Concordance of Interests in Dynamic Models of Social Partnership in the System of Continuing Professional Education

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ABSTRACT

A dynamic game theoretic model of concordance of interests in the process of social partnership in the system of continuing professional education is proposed. Non-cooperative, cooperative, and hierarchical setups are examined. Analytical solution for a linear state version of the model is provided. Nash equilibrium algorithms (for non-cooperative and cooperative setups) are identified. H.Stakelberg's1t algorithms of equilibrium solution of the game in hierarchical setup are described (in the general case). A method of building the precise discrete analogue of a continuous model is used for examining the hierarchical setup. Examples of test calculations for different data sets are provided; content interpretation of the results is discussed.

Introduction

Social partnership in continuing education is a specific system of joint activities of the education system agents characterized by trust, common objectives and values; it provides highly qualified, competitive, and mobile specialists for the labor market (Tarasenko, 2009) Many authors studied the social partnership in education (Tarasenko, 2009; Keith, 2011; Siegel, 2010), but only a few papers are dedicated to modeling of this process (Siegel, 2010; Fandel et al., 2012; Talman and Yang, 2011). The first author contributions are presented in the following works (Zaharatul et al., 2012; Dyachenko et al., 2014).

In contrast to (Zaharatul et al., 2012; Dyachenko et al., 2014), in this paper the authors consider a differential game model with payoff functionals reflecting the concordance of private and public interests in resource allocation. This problem setup is initiated by a seminal paper by Yu. B. Germeier and I. A. Vatel (Dyachenko et al., 2015) who have shown that under a specific structure of payoff

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functions the Pareto optimal Nash equilibrium of the static game in normal form exists, i.e. the complete concordance of the players' interests is achieved. Close setups were also considered in static models of the active systems theory (Germeier and Vatel, 1975) and in the information theory of hierarchical systems (Burkov and Opoitsev, 1974). Later on the problem of concordance of interests was actively developed in network games. In particular, Ch. Papadimitriou has introduced a notion of price of anarchy as a quantitative characteristic of the level of concordance of private and public interests (Gorelov and Kononenko, 2015). This idea was developed by Basar and Zhu in the dynamic case (Basar and Zhu, 2011). Dynamic counterparts of the models are reviewed in (Basar and Zhu, 2011).

In some specific cases using theoretical game model in continuous time allows to find the analytical solution. In general case, the author's method based on building the precise discrete analogue of a continuous model (Long, 2010) is used for finding the solution of a differential game.

The paper is organized as follows. In Section 1 a mathematical setup of the problem of concordance of interests in social partnership in the system of continuous professional education (CPE) is presented. The model linear state version analytical examination (Ugol'nitskii and Usov, 2013) is described in Section 2. The method of solution of the general differential game model of concordance of interests by digitization is proposed in Section 3. Test examples of numerical calculations are analyzed in Section 4. The obtained results are discussed in Conclusion.

1.1. Mathematical setup of the problem

A social partnership between three control agents: a university teacher (B), an employer (P), and a student (C) is considered. First, it is assumed that the agents are independent and make decisions independently and simultaneously. All control agents aim for maximizing their payoffs discounted at time period T. The differential game model has the following form:

$$J_i = \int_0^T e^{-\rho t}[g_i(r_i - u_i(t)) + s_i(t)c(x(t))]dt + e^{-\rho T}s_i(T)c(x(T)) \to \max,$$

$$(1.1)$$

$$0 \le u_i(t) \le r_i, \quad i \in N;$$

$$(1.2)$$

$$x = h(x(t)) + f(u_B(t), u_P(t), u_c(t)), \quad x(0) = x_0$$

$$(1.3)$$

In the model of concordance of interests it is assumed that each player i from set $N = \{B, P, C\}$ allocates his budget between two directions: share $u_i(t)$ (the player's control) is allotted for the students' professional level improvement, whereas the rest $r_i - U_i(t)$ is used for financing his private activities (business development, stocks and deposits, or consumption). Respectively, a current payoff of a player is formed by private revenues and utility from the level of professional training of the students (public good).

