Asymptote Misconception on Graphing Functions: Does Graphing Software Resolve It?

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ABSTRACT

Graphing function is an important issue in mathematics education due to its use in various areas of mathematics and its potential roles for students to enhance learning mathematics. The use of some graphing software assists students' learning during graphing functions. However, the display of graphs of functions that students sketched by hand may be relatively different when compared to the correct forms sketched using graphing software. The possible misleading effects of this situation brought a discussion of a misconception (asymptote misconception) on graphing functions. The purpose of this study is two- fold. First of all, this study investigated whether using graphing software (GeoGebra in this case) helps students to determine and resolve this misconception in calculus classrooms. Second, the reasons for this misconception are sought. The multiple case study was utilized in this study. University students in two calculus classrooms who received instructions with (35 students) or without GeoGebra assisted instructions (32 students) were compared according to whether they fell into this misconception on graphing basic functions (1/x, lnx, ex). In addition, students were interviewed to reveal the reasons behind this misconception. Data were analyzed by means of descriptive and content analysis methods. The findings indicated that those who received GeoGebra assisted instruction were better in resolving it. In addition, the reasons behind this misconception were found to be teacher-based, exam-based and some other factors.

Keywords: *Asymptote Misconception, GeoGebra, Graphing Functions*

INTRODUCTION

Functions play a crucial role during and after the late-elementary school years. They are used in many areas of mathematics education including word problems, analytic geometry (Nachlieli & Tabach, 2012), derivatives, and integrations (Thompson, Byerley, & Hatfield, 2013). In many cases, teachers present the graph of any function to visualize it and interpret its behavior under different circumstances. Some of the functions (such as $\frac{1}{x}$, $\ln x$, e^x) are considered as basic ones especially after late high school education and both teachers and students can graph them on paper by rote (Haciomeroglu & Andreasen, 2013) without wasting time.

In mathematics education, technology use is widespread in recent years. Various software, applications, and animations are used by teachers or proposed for use by mathematics educators in the classroom in order to visualize the mathematical contents (Hohenwarter, 2006) and improve students' mathematical thinking and conceptual understanding (Özgün-Koca & Meagher, 2012). Among them, the dynamic mathematics software developed purposively for learning mathematics give students opportunities to communicate and explore the relations among mathematical concepts and reason about them (Akkaya,

Tatar, & Kağızmanlı, 2011). In addition, students can visually observe the instant effects of the changes made (e.g., the effect of changing constants of a function) while working in such environment due to their dynamic feature. GeoGebra is one of these dynamic mathematics environments. Since this software elicits the relation between the algebraic and geometric forms of mathematical concepts (Haciomeroglu & Andreasen, 2013), it can be used in different grade levels from primary school to university mathematics classrooms. Therefore, this software is a useful tool for teaching and learning the graphs of the functions and is a remedy for misunderstanding in this topic.

This paper emerged due to students' rote memorization of graphing function. In this study, using GeoGebra in a calculus course to observe the behavior of basic functions made an inspiration for the researcher to discover a misconception for graphing functions. Studying the graph of a logarithmic function (ln x in that case) on GeoGebra, a student noticed how fast the graph was approaching to y-axis as the value of x approaches to 0^+ by stating that "the left arm of graph is *very very* close y-axis just after x is smaller than -2". He added that "the arms of this function is not as close as it is on GeoGebra in my sketches". This situation was considered to be a new emerging misconception on graphing functions, especially for the basic ones (e.g., $\frac{1}{x}$, $\ln x$, e^x).

With this respect the purpose of this study is to investigate students' misconception while sketching graphs of functions by rote, what is called in this study *the asymptote misconception* on graphing functions. Based on the function's characteristics, this misconception occurs if the arm/arms of the function are approaching to any asymptotes and the arm/arms sketched are relatively far from such asymptote. That means in its operational definition that if students have problems in approaching the arms of functions to the asymptotes during the sketching process, this cognition was considered as *asymptote misconception*.

