ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

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To cite this article:

Ozaltun Celik, A. & Bukova Guzel, E. (2017). Revealing Ozgur's thoughts of a quadratic function with a clinical interview: Concepts and their underlying reasons. *International Journal of Research in Education and Science (IJRES), 3*(1), 122-134.

International Journal of Research in Education and Science
Volume 3, Issue 1, Winter 2017 ISSN: 2148-9955

Revealing Ozgur's Thoughts of a Quadratic Function with a Clinical Interview: Concepts and Their Underlying Reasons

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Article Info

Article History

Received:
24 November 2016

Accepted:
09 December 2016

Keywords

Clinical interview
Conceptual understanding
Conceptual analysis
High school student
Quadratic function

Abstract

The quadratic function is an important concept for calculus but the students at high school have many difficulties related to this concept. It is important that the teaching of the quadratic function is realized considering the students' thinking. In this context, the aim of this study conducted through a qualitative case study is to reveal the concepts in a student's mind and their underlying reasons while he was analyzing a quadratic function and drawing its graph. While collecting the data, a clinical interview was conducted with Ozgur studying at Grade 11 and the interview was recorded with a camera. Then, the video camera records were verbatim transcribed for revealing Ozgur's thoughts related to the quadratic functions. At the beginning of the clinical interview, he tried to remember his existing knowledge but he could not be successful. He expressed that the given quadratic function was a linear function. Also, he did not relate the algebraic representation to graphical representation although he thought that its graph was a parabola and the parabola was opening upwards or downwards. As the interview progressed, he began to reason the concept and its properties. With the researcher's questions during interview, he noticed his misunderstanding. He understood the difference between the linear function and the quadratic function. Also he inferred the axis of symmetry and its importance for the quadratic function. Reflection of these thoughts in the teaching of the quadratic functions and the tasks related to the concept is important for conceptual learning.

Introduction

Understanding functions that requires relational thinking supports higher level mathematical thinking and reasoning. Dreyfuss (1991) stated that understanding higher-level mathematical concepts is not possible without learning the concept of function. Conceptual understanding of function is based on having the ideas, covariational reasoning and mapping. Similarly, Wilkie (2016) emphasized that change and variation of quantities were important notion for functional thinking. Thompson and Carlson (2016) expressed that students should be gained covariational and quantitative reasoning in their mathematics lessons because these reasoning types are necessary for their real lives and advanced mathematical understanding. While learning functions, the students firstly have experiences with the linear functions and then quadratic functions. Quadratic functions play a critical role in transition from linear functions to higher-degree functions (Movshovitzs-Hadar, 1993). Quadratic functions are one of the most important concepts going beyond linear functions but the students have difficulties for understanding the quadratic functions (Ellis & Grinstead, 2008). During the teaching quadratic functions, not relating the quadratic functions with the linear functions which are considered as prior concepts may lead to have these difficulties for students. A number of studies (Duarte, 2010; Eraslan, 2005; Kutluca, 2009; Sevim, 2011; Stokes Parent, 2015; Zaslavsky, 1997; Zazkis, Liljedahl, & Gadowsky, 2003) were conducted about students' understanding of quadratic functions and strengths they have in this process. Some of these studies emphasized the differences between the rate of change of a linear function and the rate of change of a quadratic function and their relations. Additionally, some of them provide an insight what should be done or not done during the teaching these function. However, we realized the necessity of focusing the question of how to teach the quadratic functions by considering students' thinking and difficulties. Also, we noticed that one of the important reasons of these difficulties was teaching the quantities related to the quadratic functions as an isolated structure. Thus, we determined that there was a gap related to designing the teaching process by relating the all quantities about the quadratic functions. Besides, Oehrtman, Carlson and Thompson (2008) recommended that teaching quadratic functions should include a greater focus on understanding covariation and develop the idea of covariation by using multiple representations.

Zaslavsky (1997) who is one of the researchers revealing that the students encountered several difficulties while learning the quadratic functions examined students' understanding and difficulties of quadratic functions asserted that the students have difficulties about graphical representations. For instance, "the graph of the quadratic function may seem as if it is limited only to the visible part that is actually drawn, although in fact it represents an infinite domain." (p. 30) and "students use "eye measurement" to determine whether certain points were or were not on the graph." (p. 30). As another difficulty about the graph of functions, Oehrtman, Carlson and Thompson (2008) stated that students generally view the graph of function as a curve or fixed object but not as a general mapping of a set of input values to a set of output values. Additionally, Zaslavsky (1997) expressed that the students have difficulties about the relations between a quadratic function and a quadratic equation, conversions different algebraic representations of quadratic functions and characteristics of these representations. Similarly, Carlson (1998) stated that students had difficulties in determining the relation between an algebraic representation of a function and an equation. Karim (2009) pointed out that students especially have difficulties while interpreting several quantities related to the quadratic functions such as extreme points, the leading coefficient and vertex, etc.

