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THE STUDENTS' PROCEDURAL FLUENCY AND WRITTEN-MATHEMATICAL EXPLANATION ON CONSTRUCTED RESPONSE TASKS IN PHYSICS

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Abstract

This study was designed to analyze the procedural fluency and written-mathematical explanation to select constructed response tasks of students in Thermodynamics problems. The study made use of 2 sections, composed of 26 students, in University Physics 1 to conclude on the research problem. It made use of the assumption that mathematical and English abilities control the students' performance to problem solving. Using Pearson-r and One-way ANOVA, it was found out that their procedural fluency on constructed response tasks is significantly related to their written-mathematical explanation ability, and a significant difference on their performance when grouped according to their mathematical and English ability. Bonferroni Correction Post Hoc Test results confirmed the assumptions of the study: the students' procedural fluency is dependent to their mathematical ability, both algebraic and trigonometric, while their written-mathematical explanation is associated to their English ability.

Keywords – *Mathematical Procedural Fluency, Written-Mathematical Explanation, Constructed Response Tasks, Problem Solving*.

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1 INTRODUCTION

Problem solving in Physics, as influenced by mathematical learning theories, is mystified as difficult over the years as students hold negative stereotype images towards the subject (Wambagu & Changeiywo, 2006; Bautista, 2012a). These stereotypical tendencies had hampered its efficacy over the years and became a concomitant factor to be addressed properly. This calls for a sound technique of decontextualized set of skills on convergent reasoning in engaging students to higher cognition activities towards the subject (Bautista, 2012a).

Students' success on constructed response tasks is the result of their mastery on procedural fluency and written-mathematical explanation to problem solving skills and strategies. This helps them hone their negative stereotype images that were drawn over the years in Physics instruction.

Physics instruction, if it is to be responsive, needs a development of students' competence in problem solving skills and strategies and explaining mathematical ideas in a scientific perspective. The students' procedural fluency in problem solving must complement their mathematical-scientific explanations in their decision making. This uses an accurate mathematical language in the teaching and learning as their knowledge and skills in problem solving, which are developed in Mathematics, are of great implication especially on the peculiarities of problems both in science, engineering and industry.

Their ability to solve problems with understanding and communicate mathematical ideas must be nurtured enabling the student-learners acquire the skills to evaluate information, to compare, to make decisions and justify effectively (Pugalee, 2004).

1.1 General Problem Solving Model

Problem solving had been mystified as one of the horrifying classroom activities that hamper its efficacy towards mastery of learning outcomes. Addressing its peculiarities, researchers like Polya, Newell and Simon and Bransford and Stein had developed models to explain the solving processes. The model was based on the assumption that problem solvers can learn abstract problem solving skills (decontextualized knowledge) by transferring these skills into a functional schema of conceptual understanding (contextualized knowledge). The Bardsford's Ideal Model (1984), as cited in Bautista (2012b), in problem solving includes the following steps:

1. Identify the problem;
2. Define the problem through thinking about it and sorting out the relevant information;
3. Explore solutions through looking at alternatives, brainstorming, and checking out different points of view;
4. Act on the strategies; and
5. Look back and evaluate the effects of your activity.

The foregoing model matches with most of the general problem solving models that are commonly used to problem solving activities among classroom instructions. This model is one of the bases of the "Inquiry" curriculum movement that led to the realization of "new" curricula towards problem solving such as "new math."

1.2 Current Problem Solving Models

Problem solving is a mental cognition that involves a series of complex set of cognitive, behavioral, and attitudinal components. In 1983, Mayer defined problem solving as a multiple step process where the problem solver must find relationships between past experiences (schema) and the problem at hand and then act upon a solution. He suggested three complex and complementary characteristics of problem solving:

1. Problem solving is cognitive but is inferred from behavior;
2. Problem solving results in behavior that leads to a solution; and
3. Problem solving is a process that involves manipulation of or operations on previous knowledge (Funkhouser & Dennis, 1992).

Mayer's model for problem-solving (1987, 1992), as cited in Bautista (2012b), consists of four phases: problem translation, problem integration, solution planning and monitoring and solution execution.

