

*Full Length Research Paper*

# The problems posed and models employed by primary school teachers in subtraction with fractions

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Students have difficulties in solving problems of fractions in almost all levels, and in problem posing. Problem posing skills influence the process of development of the behaviors observed at the level of comprehension. That is why it is very crucial for teachers to develop activities for student to have conceptual comprehension of fractions and operations involving fractions. The achievement of such conceptual comprehension can be accelerated through the use of mathematical models. For this, the aim of the study is to identify the errors in the problems posed by primary school teachers with respect to subtractions with fractions, and the models they employ to solve these problems. The present study employs both quantitative and qualitative methods together. This study was carried out with 31 primary school teachers. The teachers involved in the study were selected through random sampling. The study employs the "Problem Posing Test" comprising four items of subtractions with fractions. The test drawn up with reference to the operation of subtraction with fractions includes one item for each: subtracting a proper fraction from another proper fraction, and subtracting a mixed fraction from another mixed fraction. First of all, the answers provided by the teachers were categorized as problem, not-a-problem, or blank. Following such a classification is an analysis of the errors observed in the responses provided in the problem category. At the end, the study reveals that the rate of correct responses offered in the problem category falls as one progresses from item, one where both the minuend and the subtrahend are proper fractions, towards item four where a mixed fraction is subtracted from another mixed fraction. The fact that nine distinct types of errors were observed in the study reveals that the teachers have significant shortcomings when posing problems regarding subtraction with fractions.

**Key words:** Fractions, subtraction, primary school teachers, problem posing, modeling.

## INTRODUCTION

The studies reviewing the qualifications of teachers are often constructions built on the grounds laid by Shulman (1987). Shulman (1987) noted that various types of field

knowledge a teacher is expected to have involve references to basic concepts and principles regarding that field, the curriculum, and the relationships between

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these.

Furthermore, the teachers' behavior in the classroom and their practices during the activities are also shaped by their content knowledge (Ball and Bass, 2000). That is why content knowledge plays a major role in shaping the process of education in the classroom (Baki and Çelik, 2005; Ball et al., 2001; Çakan, 2004; Dursun and Dede, 2004; Seferoğlu, 2001). The knowledge to be provided and the means to provide that knowledge is important, for the effectiveness of the teacher is among the major factors that affect learning (Romberg and Carpenter, 1986). Fractions, in turn, are among the topics deemed necessary to teach and learn.

### Fractions

It is important to realize the conceptual understanding of fractions and operations for fractions, which present numerous problems during learning. Because mathematically, rich fractions have an important place in the learning of algebraic subjects, one of the areas of advanced learning (Redmond, 2009; Smith, 2002). However, fractions are one of the subjects that are difficult to teach because they are cognitively complex (Smith, 2002). The difficulties observed during the learning of fractions have been covered by numerous studies (Haser and Ubuz, 2002; Aksu, 1997; Başgün and Ersoy, 2000; Ersoy and Ardahan, 2003; Hanson, 1995; Wu, 1999). When looking at the literature reveals that students have some difficulties with respect to solving problems regarding fractions at almost all levels (Kocaoğlu and Yenilmez, 2010), and hence in problem posing (Işık et al., 2011).

### Problem posing

One of the main purpose of primary school mathematics classes is to instill the skills of problem posing. Problem posing can be used not only to determine the mathematical knowledge and skill levels of the students, pre-service teachers, or teachers (Işık and Kar, 2012a; Kılıç, 2013; McAllister and Beaver, 2012) but also influence the problem solving skills.

Problem posing skills provide a distinct perspective regarding the problem solving process, and therefore, help with the comprehension of the relationships the problem entails. That is why teachers and therefore pre-service teachers would benefit from a high level of problem posing skills in mathematics.

For instance, problems provided in text books may be insufficient, incompatible with the current proficiency levels of students, or unrelated with their interests or needs. In such cases, the teacher may be required to pose additional problems regarding the topic at hand, in order to for the purpose of the course (Albayrak, 2000; Korkmaz and Gür, 2006).

Furthermore, problem posing skills influence the process of development of the behaviors observed at the level of comprehension. That is why it is very crucial for teachers to develop activities for student to have conceptual understanding of fractions and operations involving fractions (Işıksal, 2006; Mack, 1990; Mok et al., 2008; Rule and Hallagan, 2006; Utley and Redmond, 2008). The achievement of such conceptual understanding, in turn, can be accelerated through the use of mathematical models.

### Modeling

Mathematics exhibits an inherently more abstract structure compared to other sciences. Direct presentation is not deemed a suitable means to ensure that students can imagine or visualize a mathematics concept (van de Walle, 2004).

The leading factor posing a problem in terms of primary school students' understanding of mathematics is the insufficient level of abstract thinking abilities on their part. Researchers consider materialization of the concepts through generalization as one of the ways to overcome this issue (Çelik and Çiltaş, 2015).

In this context, models with some certain physical and mental actions can be presented in order to construct mathematical conceptualization among students. A picture, drawing, symbol, or a concrete means entailing the relationship conveyed by a mathematical concept could be the model of that mathematical concept. Models can be employed for three distinct purposes, namely enabling students develop new concepts and relationships in their minds, helping students establish the relationships between concepts and symbols, and assessing the level of comprehension in students' mind (Olkun and Toluk Uçar, 2012). There are a number of models used for the teaching of fractions based on these objectives.

Some studies reveal the importance of using fraction models. The use of applicable and different models would expand and deepen the understanding of fractions in the minds of students as well as teachers. Three distinct models are employed for the teaching of fractions: region/area, length/measurement, and set (van de Walle, 2004). Region/area models help materialize the fraction as a portion of the region, while length models allow the comparison of lengths and measurements instead of regions. The number line model, an example of length models, presents the fraction as a real number.

In the set model, on the other hand, a portion of the objects included in the set are represented with reference to the fraction. In other words, a given set of objects refers to the whole, and a sub-set of those objects represents the fraction (Olkun and Toluk Uçar, 2012; van de Walle, 2004). For the teachers to be able to employ suitable models in the process of teaching concepts,

they should be aware of such models.

### Purpose of the study

In recent years, studies on the teaching of mathematics tend to focus on the characteristics of knowledge the teacher would need to have (Newton, 2008). Hill et al. (2005) stated that the mathematical knowledge the teachers need to have should enable them to provide explanations to students, and to analyze their responses in turn.

On the other hand, studies investigating the pedagogical content knowledge of teachers and pre-service teachers (Chick and Baker, 2005; Işık, 2011; Newton, 2008; Özmantar and Bingölbali, 2009; Toluk Uçar, 2009; Ward and Thomas, 2007) reveal that they experience numerous problems in terms of identifying the errors of students, explaining reasons, and planning teaching with a view to eliminating errors.

Difficulties in learning fractions have been the subject of many researches (Haser and Ubuz, 2002; Aksu, 1997; Başgün and Ersoy, 2000; Ersoy and Ardahan, 2003; Hanson, 1995; Wu, 1999); and it can be seen that at almost all levels, the students have encountered some difficulties in solving problems about fractions (Kocaoğlu and Yenilmez, 2010).

