

**Developing an Understanding of Quadratics through the Use  
of Concrete Manipulatives:  
A Case Study Analysis of the Metacognitive Development of a  
High School Student with Learning Disabilities**

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This case study analyzed the impact of a concrete manipulative program on the understanding of quadratic expressions for a high school student with a learning disability. The manipulatives were utilized as part of the Concrete-Representational-Abstract Integration (CRA-I) intervention in which participants engaged in tasks requiring them to multiply linear expressions and factor quadratic expressions embedded within contextualized area problems. The case study focused on a representative participant, Marcia, who demonstrated significant gains from pre- to post-intervention assessments. The qualitative analysis provided descriptive data which offered insight into the reasons for these gains. Results indicated that the manipulatives supported metacognition through strategic planning and self-regulation.

*Keywords:* Learning Disabilities, Secondary Mathematics, Metacognition

High-level mathematics courses have not historically been accessible to students with learning disabilities (LD). However, secondary mathematics expectations are increasing as a result of the Common Core State Standards (CCSS) for Mathematics. Specifically, *all* students are expected to participate in three years of rigorous high school mathematics. Students may follow the traditional pathway and take High School Algebra I in ninth grade, Geometry in 10<sup>th</sup> grade, and Algebra II in 11<sup>th</sup> grade, or they may take three years of integrate mathematics which contain the same content found in the traditional courses. These courses are considered the minimal require-

ments necessary for students to be college and career ready (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

Although 62% of secondary students with LD participate in mathematics courses in the general education setting (Newman, 2006), on average they are enrolled in less rigorous mathematics courses that focus on basic math rather than age-appropriate mathematics content (Kortering, deBettencourt, & Braziel, 2005; Wagner, et al., 2003). Additionally, students with LD take fewer mathematics courses as they progress through high school (Wagner, et al., 2003). On average, Algebra 1 is the highest level

mathematics course completed by students with disabilities (Wilson, 2008).

Students with LD may take less rigorous mathematics courses in high school because of common characteristics that impede progress in mathematics. They may lack automaticity of mathematics facts (Garnett, 1998, Geary, 2004) which then makes procedures such as factoring quadratic expressions laborious. Additionally, students with LD often have procedural deficits which impede multistep problem solving. Further, these students may have a poor understanding of concepts that underlie procedures (Geary, 2004). Students with LD may also find the abstract symbolism in mathematics confusing (Garnett, 1998) which is compounded when faced with both numerals and variables in algebra.

In addition to deficits in mathematics, students with LD also have immature metacognitive skills (Montague, 2007). Metacognition refers to a person's self-awareness of their cognitive abilities, steps and strategies used during a task, self-monitoring of task completion, and appraisal of task completion through checking the accuracy of work (Bley & Thornton, 2001; Mazzocco, 2007). Self-regulation underlies the processes and functions associated with metacognition (Montague, 2008). Self-regulation refers to monitoring and evaluating one's performance during a problem solving task (Fuchs & Fuchs, 2007). Typically, students with LD are poor self-regulators (Montague, 2007). Additionally, poor strategic planning is representative of immature metacognitive skills for students with LD. Strategic planning refers to a student's ability to develop and execute a plan of engagement with a mathematical task. Students with LD often employ immature strategies when engaging in mathematics tasks and make numerous computational errors when executing the plan (Geary, 2004). Deficits in mathe-

matical content knowledge as well as metacognition interfere with the mathematics progress for many students with LD.

The authors of the CCSS acknowledge that some students will require additional supports to meet the high school mathematics expectations. The authors suggest strategies such as extended time in mathematics, after-school tutoring, and summer instruction (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010); however, using research-based instructional practices during the regular mathematics class time may be more feasible. When used effectively, the use of concrete manipulatives during instruction has been found to be beneficial for many students with LD (Bley & Thornton, 2001; Hudson & Miller, 2006) and without (Van de Walle, Karp, & Bay-Williams, 2010). Manipulatives are physical objects that support mathematical thinking (National Research Council, 2001) and include any physical object that represents a mathematic concept. Examples include counters, beads, blocks, fraction bars, pattern blocks, Cuisenaire rods, algebra tiles, and geoboards (Maccini, Strickland, Gagnon, & Malmgren, 2008). Manipulatives create an external representation of a mathematical idea, which may help students form internal representations (Puchner, Taylor, O'Donnell, & Flick, 2008).

Despite the research supporting the potential benefits of using manipulatives in mathematics classes, manipulatives are not used frequently in the secondary classrooms. In a survey conducted by Swan and Marshall (2010), teachers reported a steady decrease in the use of manipulatives from kindergarten through middle school. Additionally, ninth grade teachers reported using manipulatives once a month or less. The following section reviews the current research in special education regarding the

use of manipulatives within the Concrete-Representational-Abstract instructional practice followed by the purpose statement for the current study.