Since this study was related to the graphs of basic functions ($\frac{1}{x}$, $\ln x$, e^x), the focus was on the arms of functions approaching to the linear (horizontal and vertical) asymptotes.

LITERATURE REVIEW

The idea of using graphing technology in mathematics learning and teaching is widespread. In various domains of mathematics including calculus (Lauten, Graham, & Ferrini-Mundy, 1994) and representation of algebraic equations (Erbas, Ince, & Kaya, 2015), graphing technologies are widely used in mathematics classrooms. In addition, the present literature and international curricula support the use of technologies in learning mathematics (e.g., Common Core State Standards Initiative [CCSSI], 2010; Heid, Thomas, & Zbiek, 2013; National Council of Teachers of Mathematics [NCTM], 2000). For example, one of the six principles in the principles and standards for school mathematics is the technology principle indicating that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). Considering that "…graphing utilities facilitate the exploration of characteristics of classes of function" (NCTM, 2000, p. 27), it would be better for students to deal with the interactive or dynamic environment while investigating the behavior of the functions on the graphs sketched (Hohenwarter, 2006).

Students may have misconceptions about sketching graphs of functions. As known, the misconceptions are considered to be incorrect conceptions and many students are prone to perceiving them as correct (Koray & Bal, 2002). The main characteristics of misconceptions are that students construct alternative definitions for the concepts studied; many of them believe in them as scientific fact, and it is very hard to change such incorrect beliefs (Fisher, 1985).

Especially in mathematics education, various misconceptions related to different topics including graphing functions. For example, Clement (1985) mentioned about two types of misconceptions about graphs of functions. These are "treating the graph as a picture" and "slope-height confusion", the latter of which is somewhat related to the present study. Similarly, Glazer (2011) emphasized that the display of graphs of functions is of great importance for interpreting them by discussing which one of creating or

interpreting a graph is more crucial. Considering these facts, students' misconceptions might be strengthened with inappropriate use of graphs in high school and university textbooks (Kajander & Lovric, 2009), role of teachers' pedagogical content knowledge during instructions (Rubel, 2002) and their use of inappropriate visual materials (Mudaly & Rampersad, 2010), and inconsistencies in students' mind while they are dealing with interrelated mathematical concepts (Tall, 1990). Taking them into consideration, the traditional instructional methods and teachers' role in teaching activities during such instructions may foster misconceptions in students' cognitions (Marek, Cowan, & Cavallo, 1994; Ubuz, 1999). Combination of these reasons may influence students' understanding and become a serious obstacle to learning.

The mathematics education researchers investigated the ways of remedying or at least diminishing negative effects of emerging misconceptions (e.g., Ellis & Grinstead, 2008; Straesser, 2001). For graphing functions, studying the problems in interactive platforms including GeoGebra, Sketchpad or graphing calculators are seen as one of the effective ways of doing so (Koklu & Topcu, 2012). If teachers use graphing software (such as GeoGebra) effectively, it is possible to deal with possible arising misconceptions related to graphing functions (Heid, Thomas, & Zbiek, 2013). This is because such platforms and the effective use of graphing software provide advantages in student learning. Some of the advantages of graphing software are "representing mathematical objects from multiple perspectives, examining and/or exploring mathematical relationships in depth, experimenting with different approaches to problem solving, forming and testing conjectures and questioning and critical thinking" (Koklu & Topcu, 2012, p. 1000).