Thus, in the model (1.1) - (1.3) $g_i(z)$ is a concave increasing function of the variable z, reflecting private revenues of the players; $x(t)$ is a level of professional training of the students (a state variable); $c(x)$ is a concave increasing function showing a financial expression of the public utility from the level of professional training; $s_i(t)$ is a share of the i-*th* player in this public good; h is a decreasing function, i.e. in the absence of investments the level of professional training diminishes; f is an increasing function of players' investments in the professional training of the students.

1.2. Analytical study of the linear state model

For analytical tractability let us consider a linear state simplified version (Dockner et al., 2000) of the model (1.1)- (1.3) in the following form

$$J_i = \int_0^T e^{-\rho t}[k_i(r_i - u_i(t))^{p_i} + s_i(t)cx(t)]dt + e^{-\rho T}s_i(T)cx(T) \rightarrow \max,$$

(2.1)

$$0 \le u_i(t) \le r_i, \quad i \in N;$$

(2.2)

$$\dot{x} = ax(t) + \sum_{i \in N} b_i u_i(t),\ x(0) = x_0$$

(2.3)

Hence, in (2.1) - (2.3) in comparison with (1.1) - (1.3) model, a linear function $c(x(t)) = cx(t)$, where $c > 0$ is a coefficient of transition from a level of professional training to the public utility, and a linear function $h(x(t)) = ax(t), a < 0$, are used. In what follows, a linear function $f(u_B(t), u_P(t), u_C(t)) = \sum_{i \in N} b_i u_i(t)$, where $b_i > 0$ is a share of the ith player contribution to the upgrading of the students' professional training level, is used for simplicity. Finally, without essential loss of generality a power function of private revenues is taken: $g_i(r_i - u_i(t)) = k_i(r_i - u_i(t))^{p_i}, k_i > 0, 0 < p_i < 1, i \in N$.

For solution of the game (2.1) - (2.3) Pontryagin's maximum principle is applicable. The i-th player's Hamiltonian function is as follows:

$$H_i(u_i(t), \lambda_i(t), x(t), t) = k_i(r_i - u_i(t))^{p_i} + cs_i(t)x(t) + \lambda_i(t)[ax(t) + \sum_{j \in N} b_j u_j(t)]$$

.

From the condition $\partial H_i / \partial u_i = 0$ with regard to non-negativity of $u_i(t)$ the following can be determined:

$$u_i^{NE}(t) = \begin{cases} r_i - \left(\dfrac{b_i\lambda_i(t)}{k_i p_i}\right)^{\frac{1}{p_i-1}}, & 0 < \lambda_i(t) < \dfrac{k_i p_i}{b_i} r_i^{p_i-1}, \\ \\ 0, & \lambda_i(t) \geq \dfrac{k_i p_i}{b_i} r_i^{p_i-1}. \end{cases}$$

(2.4)

A boundary value problem for the conjugate variable is

$$\frac{d\lambda_i}{dt} = (\rho - a)\lambda_i(t) - cs_i(t), \quad \lambda_i(T) = cs_i(T)$$

,

(2.5)

its solution

$$\lambda_i^{NE}(t) = e^{(\rho-a)(t-T)}cs_i(T) + e^{(\rho-a)(t-T)}\int_t^T cs_i(\tau)e^{(\rho-a)(T-\tau)}d\tau$$

.

(2.6)

Due to the properties of the model (2.1) - (2.3) the relations (2.4) with regard to (2.6) actually form the Nash equilibrium $u^{NE}(\cdot) = (u_B^{NE}(\cdot), u_P^{NE}(\cdot), u_C^{NE}(\cdot))$ in this model. It is known (Ugol'nitskii and Usov, 2013) that in the linear state differential games open-loop and closed-loop (Markovian) Nash equilibria coincide. The respective equilibrium path is

$$x^{NE}(t) = x_0 e^{at} + \int_0^t e^{a(t-\tau)}\left[\sum_{i\in N} b_i u_i^{NE}(\tau)\right]d\tau$$

.

