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Construction of High School Students' Abstraction Levels in Understanding the Concept of Quadrilaterals

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Abstract

The purpose of this study was to examine the abstraction thinking or the vertical reorganization activity of mathematical concepts of high school students while taking account of the abstraction that was constructed earlier, and the socio-cultural background. This study was qualitative in nature with task-based interviews as the method of collecting the data. It involved 62 high school students, and conducted for one year. The study focused on activities related to how the subjects grouped plane figures, recognized the attributes of each two plane figure, recognized the relation among them based on their attributes, defined plane figures, connected their attributes, as well as constructed the relations among plane figures. The results indicates that the abstraction level of high school students in constructing the relations among quadrilaterals consists of concrete visual level, semi-concrete visual level, semi-abstract visual level, and abstract visual level, together with indicators of each level. Therefore, the researchers suggest that it is necessary to design a learning activity that facilitates the four levels of abstraction, so that a student might increase his/her level of abstraction.

Keywords: abstraction level, geometry, two-dimensional figures, high school mathematics

1. Introduction

Abstraction has been known as the most relevant characteristics in terms of cognitive aspects that led to the failure to learn mathematics. Researchers such as Mitchelmore and White (1995), Hershkowitz, Schwarz, and Dreyfus (2001), Dubinsky (2000), Ozmantar and Roper (2004), Ozmantar and Monaghan (2007), Gray and Tall (2007), Ron et al. (2008), Simon et al. (2004), Menmun (2012), as well as Dooley (2012) state that abstraction is instrumental in mathematics, and that someone can abstract knowledge empirically. Abstraction is an activity of a vertical reorganization of previously constructed mathematical concept to become a new mathematical structure. The kinds of activity used in abstraction process involve identifying, arranging, and constructing. Reorganization is an activity to collect, to collate, and to develop mathematical elements into a new element. Vertical reorganization means an activity of reorganizing an abstract idea into a more abstract form or more formal than the original. The types of activities used in organization involve identifying, arranging, and constructing.

In this study, what we mean as recognizing is an activity of identifying the characteristics of a triangle or quadrilateral. Arranging is an activity of combining the characteristics of two triangles or quadrilaterals, while constructing is an activity of reorganizing the characteristics of a triangle or a quadrilateral into a new structure that has been possessed by students.

The purpose of this study is to establish the level of abstraction of junior high school students in comprehending the concept of quadrilateral, as well as the characteristics of each level. This study contributes to the development of the theory of abstraction, especially on the level of abstraction of junior high school students in understanding the concept of quadrilateral in Indonesia. This study also contributes to the characteristics of each level so it make possible to develop a learning model that can improve the students' level of abstraction to a higher level.

2. Theoretical Framework: The Concept Map of Quadrilaterals, and Abstraction

A definition is described as a statement that can be used to limit a concept. Quadrilaterals such as parallelogram, rectangle, square, rhombus, kite and trapezoid are examples of concepts, while "parallelogram is a quadrilateral that has two pairs of parallel opposite sides," are an example of a definition. The statement in such a definition limits the concept (Budiarto et al., 2008). Furthermore, it was stated that analytic definition was a definition that mentioned *genus proximum* (immediate family) and *differensia spesifika* (special distinction). The above definition of parallelogram is an analytic definition having "quadrilateral" as the immediate family, and "has two pairs of parallel opposite sides" as special distinction.

Genetic definition is a definition that would indicate or reveal the occurrence or formation of concepts defined, as the following example. "A kite is a quadrilateral which is formed when two isosceles triangles coincides on their similar base." There are four elements in a definition, namely, background, genus, defined term, and attributes (Soedjadi, 2000). In the case of the above definition of parallelogram, as the background is square, the genus is quadrilateral, the defined term is parallelogram, while the attribute is a pair of parallel opposite sides.

The definitions used in defining quadrilaterals have impacts on the relations among the quadrilaterals (Budiarto et al., 2011). If a trapezoid is defined as "a quadrilateral which has exactly a pair of parallel sides" or "a quadrilateral which has a pair of parallel sides," then the two different definitions will affect the relations among quadrilaterals. If the first definition is used, then the set of parallelogram and trapezoid are mutually exclusive. But, if the latter definition is used, then the set parallelogram is a subset of the set of trapezoids.