As a dynamic mathematics environment, the effect of using GeoGebra in math classrooms was investigated by many researchers (e.g., Akkaya, Tatar, & Kağızmanlı, 2011; Aydos, 2015; Carter & Ferrucci, 2009; Dikovic, 2009; Haciomeroglu & Andreasen, 2013; Hutkemri & Zakaria, 2012; Koklu & Topcu, 2012). For example, Akkaya, Tatar, and Kağızmanlı (2011) compared the effects of instruction with GeoGebra and traditional instruction for the case of teaching trigonometry. This study revealed that achievement levels of students receiving instruction with Geogebra were higher than those taught via traditional method. Similarly, Aydos (2015) studied gifted students' conceptual understanding of limit and continuity. The study mentioned the importance of the sketching and interpreting the graph of functions for increasing the conceptual understanding of such topics. In this experimental study, the students taught with Geogebra were more successful in the test measuring the conceptual understanding of limit and continuity. In addition, Carter and Ferrucci (2009) indicated that Geogebra was a helpful tool for enhancing learning and understanding the geometry topics including sketching graphs after investigating the prospective mathematics' teachers' understanding of geometry by means of Geogebra. Another study was about high school students' conceptual and procedural knowledge of functions after teaching the topic via Geogebra (Hutkemri & Zakaria, 2012). It was concluded that visual representations of the functions played a key role in improving conceptual knowledge of functions and Geogebra is a beneficial interactive environment for doing so.

Taking this suggestion into account, enhancing conceptual understanding of functions might hinder the possibility of asymptote misconception and other misconceptions related to graphs of functions. Moreover, the aid of graphing technology may diminish their negative effects. Therefore, students do not encounter further problems in understanding the relation between a function and its graph, the meaning of graphs of functions, and interpreting them.

This paper gives answers to the following questions:

1- Is there any difference in remedying students' asymptote misconception between calculus classrooms with and without GeoGebra assisted teaching environment?

2- What are the reasons for students' asymptote misconception while sketching graphs of basic functions?

METHODOLOGY

The present study is a qualitative case study investigating students' asymptote misconception and its possible reasons. With the case study design, one or more cases including a person, a clique or a group of people are examined deeply within their holistic constraints (McMillan & Schumacher, 2006). Creswell (2007)

indicates that the researcher can investigate the cases studied through *in-depth data collection* (p. 97) with multiple data sources. With its nature, this study is a multiple case study (Yıldırım & Şimşek, 2008) investigating whether there is a difference in remedying asymptote misconception with or without GeoGebra assisted instructions. Therefore, there are two cases, first of which was the students in calculus classroom receiving GeoGebra assisted instruction. The latter one is those who did not receive GeoGebra assisted instruction in calculus lesson.

Participants

In order to investigate the difference in calculus classrooms with and without GeoGebra assisted instruction, the participants were selected from two calculus classrooms. Students in these classrooms were generally second graders of a public university located in the eastern part of Turkey. The convenience sampling method was utilized in order to determine the participants of the study (Yıldırım & Şimşek, 2008).

The lecturer of one chosen calculus classroom preferred to use GeoGebra in some parts of his lessons. Although he was not strictly dependent on using GeoGebra, he used it throughout the semester. He consulted the software at least once per block scheduled lesson (merging two lessons). He used this software when there was a need for visual stimulant related to content studied during the lessons. For example, he visualized the function of max-min problem or investigated the limit of function on graph. Therefore, students attained familiarity with the correct form of any given functions. On the other hand, the lecturer of the other chosen calculus classroom did not use any graphing software or physical material to show the graphs of functions. Instead, he sketched the graphs on the board by hand. In both classrooms, the lecturers did not emphasize this misconception to students. There were 35 students enrolled in the former classroom (instruction with Geogebra aid classroom [Iw/G]); some 32 students were in the latter (control group classroom [CGC] with regular instruction implemented).

Data Collection Procedures and Tools

The calculus course contents included the limits of functions, derivative, integration, and their applications. All participants have taken the pre-calculus course providing knowledge for pre-requisites of the content of the calculus course. The course was given in different sections in the faculty of education in a public university. Before enrolling in the calculus course, it was assumed that students were taught several functions and their graphics including basic functions such as $\frac{1}{x}$, $\ln x$, e^x during their high school education. This is because they graduated from the science and mathematics track in high schools and took nationwide university entrance exam in science and mathematics, which include tests from geometry and pre-calculus mathematics.