1. Problem translation: The problem-solver translates the information given in the word problem into a mental model representative to his interpretation of the problem. The translation may be in a form of a geometrical sketch or a free-body diagram (FBD).
2. Problem integration: The problem-solver tries to combine his pieces of interpretations into a coherent structure supportive to his problem-solving plan.
3. Solution planning and monitoring: During this phase, the problem-solver formulates a solution plan composed of sequenced steps in solving the word problem. It includes indicators to self-correct the solution.
4. Solution execution phase: The problem-solver tries to carry out the planned solution and solves the problem. This includes a look-back where the problem-solver can gauge whether or not the steps used are correct or not.

Gick (1986), as cited in Bautista (2012b), also formulated a model for problem-solving which became a common model used by many in problem-solving tasks.

This model identifies a basic sequence of three cognitive activities in problem solving:

1. *Representing the problem.* This step requires the ability to call up some appropriate context knowledge and the identification of goal and relevant starting conditions for the problem;
2. *Solution search.* This includes the refinement of the goal and the development of a plan of action to reach the goal; and
3. *Implementing the Solution.* This refers to the execution of the plan of action and the evaluation of the results.

The problem solving task is linked to the problem-solver's prior knowledge and experiences (mental schema). The problem solver formulates "short-cut mechanism" to solving the problem by recalling his past experiences and tries to employ it again.

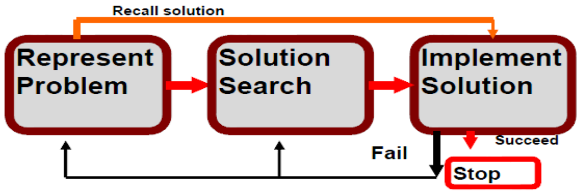

Figure 1: A Model of the Problem Solving Process (Gick, 1986)

Problems, whether simple or complex, may not be solved with a single iteration. The problem-solver may break the problem into *intermediate goals and contextual information*. The switching of this contextual information requires the development of higher order thinking skill called a *cognitive strategy*. Hence, problem solving elicits the development of higher order thinking skills among the students by a "synthesis of other rules and concepts into higher order rules which can be applied to a constrained situation" (Gagne, 1985 as cited by Graven & Stott, 2012).

The solving tasks also include behavioral and cognitive components of mental cognition since the learners must learn to want to do the task. Apparently, they have to develop a sound motivation, confidence, persistence and the belief that they can do the task. This knowledge about self plays an important drive in dealing the decontextualized input towards problem solving.

1.3 Five Strands Of Mathematics Proficiency

The five strands of mathematical proficiency by Kilpatrick, Swafford and Findell (2001) presents the interdependence of the five components of learner's mathematics proficiency in problem solving. It starts with a clear grasp and understanding of the concept/problem, efficient strategies in solving problems, meaningful procedures and justified reasoning. This results in a challenging, engaging and investigative problem solving. All are purposively to refine mathematical proficiency.

Students learn to integrate and form a functional grasp of mathematical ideas in conceptual understanding. This enables the students-learners to earn new learning schema by connecting those ideas to what they already know supportive to retention that prevents common errors. They then carry out their skills and meaningful procedures in a flexible, accurate, efficient, and appropriate manner (Procedural fluency). Strategic competence includes their ability to formulate plans that enables them to use symbolic analysis in solving problems. Using the capacity for a logical thought, the student-learners learn how to reflect, explain and justify procedures/plans/strategies in a diagnostic manner (adaptive reasoning). Corollary to the real life situations, the student-learners see the mental cognition as sensible, useful and worthwhile towards self-efficacy.

1.4 Lesh's Translational Model

Lesh's translational model starts from a sound conceptual understanding on the problem towards the translation within and among the modes of concept representation used by the student-learners. The model presents the interlocking processes of translating conceptual understanding on the problem under multiple modes: Written and verbal symbols, pictures (the free-body diagram report), real life situations and manipulatives. Understanding the problem uses their language proficiency for them to identify both the written and verbal symbols that lead them to picture out the problem through a free-body diagram. Suited in a real life situation, the students develop a plan to carry the strategies in problem solving that lead them to an answer and conclusion under several modes of manipulatives and symbolic notations of the actions to be undertaken (Kaput, 1992 as cited in Suh & Moyer-Packenham, 2007). The verbal mode forms an integral part of the model as they communicate their skills and strategies in each mode of the model. This paves the ability of the student-learners to select, apply and translate problems mathematically.