Knowing about the underlying reasons of these difficulties as well as focusing on the difficulties and how the students' thinking and reasoning progress is during the learning process are important for arranging the learning environment, tasks and interactions designed for students in teaching of the quadratic functions. The adopted learning and teaching theory is deterministic preventing students' possible difficulties and having them learn the concept of quadratic functions effectively. Arranging and improving learning and teaching processes by considering the students' approaches is effective for progressing their mathematical understanding. Piaget (1980) made emphasis on reflective abstraction as a strong explanation of conceptual learning and assumed reflective abstraction as the process by which new and more advance concepts emerge from existing concepts (Simon, Tzur, Heinz, & Kinzel, 2004). In this context, it is come to light that the tasks which emphasize necessity of students' thoughts and actions and provide that the students construct new concepts on their existing knowledge through reflective abstraction are essential. It is important for these tasks to be prepared in a way to allow for the students to do successive abstractions and to trigger their thoughts. For designing these tasks which Simon (2006) emphasized as logical-mathematical activities, hypothetical learning process including improvement of students' thinking should be determined. The National Research Council (2007) drew attention to learning trajectory by showing the researches pointing out the students' thinking ways that follow and depend on one another about a subject as the students learn the subject in a long time and work on the subject. Learning trajectories is based on Simon's (1995) definition on "hypothetical learning trajectory". Simon (1995) stated that hypothetical learning trajectory is made up three components as the learning goal, the learning activities and hypothetical learning process which refers how the students' thinking and understanding will improve in the context of the learning activities. The hypothetical learning trajectory is a cognitive tool based on constructivism (Clements & Sarama, 2004). Confrey, Maloney, Nguyen, Mojica and Myers (2009) defined learning trajectory as "a researcher conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time. Maloney and Confrey (2010) asserted that learning trajectories represent progression of cognitive in a linear or casual way.

To determine the hypothetical learning trajectory, firstly, it is necessary that students' thinking should be noticed. Also, it is important to reveal the students' prior knowledge, incorrect or incomplete understanding affecting their learning for determining the learning trajectory. In this context the previous studies about quadratic functions in the literature is a guide. Besides, questioning the students continuously for understanding their thinking related to quadratic functions and their explanations in this process will be guide. In addition to all these, the nature of the quadratic function is a major effective factor for students' learning. It should be known which concepts are connected with the quadratic function, what pre-necessities are, what properties lead to misunderstanding. By taking into account all of these, determining students' hypothetical learning trajectories supports considering students' cognitive processes in teaching the concept. Assuming that the learning is a cognitive action, conducting the teaching by considering the hypothetical learning process is important for conceptual understanding and constructing the concepts properly. Otherwise, the teaching is realized independent from the students and may not support for the students to learn conceptually.

In this context, examining what the students think, how they think and what difficulties they have will be important for determining hypothetical learning process about this concept. In this direction, before we started the teaching experiment for teaching quadratic functions, we realized several studies to determine hypothetical learning process and the tasks for them. In this study, one of the studies done for these purposes, it was aimed to reveal the concepts in a student's mind and the underlying reasons of his ideas while he was analyzing a quadratic function and drawing its graph. Thus, it was revealed how a student, named Ozgur, use from his existing understanding, to which concepts he associates, reasons of these connections and progressions of his understanding while learning the quadratic function.

The research question of the study was as follow:
"What are Ozgur's ideas about the quadratic functions in his mind and what are the reasons underlying these thoughts?"

The Clinical Interview

The clinical interview is one of the alternative assessment methods students' thinking and to improve their mathematical learning (Hunting, 1997). To respond the research question, we decided to use the clinical interview method and thought that opening this interview by a question including an algebraic expression of a quadratic function would be proper. We assumed that we could understand Ozgur's thoughts deeply by asking him to analyze the function during the clinical interview. This question was a general starting point for the interview and then we aimed to examine the quantities in his mind by means of this general question. If we asked more specific question including only some quantities such as the axis of symmetric, the vertex or the parameter, etc. we could not have evaluated Ozgur's cognitive process regarding the quadratic function as a whole. Thus, we could reveal the all concepts and ideas that Ozgur related with the quadratic functions. Besides, we used this question to limit his focus, especially the quadratic functions and their quantities during the interview. Hunting (1997) emphasized that the realizing a clinical interview with a quite well defined question was necessary because the focus of the researchers were more specific in this process. While doing the clinical interview, both the determined questions in the planning stage and the spontaneously occurred questions in flow of the interview shape the interview and provide the researchers understand students' thinking with a broad perspective. To increase the quality of the questions, it is necessary to realize a conceptual analysis of the concepts and to examine them deeply. The conceptual analysis play a critical role in terms of realizing all the related concepts and exactly determining students' thinking. Thompson (2008) stated that the conceptual analysis was realized in four ways: building models of what students actually know and what they comprehend in specific situations; describing ways of knowing that might be propitious for students' mathematical learning; describing ways of knowing that might be problematic in specific situations to students' understanding; and analyzing the coherence, or fit, of various ways of understanding a body of ideas. In this direction, the conceptual analysis was a critical guide for implementing the interview and analyzing the student's thinking.