Both classrooms followed the same curriculum. However, the difference was that the lecturer in the former classroom utilized GeoGebra in solving questions, sketching the graphs when necessary, explaining the meaning of the graphs sketched by using properties of GeoGebra. The lecturer in the latter classroom, on the other hand, presented the same content by solving questions or sketching graphs on the board.

The data were collected from two data sources. First of all, all students were expected to sketch graphs of functions of $\frac{1}{x}$, $\ln x$, e^x on a grid paper provided and to explain how they sketched them. Secondly, three students from each calculus classrooms were interviewed about sketching the graphs, possible difficulties encountered and the reasons for asymptote misconception. For both data collection tools, expert opinion was gathered from two lecturers who had experience in teaching calculus courses.

Data Analysis

Both data sources were analyzed according to the content analysis method (Creswell, 2007). Descriptive analysis was also used and frequency and percentage tables were provided to enrich findings. For the first data source, two researchers classified all students' graphs sketched under three categories by scoring them as incorrect, shape is correct (asymptote misconception), and correct. Answers indicating correct and asymptote misconception sketches were evaluated according to reference points that students

indicated. For example, for those who sketched the graph of $\ln x$, the reference point was (1, 0) (*ln1* = 0). How fast the left arm of the function approached to y-axis was compared according to distance from y-axis to the reference point (1, 0).

For frequency table, inter-rater reliability was found to be 91%, which is inside the acceptable level (Marques & McCall, 2005). (The inter-rater reliability was calculated according to the formula "total agreement"/ "total observations") The table is presented according to final consensus over all codes. The interviews were analyzed to find the reasons for asymptote misconception. First of all, students were asked to sketch the graphs. In case they fell into asymptote misconception, they were provided with the correct forms of the graphs on GeoGebra and the interviewer emphasized how fast the arms of the functions approached to the axes. Then, the reasons that students indicated were supported with direct quotations from their statements during the interviews.

<div align="right">Findings</div>

Findings related to Influence of Using GeoGebra in Remedying Asymptote Misconception

Both classrooms encountered the graphs of $\frac{1}{x}$, $\ln x$, and e^{x} during the calculus instructions. The difference was that the Iw/G observed these functions on GeoGebra software while CGC observed them on the board sketched by the lecturer. The correct forms sketched on GeoGebra are given in the following Figure 1. It was observed that the arms of the functions are rapidly approaching to the axes and asymptotes.
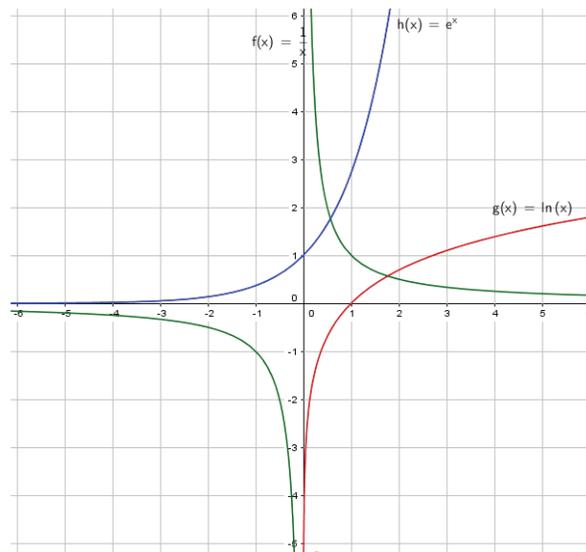

Figure 1. Graphs of functions $\frac{1}{x}$, $\ln x$**, and** e^{x} **sketched on Geogebra.**