1.5 VF Theory Of Explanation

The verbal mode, that forms an integral part of the translational model of mathematics procedural fluency, is construed by the VF Theory and the Pragmatic Theory of Explanation. The theory is essentially a theory of probing why questions that leads into the development of a diagnostic higher order thinking skills among the student-learners. This is based on the translated problem leading to a successful retrieval of cognitive knowledge and tools to be applied in a specific problem situation. The look-back, which employs the verbal mode, formulates cognition to: (1) the provision of an account of legitimate rejection of explanation-request, and (2) the accounting for the asymmetries of explanation. This delineates the empirical adequacy and accuracy of the student's procedural fluency and the mathematical-scientific explanation of the student-learners vis-à-vis with the specific problem.

2 OBJECTIVES OF THE STUDY

This study was designed to analyze the procedural fluency and written-mathematical explanation to select constructed response tasks of students in Thermodynamics problems.

Specifically, it sought to find explanations of the following:

1. What is the level of performance of the student-respondents in select constructed response task:
   1.1. procedural fluency; and
   1.2. written-mathematical explanation.
2. Is there a significant relationship between the students' competencies in procedural fluency and mathematical explanation in constructed response tasks?
3. Is there a significant difference between the students' competencies in procedural fluency and mathematical explanation in constructed response tasks when grouped according to their levels of achievement in Mathematics?

3 METHODOLOGY

The descriptive-correlational-comparative research design was used in this study. It made use of a five-item constructed response task in determining their competencies in problem solving. The written competencies were transcribed and coded to determine the students' procedural fluency in problem solving. An analytical rubric was used in the transcription process.

The subjects of the study were the 2 regular sections, composed of 26 students, handled by the author during the 3rd trimester, SY 2011 – 2012, at the Natural Sciences Department of AMA International University – Bahrain.

Using their academic performance in English and Mathematics (Algebra and Trigonometry), the student-respondents were grouped as Low, Average and High in order to establish the learning impact of English and Mathematical ability to their performance in Physics as the subject is taught in mathematical perspective both graphical, trigonometric and algebraic. The classroom instruction made use of English as the medium of instruction.

The select 5 items of the constructed response tasks included in the study are the validated items of the teacher-made achievement test of the author (Dissertation output: 2008). The validation and refinement was conducted at the Secondary Teacher Education Program, Teacher Education Institute, Quirino State College, Diffun, Quirino, Philippines, where the author was previously employed as Instructor. Items were analyzed using the chronbach's alpha. Reliability contained in the Achievement Test was determined with a coefficient reliability of 0,87. This means that the inter-correlations among the items in the test are of consistent and indicate that the degree to which the set of items measured a unidimensional latent construct.

An analytical rubric was re-formulated based on Pugale's model rubric (2004) and anchored on Mayer's model for problem solving. It provided sufficient details in differentiating the levels of performance of the students on procedural fluency and written mathematical explanations.

Scripts were assessed, analyzed and coded based on the rubric. Data were treated through SPSS. Mean, Pearson-r correlation, ANOVA and Bonferroni Correction post hoc test were conducted to conclude on the research questions especially in establishing the association of the students' procedural fluency and mathematical-scientific explanations of the student-respondents to select constructed response tasks.

0	No attempt is done to translate the problem.
1 (Not Related)	Translation through FBD is inappropriate. An attempt is made but nothing correct.
2 (Not Proficient)	Translation is correct but failed to be integrated into a coherent structure and equation. Integration is only partially correct. Something correct related to the question.
3 (Basic)	Sequenced plans are executed however plans are partially executed; dealt with all aspects but have minor errors.
4 (Proficient)	Plans are executed completely and accurately; computed values are substituted to check whether it is a correct solution or not. All required are answered.

4 RESULTS AND DISCUSSION

Table 1 presents the level of performance of the students on the constructed response tasks in terms of their procedural fluency and written-mathematical explanations. As shown in the table, the students performed better in their procedural fluency when compared to their performance in their written-mathematical explanation, 3,03 vs 1,96. This means that the students can translate problems to working diagrams, develop and execute plans to solve problems partially. However, it can be noted that they had errors in terms of integrating other concepts necessary to determine the entire problem. It can be construed, therefore, that the mathematical cognition of the respondents has a great impact on the problem-solving skills of the students.