Method

We conducted the study through a qualitative case study to reveal the concepts in a high school student's mind and their underlying reasons while examining a quadratic function and drawing the graph of it. The case study design was important to answer our research question and give an opportunity for understanding student's thinking and the underlying reasons in a deep way. In this study, the cases were the student and his explanations. For this purpose, we asked the question of "What kind of function is $f(x) = x^2 + 2x$-3 and how do you draw its graph?'' and we used from the clinical interview method for understanding the student's thinking and reasoning.

Firstly, we realized the pre-studies by examining the concept of the functions and the quadratic functions. This conceptual analysis led us about how to implement the interview and which questions should be asked. Besides, we considered the possible students' responses as if we were students and we discussed these responses with each other and noted them. In the case of that the student would give different answers unlike our expectations, we determined what kind of questions we should ask him. As we thought that the physical environment was an important factor for effective interview and for the students to explain their thoughts comfortably, we arranged the interview room which had the student feel relax. We presented an interview protocol which explained our aim and all the process in the terms of both the student and us. Hunting (1997) stated that the choice or development of the task and the protocol regarding it, was of great importance.

The Participant

In this study, we conducted the clinical interview with Ozgur studying at Grade 11 and included in teaching of the quadratic functions about six months ago and scored high marks in the math exams. The reason of selecting

Ozgur was that he got involved in the teaching process of quadratic functions. It was important that the student had ideas about the concept to be able to use from the results of the interview for determining students' hypothetical learning process. We got some information from Ozgur's teacher about him. In the direction of these information, Ozgur scored high in the exams of the mathematics lesson. We thought that revealing what he thought related to quadratic functions, which concepts he related to this concept, which ideas in his mind predominated and what all these thoughts affected would be important contributions to the design study because Ozgur was seen as a successful student at average school level. From the preliminary examinations, we understood that Ozgur was a socially extraverted student and well in with his friends. So, we also assumed that Ozgur would explain his thinking without hesitation and he would respond to the questions in detail when we asked him to express his thinking.

Before the interview, we told Ozgur that we wanted to interview with him without explaining the content of interview. We gave the clinical interview protocol to Ozgur who voluntarily accepted to interview. In the context of the protocol, we emphasized that we did not aim to grade his responses but only we tried to understand his thinking and the reasons of thoughts. We expressed that he had enough time to think, he needed to not hesitate by thinking that his response would be incorrect and we could take a break in the interview when he wanted. We warmly mentioned our aim was to learn his thinking and not to judge him.

Data Collection

We conducted a clinical interview with Ozgur for revealing his thoughts related to the quadratic function. Piaget (1976) developed the clinical interview to understand the form and functioning of children's reasoning (Mojica, 2010). According to Piaget, the main role of the clinical interview is to reveal children's independent thinking, cognitive dispositions, intellectual habits and their schemas (Confrey, 2006). Piaget used the clinical interview method alternatively to standardized test which does not enable understanding the reasons of misconceptions related to a particular concept (Pablopulos & Carolina, 2015). Ginsburg (1981) emphasized three purposes in a clinical interview. The first is to explore the student's cognitive process, the second is to define essentials of these cognitive processes and the last is to determine student's competency. In a clinical interview, the focus can be a problem to be solved, an idea to be explained or something to be thought (diSessa, 2007). The student's responses to the questions related to the concept or the problem are examined in detail and his/her thinking and their reasons are questioned during the interview. The result of clinical interviews are transferred to learning environment by having better understanding regarding how the students comprehend particular concepts and what alternative approaches should be used for the students (Engelhardt, Corpuz, Ozimek, & Rebello, 2003). We realized the clinical interview to understand how Ozgur conceived the quadratic functions, which concepts he related to the quadratic functions and which alternative approaches he used while responding the question related to the quadratic functions. The clinical interview lasted about 75 minutes. While one of the researchers was conducting the interview, the other took the notes by observing the clinical interview. Also, we recorded the interview with a camera.