As well seen in Figure 1, functions approach the x- and y-axes very fast as x approaches to plus or minus infinity or 0^{+}. On the other hand, answers of students who fell into asymptote misconceptions were shown in the Figure 2, 3, and 4.
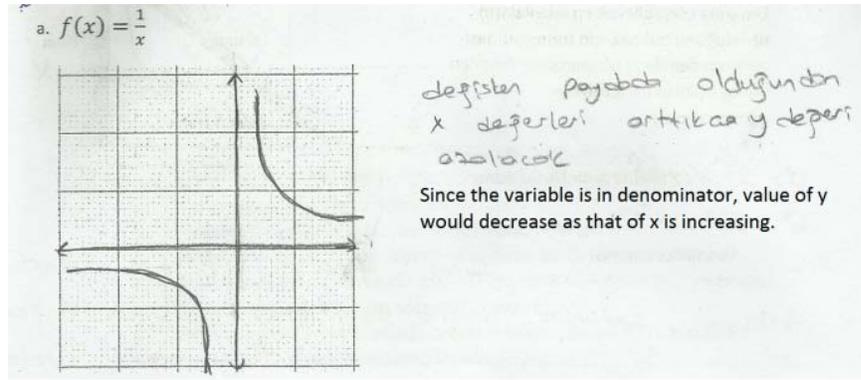
Figure 2. Students' answers indicating asymptote misconceptions for the function of $\dfrac{1}{x}$
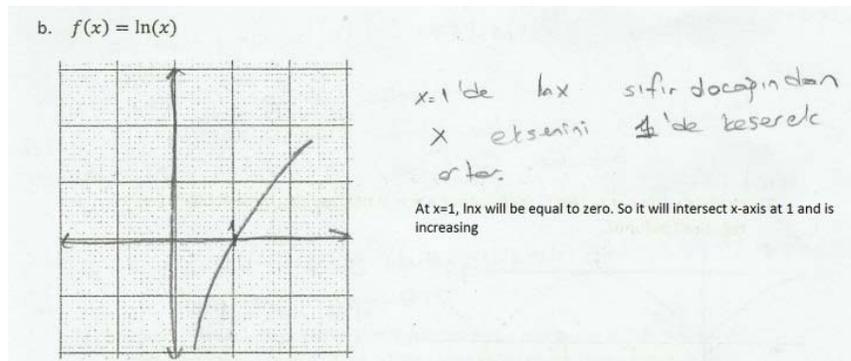

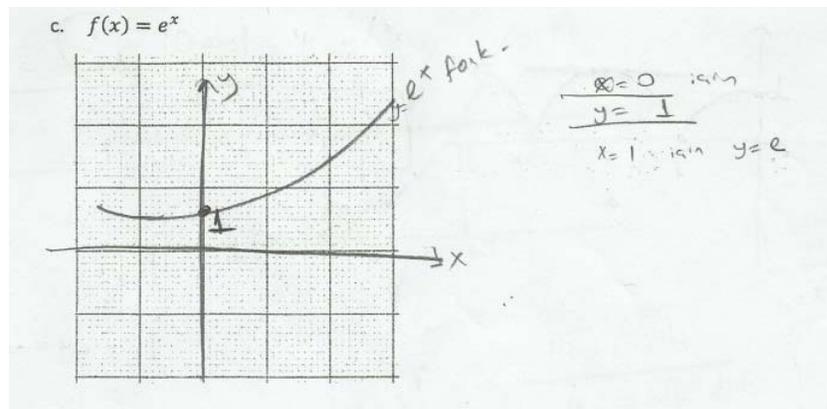Figure 3. Students' answers indicating asymptote misconception for the function of $\ln x$

Figure 4. Students' answers indicating asymptote misconception for the function of e^{x}

According to reference points that the students indicated (for example (1, 0) for $\ln x$ and (0,1) for e^{x}), the graphs sketched by students were not approaching to the axes as fast as the correct forms of the graphs of the functions shown in Figure 1. To give answer to the first research question, Table 1 showed the frequencies and percentages of whether students correctly sketched the graphs of given functions, fell into asymptote misconception while sketching them, or incorrectly sketched them.