Constructed Response Task	***Procedural Fluency***		***Written-Mathematical Explanation***	
	Mean	***Descriptive Interpretation***	***Mean***	***Descriptive Interpretation***
Task 1	*2.58*	*Basic*	*1.58*	*Not Proficient*
Task 2	*3.23*	*Basic*	*1.88*	*Not Proficient*
Task 3	*3.31*	*Basic*	*2.31*	*Not Proficient*
Task 4	*3.15*	*Basic*	*2.15*	*Not Profienct*
Task 5	*2.88*	*Basic*	*1.88*	*Not Profienct*
Average	*3.03*	*Basic*	*1.96*	*Not Proficient*

Table 1. The Students Level of Performance on Constructed Response Task in terms of Procedural Fluency and Written-Mathematical Explanation

It can be construed then that the student-respondents had a difficulty in interpreting word problems as majority of them were high school graduates of non-science curriculum in Arabic instruction. This difficulty hampered their solving tasks as they are going to integrate other facets of the problem in a given equation. Lesh' Translational Model in problem solving in Suh and Moyer-Packenham (2007) explained that understanding the problem uses one's language proficiency for them to identify both the written and verbal symbols that lead them to picture out the problem through a free-body diagram. Suited in a real life situation, the students develop a plan to carry the strategies in problem solving that lead them to an answer and conclusion under several modes of manipulatives and symbolic notations of the actions to be undertaken (Kilpatrick et al., 2001; Suh & Moyer-Packenham, 2007; Samuelsson, 2010).

		Written-Mathematical Explanation
Procedural Fluency	*Pearson Correlation*	*.992**
	Sig. (2-tailed)	*.000*
	N	*26*

**. Correlation is significant at the 0.01 level (2-tailed).*

Table 2. Relationship of the Students' Procedural Fluency to their Written-Mathematical Explanation in Problem Solving

Table 2 discerns the relationship between the students' procedural fluency and written-mathematical explanation to their problem solving skills towards constructed response tasks. Using Pearson-r correlation at 0,01 level of significance, it was found out that there is a highly significant relationship between the two variables as depicted by their r-coefficient value, 0.992, and a p-value of less than 0.001. This means that they complement each other and therefore predicts their success towards problems solving tasks (Pugale, 2004, 2005; Rusell, 2000). Complementing each other requires a balance and connection between conceptual understanding and computational proficiency (Rusell, 2000).

Moreover, Samuelsson (2010) concluded that the students who are able to use their language efficiently to discuss mathematical problems seem to have a positive effect on students' conceptual understanding, strategic competence and adaptive reasoning. Hence, the null hypothesis, of no significant relationship between the students' procedural fluency and written-mathematical explanation, is hereby rejected.

Table 3 presents the test of difference on the students' performance on constructed response tasks when grouped according to their Mathematical and English abilities. It was found out that there is a very high significant difference between and among the performances of the respondents on constructed response tasks: F-values of 9,641 and 7,753, and p-values of 0.001 and 0.003, respectively, for procedural fluency and written-mathematical explanation, at 0,05 level of significance.

		Sum of Squares	***Df***	***Mean Square***	***F***	***Sig.***
Procedural Fluency	*Between Groups*	*6.446*	*2*	*3.223*	*9.641*	*.001**
	Within Groups	*7.689*	*23*	*.334*		
	Total	*14.135*	*25*			
Written-Mathematical Explanation	*Between Groups*	*5.510*	*2*	*2.755*	*7.753*	*.003**
	Within Groups	*8.172*	*23*	*.355*		
	Total	*13.682*	*25*			

**. The mean difference is significant at the 0.05 level.*

Table 3. The Difference of the Students' Performance when grouped according to their Mathematical and English Ability

It can be construed then that the students' procedural fluency is controlled by their mathematical ability, both algebraically and trigonometrically. This means that their mathematical endowment predicts their success in evaluating problems in Physics. Hence, the null hypothesis of no significant difference on their performance on constructed response tasks is hereby rejected.