Moreover, a parallelogram can be defined as follows: (1) Parallelogram is a quadrilateral of which two pairs of opposite sides parallel; (2) Parallelogram is a quadrilateral of which two pairs of opposite sides of the same length; and (3) Parallelogram is a quadrilateral whose pair of opposite sides are parallel and equal in length. These three definitions of parallelogram are equal, and have similar extension (Soedjadi, 2000). In addition, two or more definitions having similar extension are called equivalent definition. Extension is the whole things upon which an idea can be applied, or the environment that can be designated with a concept.

The attribute used in definition (1) is "has two pairs of parallel sides," while the attributes used in definition (2) is "has two pairs of equal sides, as well as "has a pair of parallel and equal sides" in definition (3). These kinds of definitions are said as having different intention (word meaning) (Soedjadi, 2000). The meaning of parallelogram that is constructed by students is said to be accurate if it is equivalent to the above definition.

As a different case, the following is the various definition of rectangle. (1) A quadrilateral of which two pairs of opposite sides are parallel, and has one right angle; (2) A quadrilateral which has two pairs of equal opposite sides, and one right angle; and (3) A quadrilateral of which a pair of its opposite sides are parallel and equal in length, and it also has a right angle. Thus, the three definitions above have similar extension, but different intention. According to Budiarto et al. (2009), a concept map is strongly influenced by the semantic statement of the definition and the preferred relationship. The concept map of quadrilaterals can be seen on the following page.

In a concept map, a chordal quadrilateral and a trapezoid have the same position level, because both of them are defined by adding one condition to a rectangle. Likewise, a parallelogram and a kite are also in the same level, because both of them are defined by adding two conditions to a quadrilateral. A tangential quadrilateral, a rectangle and a rhombus also have the same level, because they are defined by adding three terms to a quadrilateral. In addition, a square occupies the lowest level since it is defined by adding four terms to a quadrilateral.

The creation of a concept map taking into account the position or the level of concepts will bring the following impact, if we add one term into a chordal quadrilateral, it will become a trapezoid. But, if we add three terms, it will become a rectangle. While, if we add four terms, it will be a square. Likewise, if we add an additional term to a trapezoid, it will be a chordal quadrilateral or a parallelogram. Moreover, if we add three conditions, it will become a tangential quadrilateral.

Given quadrilateral ABCD, $\overline{AB} = s_1$, $\overline{BC} = s_2$, $\overline{CD} = s_3$, and $\overline{AD} = s_4$ with successive gradients m_{s1}, m_{s2}, m_{s3}, m_{s4}. If P is the center of the incircle of quadrilateral ABCD, then dP_{s1} represents the distance from center P to side s_1. The concept map which is based on the intentions of the definition is presented in the following page.

Another important concept used in this research is "Abstraction". The term abstraction comes from the Latin *"ab"* which means "from," and *"trahere"* which means "to pull/to draw" (Gray & Tall, 2007). Grammatically, *to abstract* (a verb) constitutes a process, while *to be abstract* (an adjective) constitutes a property. In addition, *an abstract* (a noun) means abstraction, and constitutes a concept. The term *abstraction* has two meanings, first, as a process to describe a situation. Second, as a concept resulted from a process. Therefore, abstraction might mean a process as well as an output.

A student is said to recognize a triangles or a quadrilateral if he/she is able to identify the different characteristics, the same characteristics, and the meaning of a triangle or a quadrilateral (Hershkowitz et al, 2001). An attribute is said to be true if it is mathematically true or it has an equivalent on a related plane figure model.

