

Development of a Mathematical Model to Assess the Accuracy of Difference between Geodetic Heights

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ABSTRACT

The article includes the results of theoretical studies of the accuracy of geodetic height survey and marks points on the Earth's surface using satellite technology. The dependence of the average square error of geodetic heights difference survey from the distance to the base point was detected. It is being proved that by using satellite technology one can define vertical displacements of the Earth's surface with accuracy sufficient for practical application. It was revealed that the use of positioning methods after the signals of global navigation satellite systems GPS and GLONASS (GNSS) for geodetic purposes has begun in Russia in the beginning of 90-s of the last century. Their essential advantages compared to traditional survey methods were revealed. These include a wide range of accuracies (from tens of meters to millimeters), independence from weather, time of day and year, absence of necessity for mutual visibility between points, high automation and, consequently, efficiency, ability to work continuously and in motion. These qualities have led to high performance and economic efficiency of GNSS, which is particularly noticeable in remote and unpopulated areas occupying a large part of our country. Currently, according to the results of the measurements by double-frequency satellite receivers the mean square error of coordinate increment calculation was reached equaling to $3 \text{ mm} + 1 \cdot 10^{-6} D$, where D is the distance between the satellite receivers. But exceedances between the very same points can be obtained with the mean square error of 10-30 mm, which greatly increases along with the increase of the distance (D) to tens of kilometers. In traditional geodesy height and exceedances surveys are conducted relative to the surface of quasigeoid, which means that measurements are based on physical principle of measurements. As a result, geodetic networks built by traditional methods can be divided into planimetric (B and L) and vertical control H1 networks, which are almost unrelated. In this regard, for the purpose of enhancing the effectiveness of geodetic application of the satellite leveling, methodology and technological support of satellite measurements require improvement.

KEYWORDS

Satellite technology; geodetic height; coordinate increment; accuracy assessment; differential; coefficient; Earth surface; deformation parameter.

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Introduction

Numerous effects on the Earth's surface occurring during aboveground and underground construction and production of liquid and solid commercial minerals, lead to its deformation (Koneshov et al., 2016; Bol'shakov, 1997; Drobyshev, 2006;

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Nepoklonov, 2010; Schultz & Winokur, 1969; Nesenyuk et al., 1980; Dmitriev, 1997; Timochkin, 2013). Timely and expeditious detection of reliable deformation parameters of the Earth's surface allows taking effective measures for engineering protection of the objects located thereon (Klyushin & Kravchuk, 2009; Gairabekov et al., 2009; Gairabekov, 2011a; 2011b; Gairabekov & Pimshin, 2010; 2011). Information on deformation processes is necessary for the proper design of new construction sites as well (Li & Jekeli, 2008; Peshekhonov et al., 1995; Fagerlund, 1977; Hirt et al., 2010; Kudryns, 2009; Tsodokova et al., 2014; Vasil'ev et al., 1991; Troitskii, 1994). For quick data updates of the relevant accuracy advanced geodetic measurement means should be used, which include geodetic satellite technologies (Semenov, 2012).

Materials and Methods

Satellite technologies allow accurate fixation of X and Y planimetric coordinates. Thus accuracy of coordinate increments with the use of double-frequency satellite receivers equals to $3\text{ mm}+1\cdot 10^{-6}D$, where D is the distance between station points of satellite receivers. Vertical control position of points is considered to be fixed with much less accuracy, with the mean square error of 10-30 mm. However, this problem is still under-explored with both theoretical and experimental study of this problem remaining the relevant objective. We are going to undertake theoretical study of the possibility of using satellite technologies to detect vertical displacements of the Earth's surface.