Data Analysis

Firstly, we transcribed the video camera records verbatim for revealing Ozgur's thoughts. We generally examined what he thought based on the field notes taken in the interview. Then, we read and evaluated the transcription in detail by comparing the field notes. While we were examining the student's thinking, we continuously considered the related concepts and the quantities which we revealed in the context of the conceptual analysis. As the conceptual analysis included both the building blocks of the concept and students' understanding and thinking, it mainly affected our analysis process. Additionally, examining the concept by considering this comprehensive structure and thus interpreting the student's expressions provided important ideas regarding students' learning process and their learning trajectories. So, even though we did the clinical interview with only one student, the findings based on the conceptual analysis which we realized by taking into consideration all students and the quadratic functions informed us about any student's learning ways on the quadratic functions beyond those of Ozgur. Analyzing the data by examining all the concepts regarding quadratic functions was a factor providing to enhance the reliability of the analyzing process. Besides, as both of two researchers independently analyzed the student's thinking in the context of the quantities, possible different interpretations were discussed and then had a consensus about the data. Thus, the validity of the analyzing was ensured. We examined what Ozgur had prior knowledge related to the quadratic function, how the existing knowledge was related, how these knowledge affected his thoughts about the concept, what kind of meanings he attributed to the critical concepts related to the quadratic function and we determined the important excerpts in the transcription. Examining the data, we especially focused on the central concepts about the quadratic functions. So, the quantities about the concept were deterministic points for analyzing Ozgur's actions. As a result, we revealed what thoughts he had about the vertex of the parabola, how he related the axis of symmetry with the algebraic expression of the function, how he draw the graph of the function, how he considered intersections of graph with axis, how he converted the algebraic representations to graphical representations and what meaning he attributed the coefficients of a, b, c.

Findings

When the researcher started the clinical interview, Ozgur tried to remember his prior knowledge instead of reasoning because the question was related to the function which he had taught at previous grade. Being anxious not be able to remember led him to not focus on the question and he hesitantly gave the responses. In the beginning, Ozgur who did not have such an experience earlier responded the questions with one-word answers or short sentences. Later in the interview, he began to think more instead of trying to remember his memorizing knowledge and what the teacher had taught. So, he became enthusiastic about explaining his thinking in a more detail way. Also, his willingness prompted him to do more proper connections. Then, he acquired new ideas related to the concepts by noticing the situations and connections which he had not thought before. It was supported that Ozgur thought some critical concepts and connected the related concepts by the interview even though it was started in the purpose of revealing Ozgur's existing understanding and thinking.

Although Ozgur had been seen as a successful student by his teacher and he scored high in the exams related to the quadratic functions about six month ago, it was seen that Ozgur had problems about conceptual understanding of the quadratic functions during the interview. This case provided us with chance that we could evaluate the results from different perspectives by enabling to examine what concepts the student correctly or incorrectly related with the quadratic functions. So, we could focused on which concepts we should handle prior to teaching the quadratic functions and which thoughts and ideas would be critical to prevent students' misunderstanding. Ozgur's all thoughts was questioned and this questioning prompted the researchers to understand Ozgur's thinking and also provided Ozgur awareness about his own thinking.

When the researcher asked Ozgur the question of "What kind of function is "$f(x) = x^2 + 2x - 3$ and how do you draw its graph?", he immediately said that the function was a linear function without thinking. The researcher asked what the linear function meant upon Ozgur's response and realized that he gave this answer because he only remembered the linear function from the types of functions. Then, the researcher asked several questions to understand what Ozgur knew about the linear functions. As Ozgur did not have the idea that the rate of change was constant for whatever two points in a linear function, he could not response the questions. In this process, it was revealed that Ozgur did not examined the quantities such as the rate of change, the slope of the graph, etc. regarding the linear functions and so he did not have conceptual learning.

After then, the researcher asked Ozgur to draw the graph of the given quadratic function. While drawing the graph, Ozgur firstly tried to find intersections of graph with axis. He found the point of (1,0) but he could not find the other point of intersection with x-axis because he assigned several values to x to find the roots of the equation of $x^2 + 2x - 3 = 0$. He also determined the point of (0,-3) by assigning 0 to x. Ozgur who determined two points tried to draw the graph by using these points but he could not. Ozgur knew that y–value was equal to 0 to find the intersection of graph with x-axis. He tried to find the roots of the equation by using trial-and-error method. He could not determine the negative root because he did not think negative real numbers. The reasons of his difficulties were that he did not remember how to find the roots of the quadratic equation and only thought the procedural process.