Soylu and Soylu (2005) note that students have major learning difficulties regarding ordering, addition, subtraction, multiplication and division with fractions. Research shows that on the topic of difficulties not only students but also teachers and pre-service teachers have with the concept of fractions, and the operation of division with fractions (Ball, 1990; Borko et al., 1992; Carraher, 1996; Işıksal, 2006; Ma, 1999; Mok et al., 2008; Post et al., 1991; Redmond, 2009; Sharp and Adams, 2002; Tirosh, 2000; Toluk-Uçar, 2009; Yim, 2010; Zembat, 2007; Şiap and Duru, 2004; Seyhan and Gür, 2004; Soylu and Soylu, 2005).

While teachers trying to develop the conceptual meaning of fractions by students, they also have difficulties in making interpretation and making meaning with division of fractions by themselves (Utley and Redmond, 2008). That is why problem posing occupies a central position in terms of associating fractions and operations involving fractions, with real life cases (Abu-Elwan, 2002).

The studies so far focus on multiplication and division with fractions and the solution of verbal problems requiring division with fractions (Işık et al., 2011; Işık and Kar, 2012a); and often focus on pre-service teachers and students. On the other hand, the studies performed on problem posing show that the level of competence in problem posing skills leaves much to be desired (Işık et al., 2011; Işık and Kar, 2012b).

McAllister and Beaver (2012) have analyzed the errors pre-service primary school teachers had with the

problems they posed with respect to operations with fractions. Işık and Kar (2012c) on the other hand, analyzed the errors committed in the problems posed with respect to additions with fractions, in a study involving seventh year students in primary education. It is evident that the limited number of studies carried out with respect to problem posing involving addition and subtraction with fractions were more often than not executed with teachers, students, or pre-service teachers, with particular reference to addition with fractions (Kar and Işık, 2014).

Kar and Işık (2014) investigated the types of errors seventh year secondary school students had in terms of problem posing involving subtractions with fractions. Kar and Işık (2015) shed some light also on secondary school mathematics teachers' problem posing skills regarding subtractions with fractions. On the other hand, no study investigating the types of errors primary school teachers had with subtractions with fractions was observed. Yet, Primary Schools Mathematics Courses (1st, 2nd, 3rd, and 4th years) Curriculum mentions the learning outcome "Ability to solve problems requiring additions and subtractions with fractions" (Ministry of Education (MEB), 2015). Taking into account the fact that Primary Schools Mathematics Courses (1st, 2nd, 3rd, and 4th years) Curriculum envisages a learning outcome regarding problem solving with subtraction, it is clear that the analysis of the problems posed by the teachers and the identification of the models they employ with respect to the solution of the problems would enable the development of deep insights regarding potential conceptual shortcomings of teachers.

No studies investigating primary school teachers' – direct practitioners– problem posing skills regarding subtraction with fractions, and problem solving methods by modeling such problems, were observed. The present study, in turn, is crucial with a view to understanding the errors in the problems posed by the teachers, and the models they employ, with the overall objective of identifying any shortcomings on this topic. Such an endeavor would not only shed light on the efforts to eliminate any shortcomings in this context, but also contribute to teacher training. In this vein, the objective of the present study is to identify the errors in the problems posed by primary school teachers with respect to subtractions with fractions, and the models they employ to solve these problems.

### METHODOLOGY

The present study employs quantitative and qualitative methods in conjunction. In this context, the errors primary school teachers had in terms of the problems they posed with respect to subtractions with fractions were identified through qualitative analyses of the responses provided to four open-ended items. The distribution of error categories and models with respect to each of the four items in the Problem Posing Test, in turn, was investigated through a quantitative analysis.

**Table 1.** Subtraction related items of the problem posing test, and their characteristics.

Items	Characteristics
$\frac{5}{8} - \frac{2}{8} = ?$	Subtraction of a proper fraction from another
$1\frac{1}{3} - \frac{2}{3} = ?$	Subtraction of a proper fraction from a mixed fraction, to produce a proper fraction
$2\frac{4}{9} - \frac{7}{9} = ?$	Subtraction of a proper fraction from a mixed fraction, to produce a mixed fraction
$3\frac{1}{4} - 1\frac{2}{4} = ?$	Subtraction of a mixed fraction from another

### Sample

This study was carried out in spring 2015 to 2016, with 31 (18 female, 13 male) primary school teachers assigned to a number of schools in 3 district centers of Trabzon. The teachers involved in the study were selected through random sampling. The study was conducted with volunteers from selected teachers with random sampling. Each teacher who voluntarily agreed to take part in the study was assigned a code in the range T1 ... T31, with T standing for teacher. One of the teachers had an experience of 1-5 years; 10 had 6-10 years; 9 had 11-15; 6 had 16-20; 2 had 21-25; 1 had 26-30; and 2 had more than 30 years of experience in service.

Primary schools mathematics courses (1st, 2nd, 3rd, and 4th years) curriculum was updated in 2015, and started to be applied in schools since 2016 to 2017 education year. The teachers in the study group perform the teaching of operations with fractions and fractions in line with the achievements in the 4th grade "performs additions and subtractions with fractions of matching denominators" and "solves problems requiring addition and subtraction with fractions" under the "operations with fractions" category of the curriculum (MEB, 2015).

### Data collection tools and the collection of data

The study employs the "problem posing test" comprising four items regarding subtractions with fractions, as the data collecting tool. The "problem posing test" is offered to each pre-service teacher in written form. There is a total of 4 operations that require only subtraction operation with fraction to be performed in the problem posing test.

The test drawn up with reference to the operation of subtraction with fractions includes one item for each: subtracting a proper fraction from another proper fraction, and subtracting a mixed fraction from another mixed fraction, as was the case with the study by Kar and Işık (2014). These are coupled with two more items regarding the subtraction of a proper fraction from a mixed fraction. One of these latter two items produces a proper fraction while the other produces a mixed one (Table 1).

In contrast to the study by Kar and Işık (2014), however, the operations included in the "Problem Posing Test" entail fractions with matching denominator, for primary school curriculum includes operations with fractions with matching denominator.

A number of researchers employed similar questions when investigating operations with fractions (McAllister and Beaver, 2012; Toluk-Uçar, 2009). Firstly, the test prepared by the researcher was examined by an expert in doctoral dissertation in elementary mathematics education. After the necessary changes were made, the pilot study was applied to the three primary school teachers who were not involved in the actual study. As a result of the pilot study, the items in the test were given the final shape in line with

the opinions of the primary school teachers.

The test prepared in the process of collecting the data was given in written form to the teachers. At the beginning of the test, there are questions to determine the gender of teachers and to determine how many years they were working as a teacher. Then, the operations of fractions are numbered and each of them are written as one item. Appropriate gaps were left under each item in the test for teachers to work. After test being distributed to the teachers, it is required to write a verbal problem statement in the solution to the given process, which is related only to the daily life situations that can be used for that process. The respondents were told that they were required to pose appropriate problems suitable for the primary school students' level, and were recommended to leave the question unanswered, if they felt unable to pose a proper problem.