Concomitantly, the written-mathematical explanation, which involves the verbal mode of problem solving in science, is also controlled by their English ability as shown by their performances on the select constructed response tasks. It can be said that their success is dependent on their capability to understand and translate problems into a functional working schema and diagrams where they can draw solution plans and strategies in problem solving.

These results support the findings and conclusions of Kilpatrick et al. (2001), Pugale (2004, 2005), Suh and Moyer-Packenham (2007), Ginsburg (2012) and Graven and Stott (2012) when they concluded that the increase on the levels of cognitive demand, mathematical complexity and levels of abstraction complements the procedural fluency and written-mathematical explanations of students in problem solving tasks is based on their ability to use cognitive knowledge and skills in decontextualizing a problem into a functional knowledge and schema. Cognitive demand, according to Bernstein, 1966 cited in Graven and Stott (2005), may range from rudimentary (constrained skill) to elaborated tasks (flexible fluency) through conceptual understanding of the decontextualized problem.

Table 4 presents the post hoc test on the performance of the students on select constructed response tasks. The performance of the students varied significantly according to their mathematical learning abilities. It can be gleaned on the table that the scores between the high and average ability and high and low ability differ significantly: F-values of 0,743 and 1,474, and p-values of 0,032 and 0,001, respectively, at 0,05 level of significance. However, a comparable score was observed between the low and average abilities in mathematics. It can be construed then that instruction must provide a recuperating cognition to this deficiency as their performance depends on their ability to use the decontextualized skills in procedural analysis and evaluation. Also, instructions in Algebra and Trigonometry must be strengthened since they are complementary to learning and understanding Physics.

On the other hand, the written-mathematical explanation, which is believed to be dependent on their English ability, was observed to be differing significantly between the low and high abilities: F-value of -1,36 and p-value of 0,002, at 0,05 level of significance. This means that their ability to understand the word problem plays an important role in their problem solving tasks. Hence, the inability to form a conceptual understanding on a specific problem is detrimental to the verbal mode of the problem solving tasks in science especially on the establishment of a conceptual understanding on the decontextualized knowledge leading to a functional schema towards problem solving.

	(I) Math Ability	*(J) Math Ability*	*Mean Difference (I-J)*	*Std. Error*	*Sig.*
Procedural Fluency	*Low*	*Average*	*-0,731*	*.301*	*.070*
		High	*-1,474**	*.339*	*.001*
	Average	*Low*	*0,731*	*.301*	*.070*
		High	*-0,743**	*.268*	*.032*
	High	*Low*	*1,474**	*.339*	*.001*
		Average	*0,743**	*.268*	*.032*
Written-Mathematical Explanation	*Low*	*Average*	*-0,6600000*	*.3105475*	*.134*
		High	*-1,3600000**	*.3490253*	*.002*
	Average	*Low*	*0,6600000*	*.3105475*	*.134*
		High	*-0,7000000*	*.2759287*	*.055*
	High	*Low*	*1,3600000**	*.3490253*	*.002*
		Average	*0,7000000*	*.2759287*	*.055*

**. The mean difference is significant at the 0.05 level.*

Table 4. Bonferroni Correction Post Hoc Test on the Performance of the Students on Select Constructed Response Task in Thermodynamics

5 CONCLUSION

Physics instruction, as influenced by mathematical thinking and reasoning, uses conceptions of domain knowledge in mathematical thinking. It tends to emphasize metacognition, critical thinking and mathematical practices through probem solving as its forefront competency.

Students' procedural fluency is influenced by their mathematical knowledge and abilities, both algebraic and trigonometric knowledge. Their rudimentary mathematical knowledge (constrained skill) and elaborated tasks (flexible fluency) affect their conceptual understanding of the decontextualized problem. On the other hand, their written-mathematical explanation, which involves the verbal mode of problem solving in science, is controlled by their English ability. Hence, abstraction complements the procedural fluency and written-mathematical explanations of students in problem solving tasks which uses their ability to use cognitive knowledge and skills in decontextualizing a problem into a functional knowledge and schema.

The content and pedagogic classroom activities and cognition must be complementary to each other in reshaping the students' performance on procedural fluency and mathematical explanation towards problema solving. Theoretical perspectives in mathematics instruction (College Algebra and Trigonometry) and English instruction are to be translated into an alternative epistemology of problem solving skills in order to recupérate the attainment of critical aspects of problem solving among students.

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