Table 1. Comparison of students' asymptote misconception on graphing functions in calculus classrooms with and without GeoGebra assisted instruction

	Graph of $\dfrac{1}{x}$		Graph of $\ln x$		Graph of e^x	
	Iw/G (n=35)	CGC (n=32)	Iw/G (n=35)	CGC (n=32)	Iw/G (n=35)	CGC (n=32)
Incorrect	4* (11.4)**	5 (15.6)	8 (22.9)	8 (25.0)	15 (42.9)	13 (40.6)
Shape is correct (*Asymptote Misconception*)	16 (45.7)	16 (50.0)	13 (37.1)	17 (53.1)	8 (22.9)	9 (28.1)
Correct	15 (42.9)	11 (34.4)	12 (34.3)	7 (21.9)	10 (28.6)	7 (21.9)
No Answer	- -	- -	2 (5.7)	- -	2 (5.7)	3 (9.4)

* Frequency of students for the intended answer

** Percentage of students for the intended answer. Rounded to first decimal place.

As seen in Table 1, the percentages of students who fell into asymptote misconception are high both in Iw/G and CGC. However, the percentages of students who fell into asymptote misconception are slightly lower for all types of functions for the Iw/G group. In addition, percentages of students who correctly sketched the functions are slightly higher in Iw/G. One interesting finding is that students experienced difficulty in correctly sketching the graph of e^x. Highest difference was observed in the percentages of students who fell into asymptote misconception while sketching ln (x) where 13 students in Iw/G fell into asymptote misconception for this question. Comparing this finding with students in CGC, the number of students sharply increased to 17, which was more than half of all students in CGC.

These findings indicated that using GeoGebra in teaching the topic had a limited effect on resolving students' asymptote misconception. This is because it was observed that the correct answers in the GeoGebra classroom were slightly higher than those in the other classroom. Moreover, there was a small difference between students' answers indicating the misconception in Geogebra and control group classrooms. It should be pointed out that students in the GeoGebra classroom achieved better scores according to the findings summarized in Table 1. Keeping in mind its limited effect, students could observe the functions on GeoGebra and correctly sketch, therefore, interpreting correctly the basic functions such as $\dfrac{1}{x}$, $\ln x$, and e^x. This limited effect could be increased over students if the lecturer emphasized the behavior of functions on arms of their graphs. The following transcript of the interview with Aylin (pseudonym) having asymptote misconception in CGC shows this situation clearly.

Interviewer   : *Can you sketch the graph of ln(x)?* (Grid paper is provided for student)

Aylin   : (While sketching graph). *Well, first of all, I need to show the intersection point with x-axis, which is (1, 0). Because when x is equal to 1, ln1 is equal to 0. If the x goes to infinity, so does* $\ln x$ *. And the other arm of function approaches to y-axis.*

Interviewer   : *Ok. Your sketch is somewhat correct, but you have little problem on it.*

Aylin   : *Umm… I think it is correct. It intersects the x-axis at point (1, 0) and is going to minus infinity when x is approaching to zero. I think it is totally correct.*

Interviewer   : *Do you want to see the graph on graphing software, for example GeoGebra?*

Aylin   : *I wondered about it? Yes. Please show it.* (After observing the graph on GeoGebra). *Umm. It is similar to my sketch. … Hmm… Wait. It is very close to y-axis. Something like they* (left arm of function and y-axis) *are almost united. … And, the left side is getting very flatty.*

This transcript showed how important it is to emphasize the behavior of functions on any graphing software when teaching the behavior of graph of functions, especially for those having asymptotes. This student could not realize the approaching behavior of the function in the first place. After analyzing the function on GeoGebra in detail, she realized her asymptote misconception.

Findings related to Possible Reasons for the Asymptote Misconception

The interviews with students continued to discuss the possible reasons for this misconception. These interviews helped the researchers to present answers for the second research question. Based on interview findings, they can be grouped under the main themes such as teacher-related, exam-related, and some other miscellaneous reasons. Table 2 presents these reasons and respective quotations from students' explanations during the interviews.

Table 2. Reasons for asymptote misconception from students' points of view.