(2.7)

The obtained analytical solution allows for some qualitative conclusions about the social partnership in the framework of the model (2.1) - (2.3). Hence, the condition $\lambda_i(t) \geq k_i p_i r_i^{p_i-1}/b_i$ characterizes a "pure egoism" of the ith player due to which the whole budget is assigned only to his private activities. Note that a "pure collectivism" $(u_i(t) = r_i)$, when all investments are allocated for professional training and, therefore, for social partnership development in the continuing professional training system, is not attained in the equilibrium strategy (2.4) if $k_i \gg 0$.

From the equilibrium path form (2.7) it is clear that high level of professional training can hardly be sustained under the linear dynamics of the state because the first summand in (2.7) decreases exponentially $(a < 0)$.

Now consider a cooperative setup of the social partnership control problem when all agents unite and jointly maximize the summary payoff functional

$$J = \sum_{i\in N} J_i = \int_0^T e^{-\rho t}[\sum_{i\in N} k_i(r_i - u_i(t))^{p_i} + cx(t)]dt + e^{-\rho T}cx(T)$$

(2.8)

(s. t. $\forall t \sum_{i \in N} s_i(t) = 1$) in all controls (2.2) s. t. equation of dynamics (2.3). In this case we receive a Pareto optimal team solution $u^{PO}(\cdot) = (u_B^{PO}(\cdot), u_P^{PO}(\cdot), u_C^{PO}(\cdot))$, where

$$
u_i^{PO}(t) = \begin{cases} r_i - \left(\dfrac{b_i \lambda(t)}{k_i p_i}\right)^{\frac{1}{p_i - 1}}, & 0 < \lambda(t) < \dfrac{k_i p_i}{b_i} r_i^{p_i - 1}, \\ 0, \; \lambda(t) \geq \dfrac{k_i p_i}{b_i} r_i^{p_i - 1}, \end{cases} \qquad i \in N
$$

(2.9)

$$
\lambda^{PO}(t) = c e^{(\rho - a)(t - T)} + e^{(\rho - a)(t - T)} \int_t^T c e^{(\rho - a)(T - \tau)} d\tau
$$

(2.10)

Optimal cooperative path $x^{PO}(t)$ has the form (2.7) s. t. (2.9) - (2.10). A quantitative characteristic of the system losses from a non-cooperative behavior is captured by the price of anarchy [12,13]

$$
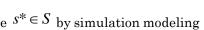PA = \frac{\sum_{i \in N} J_i^{NE}}{J^{PO}}
$$

(2.11)

Finally, consider a hierarchical setup of the social partnership control problem. Suppose that agents from the set $N = \{B, P, C\}$ with payoff functional (2.1) and constraints (2.2) - (2.3) form a lower control level, and the federal state maximizing the summary payoff functional (2.8) forms the upper control level. Assume for simplicity that federal state controls are open-loop strategies $s(\cdot) = (s_B(\cdot), s_P(\cdot), s_C(\cdot))$ satisfying the conditions

$$
s(\cdot) \in S = \{s(t) : s_i(t) \geq 0, \sum_{i \in N} s_i(t) = 1, t \in [0, T]\}
$$

(2.12)

Then there is a hierarchical differential game of type G_{1t} (Gorelov and Kononenko, 2015) with the following information structure.

1. The leader (federal state) chooses an open-loop strategy (2.12) and reports it to other players (followers) from set $N = \{B, P, C\}$.

2. Given the chosen strategy (2.12), agents from set $N = \{B, P, C\}$ play a differential game (2.1) - (2.3), the solution of which is Nash equilibrium $NE(s(\cdot))$.

3. In fact, the leader chooses a strategy (2.12) which maximizes his payoff (2.8) on the set of Nash equilibria $NE(s(\cdot))$.

Let us find a solution of the hierarchical game $s^* \in S$ by simulation modeling based on the scenarios method. The proposed algorithm has the following form.

1. Choose a set of scenarios $S_M = \{s^{(1)}(\cdot), ..., s^{(M)}(\cdot)\}$, where $s^{(j)}(\cdot) \in S, \; j = 1, ..., M$.