The article (Klyushin & Kravchuk, 2009) gives the derived formula, which allows computing the geodetic height of a point based on the results of satellite measurements:

$$H = [X^2 + Y^2 + Z^2(1 - a_1R^2r^{-3})^{-2}(1 + b_1Z^2r^{-3})^2]^{1/2} - a \left[1 - \frac{e^2Z^2R^{-2}}{Z^2R^{-2} + (1 - a_1R^2r^{-3})^2(1 + b_1Z^2r^{-3})^{-2}} \right]^{-1/2} \quad (1)$$

where $a_1 = ae^2(1 - e^2)^{3/2}$ and $b_1 = be'^2$; a and b are ellipsoid semiaxes adopted for the processing of geodetic measurements and establishing a geodetic coordinate system; e^2 and e'^2 are values of eccentricities;

$$R = \sqrt{X^2 + Y^2}$$

$$r = \sqrt{(1 - e^2)(X^2 + Y^2) + Z^2}$$

Formula (1) can be represented as follows:

$$H = P - N, \quad (2)$$

where N is the curvature of the first vertical;

$$P = \sqrt{X^2 + Y^2 + Z^2(1 - a_1R^2r^{-3})^{-2}(1 + b_1Z^2r^{-3})^2} \quad (3)$$

If we take the differential equation (2) we get:

$$dH = \frac{\partial P}{\partial X} dX - \frac{\partial N}{\partial X} dX + \frac{\partial P}{\partial Y} dY - \frac{\partial N}{\partial Y} dY - \frac{\partial P}{\partial Z} dZ - \frac{\partial N}{\partial Z} dZ \quad (4)$$

The following designations are accepted:

$$t_1 = 1 - a_1R^2r^{-3}, \quad (5)$$

$$t_2 = 1 + b_1Z^2r^{-3}, \quad (6)$$

$$t_3 = \frac{e^2Z^2}{R^2}, \quad (7), \quad (7)$$

$$t_4 = \frac{Z^2}{R^2} + \frac{t_1^2}{t_2^2} \quad (8), \quad (8)$$

Vertical curvature we present in the following way:

$$N = \frac{a}{\sqrt{1 - \frac{t_3}{t_4}}}$$

Formula (4) can be represented as follows:

$$P = \sqrt{X^2 + Y^2 + Z^2 t_1^{-2} t_2^2}$$

After some transformation of the formula (4) we get:

$$\begin{aligned}
 dH = & \left[\frac{X}{P} - \frac{3b_1(1-e^2)t_2 XZ^4}{Pt_1^2 r^5} + \frac{2a_1 t_2^2 XZ^2}{Pt_1^3 r^3} - \frac{3a_1(1-e^2)t_2^2 XZ^2 R^2}{Pt_1^3 r^5} + \frac{ae^2 XZ^2}{t_4^{1/3}(t_4-t_3)^{3/2} R^4} \right. \\
 & - \frac{at_3 XZ^2}{t_4^{4/3}(t_4-t_3)^{3/2} R^4} - \frac{2aa_1 t_1 t_3 X}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^3} + \frac{3aa_1 t_1 t_3 (1-e^2) XR^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} \\
 & \left. - \frac{3ab_1 t_1^2 t_3 XZ^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} \right] dX \\
 & + \left[\frac{Y}{P} - \frac{3b_1(1-e^2)t_2 YZ^4}{Pt_1^2 r^5} + \frac{2a_1 t_2^2 YZ^2}{Pt_1^3 r^3} - \frac{3a_1(1-e^2)t_2^2 YZ^2 R^2}{Pt_1^3 r^5} \right. \\
 & + \frac{ae^2 YZ^2}{t_4^{1/3}(t_4-t_3)^{3/2} R^4} - \frac{at_3 YZ^2}{t_4^{4/3}(t_4-t_3)^{3/2} R^4} - \frac{2aa_1 t_1 t_3 Y}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^3} \\
 & \left. + \frac{3aa_1 t_1 t_3 (1-e^2) YR^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} - \frac{3ab_1 t_1^2 t_3 YZ^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} \right] dY \\
 & + \left[\frac{Zt_2^2}{Pt_1^2} - \frac{3b_1 t_2 Z^5}{Pt_1^2 r^5} - \frac{3a_1 R^2 Z^3}{Pt_1^3 r^3} - \frac{ae^2 Z}{t_4^{1/3}(t_4-t_3)^{3/2} R^2} + \frac{at_3 Z}{t_4^{4/3}(t_4-t_3)^{3/2} R^2} \right. \\
 & \left. - \frac{3aa_1 t_1 t_3 ZR^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} + \frac{3ab_1 t_1^2 t_3 Z^3}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} \right] dZ \quad (9)
 \end{aligned}$$