Main Categories	Related Reasons	Quotations from Interviews with Students
Teacher-Related	Teachers' Misconception	*…Like us, I think they did not also realize how fast they* (graphs of functions) *were approaching to the axes. Most probably, they did not know it* (asymptote misconception) *either…*
	Time Taking	*…Sketching the graphs with original shape similar to computer sketches takes too much time. In the paper* (test applied to students for graphing functions before interviews), *I tried and it took much time. I think teachers do not want to spend such much time for sketching any graph…*
	Belief that Students do not understand	*…Maybe teachers did not want students to get confusion in mind. Instead, I mean, they are sketching them* (graphs of function) *roughly…*
	Not giving attention	*…They were roughly sketching the graphs. They were not paying attention to sketch as similar to GeoGebra does…*
Exam-Related	Nation-wide university exam	*…we were dealing with questions instead of the details of the graphs during high school education. We needed to solve as many questions as possible. So, it was enough for us to see the graphs sketchy…*
	Regular exams	*…Our focus was on the solution of the question* (during high school education). *In the practice papers and tests, we did not look whether the graph was close to axes or not. It is not useful for us to solve questions…*
Miscellaneous	Sketching by rote	*…Neither we nor teachers applied the basic steps on sketching graphs of function. If we knew where the function approaches as x goes to minus or plus infinity, or if we knew the shape of it roughly, we would sketch it by rote. It is a kind of hand habit…*
	Sketching based only on a few points	*…I'm giving some value to x and defining the points that the function pass through. Then, I'm guessing what shape the graph takes…*

According to the findings gathered from the data sources, it could be concluded that there were many possible reasons for the misconception. In general, both teachers and students ignored to sketch the graphs

as correct as GeoGebra did. Instead, they all preferred to sketch the graphs roughly. In addition, students generally referred to the high school teachers and high school education for this misconception.

Lastly, one student gave much attention to sketching the graphs correctly. The researcher asked the reason for her attention. She gave the following answer.

It is possible that teachers in high school cope with details in sketching graphs. But I will be a mathematician, I have to know them. I have to interpret the graphs. I have to know the critical points and steps of any topic in mathematics.

CONCLUSION & DISCUSSION

Based on the findings of the study, students were prone to fall into asymptote misconception. Considering students' incorrect answers exhibiting the asymptote misconception for the functions of $\frac{1}{x}$, $\ln x$ and e^x, it was observed that they roughly sketched them. They did not even give detailed explanations for their sketches or provide information about how to sketch the graphs. Their sketches (e.g., Figure 2, 3 and 4) gave evidence that they constructed an alternative belief and definition that the sketches of graphs of functions are correct if their displays are roughly similar (Fisher, 1985), which was a misconception. Alternative belief in this case was that sketching the graphs by rote represented the correct form for the indicated function. However, the graphs of functions were distinguished when compared to their correct forms as shown in Figure 1. The main concern for this misconception was that it may result in students' misinterpretation of graph of function (Glazer, 2011), misleading effect of which might cause new misconceptions for further learning due to their widespread use in many areas of mathematics (Nachlieli & Tabach, 2012). Considering the NCTM (2000) technology principle, GeoGebra is a useful platform for students to study graphs of functions to diminish the misleading effect of this misconception, because it enhances students' understanding and explorations of the functions due to its dynamic feature (Hohenwarter, 2006).

Table 1 indicated that instruction with Geogebra in the calculus classroom had slight positive effect on students' correct sketches of the graphs of functions and remedying this misconception. During the instructions, the focus was not on investigating the behavior of functions. For example, the lecturers did not emphasize how the arms of investigated functions (e.g., $\ln x$) approach to the x- and y- axes as x approaches to minus or plus infinity. However, students in the Iw/G classroom had opportunities to observe the correct form of the functions and their behaviors in a dynamic environment and explore the characteristics of functions including their graphs (NCTM, 2000). Considering the slight effect of using GeoGebra without focusing on the behavior of functions, the teachers or lecturers should beware of the presence of asymptote misconception and make an effort to resolve it with graphing software (such as Geogebra) in order not to encounter new misconceptions fostered by it in the further learning of mathematics (Heid, Thomas, & Zbiek, 2013).