2. Given $s(\cdot) \in S_M$ calculate $NE(s(\cdot))$ using formulas (2.4) - (2.5) as well as respective path (2.7).

3. Calculate the leader's payoffs $J^{ST}(s(\cdot), u^{NE(s(\cdot))}, x^{NE(s(\cdot))})$ for all $s(\cdot) \in S_M$.

4. Find

$$s*(\cdot) \in S_M : J^{ST}(s*(\cdot), u^{NE(s*(\cdot))}, x^{NE(s*(\cdot))}) = \max_{s(\cdot) \in S_M} J^{ST}(s(\cdot), u^{NE(s(\cdot))}, x^{NE(s(\cdot))})$$

1.3. Solution of the power model

Now suppose that the function of public utility depending on the level of professional training has a more general form $c(x(t)) = cx^p(t)$, $0 < p < 1$.

In the case of independent agents, a boundary value problem arises for the system of non-linear ordinary differential equations of the following form:

$$u_i^{NE}(t) = \begin{cases} r_i - \left(\dfrac{b_i \lambda_i^{NE}(t)}{k_i p_i} \right)^{\frac{1}{p_i - 1}}, & 0 < \lambda_i^{NE}(t) < \dfrac{k_i p_i}{b_i} r_i^{p_i - 1}, \\ 0, & \lambda_i^{NE}(t) \geq \dfrac{k_i p_i}{b_i} r_i^{p_i - 1}. \end{cases}$$

(3.1)

$$\frac{d\lambda_i^{NE}}{dt} = -cs_i p(x^{NE})^{p-1}(t)s_i(t) + (\rho - a)\lambda_i^{NE}(t),$$

(3.2)

$$\lambda_i^{NE}(T) = Cp(x^{NE})^{p-1}(T)s_i(T);$$

$$\frac{dx^{NE}}{dt} = ax^{NE}(t) + \sum_{i \in N} b_i u_i^{NE}(t), \; x^{NE}(0) = x_0$$

(3.3)

In the numerical methods of non-linear boundary value problems solution two approaches are differentiated (Dockner et.al., 2000).

The first one uses an approximate discrete representation of the boundary problem connected with determination of a space grid and approximation of the unknown functions. The second one is a multiple shooting method where "shooting" grid values are the solutions of a system of non-linear equations determined by the solutions of a series of Cauchie initial value problems. The second method is used to solve the problem (3.1) - (3.3).

In the cooperative case all the agents unite, and the optimal solutions for optimal power team problem are found from the following boundary value problem:

$$u_i^{PO}(t) = \begin{cases} r_i - \left(\dfrac{b_i \lambda^{PO}(t)}{k_i p_i} \right)^{\frac{1}{p_i-1}}, & 0 < \lambda^{PO}(t) < \dfrac{k_i p_i}{b_i} r_i^{p_i-1}, \\ \\ 0, & \lambda^{PO}(t) \geq \dfrac{k_i p_i}{b_i} r_i^{p_i-1}. \end{cases}$$

(3.4)

$$\frac{d\lambda^{PO}}{dt} = -Cp(x^{PO})^{p-1}(t) + (\rho - a)\lambda^{PO}(t), \quad \lambda^{PO}(T) = Cp(x^{PO})^{p-1}(T)$$

(3.5)

$$\frac{dx^{PO}}{dt} = ax^{PO}(t) + \sum_{i \in N} b_i u_i^{PO}(t), \ x^{PO}(0) = x_0$$

(3.6)

The problem (3.4) – (3.6) is once again solved by the multiple shooting method (Dockner et al., 2000).

Finally, let us analyze a hierarchical setup of the problem. The algorithm of hierarchical problem study coincides with the one described in the previous section; the problem is solved by the scenarios method. The method of transition from a differential control model to its precise multistep analogue (Long, 2010) based on the following hypothesis is used.