### **CRA Instruction**

Although there is a paucity of research in the area of algebra interventions for secondary students with disabilities, four studies have investigated the effects of manipulatives and the algebra content. In these studies, manipulatives were an essential component within the Concrete-Representational-Abstract (CRA) instructional practice. The CRA instruction involves teaching algebra content using concrete manipulatives (i.e., algebra tiles), representations of manipulatives (i.e., drawings of tiles), and abstract notation (i.e., numbers and variables). Two studies (Scheuermann, Deshler, & Schumaker, 2009; Witzel, Mercer, & Miller, 2003) utilized a graduated CRA sequence, in which participants mastered the algebraic task using concrete manipulatives then progressed to using representations of the manipulatives. After demonstrating mastery of completing the task using representations, participants completed the algebra task by using abstract notation only. Witzel and colleagues (2003) investigated the effects of the CRA sequence on the ability of sixth and seventh grade students with disabilities or at risk for algebra failure to transform linear equations. The authors reported that the students who received CRA instruction significantly outperformed a comparison group who received instruction using abstract notation only.

Similarly, Scheuermann and colleagues (2009) incorporated the CRA sequence into an instructional package entitled Explicit Inquiry Routine (EIR) to teach one-variable equations embedded in word problems to 14 middle school students with LD. EIR included three components: (a) explicit sequencing of skills; (b) scaffolded instruction in which students first told the teacher how to illustrate and manipulate

the problem, followed by students telling a peer and telling themselves; and (c) the CRA sequence. The researchers found that students made significant improvements after receiving this intervention.

Similarly to the CRA instructional sequence, two studies (Strickland & Maccini, 2013; Strickland & Maccini, in press) investigated the impact of the Concrete-Representation-Abstract-Integration (CRA-I) strategy on multiplying linear expressions and factoring quadratic expressions. The CRA-I strategy modifies the CRA sequence by simultaneously using concrete manipulatives, sketches of the manipulatives, and abstract notation. Additionally, students may move between these representations based on their individual needs, rather than progressing through each phase in a linear fashion. Algebra Lab Gear (Picciotto, 1995) was the manipulative program utilized in both of these studies. Participants used blocks representing constants (whole numbers), linear terms ( $x$ -bars) and quadratic terms ( $x^2$  blocks) to multiply linear expressions (Strickland & Maccini, 2013) and to factor quadratic expressions (Strickland & Maccini, in press) embedded within an area contextualized task. Additionally, participants used a graphic organizer which resembled the manipulatives to support their transition to abstract notation only. Participants made significant gains in both of these studies, with all participants demonstrating proficiency of the content as evidenced by posttest scores ranging from 78% - 100% accuracy.

The purpose of the present study is to provide descriptive data as to how participants in the Strickland and Maccini study (in press) made significant academic gains. A qualitative analysis of the video recorded instructional sessions revealed a heavy reliance on the manipulatives as participants engaged in the algebraic tasks. The manipulatives provided an avenue for students to plan strategically as well as self-

regulate while executing the strategies. Therefore, this study focused on the impact of the Algebra Lab Gear (ALG) manipulatives on the metacognitive development of a high school female with LD.

### **Method**

The qualitative method used in this design was a case study focusing on one critical case, Marcia, who provided a rich data source that was representative of the group (Creswell, 2007). Specifically, Marcia's data provided insight into why all participants demonstrated significant gains on the domain probes from pre-intervention to post-intervention. The case study focused on Marcia's thinking and understanding of quadratic expressions through the instructional practices and tools embedded within the intervention. The following section describes (a) participants and setting; (b) the intervention; (c) data collection; (d) data analysis; and (e) data validation of the case study of Marcia.

### **Participants and Setting**

Marcia was a 16 year old white female who met the state's criteria for a learning disability and was also identified as having Attention Deficient with Hyperactivity Disorder (ADHD). Although her Individualized Education Plan (IEP) did not explicitly state that she had a mathematics learning disability, her IEP contained goals and objectives targeting mathematics. Marcia completed the intervention in a small group with two additional students, Sasha and Anna, who were white females, ages 16 and 17, respectively. Sasha was identified as LD and ADHD while Anna was awaiting an educational evaluation to determine the presence of a learning disability. All three

participants were participating in an Algebra II course; however, all were at risk for failing the course. Additionally, all three participants had a history of mathematics difficulties and were consistently placed in the lowest level mathematics course since they began attending the school in 7<sup>th</sup> grade.

The study took place in a private high school located in a city in the Mid-Atlantic region of the United States. Participants were removed from their current mathematics class to receive the intervention. The author assumed the role of teacher-researcher for the duration of the study. Additionally, the author had a pre-existing relationship with Marcia as her seventh-grade teacher. Although four years had passed since Marcia and the author were together, their relationship may have impacted Marcia's comfort level and her ability to articulate her thoughts throughout the intervention.

### **Intervention**

The intervention consisted of the CRA-I Strategy in which participants explored quadratic expressions by simultaneously using ALG, drawings of ALG, and abstract notation. The ALG is a manipulative program that consists of algebra blocks representing constants (whole numbers), linear variables to the first degree ( $x$ ), and quadratic variables to the second degree ( $x^2$ ). An area model is incorporated in ALG program when teaching quadratics. Specifically, the linear expressions represent the length and width while the quadratic expression represents the area. Therefore, tasks within the instructional unit consisted of area word problems (see Figure 1).

Figure 1.