Ozgur who could not find all the intersections with axis drew parabolas by his hand and these parabolas opened downward. When the researcher asked questions to understand whether he accidentally drew these parabolas, she determined that Ozgur consciously tried these drawings and that the reason of drawing the parabolas opened downward was derived from the c-coefficient which value was -3. Ozgur was confused about meaning of the coefficient. He thought the effect of c-coefficient which represents the intersection point of the parabola with y-axis as the effect of a-coefficient which determines whether the parabola opens upward or downward. Although he confused these quantities, Ozgur stated that he should determine the vertex to draw the graph and the vertex was the maximum or minimum value of the function. Ozgur did not know the relation between the vertex of the parabola and the axis of symmetry which is an important quantity related to parabola and was not aware of that the graph is symmetrical. Although the researcher asked many questions regarding these thoughts, he could not respond. One of these questions was "If you had known the vertex as well as the points of (1,0) and (0,-3), could you have drawn the parabola? However, Ozgur gave the answer "If I find other intersection point of the parabola with x-axis, I can draw the graph." That is, even though he mentioned the vertex, he was unaware of the axis of symmetry which was an important quantity and passed through the vertex. Then, he again tried to remember his prior knowledge to find the intersections of the graph with x-axis and he expressed that he should use the discriminant (Δ) of the equation. As noted in the excerpt below, Ozgur remembered a relation between intersections of the parabola with x-axis and the discriminant but he made incorrect explanations about the discriminant. Ozgur did not know what the discriminant formula was and how it would be used to find the roots of a quadratic equation. When he tried to remember its formula, he expressed a different formula every time. Ozgur did not have a conceptual understanding about a quadratic equation, its degree, the existence of two roots, and the role of discriminant in finding the roots of a quadratic equation.

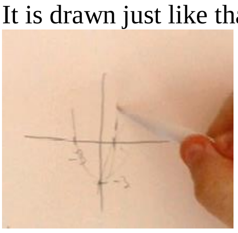
Researcher: Why do you need to find one more x-intercept of the graph to draw it?
Ozgur: There are three options for drawing. The graph does not touch x-axis or its vertex is on the x-axis or it has two x-intercepts. Δ tells us how we draw the graph.
Researcher: What do you mean by Δ?
Ozgur: Immmm.. Δ=a-4b.c or Δ=4a-b.c

ooo

Researcher: What is Δ?
Ozgur: I suppose -4a.b-c.

Ozgur could not find the other root of the equation $x^2 + 2x - 3=0$ by using Δ but he thought that the root could be negative and found "x=-3" by trial and error. So, he obtained three critical points of the function and drew the graph opening upward by using these points. Ozgur noticed that his idea regarding what direction the parabola opened was incorrect and expressed that the last drawing was correct. Ozgur asserted that the intersection point of the graph with y-axis was the vertex during the clinical interview for a while. Upon the researcher's questions, Ozgur was be confused and then noticed that the intersection point of the graph with y-axis did not have to be the vertex. Later on, he tried to remember the formula of the vertex, he could not show the vertex on the graph because he could not remember it. The researcher asked him to think what the vertex meant instead of trying to remember what formula was used. But Ozgur could not make any inference except that it represented the maximum or minimum value of the function. We realized that the quantity of the vertex was related to the maximum or minimum value of the quadratic function in Ozgur's mind.

Ozgur: It is drawn just like that.



Researcher: Which point is this? [by showing the point which he drew as vertex]
Ozgur: The vertex of the parabola

ooo

Researcher: Why?
Ozgur: It cannot be. There is a rule to determine it [he is trying to remember] and minimum point of the parabola.
Researcher: What does the minimum point mean?
Ozgur: That is, the value of function is minimum.
Researcher: Is there any characteristic of vertex?
Ozgur: Nothing else.

Ozgur drew the graph of the function by using the intersection points of the graph with axis and then he stated that he could not draw the graph because he could not find the vertex and he should find it. He had difficulties because of his thinking that the vertex could not be the point of (0, -3) and he expressed that he could not exactly draw the graph. The researcher asked Ozgur what kind of function y=x+1 was and whether he drew its graph. By this question, as the researcher thought to be able to reveal the reason of these difficulties and to support him to think effectively while drawing the graph of the linear function. Ozgur stated that the function of f(x)=x+1 was a linear function, thus, the researcher asked him rethink by comparing this function with the quadratic function of $f(x) = x^2 + 2x - 3$. Ozgur insisted on his thoughts that both functions were linear functions. He remarked that he could find the intersections of the graph with axis by assigning 0 to x and y in the equation while drawing the graph. The thing that Ozgur remembered from his prior knowledge was that a function was a transformation mapping between x and y. As it can be seen in the excerpt, Ozgur also stated that there was no difference between an equation and a function.