In any real control system the agents are unable to change their strategies in any arbitrary desired moment of time. Their strategies remain constant for a certain time period due to objective inertia. Without losing the generality of analysis one may suppose that control strategies of all agents are constant at equal time periods, i.e.

$$w(t) = \begin{cases} w_1, & \text{if} \quad 0 \leq t < t_1, \\ w_2, & \text{if} \quad t_1 \leq t < t_2, \\ \quad \dots \\ w_M, & \text{if} \quad t_{M-1} \leq t < T, \end{cases}$$

(3.7)

where $w_i = const;\ t_i = i\,\Delta t;\ \Delta t = T/M;\ M$ is a number of intervals of the constancy of control variables; $w_i(t)$ is a control variable of one of the followers (B, P, C) or of the leader.

In the case when the leader maximizes a total payoff functional similar to (2.8), the problem (3.1) – (3.3) takes the following form:

- the leader's payoff function

$$J((s_{ik})_{\substack{k=1 \\ i \in N}}^{M}, (u_{ik})_{\substack{k=1 \\ i \in N}}^{M}) = \sum_{k=1}^{M} \sum_{i \in N} k_i (r_{ik} - u_{ik})^{p_i} (e^{-\rho t_{k-1}} - e^{-\rho t_k}) + \int_{0}^{T} e^{-\rho t} c x^p(t) dt +$$

$$e^{-\rho T} c x^p(T) \to \max; t_k = k\Delta t; \Delta t = T/M.$$
$$(3.8)$$

- the followers' payoff function

$$J_i((s_{ik})_{k=1}^{M}, (u_{ik})_{k=1}^{M}) = \sum_{k=1}^{M} (k_i (r_{ik} - u_{ik})^{p_i} (e^{-\rho t_{k-1}} - e^{-\rho t_k}) + \int_{0}^{T} e^{-\rho t} s_i(t) c x^p(t) dt +$$

$$e^{-\rho T} s_i(T) c x^p(T) \to \max; \quad i \in N$$
$$(3.9)$$

- the control constraints for the follower and the leader, respectively

$$0 \le u_{ik} \le r_{ik}, \quad i \in N; k = 1,2,...,M$$
$$(3.10)$$

$$0 \le s_{ik} \le 1, \sum_{i \in N} s_{ik} \quad i \in N; \ k = 1,2,...,M$$
$$(3.11)$$

Here $f_{ik} = f_i(k\Delta t), i \in N; f_{ik}$ is a function of time. A dynamics equation has the form

$$\frac{dx}{dt} = ax(t) + \sum_{i \in N} b_i u_i(t), \ x(0) = x_0$$
$$(3.12)$$

Notice that the multistep-differential model (3.8)— (3.12) is equivalent to the differential model (1.1) – (1.3), (2.8) in the following sense. If the set of functions $\left(u_i^*(\cdot), s_i^*(\cdot); i \in N\right)$ is the equilibrium of the differential model then due to the constancy of all control strategies at time periods

$$u_i^*(t) = \begin{cases} u_{i1}^*, & \text{if} \quad 0 \le t < t_1, \\ u_{i2}^*, & \text{if} \quad t_1 \le t < t_2, \\ \quad\quad ... \\ u_{iM}^*, & \text{if} \quad t_{M-1} \le t < T, \end{cases}$$

$$s_i^*(t) = \begin{cases} s_{i1}^*, & \text{if} \quad 0 \le t < t_1, \\ s_{i2}^*, & \text{if} \quad t_1 \le t < t_2, \\ \quad\quad ... \\ s_{iM}^*, & \text{if} \quad t_{M-1} \le t < T. \end{cases}$$

Then the set $\{u_{it}^*\}_{t=0}^{M}$, $\{s_{it}^*\}_{t=0}^{M}$, $i \in N$ gives the solution of the problem (3.8) – (3.12). In contrast to (1.1) – (1.3), the system (3.8) – (3.12) forms the problem of

optimization for functions of many variables (not functionals!) with consideration of the hierarchical relations between the control agents.

Thus, a solution of the problem (1.1) – (1.3) was reduced to the investigation of the model (3.8) – (3.12). Notice again that (3.8), (3.9) are objective functions (not functionals) depending on 2M variables which are maximized in M variables.

The following algorithm of solution of the problem (3.8) – (3.12) is proposed.