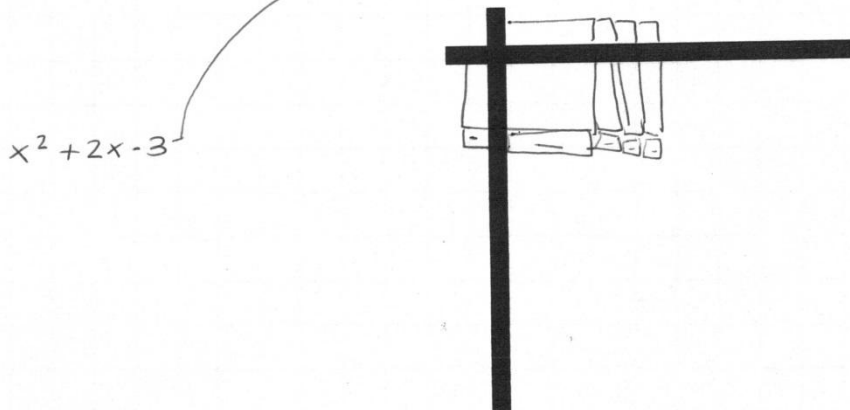
Sample task from Lesson 2 of the instructional unit completed by Marcia.

The offices in Jackson's current building are all square shaped. His office in the company's new building will be 3 feet longer in length and 1 foot shorter in width.

Fill in the table to represent the current and new dimensions and areas of offices in Jackson's building.

	Side of original office in feet	New length in feet	New width in feet	New Area in square feet
Jackson	6	$+3 = 9$	$-1 = 5$	45
Sue	7	10	6	60
Any square office	x	$x+3$	$x-1$	→

Use algebra blocks to determine the area of any employee's office in the new building. Sketch the blocks.



The instructional unit consisted of an introductory lesson on the use of the ALG and nine lessons targeting the algebra content of multiplying linear expressions and factoring quadratics. Each lesson contained a teacher-facilitated task which required students to engage in discourse that demonstrated their thought processes. The first four lessons focused on multiplying linear expressions embedded in an area context, while lessons 5 through 9 focused on factoring quadratic expressions embedded in an area context. The total intervention consisted of thirteen 45-minute sessions. Additionally, participants completed a series of researcher-developed pretests and posttests, as well as a transfer test immedi-

ately following the posttests and a maintenance test four weeks after intervention. Marcia's performance on these assessments demonstrated significant growth, as her average pretest score was 1% accuracy while her average posttest score was 94% accuracy. Additionally, Marcia scored 100% accuracy on the transfer measure and on the maintenance assessment, which was administered six weeks after intervention. See Strickland and Maccini (in press) for quantitative data for all participants.

#### Data Collection

Qualitative data were collected through: (a) transcriptions of video recorded sessions; (b) work samples; (c) investigator field notes of direction observations (Creswell,

2007). All instructional sessions were video recorded. After viewing all recordings, segments that describe the participants' cognitive processes were transcribed. Video recordings were transcribed to document: (a) participants' spoken words verbatim; and (b) participants' behaviors (i.e., manipulation of algebra blocks). Furthermore, work samples were collected from Marcia for analysis. In addition, the investigator wrote field notes after each section to address Marcia's progress and participation during the intervention sessions.

### **Data Analysis**

Data analysis methodology was based on Creswell's (2007) data analysis procedure. Specifically, the researcher progressed through four stages of data analysis: (a) data managing; (b) reading and memoing; (c) describing, classifying, and interpreting the data; and (d) representing the data. To manage the data, relevant sections of all instructional sessions were transcribed verbatim. Next, transcripts of Marcia's group were read and re-read while making notes (i.e., memoing), which reflected initial analysis and possible codes and/or themes. Throughout this stage, the researcher continually triangulated (i.e., cross-checked) memos with field notes and with Marcia's work samples. In the describing, classifying, and interpreting phase, possible codes were developed based on the memos. Specifically, codes focused on the multiple representations (i.e., ALG and Box Method) included in the intervention and the impact of the intervention on metacognition (i.e., self-regulation and strategic planning). Reliability, or dependability, of codes was established through confirmation from a second coder (Creswell & Clark, 2011). Based on discussions with the second coder and the support from transcripts, field notes, and Marcia's work samples, data were organized into codes. Through interpretation of codes, themes regarding the use of the manipulative emerged.

### **Data Validation**

In qualitative research, validation refers to the attempt to assess the accuracy of the findings as described by the researcher and the participant (Creswell, 2007; Creswell & Clark, 2011). The current study utilized three validation strategies based on Creswell's procedures of validation. Specifically, through triangulation, evidence of themes was also found in the transcripts from field notes and Marcia's work samples. Additionally, throughout the data analysis process, the researcher continually engaged in peer debriefing sessions with an expert in the field of mathematics special education. Lastly, an external auditor examined both the process and the product of the account to assess for accuracy. The external auditor had no connections to the study, but had experience with mixed methods research designs.

### **Results**

The following sections describe a major theme that emerged from the data analysis revolving around the use of the manipulative program, Algebra Lab Gear (ALG). Specifically, ALG supported Marcia's metacognition development via strategic planning and self-regulation.

#### **Metacognition**

Marcia demonstrated metacognitive development in strategic planning and self-regulation as she progressed through the intervention. Several interpretations emerged from each category as described below.

**Strategic Planning.** Strategic planning refers to developing a plan to engage in a task and executing the plan to successfully complete the task. Development and execution of plans of action occurred simultaneously and therefore are described concurrently below in the order in which the tasks occurred within the instructional unit.