Then, they are asked to solve each problem by modeling the problems they had pose for each of these items. No time limitations have been made to the teachers in this whole process. Teachers were warned not to use any device such as calculator, computer, or phone in this process because there is only one operation in each question. Also, it is stated that the teachers do not influence each other when responding to the test and that each teacher should do it herself/himself.

### Data analysis

First of all, the answers provided by the teachers were categorized as problem, not-a-problem, or blank. Such a categorization was employed in previous research as well (Işık and Kar, 2012b; Kar and Işık, 2014; Leung, 2013; Silver and Cai, 2005).

This approach aimed to discern answers which cannot be associated with daily life, or which do not contain a question phrase. The not-a-problem category contains the responses where only a description was provided in one or more sentences, which do not contain a question form, and which cannot be associated with elements of daily life. The discussion based on the examples of responses provided for the not-a-problem category is presented in the findings section. Also, those who found a solution were described as "problem".

The ones that the teacher has typed as a problem are re-analyzed. This was done in order to determine the errors that teachers made in the problems they had established. Thus, the analysis of the errors in the answers in the problem category was made and classified. The classification of the primary school students' errors in posing problems with subtraction with fractions was based on 12 types of errors.

Kar and Işık (2014) identified the problems in seventh year primary school students posed with reference to subtractions with fractions. Kar and Işık (2014) identified the following types of errors: expressing the subtrahend fraction over the remainder of whole (Error 1(E1)), not being able to establish part-whole relationship

**Table 2.** Problem states.

Items	Problem		No problem		Blank		Total
	N	Percentage	N	Percentage	N	Percentage	N
Item 1	30	96.7	1	3.3	-	-	31
Item 2	28	90.3	2	6.4	1	3.3	31
Item 3	28	90.3	1	3.3	2	6.4	31
Item 4	25	80.8	3	9.6	3	9.6	31
Total	111	89.6	7	5.6	6	4.8	124

(E2), attributing natural number meaning to the result of the operation (E3), confusion about units (E4), attributing natural number meaning to the fractions (E5), failure in expressing the operation in the question root (E6), not being able to express whole parts of mixed fractions (E7), representing the mixed fraction as numerator and denominator (E8), attributing a value to the whole (E9), expressing the subtrahend fraction as a certain amount of minuend fraction (E10), logical error (E11), and expressing the fractions over the different wholes (E12). The statements regarding such error types are presented in the findings section.

The problems posed by the primary school teachers were analyzed using the qualitative descriptive analysis method, with reference to the problem types categories identified by Kar and Işık (2014). Each problem sentence posed by primary school teachers was read and reviewed carefully, and a classification was sought by coding the sentences.

Coding of data was carried out by an expert in doctoral dissertation of elementary mathematics education and the researcher. The doctoral student was trained by the researcher. To examine the inter-rater reliability, the researcher coded all primary school teachers' responses and then an expert in doctoral dissertation of elementary mathematics education independently coded them. The rate of agreement on the coding of the responses was 92%. Then, by comparing the analyses, the same error categories were taken directly and a common decision was reached by discussing different ones. As a result, the category of each problem has been determined. These categories are included in the findings section. In the problems teachers set up, more than one type of error can be found at the same time. The distributions of error categories for each of the four items in the problem building test were formed by quantitative analyzes.

In this process, the percentage and frequency tables were formed taking into account the number of faults in each problem setting item. As a result of the analysis, the determined categories are detailed in the findings together with the frequency and percentage table. Thereafter, the analyses by the author and the researcher were compared. The responses on which the disagreement occurred were reread, and an agreement was reached. As a result, the category of each problem has been determined. These categories are included in the findings section. The problems posed by the teachers can present more than one error type simultaneously. The distribution of error categories with respect to each of the four items in the Problem Posing Test, in turn, was developed through quantitative analyzes.

In this process, the number of errors regarding each problem posing item was used in developing the percentage and frequency tables. As a result of the analysis, the determined categories are detailed in the findings together with the frequency and percentage table.

The models used in fraction teaching are considered in the process of determining the models that teachers use to solve the problems they have posed. For this reason, the models teachers employ when solving problems they posed are categorized as

region/area, length, set, concrete model, or blank. The region/area models were further classified using the sub-categories of square, rectangle, circle, and triangle. The models used by the teachers were analyzed by two distinct researchers, followed by a comparison of their assessments. The ones where they concurred were included directly, whereas further reconciliation was sought for the ones that differed. In conclusion, percentile and frequency tables were drawn up, taking into account the numbers of models used to solve each problem.

## RESULTS

The answers primary school teachers enrolled in the study provided with respect to four basic operations entailing subtraction with fractions were categorized as problem, not-a-problem, or blank. The findings regarding the categorization are provided in Table 2.

Table 2 reveals that 111 (89.6%) of all 124 responses provided by primary school teachers were problems, while 7 (5.6%) were not, and 6 (4.8%) were blanks. Furthermore, in the problem category, the largest group of responses entailed the first item, where a proper fraction was subtracted from another proper fraction, whereas the smallest group of responses entailed the fourth item, where a mixed fraction was subtracted from another mixed fraction, to produce yet another mixed fraction. Against this background, one can argue that the teachers' level of success in providing responses in the problem category decreases as they proceed from the first item to the fourth one.

Seven (5.6%) responses provided by the teachers were categorized in the not-a-problem category. For instance, the response provided by T30 for item 1 reads "Ali spent a part of  $\frac{5}{8}$  of his cash. The amount he spent is equal to  $\frac{2}{8}$  of his cash." In his response, the teacher attempted to express the fractions verbally, but failed to reflect the subtraction operation in the question form. The response provided by T3 for item 4, on the other hand, reads "I paint  $1\frac{2}{4}$  of the fraction  $3\frac{1}{4}$  to blue. Please depict the not-blue section using a model employing operations." Again, the response provided by the teacher coded T3 does not contain a question form. For item 3, T3 again responded with the phrase "What is  $\frac{7}{9}$  less than fraction  $2\frac{4}{9}$ ?", asking the calculation of an operation's result, rather than

**Table 3.** Classification of the problems posed by the teachers, with reference to the types of errors.

Errors	Item 1 $(\frac{5}{8} - \frac{2}{8})$		Item 2 $(1\frac{1}{3} - \frac{2}{3})$		Item 3 $(2\frac{4}{9} - \frac{7}{9})$		Item 4 $(3\frac{1}{4} - 1\frac{2}{4})$		Total	
	N	%	N	%	N	%	N	%	N	%
E1	1	2.3	-	-	-	-	-	-	1	0.4
E2	-	-	8	15.7	10	17.2	9	18.4	27	13.4
E3	1	2.3	-	-	1	1.7	1	2	3	1.5
E4	-	-	-	-	-	-	-	-	-	-
E5	-	-	-	-	-	-	-	-	-	-
E6	18	41.9	18	35.3	17	29.4	14	28.7	67	33.4
E7	-	-	-	-	-	-	-	-	-	-
E8	-	-	-	-	7	12.1	3	6.1	10	5
E9	5	11.6	4	7.8	4	6.8	3	6.1	16	8
E10	17	39.6	21	41.2	18	31.1	16	32.7	72	35.9
E11	1	2.3	-	-	1	1.7	1	2	3	1.5
E12	-	-	-	-	-	-	2	4	2	0.9
<b>Total</b>	43	100	51	100	58	100	49	100	201	100

formulating a problem that can be associated with daily life.