1. As a result of parametrical optimization of N functions of $2M$ variables (M parameters $\{s_{it}\}_{t=1}^{M}; i \in N$) (3.9) s. t. (3.10) in M variables $\{u_{it}\}_{t=1}^{M}$ the followers' optimal strategies depending on the leader's controls, i.e. values $\{u_{it}^{*}\}_{t=1}^{M} = \{u_{it}^{*}(s_{it})\}_{t=1}^{M}; i \in N$, are defined. The equation (3.12) is solved analytically or numerically by Runge-Kutta method.

2. The values found at the first stage of the algorithm $\{u_{it}^{*}\}_{t=1}^{M} = \{u_{it}^{*}(s_{it})\}_{t=1}^{M}; i \in N$ are substituted in (3.8). The values $\{s_{it}^{*}\}_{t=1}^{M}; i \in N$ which provide maximum (3.8) are optimal for the leader.

3. Let us cal the set $\left(\{s_{it}^{*}\}_{t=1}^{M}, \{u_{it}^{*}(s_{it}^{*})\}_{t=1}^{M}\right)$ an equilibrium in the Stakelberg game G_{1t}.

Implementation of the mentioned algorithm by simulation modeling is based on the method of direct ordered enumeration with a constant step (14) and consists of the following.

1. Type and values of input functions and the model's parameters are assigned.

2. Current leader's strategy, i.e. the grid function $\{s_{it}\}_{t=1}^{M}; i \in N$ is assigned.

3. By direct ordered enumeration of the followers' potential reactions to the current leader's strategy with a constant step the best followers' replies (grid functions $\{u_{it}^{*}(s_{it})\}_{t=1}^{M}$) are found. These replies provide maximization of objective functions (3.9) in the set of enumeration. The differential equation (3.12) is solved numerically by a finite differences method or Runge-Kutta method.

4. If the number of iterations for the leader is not exhausted then a new strategy is chosen by a new scenario examination or by perturbation of the current strategy. Then go to step 3.

5. Hence, an approximation to the system equilibrium is determined, i.e. the set of values $\left(\{s_{it}^{*}\}_{t=1}^{M}, \{u_{it}^{*}(s_{it}^{*})\}_{t=1}^{M}; i \in N\right)$.

When choosing a new current leader's or a follower's strategy methods of direct ordered enumeration with constant or variable step (Basar T. and Zhu Qu, 2011) can be used. In this case the interval of uncertainty is equal to $2\dfrac{b-a}{K+1}$, where K is a number of points of the partition of feasible controls domain of the

respective agent, a, b are boundaries of the domain. The error of determination

$$\varepsilon = \frac{r_0}{K+1}$$

of the leader's optimal strategies is .

Thus, in a finite number of iterations the proposed algorithm of simulation modeling permits to build an approximate solution of the model (1.1) — (1.3) or to conclude that the equilibrium does not exist. Credibility and efficiency of the algorithm follow from the respective properties of the method of direct ordered enumeration with a constant step during the simulation calculations.

1.4. Model calculations

Let us describe the results of numerical calculations for the linear $(p = 1)$ and power $(0 < p < 1)$ models for some typical test input data sets with the presence of the federal state as a control agent (hierarchical setup) and in its absence (independent setup).

Example 1 (without taking into account the federal state). Assume $\rho = 0.1;\ T = 5\,years$; $r_B = 175;\ r_P = 250; r_C = 150;\ k_B = k_P = k_C = 10$; $p_B = p_P = p_C = 0.1;\ b_B = b_P = b_C = 1/3;\ a = -0.05; x_0 = 420; c = 2;$ $s_B = 0.6;\ s_P = s_C = 0.2$. The results for the linear $(p = 1)$ and the power $(0 < p < 1)$ models are presented in Table 1. In what follows $u_i^{(k)}; i = C, P, B$ are the optimal strategies of the players in a cooperative case.

Example 2 (without taking into account the federal state). For input data from the Example 1 and $s_B = 0.9;\ s_P = s_C = 0.05$ the results are given in Table 2.

Example 3 (without taking into account the federal state). For input data from Example 1 and $s_B = 0.05;\ s_P = 0.05; s_C = 0.9$ the results are given in Table 3.