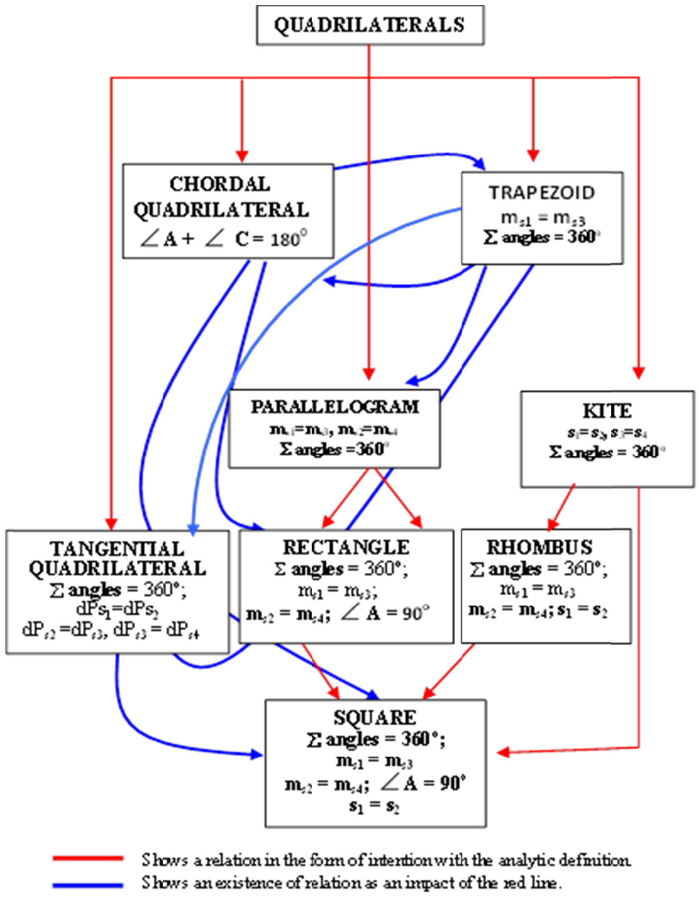

Figure 1. The concept map of quadrilaterals

To identify an abstraction as a process, one can use three epistemic actions, which are recognizing, stringing or 'building-with' and, constructing activities. Epistemic action is a mental action that exists when knowledge is used. Recognizing is the activity in identifying the characteristics of a triangle or a quadrilateral. Stringing is the activity in combining two triangles or quadrilaterals, while constructing is the activity in reorganizing the characteristics of triangles and rectangles into a new structure that is not possessed by the students. The activities to identify, to assemble, and to construct are not always in a linear form, but they can be in a nested form which means that the three activities occur together. In order to identify an object as an example of abstraction, someone who wants to know the object must already have the abstraction albeit slightly (Ohlsson & Regan, 2001). Thus, an abstraction starts from the initially abstract entity toward the complex structures.

Cognitive mechanism of abstraction is the organizing of the existing ideas to become the more complex ones. Abstraction is an activity of vertical reorganization of mathematical concept which has previously been constructed into a new mathematical structure. The term abstraction is used for both the process and the outcome (Hershkowitz et al., 2001). Many researchers developed an idea of abstraction in a context by examining in such a way that the knowledge can be constructed in a social setting. They argued that abstraction is a vertical reorganization activity of mathematical concept that has been constructed earlier to a new mathematical structure. On the other hand, Ozmantar (2007) stated that horizontal mathematization is associated with the relationship between the non-mathematical situation and mathematical ideas. Moreover, he stated that vertical mathematization is an activity of preparing, collecting, organizing, and developing mathematical elements into a new element, often in the form of a more abstract or a more formal than the original. The mathematical terms that have previously been constructed refer to two cases. Firstly, the results of the previous abstraction process can be used on the current abstraction; Secondly, the current activities are starting from an early form of abstraction. Reorganizing into a new structure implies the mathematical relationship that includes (a) creating a new hypothesis, (b) inventing or re-inventing a mathematical generalization, proof or new strategies to solve a problem. Such an activity requires a high level of theoretical thinking, namely, synthesizing, analyzing, and evaluating, without abandoning empirical thinking.

In terms of origin of the incidence of abstraction, it was proposed that abstraction takes place in three stages (Hershkowitz et al., 2001). First, abstraction stems from an early form, simple, undeveloped, no need to be consistent both internally and externally. Second, the development of abstraction takes place from analysis of early stages of abstraction to synthesis. Third, it ends with a consistent and complicated final form. It was suggested that abstraction does not run from the concrete to the abstract, but from undeveloped abstract forms into developed abstract forms which emphasizes the new characteristics of the new ones.

The abstraction was also defined as an activity which is in line with mathematical construction (Tsamir & Dreyfus, 2002). This definition gives an advantage in the term of activity and irreversibility which means that activities on mathematics components are combined together, restructured, organized, and constructed in such a way that it becomes more abstract or more formal. The power of abstraction depends on the context, the background of the learner, and the possible artifacts for them as well as the structure of the inner of the individual. An abstraction form has a lot of meaning in mathematics, for example, to replace an abstraction process, one can use the process of constructing knowledge.