89.6% of the responses provided by the teachers can be categorized as problems; however these problems have also their share of certain types of errors. The findings regarding the distribution of such error categories are provided in Table 3.

A glance at Table 3 reveals that the teachers committed the largest number of errors when responding to item 3 where a mixed fraction was produced when a proper fraction was subtracted from a mixed one (58), followed by item 2 where a proper fraction was produced when a proper fraction was subtracted from a mixed one (51). These are followed by item 4 entailing the subtraction of a mixed fraction from another mixed fraction (49 errors), and item 1 entailing the subtraction of a proper fraction from another (43 errors). The most frequent error committed by the teachers was E10 (35.9%). It is followed by E6 (33.4%), E2 (13.4%), E9 (8%), and E8 (5%). E1, E3, E11 and E12, on the other hand, were committed very rarely. Furthermore, the teachers who took part in the study did not commit E4, E5, and E7 at all.

### Item 1

A total of 43 errors were observed in 30 responses considered problems pertaining to the operation  $\frac{5}{8} - \frac{2}{8} = ?$  entailing the subtraction of a proper fraction from another one. The average number of errors committed per problem is 1.4. Among all errors, the most common ones

were E6, E10, and E9, while E1, E3, and E11 were rarest. E2, E4, E5, E7, E8 and E12, on the other hand, were not observed at all. Among these, E7 and E8 were not observed in this item as they characteristically pertain to mixed fractions.

For item 1, T4 posed the problem "Ayşe keeps  $\frac{5}{8}$  of her allowance unspent. She saves  $\frac{2}{8}$  of the unspent amount in her piggy bank. How much money Ayşe has left out of her unused allowance?" The problem posed by the teacher is marred by a failure in expressing the operation in the question root (E6). The problem asks for the calculation of  $\frac{2}{8}$  of  $\frac{5}{8}$ , thus, requires multiplication. Therefore, the subtraction was not reflected on the question root in the problem posed. Moreover, the fraction produced in the problem thus posed is expressing the subtrahend fraction as a certain amount of minuend fraction, hence contains a E10 type error as well.

One of the errors committed in item 1 is E9. T31 posed the problem "Ayşe ate  $\frac{5}{8}$  of 40 walnuts. Hasan ate  $\frac{2}{8}$  of 40 walnuts. What is the difference between the amount of walnuts they have consumed?" Even though the teachers were asked to pose a problem which can be solved with the operation specified, some attributed a value to the whole and considered the fractions involved in the operation, certain amount of that whole.

Among the teachers who took part in the study, only T14 committed the error E1 when posing a problem. That problem reads "An automobile covered  $\frac{5}{8}$  of a distance in the morning. In the afternoon, it covered  $\frac{2}{8}$  of the remaining

remaining distance. What is the difference between the distances the automobile covered in the morning and in the afternoon?" In that problem, the teacher assigned the term road covered in the morning to  $\frac{5}{8}$ , while noting that  $\frac{2}{8}$  of the remainder was covered in the afternoon. In the problem, the teacher presented the fraction with reference to the remaining amount of distance rather than the initial distance involved.

## Item 2

A total of 51 errors were observed in the 28 answers provided in the problem category with reference to the operation  $1\frac{1}{2} - \frac{2}{3} = ?$  entailing a proper fraction produced by subtracting a proper fraction from a mixed fraction. The average number of errors committed per problem is 1.8. Among all errors, the most common ones were E10, E6, and E2. Error types E1, E3, E4, E5, E7, E8, E11, and E12, on the other hand, were not observed at all in this item. The rarest type of error observed with this problem posing item, on the other hand, is E9. The problem posed by T22 for item 2 entailed an error in E10 category. The problem formulated by the teacher reads "Nur ate  $\frac{2}{3}$  of a  $1\frac{1}{3}$  pizza. What is the proportion of the remaining pizza?", where the teacher expressed the subtrahend fraction as a certain amount of minuend fraction, rather than of the whole. This constitutes a E10 error on part of the teacher. This also posed an obstacle in terms of expressing the operation in the problem root, hence led to an H6 error as well.

Another type of error committed with the problems posed with reference to item 2 was E2 where a failure to establish the link between the part-whole exists. The problem posed by teacher T12 also exhibits E2 error. The problem reads " $\frac{2}{3}$  out of  $1\frac{1}{3}$  of a plot is left to fallow. What is the size of the cultivated section?" E2 error refers to problems which were posed in ignorance of the fact that the fractions involved in the operation or the fraction produced as a result of the operation was larger than the whole. In this problem, even though we have only a single plot, the question refers to a  $1\frac{1}{3}$  portion. Furthermore, E10 error is also present as the fraction produced in this problem is expressed a certain amount of minuend fraction, not to mention E6 due to the failure to express the operation in the problem root.

## Item 3

A total of 58 errors were observed in 28 responses considered problems pertaining to the operation  $2\frac{4}{9} - \frac{7}{9} = ?$  entailing the subtraction of a proper fraction from a mixed fraction, to produce another mixed fraction. The average number of errors committed per problem is 2.07.

Among all errors, the most common ones were E10, E6, E2, E8, and E9, while E3 and E11 were rarest. Error types E1, E4, E5, E7, and E12, on the other hand, were not observed at all.

One of the errors committed under this item is E2. For instance, the problem posed by teacher T21 reads "How much of the melon would remain after eating  $\frac{7}{9}$  of the melon  $2\frac{4}{9}$  of which had remained?" entails such a error. In this problem, the melon stipulated by the teacher is larger than a whole.

When posing a problem with respect to item 3, T16 committed the error E8. The problem posed by the teacher reads "Assuming that we have painted 2 walls of equal size at the school, and  $\frac{7}{9}$  of a wall which is equal to its  $\frac{4}{9}$ , what portion of the walls would remain without paint?" The result of the operation to solve the problem is  $1\frac{6}{9}$ . However, the problem form tried to present the mixed fraction with the phrase "what portion of the walls would remain without paint". The phrase "what portion" would be acceptable in case of proper fractions, but not so with mixed ones, taking into consideration its reference to the relationship between the whole and its parts. That is why it is a E8 error to refer to the mixed fraction produced through the operation, using the phrase "what portion".