Marcia's scores on her pretests were extremely low (0% - 4%), partially because she was unable to develop a plan to engage

in the tasks. When presented with a word problem and table of data, she wrote on her pretest, "I think if it was broken down I would be able to do it. The problem is that there are a lot of words and a lot of steps and once I understand what to do with one part I forget the other – I guess I'm not good at blending the steps." Additionally, when asked to transform a quadratic expression from standard form to factored form, Marcia wrote, "as I said this kind of stuff turns me off BUT I think that parts of it I really might know so again if it was explained and broken down I think there may be some hope." On an additional pretest domain probe, Marcia also wrote "I can't break it down." However, during the intervention, she stated that the ALG helped her to break down the tasks and develop of a plan of action that she executed to successfully complete the tasks. Examples of this process are described below. The ALG served as a tool for "breaking up" the procedure of multiplying linear expressions, as Marcia described below while multiplying  $(x + 3)(x + 2)$ .

I'm writing out my problem over here (*pointing to the manipulatives*). I have an equation and I am breaking it up and multiplying because this is a multiplying bar (*pointing to the corner piece*) and this is  $x$  and so I have  $x$  plus 3 so  $x$  plus 3 times, and this is timesing it,  $x$  plus 2 and that's going to equal  $x$  squared. So now its  $x$  plus 3 times  $x$  plus 2 equals (*manipulating the blocks*)  $x$  squared plus  $5x$  plus 6.

Marcia used the Lab Gear to both develop and execute a plan for multiplying linear expressions. First she represented her dimensions (i.e., linear

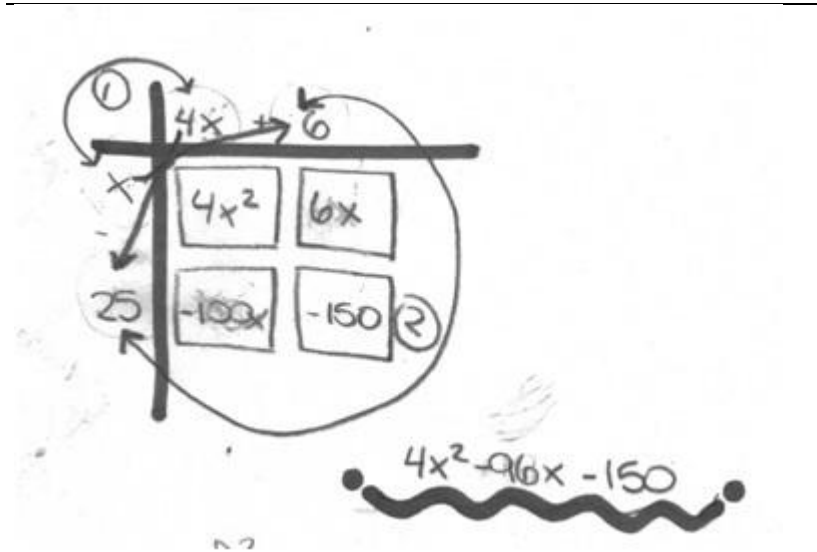
expressions) using the manipulatives and placed them on the outside of the corner piece. Then she filled in the corner piece with the appropriate manipulatives to form the required rectangle to correctly determine the area (i.e., quadratic expression). Marcia was pleased with her ability to multiply linear expressions using the ALG and therefore resisted giving up the manipulatives. When told that we were moving away from the blocks to use only abstract symbols, Marcia replied:

It's so much more hard because it's not broken up then. Like what I do is I see this (*pointing to  $x$ -bar*) and this (*pointing constant blocks*) and I read it and I write it then I move it. And then it's all broken up and I see the whole problem happening. But when it's all numbers then I forgot where to break it up and what's what.

Although resistant to giving up the manipulatives, Marcia developed a graphic organizer (i.e., the Box) that was closely linked to the ALG representation which further assisted with strategic planning. Figure 2 illustrates the connection Marcia established between the ALG and her graphic organizer (see Figure 2).

Figure 2.

*Marcia's graphic organizer for multiplying linear expressions*



The arrows provide additional insight into her strategic planning development that she successfully executed to find the product of  $(x - 25)(4x + 6)$ .

Strategic planning was also evident when presented with the task of multiplying  $(-13 + 2x)(10 + x)$ :

Marcia: Can I do the numbers after the  $x$ 's?

TS: Show me what you mean.

Marcia: Can I do  $2x - 13$ ? The  $x$ 's are always in this box (*pointing to the top left box of her graphic organizer*).

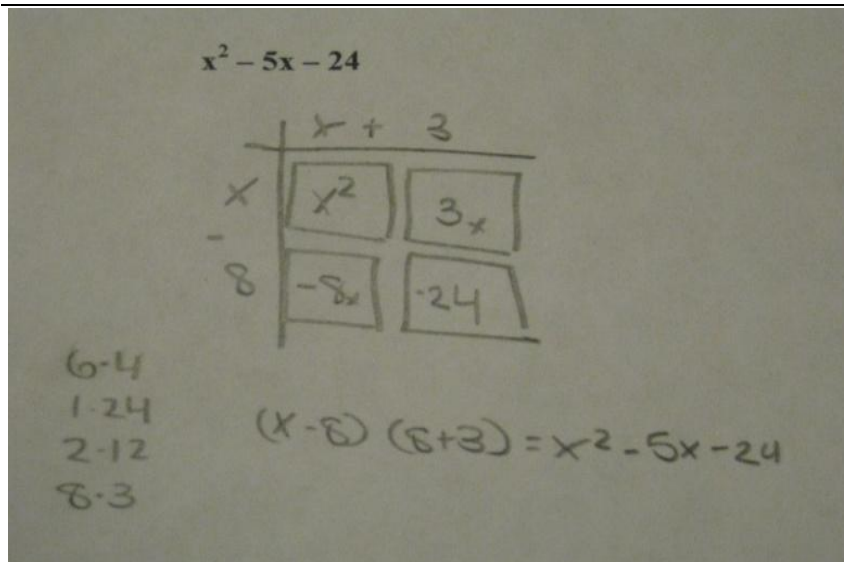
Marcia was able to develop her own plan of action and switch the order of the terms so that the terms with the variables were always in the position of the manipulative

representation. She then was able to successfully complete the task.