Budiarto et al. (2009) stated that an attribute was said to be non-routine if it was not an attribute to be used as a necessary and sufficient condition for establishing a definition. Moreover, an attribute was said to be irrelevant if it generally did not build understanding of quadrilateral. For example, the attribute "it has an acute angle", "it has an oblique side", "it resembles a diamond," or "adjacent sides are not equal." The definition of a quadrilateral is accurate if the used attributes denote necessary and sufficient condition for establishing a definition. Then, analytical definition is said to be accurate analytical definition if it is analytical and accurate. If the results of a series of two figures and the accurate analytical definition of a subject are more than any other subject, it is said that subject has a better abstraction. A student is said to string the characteristics of two quadrilaterals, if he/she has combined the characteristics of the two quadrilaterals. Thus, a student is said to have constructed knowledge, if he/she has reorganized the characteristics of two quadrilaterals into a new structure that has not been possessed by the student.

3. Methodology

3.1 Subjects and Data Sources

This research is descriptive explorative in nature. The data were collected using task-based interviews. The researchers conducted clinical interviews which were recorded using audio-visual recorder. In performing the task-based interview, the researchers took into account the objectivity and neutrality. Objectivity refers to the relationship between the interviewer and the subject, while neutrality refers to the psychological relationship between respondents' answers or opinions. As the subjects of this study were 62 junior high school students in Surabaya. In addition, this study took two years to complete.

3.2 Instruments, Data Collection, and Data Analysis

Through task-based interviews, the researchers collected the data of how students recognizing the attributes of a plane figure, defining, stringing and finally constructing the relationship among quadrilaterals. Then, the data obtained were characterized in such a way that resulted in the characteristic of abstraction model of each subject. Having acquired the characteristics of abstraction model for a geometric concept, then it followed with the process of theorizing, which aimed to determine the characteristics of each level in the abstraction path. Such a process consisted of four stages. First, the comparison of events applicable to each category; Second, the integration and the sector/region; Third, restriction of the theory; and fourth, writing the theory. In the first phase, the researchers encoded each datum that appears to fit the category. Furthermore, it was compared with the previous events in the same group and the different groups, which were coded in the same category. The coding forwarded, so did the comparisons between events, until the researchers got an accumulation of knowledge regarding the sector/region of a category that was ready to be integrated (i.e. the second stage). The integration is carried out toward one category to other categories, such that a theory developed after various categories with their sector/region tend to be integrated. These activities started at the stage of data collection and data analysis.

In the third stage, which is the stage for theory limitation, the researchers removed the irrelevant things, and then integrated the small sectors/regions into a framework of interrelated categories. Afterward, the researchers carried out reduction. This reduction could also be done by comparing with the results of previous studies, references, or expert opinions. Finally, writing the theory as the last stage. It meant arranging precise and reasonable statements as the result of the previous stages. Such statements are arranged in the form of a framework that can be used by other researchers in the same field. Thus, it resulted with a new theory called Abstraction Levels of Junior High School Students in Constructing the Concept of Plane Figures.

4. Result and Findings

Based on the results of interview with 62 subjects, the researchers compared the occurrences applicable to each category in the form of an inter-relationship diagram, which provide the following characteristics of plane figures. The first important thing to explain is clarification the geometric figures according to their observed attributes. These observed attributes consist of the shape, the length of the side, the numbers of sides, the angles, the slope of the sides, the area, the name of figure, the numbers of fold symmetry, and the many levels of rotational symmetry. The subject activities in grouping two-dimensional figures consisted of using physical model, model image, spoken language or symbolic language, which then integrated in the region by activities of identifying, compiling, and constructing to observe whether the used attributes belonged to routine, irrelevant, relevant, meaningful, meaningless, inaccurate or excessive attributes. Furthermore, the process of integration took into account how the subject grouped two-dimensional figures, stated the relations among two- dimensional figures based on recognized attributes, defined a two-dimensional figure, developed a logical proof, and constructed geometric model of two-dimensional figures.

The results of the integration can be described as follows. The subject grouped the model of two-dimensional figures into triangles and rectangles. In addition, the subject conducted such activities as recognizing triangular and quadrilateral characteristics based on the numbers of their sides. The subject also conducted activities as recognizing the name of the quadrilateral, and then he/she classified the quadrilateral models into groups of parallelogram, rectangle, rhombus, square, kite, trapezoid, and arbitrary quadrilateral based on the figure names, the figure shapes or the observed characteristics. In terms of the attributes used to construct definition, the subject utilized routine, irrelevant, relevant, meaningful, meaningless, accurate and excessive attributes. It also evidenced that there were several analytic definitions constructed for one definition or more.