Since there is a small quantity of difference between t_1 and t_2 coefficients and the unit, formula member (9) $\frac{Zt_2^2}{P_1^2 t_1^2}$ can be represented as follows:

$$\frac{Zt_2^2}{P_1^2 t_1^2} = \frac{Z(1-a_1 R^2 r^{-3})^2}{P(1+b_1 Z^2 r^{-3})^2} \approx \frac{Z}{P} - \frac{2a_1 ZR^2}{Pr^3} - \frac{2b_1 Z^3}{Pr^3} \quad (10)$$

And given the (10) we present the formula (9) as follows:

$$dH = \left(\frac{X}{P} + A\right) dX + \left(\frac{Y}{P} + B\right) dY + \left(\frac{Z}{P} + C\right) dZ, \quad (11)$$

where

$$\begin{aligned}
 A = & -\frac{3b_1(1-e^2)t_2 XZ^4}{Pt_1^2 r^5} + \frac{2a_1 t_2^2 XZ^2}{Pt_1^3 r^3} - \frac{3a_1(1-e^2)t_2^2 XZ^2 R^2}{Pt_1^3 r^5} + \frac{ae^2 XZ^2}{t_4^{1/3}(t_4-t_3)^{3/2} R^4} \\
 & - \frac{at_3 XZ^2}{t_4^{4/3}(t_4-t_3)^{3/2} R^4} - \frac{2aa_1 t_1 t_3 X}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^3} + \frac{3aa_1 t_1 t_3 (1-e^2) XR^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5} \\
 & - \frac{3ab_1 t_1^2 t_3 XZ^2}{t_2^2 t_4^{4/3}(t_4-t_3)^{3/2} r^5}; \quad (12)
 \end{aligned}$$



$$B = -\frac{3b_1(1-e^2)t_2YZ^4}{Pt_1^2r^5} + \frac{2a_1t_2^2YZ^2}{Pt_1^3r^3} - \frac{3a_1(1-e^2)t_2^2YZ^2R^2}{Pt_1^3r^5} + \frac{ae^2YZ^2}{t_4^{1/3}(t_4-t_3)^{3/2}R^4} \\ - \frac{at_3YZ^2}{t_4^{4/3}(t_4-t_3)^{3/2}R^4} - \frac{2aa_1t_1t_3Y}{t_2^2t_4^{4/3}(t_4-t_3)^{3/2}r^3} + \frac{3aa_1t_1t_3(1-e^2)YR^2}{t_2^2t_4^{4/3}(t_4-t_3)^{3/2}r^5} \\ - \frac{3ab_1t_1^2t_3YZ^2}{t_2^2t_4^{4/3}(t_4-t_3)^{3/2}r^5}; \quad (13)$$

$$C = -\frac{2a_1ZR^2}{Pr^3} - \frac{2b_1Z^3}{Pr^3} - \frac{3b_1t_2Z^5}{Pt_1^2r^5} - \frac{3a_1R^2Z^3}{Pt_1^3r^3} - \frac{ae^2Z}{t_4^{1/3}(t_4-t_3)^{3/2}R^2} + \frac{at_3Z}{t_4^{4/3}(t_4-t_3)^{3/2}R^2} \\ - \frac{3aa_1t_1t_3ZR^2}{t_2^2t_4^{4/3}(t_4-t_3)^{3/2}r^5} + \frac{3ab_1t_1^2t_3Z^3}{t_2^2t_4^{4/3}(t_4-t_3)^{3/2}r^5} \quad (14)$$

Next we can move from differentials to finite increments and then to mean square errors, obtaining the following from (11):

$$m_H^2 = \left(\frac{X}{p} + A\right)^2 m_X^2 + \left(\frac{Y}{p} + B\right)^2 m_Y^2 + \left(\frac{Z}{p} + C\right)^2 m_Z^2 \quad (15)$$

The analysis of accuracy of point coordinates survey using modern satellite technologies reveals that generally $m_X = m_Y = m_Z = m_K$. Therefore, the formula of accuracy assessment of geodetic height calculation of a point can be presented as follows:

$$m_H = m_K \sqrt{\frac{X^2 + Y^2 + Z^2}{p^2} + \frac{2AX}{p} + \frac{2BY}{p} + \frac{2CZ}{p} + A^2 + B^2 + C^2} \quad (16)$$

Results and Discussion

The performed formula analysis (16) showed that with an error not exceeding 4%, it can be written as:

$$m_H = m_K$$

Since, $\frac{X^2+Y^2+Z^2}{p^2} \approx 1$ the remaining members are small.