## Item 4

A total of 49 errors were observed in the 25 answers provided in the problem category with reference to the operation  $3\frac{1}{4} - 1\frac{2}{4} = ?$  entailing a mixed fraction produced by subtracting a mixed fraction from a mixed fraction. The average number of errors committed per problem is 1.9. Among all errors, the most common ones were E10, E6, and E2. Error types E1, E4, E5, and E7, on the other hand, were not observed at all with this item. The rarest types of error observed with this problem posing item, on the other hand, are E3, E11, and E12.

Another error committed with item 4, entailing the subtraction of a mixed fraction from another mixed fraction is E3, which refers to attributing natural number meaning to the result of the operation. T19's problem with a E3 error was posed as "A student who read  $3\frac{1}{4}$  of her books prepared summaries of  $1\frac{2}{4}$  of the books she read. How many books remain without a summary?" Even though the result of the operation was a mixed fraction, T19 formulated the question as if the result would be a natural number.

T7, on the other hand, committed E11 error with a logical error when posing a problem for the operation provided. The problem posed by T7 reads "I drank  $1\frac{2}{4}$  L out of  $3\frac{1}{4}$  L milk. How much water do I have left?" The problem mentions milk in the part presenting the data,

**Table 4.** Models employed by teachers for each item.

Items	Area model				Concrete model	Number Line	Blank	Total
	Square	Rectangle	Circle	Triangel				
Item 1	2	23	5	-	-	-	1	31
Item 2	4	16	6	1	1	1	2	31
Item 3	5	22	1	-	1	1	1	31
Item 4	6	13	10	-	-	1	1	31
Total	17	74	22	1	2	3	5	124

and refers to water when asking the question. Therefore, the data presented and the result asked are not compatible.

The problem posed by T23 for item 4 entails a E12 error. The problem posed by the teacher reads "*Ahmet used  $3\frac{1}{4}$  plates for the roof of his home, while Mehmet used  $1\frac{1}{4}$ . What is the amount of plates Ahmet used in excess of the amount used by Mehmet?*" This error type refers to the problems where the fractions involved in subtraction were represented over unequal wholes. The problem posed by T23, in this context, mentions the roofs of different houses.

The teachers were asked to employ models for solving the problems they posed. Table 4 presents the models the teachers employed for each item. The teachers have resorted to modeling for each item, even though they did not pose a problem. Table 4 presents the models employed by the teachers for the items provided. Teachers left 5 out of a total of 124 items blank, without employing any models at all.

Furthermore, in 116 cases, the teachers employed the area model. When employing the area model, the teachers used a number of distinct geometric shapes and images of concrete models. In this process, 17 items were posed with squares, 74 with rectangles, 22 with circles, and 1 with triangles. Two items, in turn saw the use of concrete models.

Among the two, one employed the area model with a torus, and the other with a bottle drawing. Just 3 items saw the use of number lines, whereas none of the teachers who took part in the study employed the set model to represent the fractions.

Figure 1 below presents the area models provided by teachers for items 1, 2, and 4. Teachers T1 and T26 employed geometric shapes in the area model, whereas T9 opted with the use of a concrete model. T1 used a rectangle, T29 a circle, and T9 a torus.

Figure 2 depicts the number line model T12 used for item 3. The teacher first presented the subtracted fraction by converting it to an improper fraction over number line, followed by the conversion of the subtracted fraction to an improper fraction. Then, the teacher referred to the difference between the two.

## DISCUSSION

Posing a problem is one of the alternative assessment tools used to identify conceptual understanding skills, misconceptions or errors (Ticha and Hospesova, 2009). The primary school teachers who took part in the study were presented with four operations entailing subtraction with fractions. The teachers were then asked to pose verbal problems regarding these operations, and to solve the problem thus posed, through modeling. The responses provided by the majority of the teachers involved in the study, with respect to the operations provided, meet the criteria for problems.

The study reveals that the rate of correct responses offered in the problem category falls as one progresses from item one where both the minuend and the subtrahend are proper fractions, towards item four where a mixed fraction is subtracted from another mixed fraction; a result confirming the findings of Kar and Işık (2014). One of the reasons for this may be the difficulties teachers have with mixed fractions.

In this context, one could argue that the increase in the number of mixed fractions involved in the operation regarding which a problem is to be posed have a negative impact on the teachers' ability to pose correct problems (Kar and Işık, 2014). Even though most of the given answers have problem property, the verbal problems posed by the teachers entail certain errors.

Kar and Işık (2014) identified 12 types of errors seventh year secondary school students had in terms of problem posing involving subtractions with fractions. However, the present study observed just 9 of these. The types of errors identified in the present study are E1, E2, E3, E6, E8, E9, E10, E11 and E12. On the other hand, in contrast to what Kar and Işık (2014) found, the present study did not come across error types E4, E5, and E7. Error types E1, E4, E10, E11, and E12 observed in this study are comparable to what Kar and Işık (2015) found with secondary school mathematics teachers, while error types E8, E9, E10, E11, and E12 are similar to the types of errors McAllister and Beaver (2012) observed in the problems pre-service primary school teachers posed with respect to the subtraction operations.