When factoring quadratic expressions, the ALG also supported Marcia's plan of action, which she said was to "go backward." She was able to arrange the blocks into a rectangle inside the corner piece and visualize, or as Marcia stated "see" the dimensions of this area. When transitioning to the abstract notation, she again used her Box method to develop her plan of action. She always placed the quadratic term in the top left of her organizer and the constant in the bottom right square. She then wrote out all of the factors of the constant to find a pair that equaled the coefficient of the linear term (see Figure 3).

Figure 3.

*Marcia's graphic organizer for factoring a quadratic expression.*



The ALG and the Box Method also served as valuable tools when Marcia completed her Transfer measure. Despite being presented with tasks that differed from those in the instructional unit, Marcia developed a plan of action and successfully solved each task using the manipulatives and

the Box Method representations. In the first transfer task, Marcia relied on the Box Method for planning and executing her solution strategy by completing a table of data for determining the perimeter and volume for specified numbers and for a generalized statement (see Figure 4).



Figure 4.  
Marcia's Transfer task 1

**Directions: Complete the following tasks. Show all of your work.**

All of the houses in the Sun Shine Valley neighborhood have rectangular swimming pools in which the length is 4 feet longer than the width and the height that is 3 feet longer than the width. Fill in the table below to determine the perimeter and the volume of the pools.

Remember:  
 $length \cdot width \cdot height = volume$   
 $2(length) + 2(width) = Perimeter$

	Length in feet	Width in feet	Height in feet	Volume in cubic feet	Perimeter in feet
Berger's Pool	14	10	13	1820	48
Joffe's Pool	16	12	15	2880	56
Any pool in Sun Shine Valley	$x+4$	$x$	$x+3$	$(x+4)(x)(x+3)$	$4x+8$

Figure 5.  
Marcia's Transfer task 2.

For the second transfer task, Marcia also used the Box Method to develop and implement a plan for multiplying a trinomial by a four-term polynomial (see Figure 5).

At first, she sketched the  $3x^2$  inside the corner piece, as evidenced by the sketch in the upper left. Marcia realized that this was a multiplication problem so the polynomials must be on the outside of the corner piece, which lead to the bottom representation. After distributing all of the terms in that

**Multiply the polynomial expressions and explain your strategy using words, pictures symbols.**

$$(3x^2 + 2x + 7)(4x^4 - 2x^3 + 3x + 2) = 12x^6 + 2x^5 + 24x^4 - 5x^3 + 12x^2 + 2$$

sketch, Marcia was confused about how to combine terms. She then drew her representation on the top right and she recognized that she combined terms that were diagonal (i.e., the  $x$ -terms). Marcia used that process of looking at diagonal terms to begin the process of simplifying like terms.

When completing the third task on the transfer measure, Marcia initially attempted to use the Box Method to factor a quadratic

expression with a coefficient of 3. She chose to use the template graphic organizer, rather than her unique form. When Marcia realized that “having the 3 doesn’t let us just add anymore” she abandoned the Box Method and instead sketched the ALG to successfully find the dimensions (see Figure 6).

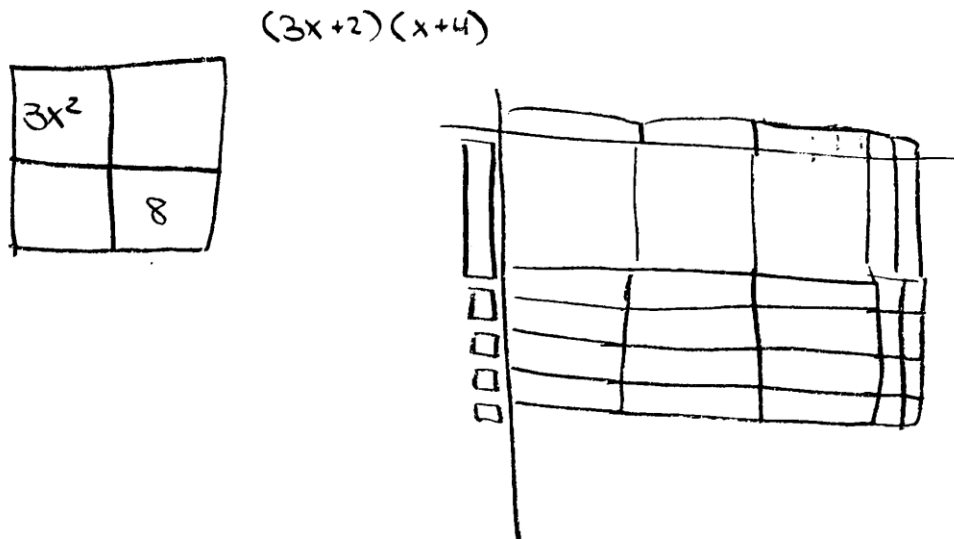
Figure 6.