By way of example, we get the following values in the middle line:

$$B = 45^\circ; L = 37; H = 200\text{m}$$

$$X = 3608020 \text{ m}; Y = 2718839 \text{ m}; Z = 4487489 \text{ m}$$

$$\frac{X^2 + Y^2 + Z^2}{p^2} = 1.023$$

$$\frac{2AX}{p} = -0,0048$$

$$\frac{2BY}{p} = 0,0089$$

$$\frac{2CZ}{p} = 0,0397$$

$$A^2 = 0,000018; K = 1,0237$$

$$B^2 = 0,000108$$

$$C^2 = 0,000791$$

At the equator we get; $B = 0^\circ; L = 37^\circ; H = 200\text{m}$

$$X = 5093965 \text{ m}; Y = 3838578 \text{ m}; Z = 0 \text{ m};$$

$$\frac{X^2 + Y^2 + Z^2}{P^2} = 1$$

$$\frac{2AX}{P} = 0$$

$$\frac{2BY}{P} = 0$$

$$\frac{2CZ}{P} = 0$$

$$A^2 = 0$$

$$B^2 = 0$$

$$C^2 = 0$$

In the area of the Arctic covered settings will have values: $B = 72^\circ; L = 37^\circ; H = 200 \text{ m}$

$$X = 1578909 \text{ m}; Y = 1189793 \text{ m}; Z = 6043875 \text{ m};$$

$$\frac{X^2 + Y^2 + Z^2}{P^2} = 1,0012$$

$$\frac{2AX}{P} = 0,0027$$

$$\frac{2BY}{P} = 0,0036; K = 1,0593$$

$$\frac{2CZ}{P} = 0,1163$$

$$A^2 = 0,0060$$

$$B^2 = 0,00009$$

$$C^2 = 0,0037$$

It should be noted that the geodetic heights are tied to point coordinates identified by means of satellite technologies and the chart of complex and non-linear form. However under the equally accurate Cartesian coordinates, mean square error of geodetic height calculations is essentially independent of a point position.

To assess the accuracy of the results of satellite measurements, considering formula (2), we state:

$$h = H_2 - H_1 = P_2 - P_1 + N_1 - N_2 \quad (17)$$

where

$$P_1 = \sqrt{X_1^2 + Y_1^2 + Z_1^2 \frac{(1 + b_1 Z_1^2 r_1^{-3})^2}{(1 - a_1 R_1^2 r_1^{-3})^2}}$$

$$P_2 = \sqrt{X_2^2 + Y_2^2 + Z_2^2 \frac{(1 + b_1 Z_2^2 r_2^{-3})^2}{(1 - a_1 R_2^2 r_2^{-3})^2}}$$

$$N_1 = \frac{a}{\sqrt{1 - \frac{e^2 Z_1^2 R_1^{-2}}{Z_1^2 R_1^{-2} + \frac{(1 - a_1 R_1^2 r_1^{-3})^2}{(1 + b_1 Z_1^2 r_1^{-3})^2}}}}$$

$$N_2 = \frac{a}{\sqrt{1 - \frac{e^2 Z_2^2 R_2^{-2}}{Z_2^2 R_2^{-2} + \frac{(1 - a_1 R_2^2 r_2^{-3})^2}{(1 + b_1 Z_2^2 r_2^{-3})^2}}}}$$

Here below we give the expression of a total differential of geodetic heights difference:

$$dh = \left(\frac{X_1}{P_1} + A_1\right) dX_1 + \left(\frac{Y_1}{P_1} + B_1\right) dY_1 + \left(\frac{Z_1}{P_1} + C_1\right) dZ_1 - \left(\frac{X_2}{P_2} + A_2\right) dX_2 - \left(\frac{Y_2}{P_2} + B_2\right) dY_2 - \left(\frac{Z_2}{P_2} + C_2\right) dZ_2 \quad (18)$$