In terms of grouping figures based on its properties, the subject mentioned the followings: quadrilateral of which two pairs of their opposite sides are parallel, quadrilateral whose four angles are right-angled, and quadrilateral whose diagonals are perpendicular to each other. Moreover, in terms of the relationship between two or more figures, the subjects mentioned the following cases: attributes of a figure owned by the others, and attribute of a figure not owned by the others. The, the subject also using relation of subsets consisted of the following cases: rectangle is a parallelogram, a square is a rectangle, or a square is rhombus. In visualizing geometric figures according to their verbal descriptions, the subjects drew quadrilaterals with or without a label. Finally, to sum up further information based on visual observations (using the properties of transitive relation), the student mentioned the followings: A square is a rectangle; A rectangle is a parallelogram. Thus, a square is a parallelogram as well.

Based on the comparison of applicable occurrences toward each category, the integration and the region, then the researchers carried out restriction of the theory which found that the indicators that can be used to build a level of abstraction consisted of (1) Clarifying the geometric figures by using relevant attributes, irrelevant attributes, and routine attributes; (2) Constructing definitions by using routine, irrelevant, relevant, meaningful, meaningless, inaccurate, or excessive attributes; (3) Determining any relationship between figures based on characteristics observed, most relationships of subsets, or all relationships of subsets to determine the relationship between the two figures; (4) Visualizing geometric figures according to their verbal descriptions, whether with labels or without labels; (5) Summing up parts or all further information based on visual observation. (by using the properties of transitive relation); and (6) Creating a connection diagram by using all observed attributes, large parts or little parts of the observed attributes, or by not using the observed attributes).

Based on indicators and characteristics described above, then the researchers drafted abstraction level of junior high school students in constructing the relationship among two-dimensional figures as follows: (1) Concrete

Visual Level, with the following characteristics: Clarify geometric figures using irrelevant attributes, construct definition using irrelevant attributes, cannot make an analytic definition, mention meaningless attributes, mention inaccurate attributes, cannot determine the relationship between two figures by using observed attributes, unable to use the relation of subsets to determine the relationship between the two figures, cannot visualize the geometric figures according to their verbal description, cannot sum up further information based on visual observations, and do not use the observed attributes in creating a connection diagram.

(1) Semi-concrete Visual Level, having the following characteristics: Clarify geometric figures using routine attributes, construct definition using excessive attributes, can make an analytic definition but it is inaccurate and it has excessive attributes; Be able to determine a little part of the relationship between two figures based on observed attributes; Be able to visualize a little part of geometric figures according to their verbal description; Be able to sum up a little part of further information based on visual observations; Use a little part of the observed attributes to create the connection diagram.

(2) Semi-abstract Visual Level, with the following characteristics: Clarify geometric figures using parts of relevant attributes; Construct definitions using excessive attributes; Be able to create an accurate analytic definition; Be able to use most parts of subset relations to determine the relation between two figures; Be able to visualize most parts of geometric figures according to their verbal description with labeling; Be able to sum up most parts of further information based on visual observations; and use most parts of observed attributes to create a connection diagram.

(3) Abstract Visual Level, with the following characteristics: Clarify geometric figures using all relevant attributes; Construct definitions using attributes; Be able to create more than one accurate analytic definition; Be able to determine all relations between two figures based on observed characteristics; Be able to use all relations of subset to determine the relation between two figures; Visualize all geometric figures according to their verbal description with labeling; Be able to sum up all further information based on visual observations; And use all observed attributes to create a connection diagram.

5. Discussion

The results of this study showed that some subjects tend to use models of plane figures. As a result, a rectangle should not be called a parallelogram. Based on the fact that characteristics of parallelogram could be found in a rectangle, the subject stated that a rectangle is a parallelogram, but a rectangle should not be called as a parallelogram. The students' inconsistent answer is a kind of partially correct constructs (Ron et al., 2008), that are the constructs that only partially matched the relevant underlying mathematical principles and their elements.

A mathematical principle is then considered a partially correct construct if only some of its elements have been constructed and are recognized as relevant when appropriate, whereas others may be lacking or may themselves be partially correct constructs. In this study, some subjects' disabilities to relate and then distinguish the characteristics of rectangle and parallelogram indicate the existence of partially correct construct.