The fact that nine distinct types of errors were observed



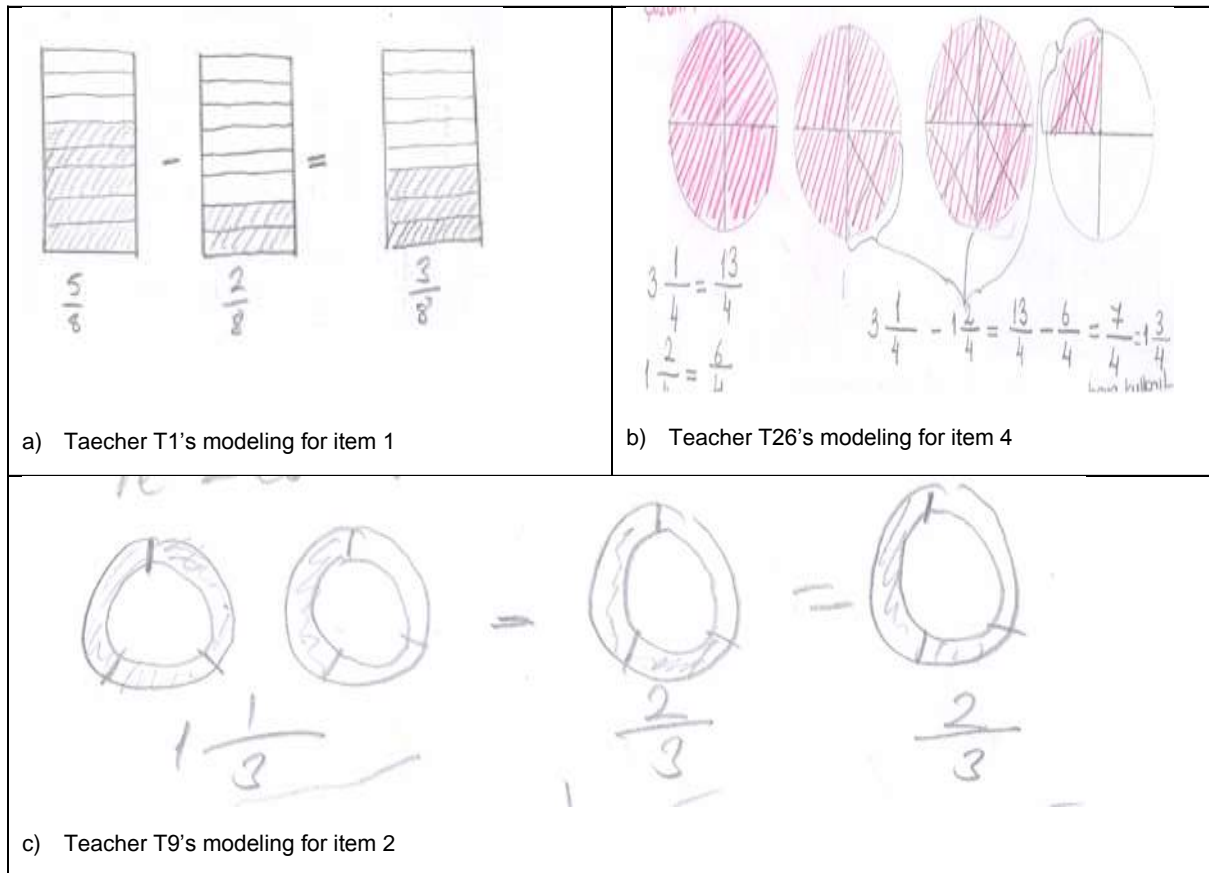


Figure 1. Area models employed by the teachers.

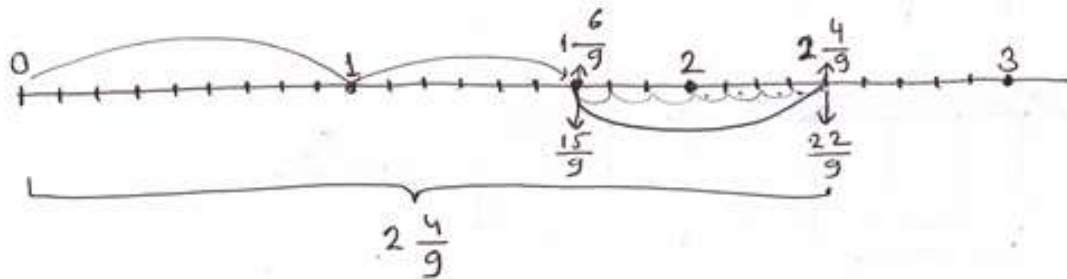


Figure 2. T12's modeling for item 3.

in the study reveals that the teachers have significant shortcomings when posing problems regarding subtraction with fractions. Kar and Işık (2015) through an analysis of the types of errors and the statements provided, noted three issues in the heart of the errors. These issues are

1. Difficulties in terms of language
2. Difficulties in terms of conceptualization, and
3. Difficulties caused by the teachers' habits regarding

the teaching process (Kar and Işık, 2015).

These issues may lie at the core of the errors committed by the primary school teachers involved in the present study. The failure to grasp the relationship between the part and the whole, the inability to use relevant units with the fractions, and the statement of fractions with reference to different wholes are but a few of the errors which may be caused by the conceptual difficulties teachers have with the concept of fractions. Such errors

observed at the conceptual level regarding the operations with fractions have roots extending back to the fractions concept (Charalambous et al., 2010).

The conceptual shortcomings observed in teaching of subtraction with fractions can have negative connotations regarding the education processes themselves as well. For content, knowledge is one of the most significant determinants of teaching of mathematics (Ball et al., 2008; Ball et al., 2001).

One of the important reasons for the errors in the problems that teachers posed in this research might be the difficulties in the linguistic dimension. Because, in the study conducted by Kar and Isik (2015), the teachers stated that they had difficulties in expressing their thoughts while posing problems during the focus group interviews. On the other hand, expressing the resulting fraction with reference to the remainder of the whole, or as a certain portion of the minuend, the failure to reflect the subtraction to the question root, assignment of a value to the whole, logical fallacies, and the use of "what portion" phrase with reference to mixed fractions may have something to do with the difficulties experienced at a conceptual level, as well as deficiencies in verbal language skills (Kar and Işık, 2014).

Besides, it is seen that the difficulties experienced in the linguistic dimension are more frequent in the items in which at least one of the mixed fractions is minuend or subtrahend. Another important reason for the errors in the posed problems is the routines of the teachers in the teaching process. Teachers, although they all know that wholes have to be equal but they do not give expressions about this situation in their problems. Problems that teachers pose such as providing opportunities for students to learn can also lead to misconceptions (Isik and Kar, 2012b; Kar and Işık, 2015).

Therefore, such deficiencies in teachers can be seen as an important reason for the misconceptions of students at the conceptual level. For example, if the teachers do not emphasize enough that all of wholes have to be equal, that may cause the students to do operations on fractional numbers which contain non-equal wholes (Kar and Işık, 2015).

The most common types of error observed with the problems posed by the primary school teachers who took part in the study are expressing the subtrahend fraction as a certain amount of minuend fraction (E10), failure in expressing the operation in the question root (E6), and not being able to establish part-whole relationship (E2), in the respective order.

These results are parallel to the study of Kar and Işık (2014). But Kar and Işık (2015) study with secondary school mathematics teachers, in contrast, express the fractions over the different wholes (E12), logical error (E11), and unit confusion (E7) as the most common errors. Among these, only H10 is specific to subtractions with fractions. Existing studies (Işık and Kar, 2012c; McAllister and Beaver, 2012) suggest that unit confusion,

attributing natural number meaning to the result of the operation, not being able to establish part-whole relationship and failure in expressing the operation in the question root as highlighted by the present study, are more common in the problems posed with reference to fraction operations, compared to others. This is perhaps due to the teachers having more difficulty in these error types, and the roots of such errors lying in the shortcomings regarding the concept of fractions. The most common error the teachers committed with respect to E6, in turn, was to pose the problems as representations of  $a*b$ , rather than  $a-b$ . In this error type, the teachers considered the subtrahend of the fraction as a certain part of the minuend of the fraction, rather than that of the whole.

The average number of errors the problems provided by primary school teachers in response to each item of the problem posing test entails is greater than one. It is clear that the teachers have significant difficulties in terms of their problem posing skills. This finding is in parallel to those of Kar and Işık (2014). Taking into account the average number of errors regarding each problem posing item, the lowest average number of errors (1.4) was observed with item one, where both the minuend and the subtrahend were proper fractions.

On the other hand, the average number of errors rises to 2.07 with item three, where, particularly the result of subtracting a proper fraction from a mixed fraction was to be another mixed fraction. In other words, on average, each problem posed contains more than two errors. This may be due to the teachers' less than stellar abilities regarding posing problems related with daily life, with reference to subtraction with fractions.

Herman et al. (2004) stated that elementary school students can explain the addition operation with fraction process with symbolic representations, while a few can create descriptions or stories about the process of addition operation with fractions. The results of this study also show that the students have a lower chance of posing problems related to daily life situations for subtraction operation with fractions.