The coordinates of the second point (X_2, Y_2, Z_2) can be defined using the coordinates of the first point (X_1, Y_1, Z_1) and coordinate increments $(\Delta X, \Delta Y, \Delta Z)$ phase measurements. Accordingly,

$$dX_2 = dX_1 + d\Delta X; \quad dY_2 = dY_1 + d\Delta Y; \quad dZ_2 = dZ_1 + d\Delta Z, \quad (19)$$

$$dH = \left(\frac{X_1}{P_1} - \frac{X_2}{P_2} + A_1 - A_2\right) dX_1 + \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} + B_1 - B_2\right) dY_1 + \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} + C_1 - C_2\right) dZ_1 - \left(\frac{X_2}{P_2} + A_2\right) d\Delta X - \left(\frac{Y_2}{P_2} + B_2\right) d\Delta Y - \left(\frac{Z_2}{P_2} + C_2\right) d\Delta Z. \quad (20)$$

Next we go from differentials to finite increments, and to mean square errors afterwards, so that we get the following from (20):

$$m_h^2 = \left(\frac{X_1}{P_1} - \frac{X_2}{P_2} + A_1 - A_2\right)^2 m_{X_1}^2 + \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} + B_1 - B_2\right)^2 m_{Y_1}^2 + \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} + C_1 - C_2\right)^2 m_Z^2 + \left(\frac{X_2}{P_2} + A_2\right)^2 m_{\Delta X}^2 + \left(\frac{Y_2}{P_2} + B_2\right)^2 m_{\Delta Y}^2 + \left(\frac{Z_2}{P_2} + C_2\right)^2 m_{\Delta Z}^2 \quad (21)$$

Subject to equally accurate positioning of point 1 $m_{X_1}^2 = m_{Y_1}^2 = m_{Z_1}^2 = m_k^2$ and coordinate increments $m_{\Delta X}^2 = m_{\Delta Y}^2 = m_{\Delta Z}^2 = m_{\Delta}^2$ formula (21) is as follows:

$$\begin{aligned}
 & m_h^2 \left[\left(\frac{X_1}{P_1} - \frac{X_2}{P_2} \right)^2 + \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} \right)^2 + \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} \right)^2 + 2(A_1 - A_2) \left(\frac{X_1}{P_1} - \frac{X_2}{P_2} \right) + 2(B_1 - B_2) \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} \right) + \right. \\
 & \left. 2(C_1 - C_2) \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} \right) + (A_1 - A_2)^2 + (B_1 - B_2)^2 + (C_1 - C_2)^2 \right] m_k^2 + \left[\left(\frac{X_2}{P_2} + A_2 \right)^2 + \left(\frac{Y_2}{P_2} + B_2 \right)^2 + \right. \\
 & \left. \left(\frac{Z_2}{P_2} + C_2 \right)^2 \right] m_\Delta^2 \quad (22)
 \end{aligned}$$

Our formula (16) analysis shows that with high level of accuracy the value is

$$\left(\frac{X_2}{P_2} + A_2 \right)^2 + \left(\frac{Y_2}{P_2} + B_2 \right)^2 + \left(\frac{Z_2}{P_2} + C_2 \right)^2 \approx 1. \quad (23)$$

With this in mind, the formula (22) can lead to:

$$\begin{aligned}
 m_h^2 \left[\left(\frac{X_1}{P_1} - \frac{X_2}{P_2} \right)^2 + \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} \right)^2 + \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} \right)^2 + 2(A_1 - A_2) \left(\frac{X_1}{P_1} - \frac{X_2}{P_2} \right) + 2(B_1 - B_2) \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} \right) + \right. \\
 \left. 2(C_1 - C_2) \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} \right) + (A_1 - A_2)^2 + (B_1 - B_2)^2 + (C_1 - C_2)^2 \right] m_k^2 + m_\Delta^2. \quad (24)
 \end{aligned}$$