*Marcia’s Transfer task 3*

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**three. Complete the following area equation:  $area = length \cdot width$ . Explain your strategy using words, pictures, or symbols.**

$3x^2 + 14x + 8 =$  \_\_\_\_\_



This exemplified Marcia’s ability give up a faulty plan and develop and execute an appropriate revised plan of action.

Initially, Marcia was unable to develop a plan to complete tasks on the pretest domain probes stating “I don’t know how to break it down.” Throughout the instructional unit, she used the ALG and the Box Method as tools for strategic planning. These tools provided Marcia with the means for “breaking down” the tasks on the posttest domain probes, which she stated

that she needed. Additionally, she used the ALG and her Box to successfully complete tasks on the transfer test.

**Self-regulation.** Self-regulation refers to monitoring and evaluating one’s performance during a problem solving task (Fuchs & Fuchs, 2007). Marcia displayed self-regulation behaviors as she routinely checked the accuracy of her work and revised as necessary and monitored her performance using the ALG. These two

themes are discussed below in the order in which they occurred in the intervention.

**Evaluating solutions.** Marcia often made faulty evaluations of the accuracy of her solutions. For example, she made frequent comments such as “I’m not good at that” and “I don’t know if I am doing this right,” yet Marcia often had an accurate solution and was able to justify her answer. For example, during Lesson 4 Marcia was transitioning from the ALG to using abstract symbols only with the Box Method.

TS: Marcia, what do we have to do to find the area of something?

Marcia: Multiply. So  $x$  times  $x$  is  $x$  squared.

TS: Well, do it down here using the box.

Marcia: Oh, the parenthesis. Oh you do the inside outside. This is supposed to be  $x - 3$  times  $x$ . I don’t get this.

*Marcia accurately completes the Box.*

Marcia: This is all wrong (*handing me her paper*)

TS: This is all right!

*Marcia looks at me disbelieving.*

TS: I’m serious.

Marcia: No way!

She often needed confirmation from me before she would acknowledge that she successfully completed a task. I regularly encouraged Marcia to rely on the tools more than me; however, she was resistant and accused me of not helping her. Marcia was often surprised by her success as exemplified in the above transcript.

**Monitoring performance.** Throughout the intervention, Marcia consistently monitored the accuracy of her solutions by using the ALG or a representation of the ALG (i.e., the Box Method). For example, Marcia used the visual cues embedded in the manipulatives to determine if she correctly multiplied binomials (e.g., blocks must form a perfect

rectangle) and referred to this process as “making a picture.” Additionally, Marcia frequently returned to the ALG for verification of solutions to tasks involving abstract notation. For example, when using the Box Method to multiply  $(3x + 15)(x - 2)$ , Marcia confirmed that  $3x$  times  $x$  equaled  $3x^2$  by setting up the ALG. Additionally, she wanted to explore other examples of multiplying algebraic terms with coefficients other than one by using the ALG.

Marcia: I have a question.

TS: Yes

Marcia: So if I add more here (*she places two  $x$ -bars on each side of corner piece*) I would multiply and get  $4x^2$ ?

TS: Yes, that’s exactly right. You got it.

Marcia: ok (*pushing away the blocks*)

In this situation, Marcia reverted to using the manipulatives to confirm the process for multiplying linear expressions with coefficients other than one. After determining that her responses were correct, Marcia returned to working in symbolic notation.

Marcia frequently moved back and forth between the ALG and the abstract notation when monitoring the accuracy of her solutions. For example, when multiplying  $(x + 3)(x + 5)$  using only abstract symbolism, she first responded  $x^2 + 15$ . When asked to explain her response using the blocks, she realized her solution was incorrect and revised her solution to  $x^2 + 8x + 15$ . Therefore, the ALG also provided Marcia with a way to check the accuracy of her work and to revise incorrect solutions.

When factoring quadratic expressions, Marcia monitored her solution by analyzing visual cues in the ALG. She relied on visual cues from the ALG or sketches of ALG. Marcia stated that she, “made a rectangle and then fit blocks up top and to the side” of the corner piece to factor a quadratic expression. Although “making a rectangle”

did not link to algebraic reasoning, she later used the Distributive Property to check her factoring when using the ALG, which also transferred to the Box Method. After using the Box Method to factor  $x^2 - 4x - 5$ , she checked her work by using the Distributive Property.

TS: Explain how you got this? ( $x^2 - 4x - 5 = (x - 5)(x + 1)$ )

Marcia: It checked out. X times x is x squared. X times one is one x. Negative 5 times x is negative 5x. Negative 5 times positive 1 is negative 5.

Throughout this explanation, Marcia pointed to the squares within the Box template. She demonstrated that multiplying the binomials was an appropriate method for checking her factoring. This explanation from Marcia demonstrates her ability to make a connection between the representations of the ALG and the Box Method to the importance mathematical concept of the Distributive Property. Throughout the intervention, Marcia made additional connections between the instructional practices and the algebra content, which are described in the following sections.