The teachers have resorted to modeling for each item. Teachers left only a few items blank, without employing any models at all. Furthermore, the majority of the teachers employed the area model, and while doing so, made use of the images of various geometric shapes and concrete models. The most widely used geometric shapes were squares, rectangles, and circles. Tabak et al. (2010) found that 4 and 5th year primary school students had substantial success in using the area model for expressing fractions. They concluded, with reference to the students' ability to interpret fractions using various geometric shapes (squares, rectangles, triangles, parallelograms, circles, and right trapeziums under the area model, that they had been highly successful with circles, rectangles, circles, and parallelograms (Tabak et al., 2010).

The primary school teachers employed the number line only with a few items; the set model, on the other hand, was not used at all. The teachers' preference for the area model may stem primarily from the fact that they would be dealing with years 1 to 4. For the students in such years are yet capable almost only of concrete thought; the use of the area model may be an easier route for the teachers to explain. The lack of use of the set concept, in turn, may be explained by the lack of reference to that topic at the primary school level. On the other hand, the teachers may be simply experiencing difficulties with the use of number lines and set models. Tabak et al. (2010) found in this context, that the students had only minor success with the number line model.

## CONCLUSION AND RECOMMENDATIONS

The present study was based on four items developed with reference to subtraction with fractions. The fractions used in these items were chosen to common denominators, as the study group was primary school teachers. As a means to build on this study, similar ventures can be tried on operations involving fractions with different denominators, as well as on different operations involving fractions. The study was carried out with 31 teachers employed at three schools. In this context, similar studies can be executed with ever larger samples. This would help improve the generalization ability of the results, through nothing but access to wider audiences.

The present study gathered data through the problem posing test developed for the study. Future studies may employ interviews as well, with a view to obtaining in-depth insights into the errors committed by the teachers. Moreover, the types of errors identified in the present study suggest roles played by difficulties experienced on the language front, problems regarding conceptualization, and issues with the teachers' habits regarding the teaching process. The interviews may help identify the causes of the types of errors committed by the teachers, and eliminate them through workshops and in-service trainings etc. Moreover, it is interesting that the teachers who took part in the study mostly made use of the area model. The interviews may try and investigate why the area model had been thus dominant.

## CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

## REFERENCES

- Abu-Elwan R (2002). Effectiveness of problem posing strategies on prospective mathematics teachers' problem solving performance. *J. Sci. Math. Educ. S.E. Asia* 25(1):56-69.
- Aksu M (1997). Student performance in dealing with fractions. *J. Educ.*

- Res. 90(6):375-380.
- Albayrak M (2000). *İlköğretimde Matematik ve Öğretimi*. 2. baskı, Ankara: Aşık Matbaası.
- Baki A, Çelik D (2005). Grafik hesap makinelerinin matematik derslerine adaptasyonu ile ilgili matematik öğretmenlerinin görüşleri. *Turk. Online J. Educ. Technol.* 4(4):146-162.
- Ball DL (1990). Pre-service elementary and secondary teachers' understanding of division. *J. Res. Math. Educ.* 21(2):132-144.
- Ball DL, Bass H (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In Jo Boaler (Ed.), *Multiple Perspectives on Teaching and Learning*. Westport, CT: Ablex Publishing. pp. 83-104.
- Ball DL, Lubienski S, Mewborn DS (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching*, 4:433-456. Washington, DC: American Educational Research Association.
- Ball DL, Thames MH, Phelps G (2008). Content knowledge for teaching what makes it special?. *J. Teacher Educ.* 59(5):389-407.
- Başgün M, Ersoy Y (2000). Sayılar ve Aritmetik I: Kesir ve Ondalık Sayıların Öğretmesinde Bazı Güçlükler ve Yanılgılar, IV. Fen Bilimleri Eğitimi Kongresi Bildiri Kitabı: MEB Yayınları, Ankara 604-608.
- Borko H, Eisenhart M, Brown C, Underhill R, Jones D, Agard PC (1992). Learning to teach hard mathematics: Do novices and their instructors give up too easily?. *J. Res. Math. Educ.* 23(3):194-222.
- Carraher DW (1996). Learning about fractions. In L. P. Steffe, P. Nesher, G. Goldin, P. Cobb & B. Greer (Eds.), *Theories of Mathematical Learning* (pp. 241-266). Hillsdale, NJ: Erlbaum.
- Charalambous CY, Delaney S, Hsu HY, Mesa V (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathe. Thinking Learn.* 12:117-151.
- Chick HL, Baker MK (2005). Investigating teacher's responses to student misconceptions. In H. L. Chick & J. L. Vincent (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education Melbourne, Victoria, Australia: PME.* 2:249-256.
- Çakan M (2004). Assessment-Evaluation Applications of Teachers and their Qualification Levels: Primary and Secondary Education. *Ankara University J. Educ. Sci.* 37:99-114.
- Çelik B, Çiltaş A (2015). Beşinci sınıf kesirler konusunun öğretim sürecinin matematiksel modeller açısından incelenmesi. *Bayburt Üniversitesi Eğitim Fakültesi Dergisi* 1:10.
- Dursun Ş, Dede Y (2004). Öğrencilerin matematikte başarısını etkileyen faktörler: matematik öğretmenlerinin görüşleri bakımından. *Gazi Üniversitesi Gazi Eğitim Fakültesi Dergisi* 24(2):217-230.
- Ersoy Y, Ardahan H (2003). İlköğretim okullarında kesirlerin öğretimi II: Tanıya yönelik etkinlikler düzenleme, <http://www.matder.org.tr/bilim/ioko/2tyed.asp?ID=49>, Download:27.04.2015.
- Hanson D (1995). Understanding Fractions (Grades 5 to 8). <http://mathcentral.uregina.ca/RR/database/RR.09.95/hanson4.html>
- Haser Ç, Ubuz B (2002). Kesirlerde kavramsal ve işlemsel performans. *Eğitim ve Bilim* 27(126):53-61.
- Herman J, Ilucova L, Kremsova V, Pribyl J, Ruppeldtova J, Simpson A, Stehlikova N, Sulista M, Ulrychova M (2004). "Images of fractions as process and images of fractions in processes". *Proceedings of the 28th Conference of the International Group of the Psychology of Mathematics Education, Bergen.*
- Hill HC, Rowan B, Ball DL (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *Am. Educ. Res. J.* 42(2):371-406.
- İşık C (2011). İlköğretim matematik öğretmeni adaylarının kesirlerde çarpma ve bölmeye yönelik kurdukları problemlerin kavramsal analizi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi* 41:231-243.
- İşık C, Kar T (2012a). İlköğretim matematik öğretmeni adaylarının kesirlerde bölmeye yönelik kurdukları problemlerde hata analizi. *Kuram ve Uygulamada Eğitim Bilimleri (KUYEB)* 12(3):2289-2309.
- İşık C, Kar T (2012b). Sınıf öğretmeni adaylarının problem kurma becerileri. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi* 23:190-214.
- İşık C, Kar T (2012c). 7. sınıf öğrencilerinin kesirlerde toplama işlemine kurdukları problemlerin analizi. *İlköğretim Online* 11(4):1021-1035.