Example 4 (hierarchical setup). For input data from Example 1 (variables s_B, s_P, s_C are desired leader's controls) the results of calculations for the linear $(p = 1)$ and the power $(0 < p < 1)$ models are presented in Table 4. If lower level control agents cooperate then the hierarchy does not make sense.

Example 5 (hierarchical setup). For input data from Example 4 and $C = 0.003$ the numerical results are given in Table 5.

Example 6 (hierarchical setup). For input data from Example 4 and $k_P = k_C = k_B = 100$ the numerical results are given in Table 6.

Table 1. Results of calculations for Example 1

P	J_C	J_P	J_B	J	PA
1	1547	1547	4548	7667	0.997
0.9	806	806	2314	3958	0.992
0.7	242	242	605	1127	0.966
0.5	105	107	181	416	0.966
0.3	75	79	98	253	0.996
0.1	68	71	75	215	0.996

Table 2. Results of calculations for Example 2

P	J_C	J_P	J_B	J	PA
1	419	419	6657	7667	0.977
0.9	237	237	3299	3958	0.953
0.7	103	104	749	1127	0.848
0.5	74	78	235	416	0.93
0.3	67	70	114	253	0.992
0.1	65	69	80	215	0.95

Table 3. Results of calculations for Example 3

P	J_C	J_P	J_B	J	PA
1	6657	419	419	7667	0.977
0.9	2297	237	237	3958	0.70
0.7	731	104	103	1127	0.832
0.5	232	78	75	416	0.925
0.3	113	71	68	253	0.996
0.1	79	69	66	215	0.995

Table 4. Results of calculations for Example 4

P	J_C	J_P	J_B	J_0
1	2551	2551	2551	7667
0.9	1315	1315	1315	3958
0.7	370	370	370	1127
0.5	160	86	160	406
0.3	89	73	90	252
0.1	70	73	71	215

Table 5. Results of calculations for Example 5

P	J_C	J_P	J_B	J_0
1	67	70	67	204
0.9	66	69	66	201
0.7	65	68	66	200
0.5	65	68	65	200
0.3	65	68	65	199
0.1	65	68	65	199

Table 6. Results of calculations for Example 6

P	J_C	J_P	J_B	J_0
1	2899	2899	2899	8856
0.9	1697	1697	1697	5221
0.7	851	875	861	2674
0.5	707	741	717	2166
0.3	667	701	677	2045
0.1	654	688	664	2008

The following conclusions can be done based on the conducted calculations:

In the model setup for a sufficiently representative class of input data the interests of agents are well compatible, the price of anarchy is close to one, and hierarchical control is not required.

If lower control level agents cooperate then the leader is not at all required due to the input functions of the model.

If a share in the public good of one player is equal to one then the price of anarchy with a growth of power decreases and tends to one when p tends to zero.

If the shares in public good of all players are approximately equal then the price of anarchy is close to one and depending on index of power p varies from 0.95 to 0.99.

When the coefficient of transition from a level of professional training to the public utility increases, the payoffs of all players also increase.

When the coefficents increase, it is profitable to the players to extend investments in their private activity.

In contrast, when the coefficient of transition from a level of professional training to the public utility increases, it is profitable to make more investments in the public good.

When p increases, the payoffs of all players also increase and obtain the maximal value in the case of linear model of transition from a level of professional training to the public utility.

Conclusion

A differential game theoretic model of the social partnership in the CPE system is built. A linear state model and a model with power payoff functions are examined. Nash equilibria, team solution, and hierarchical solution for the game G1t are found.

Some preliminary conclusions presented above are made on the basis of analytical and numerical calculations results analysis. Notice the main idea: in this model setup interests of different agents are well compatible, the price of anarchy is close to one, and there is no need in additional control levels. A transition from a linear state model to the power one does not change the optimal strategies, only payoffs change.

Certainly, the results of analysis of the test examples are quite conventional but allows for a qualitative comparison of different methods of organization of the social partnership in the CPE system. In future, it is planned to try the proposed approach using opinion poll findings.

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