## Discussion

The purpose of this study was to explore the impact of the Algebra Lab Gear (ALG) manipulatives on the metacognitive development of one critical case. The analysis provided descriptive data to hypothesize why the CRA-I Strategy that incorporated the use of the ALG manipulative program produced positive achievement outcomes for the participants in the study by Strickland and Maccini (in review). Marcia was identified as the critical case who was representative of the group, as she provided a rich data source. Although a causal relationship between the manipulatives and the participants' achievement cannot be established, this study elucidates potential benefits of using a manipulative program at the high school

level. The results point to favorable findings for the use of manipulatives with high school students with LD.

First, the manipulatives supported metacognition in regard to strategic planning. Marcia indicated on her pretest domain probes that she did not know how to break up the task into steps that would enable her to reach a solution. On three of the four pretest domain probes, she did not attempt to solve any of the tasks. This is typical behavior of students with LD as they are characteristically passive in their learning and do not actively attack a problem (Gagnon & Maccini, 2001; Hudson & Miller, 2006). An explanation for this may be that students with LD have procedural and working memory deficits (Geary, 2004) which interfere with strategic planning.

During the intervention, Marcia stated that the ALG helped her break down the tasks and develop a plan of action that she executed to successfully complete the tasks. Additionally, she was able to incorporate her knowledge from the instructional unit to develop and execute strategic plans for solving tasks on the transfer measure. This is an important finding as students with LD (Bley & Thornton, 2001; Fuchs & Fuchs, 2007) and without LD (Greeno, Collins, & Resnick, 1996) typically struggle to transfer learned material to novel situations. However, Marcia used multiple ways of expressing the algebraic content (i.e., sketches of ALG and the Box Method) which supported her strategic planning (Center for Applied Special Technology, 2008).

Second, Marcia demonstrated self-regulation when monitoring her performance on tasks and when evaluating her solutions. Specifically, she relied on visual cues from the ALG to help monitor her performance on tasks involving multiplication of linear expressions. After transitioning to using only abstract symbols in the Box Method,

she frequently returned to the blocks to verify the answer from the Box Method. The integration of the concrete and abstract representation is recommended in the mathematics literature (Pashler, et. al., 2007), although previous research has shown that a graduated approach from the concrete, semi-concrete, to abstract representations is also beneficial (Witzel, et al., 2003; Scheuermann, et. al., 2009). Additionally, Gersten and colleagues (2009) recommend that use of manipulatives with older students should be expeditious as the goal should be fluidity in abstract symbolism. Therefore, there are benefits to both the graduated and the integrated approach to CRA instruction and the determination of which approach to use should depend on the characteristics of the students and the mathematics topic.

Another component of self-monitoring involved the evaluation of accuracy of one's solutions. Marcia often made faulty evaluations of the accuracy of her performance and would often say, "This is all wrong" and yet she would have an accurate solution and be able to justify her answer. This is consistent with previous research which reported that students with mathematics LD were less accurate than their non-disabled peers when evaluating the accuracy of their solutions (Mazzocco, 2007).

### **Limitations and Future Research**

A possible limitation of the qualitative method involved the analysis of only Marcia's data. Case studies typically include more than one participant (Creswell, 2007). Marcia provided a rich source of data which the authors feel was representative of the group of participants. However, each participant experienced the intervention in her own way, thus themes that emerged from analyzing Marcia may not be generalizable to all participants. Future qualitative research should include a larger sample so that common themes among participants may emerge.

### **Implications for Practice**

It is critical to bear in mind that the manipulatives used in this study were part of the CRA-I strategy. Initially, Marcia explored algebra tasks involving multiplication of linear and expression and factoring of quadratic expressions by simultaneously using the ALG, sketching the ALG representation, and writing the abstract notation in terms of the area formula. As the intervention progressed, Marcia transitioned to using abstract notation only; however, this was supported by the graphic organizer (i.e., the Box Method) which was visually linked to the manipulatives. Although initially reluctant to give up the ALG, she eventually demonstrated proficiency of the algebra content using abstract notation only. This is the goal when using manipulatives; however, students with LD often have difficulties transitioning to abstract notation only (Hudson & Miller, 2006). Therefore, the use of additional tools, such as graphic organizers, may be necessary to support students with LD as they transition to abstract notation.

Additionally, the ALG was utilized as a tool for exploring the algebra content. For example, Marcia discovered the rules for factoring quadratics through exploring the changes that occur in the ALG representation when changing the constant and linear coefficients of given quadratics expressions. The teacher-researcher acted as a facilitator during these activities and minimalized direct instruction. Having students mimic the teacher's use of manipulatives is an ineffective use of manipulatives because students may mindlessly move the blocks around without making connections to the mathematics content (Van de Walle, et al., 2010).

### **Conclusion**

The use of manipulatives is a promising instructional practice for students with

LD as it addresses various areas of deficit. For example, manipulatives provide students with a referent to the abstract symbolism of mathematics (Reys, Suydam, & Lindquist, 1992). However, the current research supports the use of manipulatives within the instructional practice of the CRA sequence or the CRA-I strategy. Through CRA instruction, manipulatives develop conceptual knowledge (Hudson & Miller, 2006) and provide a bridge to the development of abstract ideas (Reys, et al., 1992). Additionally, manipulatives provide students with opportunities for active engagement as they explore mathematic relationships (Gurganus, 2007). Further, the use of manipulatives has been found to support retention of mathematical ideas (Reys, et al., 1992).