Teachers' emphases on resolving this misconception served the purpose. As an interactive platform, it is known that GeoGebra helps students to construct relations between algebraic and geometric representations of mathematical concepts (Haciomeroglu & Andreasen, 2013) and conceive the relations among them (Koklu & Topcu, 2012). In this study, it was observed during the interviews with students that focusing on this misconception by using GeoGebra provided them with better understanding of correct forms of the functions. Interviews with students who had such misconceptions (e.g., interview with Aylin) showed that emphasizing the asymptote misconception by means of GeoGebra helped them to resolve the misconception.

Students' responses to the interview questions revealed the possible reasons for the asymptote misconception. The possible reasons in line with students' opinions were grouped under three categories including teacher-based, exam-based and other miscellaneous reasons. These findings were supported by the Kajander and Lovric (2009) and Rubel (2002) studies. Considering the possible negative effects on analyzing and interpreting the graphs of functions for students' further learning, teachers (or lecturers) may

pay attention to resolve the appearing misconceptions (including asymptote misconception and other possible ones related to graphing functions) on time (Heid, Thomas, & Zbiek, 2013). For example, Clement (1985) mentioned about students' possible confusion in interpreting the correct graphs of speed-time and word problems.

Based on students' opinions, this misconception appeared among students mainly for teacher-related reasons. Some students thought that teachers also had this misconception and applied the incorrect sketches during teaching activities. Before teaching the mathematical content in the classroom, teachers had various misconceptions and incorrect beliefs because of their past learning experiences and tended to bring them into their classroom instructional practices (Yanık, 2011). This might result in sketching the graphs by rote, which was another reason for asymptote misconception. In this case, teachers should be open to new ideas for improving their teaching skills. Considering the graphing graphs topic, using graphing software (such as GeoGebra) in classrooms may inspire teachers' teaching methods and diminish the possibility of misconception occurrence, because both teachers and students would see the graphs of functions in the correct form.

Some students thought that although teachers were aware of this misconception, they thought that graphing function is a time-consuming activity, and did not give enough attention and sketched the graphs roughly in the classroom. This was because they had to give attention to the contents taught instead of the graphs of functions. However, it was evident that the incorrect form of graphs of functions might result in students' misinterpretation of the graphs of functions and influence the understanding of the topic studied or topics in further learning (Carter & Ferrucci, 2009). With correct and well-organized use of graphing software (such as GeoGebra) in classrooms, the possibility of such misinterpretations could be diminished and correct cognition could be maintained due to software features enabling visualization of the functions and the dynamic environment for constructing relations between algebraic and geometric forms (Hohenwarter, 2006; Koklu & Topcu, 2012).

The nationwide university exams and grade level exams were considered to be another reason for this misconception. Students thought that their focus was on solving the questions instead of dealing with sketching the exact form of the graphs of functions. The key point was that teachers were prone to accepting their sketches indicating this misconception as correct especially in regular math exams. At this point, teachers should beware of this misconception and emphasize the correct form to students.

One interesting finding was that students experienced difficulty in sketching the graph of e^x in both classrooms although they encountered it during lessons and in textbooks. In fact, one of the purposes of this study was to examine the reasons for a particular misconception (asymptote misconception). However, this finding showed that there was a need for deep investigation to study the reasons (e.g., cognitive or readiness reasons) for students' inability to sketch this and other particular functions. This is because even university level students encountered difficulty in sketching a basic function (e^x in this case).

This study specifically investigated students' asymptote misconception on particular functions. The focus was only on students' asymptote misconception during graphing basic functions of $\frac{1}{x}$, $\ln x$, and e^x. Considering that their asymptotes were either horizontal or vertical, the findings of this study were shaped accordingly. This brought a discussion on whether investigating asymptote misconception on different functions with other asymptote types may reveal new findings and implications. Therefore, the research can be extended to deeper investigation of graph-related misconceptions including the asymptote misconception for other functions with horizontal, vertical and oblique asymptotes.

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