Researcher:	What is the difference between them [She showed the quadratic function and this linear function] Why do you think that both of them are linear functions? Are you sure about that?
Ozgur:	I think so.
Researcher:	Is this linear? [she showed $y = x + 1$]
Ozgur:	This is also linear [He showed $f(x) = x^2 + 2x - 3$]. This [$f(x) = x^2 + 2x - 3$] is a second degree equation. This [$y = x + 1$] is first-degree.
Researcher:	Is this [$f(x) = x^2 + 2x - 3$] an equation or a function?
Ozgur:	[He is thinking] Function.
Researcher:	Why is not an equation?
Ozgur:	It is both an equation and a function.
Researcher:	What is the difference between them?
Ozgur:	Nothing.
Researcher:	Is not there any difference?
Ozgur:	No.

Even though Ozgur used the terms of the second-degree equation and the first-degree equation, he did not change his idea that both function were linear functions. Researcher asked him “can you the graph of an equation?” and he gave answer “No, it cannot be drawn.” and also he expressed that an equation was constructed by assigning values to y in a function.

When the researcher asked him draw the graph of $\mathrm{f(x) = x + 1}$, Ozgur tried to draw curves using two intersection points by his hand after he determined these points. The researcher understood that Ozgur thought the linear function as a function of $\mathrm{f(x) = ax^2 + bx + c}$ in the way that a-coefficient equals to 0. When he assigned 0 to a-coefficient, he did not think to eliminate the term of $\mathrm{x^2}$ and he ignored the meaning of this quantity. He tried to justify her explanations without relating the quantities.

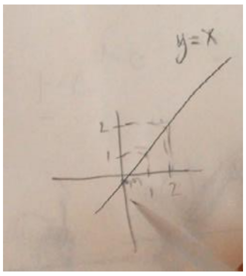
Ozgur:

As Ozgur had confusions while drawing the graph of $\mathrm{f(x) = x + 1}$, the researcher asked him to draw the graph of the function of f(x)=x. Ozgur determined the values of y by assigning the values to x. Thus, firstly he found that y was equal to 1 in case that x was equal to 1, then y was equal to 2 in case that x was equal to 2. However, he drew the graph by indicating that points of (1,0) and (0,1) instead of (1,1) were on the graph of the function and similarly that points of (2,0) and (0,2) instead of (2,2) were on the graph of the function. In doing so, he had problems in determining the points on the coordinate system and he thought that there were many graphs related to the function. He did not have the idea that each function has only one graph.

Ozgur:

The researcher asked him whether the points of (1,0) and (0,1) are on the graph of the f(x)=x and thus, Ozgur understood that these points could not be on the graph. He began to rethink systematically and to write the values in ordered pair and then he could draw the graph. After he drew the graph, he stated that this function was a linear function but the function of $f(x) = x^2 + 2x - 3$ was not a linear function.

Ozgur:



Researcher: What kind of graph is this?

Ozgur: How?

Researcher: If you look at this graph, what kind of graph?

Ozgur: A linear graph.

Researcher: Why is this linear?

Ozgur: It is like a line.

Researcher: Because it is like a line. But you said that this was a linear function. [She showed the function of $f(x) = x^2 + 2x - 3$]

Ozgur: In this case, it isn't.

Researcher: You said that this function [$f(x) = x^2 + 2x - 3$] was a linear function. What is the difference between these functions? Why is this function [$f(x) = x$] linear? Why isn't this function linear [$f(x) = x^2 + 2x - 3$]?

Ozgur: This isn't linear.

Researcher: Why?

Ozgur: This graph is not linear.

Researcher: What kind of function is it?

Ozgur: [He is thinking] This is like a line.

Researcher: Is this a linear function or not [She is showing $f(x) = x^2 + 2x - 3$]? Why?

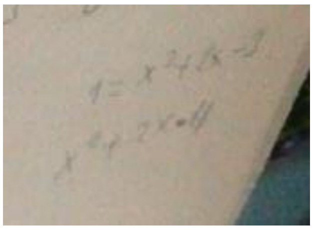
Ozgur: Because, this is a curve.

Researcher: You said that this was not linear.

Ozgur: Imm, yes.

Ozgur noticed that this difference was based on the degree of x. He began to use the term of the second-degree function for $f(x) = x^2 + 2x - 3$ because it included the term of x^2 and also began to use the term of the first-degree function for $f(x) = x$. Additionally, he said that he could draw the graph of the quadratic function by determining points. He assigned value of 1 to y and wrote the equation of $1 = x^2 + 2x - 3$. Then, he expressed that its roots were complex number if the equation was not be factorized. With this statement, it was revealed that Ozgur considered the relation between finding the roots of equation and factorizing. His expressions as follows:

Ozgur: $1 = x^2 + 2x - 3$ [He assigned 1 to y.]
$x^2 + 2x - 4$



No way out.

Researcher: Why?

Ozgur: The roots cannot be found.

Researcher: Why?

Ozgur: Because, the roots are complex number.

Ozgur tried to find intersections of graph with axis and then he drew the graph. He showed that the vertex was on the third quadrant of the coordinate plane. But, he could not determine the coordinates of the vertex because he did not remember the related formula.