Furthermore, the subject sharply distinguished between the names of figures and the relationships of the same characteristics of plane figures. However, when determining the relationship among trapezoid and parallelogram, rectangle, square, and rhombus, the subject said that a rectangle may be called as a trapezoid, and that a rectangle is also a trapezoid. These results indicated that there was inconsistency of this subject's abstraction. The students' incorrect answers indicate knowledge constructs that are partly but not fully appropriate to the mathematical problem situation at hand. Because the students' constructs only partially fit the mathematical principles underlying the learning context, then they were also named as partially correct constructs (Ron et al., 2008).

The researchers had not explored why it happened yet. It should be noted that this subject used informal language which made him easier to understand the concepts of geometry. The language used in the student book, or teacher book, sometimes might be a barrier for students to understand the concepts. Even when students understood geometry in a formal language, they talked about it informally.

The other subjects did not reveal the non-routine attributes, instead, they expressed irrelevant attributes to parallelogram and rectangle, that was adjacent sides were not equal in length. For a rhombus, they said that it resembled a diamond; for a kite, they said that it resembled "kites," and for a trapezoid, they said that the parallel sides had different length. The subject also mentioned a non-routine attribute, which was the opposite angles were equal, but he did not mention irrelevant attributes. In addition, the subject suggested one meaningless attribute, that was a parallelogram has an acute angle, and he put forward four non-routine attributes, namely, mutually perpendicular diagonal, many axes of symmetry, the degree of rotational symmetry, and that opposite angles are equal. If the numbers of relevant attributes and non-routine attributes raised by a subject become indicators of the level of abstraction, then the abstraction level is directly proportional to the subject's mathematics ability.

Viewed from didactic standpoint, first, some subjects still used the model of plane figure in abstraction. Therefore, in studying geometry, such a subject still needs a learning tool. Second, in creating the network of relations among quadrilaterals, there are six different network models. Such a difference was due to a different understanding of the trapezoid, which is a quadrilateral having only a pair of parallel sides, or a quadrilateral having a pair of parallel sides. Third, the attribute "like a kite" or "like a rhombus" are irrelevant attributes to construct the insight of a kite or a rhombus. These attributes indicate that around 7% of the subjects are at the lowest level compared to the others. Subjects who have these characteristics should receive special attention in geometry learning. On the other hand about 35% of the subjects worked on two levels consistently. They responded to questions from the interviewer and analyzed them. These subjects tried to find the hidden relations between the areas of a trapezoid to the area of other quadrilateral figures having the same characteristics with that of a trapezoid. In addition, the subjects can reinvent the formula of area of a parallelogram, rectangle, square and rhombus derived from the area of a trapezoid. The supportive environment for learning geometry of these subjects was not primarily physical environment but tend to be non-physical, such as geometry teacher who teaches them with a lot of smiles, a lot of listening, respect the opinion of learners, speak a polite language that foster a sense of comfort for the learners to learn. Avoid build prestige by taking refuge in a difficult geometry. If the learning environment is created, then the creativity will grow by itself.

Judging from the theoretical point of view, the subjects' abstraction activities indicated the nested process of abstraction, in which the subject recognized the structures previously, and then he or she assembled the structures to meet the requirements of his/her tasks. This is parallel to the idea of "vertical mathematization" (Treffers & Goffree, 1985), an idea which has brought forward by Freudenthal to mathematics education in general, and to mathematical abstraction in particular. Vertical mathematization points to a process of constructing by learners that typically consists of the reorganization of previous mathematical constructs within mathematics and by mathematical means. This process interweaves previous constructs and leads to a new construct. The design of interview in this study aimed to create a network of relations among quadrilaterals, and offered opportunities to continue beyond the constructed knowledge to construct new structures for the subject, with the forms of activities as recognizing, assembling, and constructing the relations between two quadrilaterals like a nest.

6. Conclusion

The level of abstraction of junior high school students in understanding the concept of quadrilateral consisted of concrete visual level, semi-concrete visual level, semi-abstract visual level, and abstract visual level, with the following characteristics: Clarifying quadrilaterals; Constructing a definition; Determining the relationship between two quadrilaterals based on their observed characteristics; Using the subset relation to determine the relationship between two quadrilaterals; Visualizing geometric figures according to verbal description; Concluding further information based on visual observations, and creating a relationship diagram by using all observed attributes. The construction of levels in this study is still rough with limited characteristics. It needs other research to refine the levels and to expand the characteristics that may arise.

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