to the improvement of future teacher education programs. Because teacher first introduces children to the world of mathematics so courses should be given to teacher candidates which improves problem posing skills of them. In light of this research, further studies may focus on the problems posing success of teacher candidates using different strategies. Similar researches should be done to determine in-service teachers' knowledge about problem solving strategies through problem posing. Students should be encouraged to pose more creative problems beginning from the elementary school to university. Besides teachers can assess the problems posed by secondary school students and provide support.

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