### References

- Bley, N.S., & Thornton, C. A. (2001). *Teaching mathematics to students with learning Disabilities* (4<sup>th</sup> ed.). Austin, TX: PRO-ED, Inc.
- Center for Applied Special Technology (2008). *Universal design for learning guidelines version 1.0*. Wakefield, MA: Author.
- Creswell, J.W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2<sup>nd</sup> ed.). Thousand Oaks, CA: Sage Publications.
- Creswell, J.W., & Clark, V.L.P. (2011). *Designing and conducting mixed methods research* (2<sup>nd</sup> ed.). Thousand Oaks, CA: Sage Publications.
- Fuchs, L.S., & Fuchs, D. (2007). Mathematical problem solving: Instructional intervention. In D.B. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children?* (pp.397-414). Baltimore, MD: Paul H. Brookes Publishing, Co.
- Gagnon, J.C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children*, 34(1), 8-15.
- Garnett, K. (1998). Math learning disabilities. *Division for Learning Disabilities Journal of CEC*. Retrieved from <http://www.ldonline.org/article/5896/>.
- Geary, D.C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37(1), 4-15.
- Gersten, R., Chard, D.J., Jayanthi, M., Baker, S.K., Morphy, P., Flojo, J. (2009). *A meta-analysis of mathematical instructional interventions for students with learning disabilities: Technical report*. Los Alamitos, CA: Instructional Research Group.
- Greeno, J. G., Collins, A. M., & Resnick, L. (1996). Cognition and learning. In D. C. Berliner & R. C. Calfee (Eds.) *Handbook of Educational Psychology* (pp. 15-46). New York: Macmillan.
- Gurganus, S.P. (2007). *Math instruction for students with learning problems*. Boston: Pearson Education, Inc.
- Hudson, P. & Miller, S.P. (2006). *Designing and implementing mathematics instruction for students with diverse learning needs*. Boston: Pearson Education, Inc.
- Kortering, L.J., deBettencourt, L.U., & Braziel, P. M. (2005). Improving performance in high school algebra: What students with learning disabilities are saying. *Learning Disability Quarterly*, 28, 191-203.
- Maccini, P., Strickland, T., Gagnon, J.C., & Malgren, K. (2008). Accessing the general education math curriculum for secondary students with high-incidence disabilities. *Focus on Exceptional Children*, 40(8), 1-32.
- Mazzocco, M.M.M.(2007). Defining and differentiating mathematical learning disabilities and difficulties. In D.B. Berch & M.M.M. Mazzocco (Eds.), *Why*

- is math so hard for some children?* (pp. 29-48). Baltimore, MD: Paul H. Brookes Publishing, Co.
- Montague, M. (2007). Self-regulation and mathematics instruction. *Learning Disabilities Research and Practice*, 22(1), 75-83.
- Montague, M. (2008). Self-regulation strategies to improve mathematical problem solving for students with learning disabilities. *Learning Disabilities Quarterly*, 31, 37-44.
- National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards Mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
- Newman, S. (2006). *General education participation and academic performance of students with learning disabilities* (NCESR 2006-3001). National Center for Special Education Research, Institutes of Education Sciences, U.S. Department of Education, Washington, DC.
- Pashler, H., Bain, P.M., Bottge, B., Graesser, A., Koedinger, K., McDaniel, M., & Metcalfe, J. (2007). *Organizing instruction and study to improve student learning*. U.S. Department of Education: Washington, DC.
- Picciotto, H. (1995). *The algebra lab: ALG activities for algebra I*. Mountain View, CA: Creative Publications.
- Punchner, L., Taylor, A., O'Donnel, B., & Flick, K., (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.
- Reys, R.E., Suydam, M.N., & Lindquist, M.M. (1992). *Helping children learn mathematics* (3<sup>rd</sup> ed.). Needham Heights, MA: Allyn and Bacon.
- Scheuermann, A.M., Deshler, D.D., & Schumaker, J.B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, 32, 103-120.
- Swan, P. & Marshall, L. (2010). Revisiting mathematics manipulative materials. *Australian Primary Mathematics Classroom*, 15(9), 13-19.
- Strickland, T.K., & Maccini, P. (2013). The effects of the concrete-representational-abstract-integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education*, 34(3), 142-153.
- Strickland, T.K., & Maccini, P. (in press). Exploration of quadratic expressions through multiple representations for students with learning disabilities. *Learning Disabilities: A Multidisciplinary Journal*.
- Van de Walle, J.A., Karp, K., & Bay-Williams, J.M. (2010). *Elementary and middle school mathematics*. Boston: Allyn & Bacon.
- Wagner, M., Marder, C., Blackorby, J., Cameto, R., Newman, L., Levine, P., Davies-Mercer, E., Chorost, M., Garza, N., Guzman, A.M., & Sumi, C. (2003). *The achievements of youth with disabilities during secondary school: A*

*report from the National Longitudinal Transition Study-2 (NLTS2)*. Menlo Park, CA: SRI International.

- Wilson, M.G. (2008). *Math course taking and achievement among secondary students with disabilities: Exploring the gap in achievement between students with and without disabilities*. Unpublished doctoral dissertation, University of Maryland, College Park.
- Witzel, B. S. Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students

with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice, 18*, 121-131.

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**Author's Note**

This study was funded by a U.S. Department of Education Preparation of Secondary and Transition Leadership Personnel Grant, H325D080058.