- Işık C, Işık A, Kar T (2011). Analysis of the problems related to verbal and visual representations posed by pre-service mathematics teachers. Pamukkale University J. Educ. 30(2):39-49.
- Işıksal M (2006). A study on pre-service elementary mathematics teachers' subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions. Unpublished doctoral dissertation, Middle East Technical University, Turkey.
- Kar T, Işık C (2014). Analyzing problems posed by seventh grade middle school students for subtraction operation with fractions. Elem. Educ. Online 13(4):1223-1239.
- Kar T, Işık C (2015). The Investigation of Elementary Mathematics Teachers' Views about Problems Posed by Students: Addition Operation with Fractions. Hacettepe Üniversitesi Eğitim Fakültesi Dergisi (H. U. J. Educ.) 30(1):122-136.
- Kılıç Ç (2013). Determining the performances of pre-service primary school teachers in problem posing situations. Educational Sciences: Theory Pract. 13(2):1195-1211.
- Kocaoğlu T, Yenilmez K (2010). Beşinci sınıf öğrencilerinin kesir problemlerinde yaptıkları hatalar ve kavram yanlışları. Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi 14:71-85.
- Korkmaz E, Gür H (2006). Determining of prospective teachers' problem posing skills. Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi 8(1):64-74.
- Leung SS (2013). Teachers implementing mathematical problem posing in the classroom: challenges and strategies. Educ. Stud. Mathe. 1-14.
- Olkun S, Toluk Uçar Z (2012). İlköğretimde etkinlik temelli matematik öğretimi. Ankara, Anı yayıncılık.
- Ma L (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States, Erlbaum: Mahwah, NJ.
- Mack NK (1990). Learning fractions with understanding building on informal knowledge. J. Res. Math. Educ. 21(1):16-32.
- McAllister CJ, Beaver C (2012). Identification of error types in preservice teachers' attempts to create fraction story problems for specified operations. School Sci. Math. 112(2):88-98.
- Millî Eğitim Bakanlığı, Talim, Terbiye Kurulu Başkanlığı (MEB) (2015). İlkokul Matematik Dersi (1, 2, 3 ve 4. Sınıflar) Öğretim Programı. Ankara.
- Mok IAC, Cai J, Fung ATF (2008). Missing learning opportunities in classroom instruction: Evidence from an analysis of a well-structured lesson on comparing fractions. Math. Educator 11(1/2):111-126.
- Newton KJ (2008). An extensive analysis of pre-service elementary teachers' knowledge of fractions. Am. Educ. Res. J. 45(4):1080-1110.
- Özmantar MF, Bingölbali E (2009). Sınıf öğretmenleri ve matematiksel zorlukları. Gaziantep Üniversitesi Sosyal Bilimler Dergisi 8(2):401-427.
- Post TR., Harel G, Behr MJ, Lesh R (1991). Intermediate teachers' knowledge of rational number concepts. In E. Fennema, T.P. Carpenter & S.J. Lamon (Eds.), Integrating research on teaching and learning mathematics (177-198). Albany: State University of New York Press.
- Redmond A (2009). Prospective elementary teachers' division of fractions understanding: A mixed methods study. Unpublished doctoral dissertation, Oklahoma State University.
- Romberg TA, Carpenter TP (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. C. Witrock (Ed.), Handbook of Research on Teaching, New York: Macmillan, 850-873.
- Rule AC, Hallagan JE (2006). Preservice elementary teachers use drawings and make sets of materials to explain multiplication and division by fractions. Paper presented at the 2nd Annual Preparing Mathematicians to Educate Teachers (PMET) Conference at Oswego, New York.
- Seferoğlu SS (2001). Elementary school teachers perceptions of professional development. Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 20:117-125.
- Seyhan G, Gür H (2004). İlköğretim 7. ve 8. sınıf öğrencilerinin ondalık sayılar konusundaki hataları ve kavram yanlışları. Matematikçiler Derneği. www.matder.org.tr
- Shulman LS (1987). Knowledge and teaching: Foundations of the new reform. Harvard Educ. Rev. 57:1-22.
- Silver EA, Cai J (2005). Assessing students' mathematical problem posing. Teach. Children Math. 12(3):129-135.
- Smith JP (2002). The Development of Students' Knowledge of Fractions and Ratios. In B. Litwiller and G. Bright (eds.), Making Sense of Fractions, Ratios, and Proportions, Reston, Virginia, NCTM, pp. 3-17.
- Soylu Y, Soylu C (2005). İlköğretim besinci sınıf öğrencilerinin kesirler konusundaki öğrenme güçlükleri: Kesirlerde sıralama, toplama, çıkarma, çarpma ve kesirlerle ilgili problemler. Erzincan Eğitim Fakültesi Dergisi, 7(2):101-117.
- Sharp J, Adams B (2002). Children's constructions of knowledge for fraction division after solving realistic problems. J. Educ. Res. 95(6):333-347.
- Şiap İ, Duru A (2004). Kesirlerde geometriksel modelleri kullanabilme becerisi. Gazi Üniversitesi Kastamonu Eğitim Dergisi, 12(1):89-96.
- Tabak H, Ahi B, Bozdemir H, Sarı MH (2010). İlköğretim 4. ve 5. sınıf öğrencilerinin matematik dersinde kesirleri modelleme becerileri. e-J. New World Sci. Acad. Educ. Sci. 5(4):1513-1522.
- Ticha M, Hospesova A (2009). Problem posing and development of pedagogical content knowledge in pre-service teacher training. Paper presented at the meeting of CERME 6, Lyon.
- Tirosh D (2000). Enhancing pre-service teachers' knowledge of children's conceptions: The case of division of fractions. J. Res. Math. Educ. 31(1):5-25.
- Toluk-Uçar Z (2009). Developing pre-service teachers understanding of fractions through problem posing. Teach. Teacher Educ. 25:166-175.
- Utlej J, Redmond A (2008). Prospective elementary teachers attitudes towards and knowledge of the division of fractions. Paper presented at the annual meeting of the Research Council on Mathematics Learning, Oklahoma City, OK.
- Van de Walle JA (2004). Elementary and middle school mathematics: teaching developmentally (5th ed.). Boston, MA: Pearson.
- Ward J, Thomas G (2007). What do teachers know about fractions? In Findings from the New Zealand Numeracy Development Projects Wellington, New Zealand: Learning Media. pp. 128-138.
- Wu H (1999). Some remarks on the teaching of fractions in elementary school. Available at: <http://math.berkeley.edu/~wu/fractions2.pdf>
- Yim J (2010). Children's strategies for division by fractions in the context of the area of a rectangle. Educ. Stud. Math. 73:105-120.
- Zembat İÖ (2007). Working on the same problem-concepts; with the usual subjects pre-service elementary teachers. Elem. Educ. Online 6(2):305-312.