The analysis of the members constituting the formula (13) was performed under the condition that $P_2 = P_1 + \Delta P_1$ which provided a minimum correction value of ΔP . After the appropriate conversions one can get the value ΔP equaling to:

$$\Delta P = \frac{X_1 \Delta X}{P_1} + \frac{Y_1 \Delta Y}{P_1} + \frac{Z_1 \Delta Z}{P_1} + \frac{Z_2^2 (\delta_2 - \delta_3)}{P_1} + \frac{\Delta X^2 + \Delta Y^2 + \Delta Z^2}{2P_1}, \quad (25)$$

then the formula (24) is written as follows:

$$m_h^2 = \frac{S^2}{P_1^2} m_k^2 + m_\Delta^2 + \Delta_m, \quad (26)$$

where

$$\begin{aligned}
 \Delta_m = 2(A_1 - A_2) \left(\frac{X_1}{P_1} - \frac{X_2}{P_2} \right) + 2(B_1 - B_2) \left(\frac{Y_1}{P_1} - \frac{Y_2}{P_2} \right) + 2(C_1 - C_2) \left(\frac{Z_1}{P_1} - \frac{Z_2}{P_2} \right) + (A_1 - A_2)^2 + \\
 (B_1 - B_2)^2 + (C_1 - C_2)^2 + \frac{2X_2 \Delta P \Delta X}{P_1} + \frac{2Y_2 \Delta P \Delta Y}{P_1} + \frac{2Z_2 \Delta P \Delta Z}{P_1} + S^2 \Delta P^2, \quad (27)
 \end{aligned}$$

$$S^2 = \Delta X^2 + \Delta Y^2 + \Delta Z^2 \quad (28)$$

The Δ_m amount is negligibly small so we neglect it. For example, if $B = 45^\circ$; $L = 37$; $H = 200\text{m}$ $\Delta_m = 0,000674$

Consequently, the expression of accuracy assessment of geodetic heights difference can be written as:

$$m_h = \sqrt{\frac{S^2}{R_3^2} m_k^2 + m_\Delta^2}, \quad (29)$$

where $P_1 \approx R_3$, R_3 is the mean radius of the Earth.

On the basis of the accuracy analysis we conclude that at relatively large distances between defined points, base station coordinate error has a significant impact on the calculated values of the difference between the geodetic heights. The coordinates of the base station are calculated with the mean square error equaling $m_k = 5\text{ m}$, 5 m , and coordinates increments are calculated with the mean square error $m_\Delta = 5\text{ mm}$. The dependence of the accuracy of difference between geodetic heights calculations and differences of the coordinates is presented in Figure 1. The diagram



shows that with the distances from a base station not exceeding 5-6 km coordinate error has a significantly less effect on the accuracy of heights difference calculations than mean square error of coordinate increments calculation.

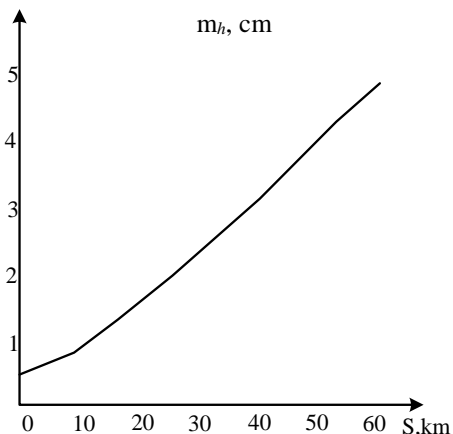


Figure 1. Diagram of dependence of the mean square error of identifying the difference between geodetic heights distance from the distance to base station

As it follows from the formula (18), the mean square error of identifying vertical displacements of points equals to the mean square error of identifying coordinate increments of this point. In the case we are considering it equals 5 mm. With the use of modern satellite double-frequency receivers of coordinate increment one can identify with the mean square error of 2-3 mm (Gairabekov et al., 2015a; 2015b), hence vertical displacements (precipitations) are calculated with the same accuracy. Such accuracy is sufficient to solve the above-mentioned problems:

- calculations of vertical displacements of the Earth's surface of anthropogenic nature as a result of overground and underground construction;
- calculations of vertical displacements of the Earth's surface during the extraction of liquid and solid mineral resources;
- calculations of vertical offsets of seismotectonic nature.

Conclusion

Thus, the accuracy of calculation of vertical displacements by means of satellite technologies with distances between points up to 6 km is commensurate with the accuracy of calculation of horizontal displacements. Therefore, deformations (including vertical displacements) can be calculated with use of satellite technologies with precision sufficient for practice in a relatively short period of time, which allows taking that prompt and effective measures for protection of engineering facilities located at the territories exposed to deformation processes, as well as obtaining information necessary for the design of new facilities.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

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