Ozgur:

The researcher asked questions which revealed what Ozgur related the vertex to the axis of symmetry. Thus, she tried to understand whether Ozgur related these concepts and supported him to construct several ideas about the axis of symmetry. In this context, she asked him to determine the ordered pair whose values of y were same. Firstly, he stated that the vertex was connected with the midpoint of the intersections of the graph with x-axis. Then, he draw the perpendicular line on x-axis by using the midpoints of x-values for different values of y. So, he realized the axis of symmetry and concluded that the vertex was on this line.

Researcher: Is it only the midpoint of the intersections of the graph with x-axis?
Ozgur: The midpoint of x-values when y-values are same.

ooo

Researcher: Can you determine different points whose y-values are same? What else?
Ozgur:

ooo

Ozgur:

[he drew the perpendicular line on axis of x by using midpoints of values of x]

Researcher: Why did you draw?
Ozgur: Because I can see whether the vertex is midpoint.

Ozgur initially could not explained the properties of the vertex, axis of symmetry of the function, and the relation with the graph but as the interview progressed, he noticed his incomplete knowledge and he made appropriate inferences by thinking on the researcher’s questions and by realizing his own mental actions. He stated that the points whose the values of y were the same had equal distances from the line of symmetry and so, he noticed the meanings of the vertex beyond its formula. Then, he expressed that he could find the value of x of the vertex by using the midpoints of any two points. Similarly, he stated that he could determine y-value of the vertex by substituting x-value into the function.

Researcher: Could you find the vertex without knowing the formula?
Ozgur: Yes, I could.
Researcher: How?
Ozgur: I would have found the midpoint, then I would have substituted this value into function.

Results, Discussion and Implementations

During the clinical interview, Ozgur noticed the difference between the linear function and the quadratic function. Also, he understood that how the linear function and its graph could be interpreted differently from the quadratic function. He could drew the graph of the quadratic function and inferred that the graph had axis of symmetry. He understood that the axis of symmetry is concept was a critical point for the quadratic functions and he could find the vertex and different points by using the axis of symmetry. Besides all these, he realized that he should not try to remember same formulas, but he should think about the concept. The concepts and connections which Ozgur rightly or wrongly presented during the clinical interview were given in Figure 1:

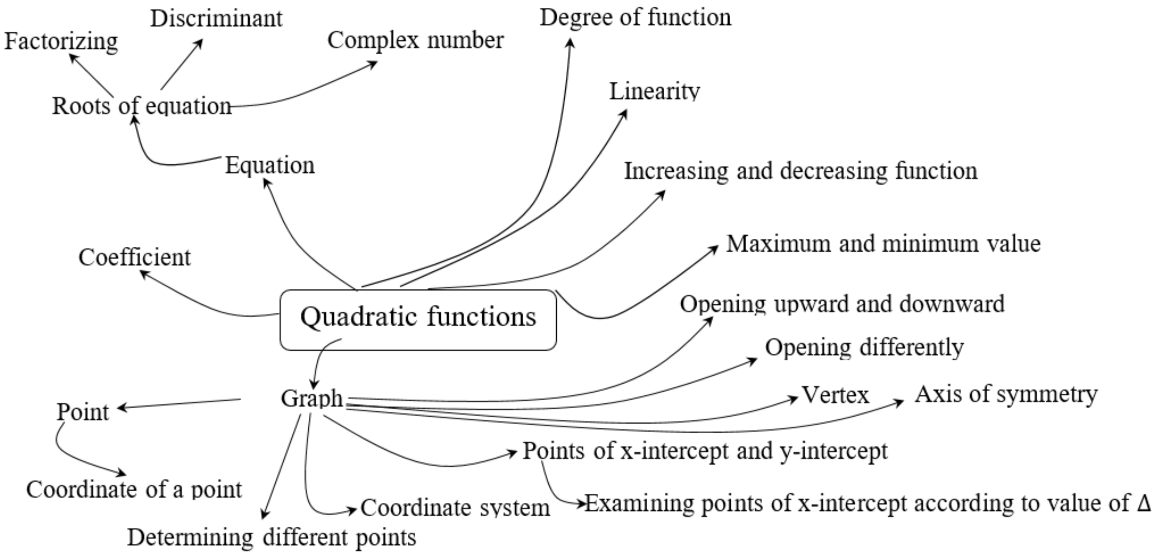


Figure 1. Ozgur's mind map related to the quadratic functions

Ozgur did not relate the algebraic representation to graphical representation although he thought that the graph of the quadratic function was a parabola and parabola was opening upwards or downwards. It can be said that the main reasons of Ozgur's difficulties were his limited understanding, memorizing the rules and properties without conceptual understanding about functions and their graphs. However, If Ozgur had tried to reason, he would have associated critical concepts or ideas related to the quadratic functions.

It was understood that Ozgur forgot the related concepts because he only had procedural understanding although he was aware of which the graph of the function would change based on changing values of coefficients of a, b and c of the function. Similarly, it can be inferred that the teaching approach had been conventional from that Ozgur who was seen as a successful student could not relate the vertex to the axis of symmetry. If Ozgur had had conceptual understanding, there would not been this kind of difficulties and he would tried to reason instead of remember his prior knowledge.

Ozgur continuously tried to associate to linear functions but he could not properly. In this direction, it is revealed that enabling the students to construct a relation between linear functions and quadratic functions by designing tasks is necessary for learning the quadratic functions. The students should firstly know what the function means, in the other ways, they should understand that the function is a mapping two sets based on a specific rule and is a concept representing the covariation of two quantities. The effective tasks should be used for students to be able to convert the algebraic representations to graphical representations and vice versa properly. Although there are different representations of the functions, for quadratic functions, particularly algebraic and graphical representations are of great importance (Huang, Li, & An, 2012).

When considering the quadratic function of $f(x) = ax^2 + bx + c$ whose domain is all real numbers, it could be expressed that all the quadratic functions cannot be written as a product of two linear functions whose domains are all real number. However, according to constructivism, the students should use from the concepts they had already known and experienced for learning quadratic functions. For constructivism, the students do not acquire the knowledge through transferring it by the teacher, in contrast, they have to construct knowledge for themselves (von Glasersfeld, 1982). As the linear function is a quantity which students know, it becomes meaningful for the students to relate this concept with the quadratic functions. They understand that the product of y values of the points whose x values are same for two linear functions is the y value of the point whose x value is same for the quadratic function. It will be easy for students to construct the meaning of the quadratic functions by relating the quantities. Thus, the students should firstly have conceptually understand related to the linear functions. They need to know the function of $f(x) = ax + b$ and what the quantities of a, b and f(x) mean and need to analyze the variation these coefficients and variables. It is of great importance for the students to understand that coefficient of a in algebraic representation of a linear function is the rate of change and to explain that this rate is constant and corresponds to the slope of the graphical representation to have conceptual understanding of the linear functions. Understanding that coefficient of a in the algebraic expression corresponds to the slope is pre-necessity for understanding that coefficient of a of $y = ax^2 + bx + c$ does not mean the slope. The students who constructed this inference could comprehend that the rate of change in a quadratic function is not constant, the rate of change of change is constant and this property leads to what kinds of changes related to quadratic functions. They could notice that a-coefficient effects that the parabola opens upward or downward by examining different functions and their graphs. Otherwise, it is possible that the students could reach an incorrect generalization the knowledge which the coefficient of a in the quadratic function represents the slope as a linear function.

The students can notice the shape of parabola by determining different points and so, by perceiving the structure of the graph. They can understand that the rate of the change is not constant because the value of the quadratic function is found through the product of y values of the points whose x values are same in the two linear functions. So, they can also comprehend that short segments of a parabola are not linear by considering the rate of change. Students do not have misunderstanding because they know what the linearity means and relate this meaning to the quadratic functions. When they use from the product of two linear functions, it will be meaningful for them to construct the function of y$= ax^2 + bx + c$ and understand the shape of its graph.

It is a critical point that the students understand that there are two x values per y value by noticing that the graph is a symmetrical while examining it. For the students to notice the maximum or minimum value of the function and to understand that this value correspond to only one x value are points to emphasize during the teaching quadratic functions. The vertical line which goes through the vertex is the axis of symmetry of the parabola. The students need to find the vertex by using the axis of symmetry. In contrast, the students do not have conceptual understanding related to the vertex and tend to remember the formula and use this formula of finding the vertex. This approach is not the required goal. One of the students' confusions about this concept is that two parabolas having the same x-value of their vertices will have the same vertex because the y-coordinate is ignored (Zavlasky, 1997). They will understand the importance of the value of y in determining the place of the vertex when they comprehend the relation between the axis of symmetry and the vertex.

Using the different algebraic representations for a quadratic function will be meaningful and useful for different conditions. Thus, it is necessary for students to convert the standard forms to factorized forms or the vertex forms and to relate them with the graphical properties for improving their thoughts related to quadratic functions. Writing the function differently by using the vertex of the parabola and the intersection points of the parabola with x-axis provides the students with interpreting the function in different ways.
It will be easy for the student to understand to which changes the translations of the parabola lead for the function expression when they comprehend the critical points related to the quadratic functions. So, prompting the students to learn the quadratic functions based on previous concepts not uncomprehendingly supports them for being able to do necessary practices in transformations of the parabolas.

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