International Education Studies; Vol. 10, No. 1; 2017
ISSN 1913-9020 E-ISSN 1913-9039
Published by Canadian Center of Science and Education

Thinking Process of Naive Problem Solvers to Solve Mathematical Problems

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Received: April 6, 2016 Accepted: May 8, 2016 Online Published: December 24, 2016

doi:10.5539/ies.v10n1p1 URL: http://dx.doi.org/10.5539/ies.v10n1p1

Abstract

Solving problem is not only a goal of mathematical learning. Students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations by learning to solve problems. In fact, there were students who had difficulty in solving problems. The students were naive problem solvers. This research aimed to describe the thinking process of naive problem solvers based on heuristic of Polya. The researcher gave two problems to students at grade XI from one of high schools in Palangka Raya, Indonesia. The research subjects were two students with problem solving scores of 0 or 1 for both problems (naive problem solvers). The score was determined by using a holistic rubric with maximum score of 4. Each subject was interviewed by the researcher separately based on the subject's solution. The results showed that the naive problem solvers read the problems for several times in order to understand them. The naive problem solvers could determine the known and the unknown if they were written in the problems. However, they faced difficulties when the information in the problems should be processed in their minds_to construct a mental image. The naive problem solvers were also failed to make an appropriate plan because they did not have a problem solving schema. The schema was constructed by the understanding of the problems, conceptual and procedural knowledge of the relevant concepts, knowledge of problem solving strategies, and previous experiences in solving isomorphic problems.

Keywords: problem solving, mathematical problem, naive problem solver, thinking process

1. Introduction

Problem solving is the goal of students in learning mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Students learn mathematical knowledge (facts, concepts, procedures, or principles) in order to solve mathematical problems. Problem itself is an unfamiliar situation, challenging but is still required to solve which a means of solution is not immediately visible to the students (Shumway, 1980; Sakshaug, M. Olson, & J. Olson, 2002; Krulik, Rudnick, & Milou, 2003). Solving problems provides students to use mathematical concepts meaningfully (Marzano, Pickering, & McTighe, 1993). When the students learn the concepts but do not use them meaningfully, it makes the concepts unrelated to the prior knowledge existing in their minds. On the contrary, the meaningful concepts retain longer in the students' minds (Skemp, 1982).

Problem solving also encourages students to have high order thinking skills (NCTM, 2000; King, Goodson, & Rohani, 2016). In general, Krulik et al. (2003) divide the thinking skills into low and high order thinking skills. Low order thinking skills are recall and basic thinking. Meanwhile, high order thinking skills are critical and creative thinking. Critical thinking is the ability to collect, organize, remember, and analyze information used to solve mathematical problems (Siswono, 2008; King et al., 2016). Creative thinking involves synthesis and development of ideas to solve problems, and come up with some answers or new means of solutions. Therefore, students with higher order thinking skills should have an ability to solve mathematical problems, and achieve the goal of learning mathematics.

Furthermore, NCTM (2000) states that students acquire ways of thinking, and positive attitudes by learning to solve mathematical problems. The attitudes are habits of persistence, and confidences in unfamiliar situations. Mathematical problems itself can be defined as unfamiliar situations. The attitudes are factors affecting students' ability to solve the problems (Lerch, 2004; Pimta, Tayruakham, & Nuangchalerm, 2009). Students who have the attitudes to solve mathematical problems are also expected to have the attitudes to solve everyday life problems.

Students' abilities to solve mathematical problems can be classified as naive, routine and good/sophisticated problem solvers (Muir, Beswick, & Williamson, 2008). The abilities are determined by using a problem solving rubric. The rubric score is 0 to 4 (Charles, Lester, & O'Daffer, 1997; Bush & Greer, 1999). The score can be connected to the classification of abilities to solve problems. The students with score of 0 or 1 can be classified as naive problem solvers. The students with score of 2 or 3 are routine problem solvers. Finally, the students with score 4 are good problem solvers.

Teachers expect their students to be good problem solvers. In fact, most of students were naive or routine problem solvers. The researcher gave two mathematical problems to 124 students from one high school in Palangka Raya, Central Kalimantan, Indonesia in November 2015. The result showed that 43% of the students got score of 0 or 1 (naive problem solvers), 51% of the students got score of 2 or 3 (routine problem solvers), and 6% of the students got score of 4 (good problem solvers). Thus, 94% of the students were not good problem solvers.

This condition is required to solve. The teacher should improve students' abilities to solve problems especially for who are classified as naive problem solvers. The first step of the solution is to understand the students' thinking process to solve problems. The process can be described in problem solving phases. What do the students think when they understand the problem, devise plan, carry out the plan, and look back? The second step is the teacher develops learning plans aimed to improve the abilities of the naive and routine problem solvers.

Problem solving itself is defined as thinking aimed to obtain the answers of problems (Shumway, 1980; Polya, 1981; Solso, 1995; Krulik et al., 2003). The first term requiring to be explained from the definition is "thinking". Thinking is an internal process occurring in the students' minds involving some manipulation of knowledge in cognitive systems (Solso, 1995). Although the process occurs in minds, it can be inferred from external representations generated by the students. The representations can be in the forms of written, verbal language (words/phrases), or gestures.

Based on the previous definition, problem solving is thinking. Since thinking itself is a process, so problem solving also can be defined as a process. Therefore, students' thinking process to solve problems is more important than the answers. A learning implication is teachers should pay more attention to how the students' thinking process works to solve the problems than the answers. Accordingly, the teachers should not only ask the students' answers but also confirm their thinking process to get the answers.

The second term is "answer". It is something obtained at the end of problem solving process. A process to arrive at the answer from the beginning is called "solution". Thus, the answer and solution of problem are two different terms.

One of theories explaining phases to solve mathematical problems is heuristic of Polya. The heuristic is understanding problems, devising plans, carrying out the plans, and looking back (Polya, 1973, 1981; Schoenfeld, 1985; Krulik et al., 2003). Understanding problems involves two stages. First, the students pay attention to relevant information by ignoring irrelevant information. Second, the students determine how to represent the problems in concrete objects. The effective ways can be in the forms of providing symbols, tables, matrices, hierarchical tree diagrams, graphs, or pictures (Matlin, 1994).

Students can devise plans if they understand the problems, and have an appropriate problem solving schema. The schema is constructed by an understanding of the problems, relevant mathematical knowledge, previous experiences to solve problems, and knowledge of problem solving approaches and strategies. Mathematical Olympiad medallists (good problem solvers) formulate some plans by using isomorphic problems had solved previously as a means of solutions as well as relevant mathematical concepts (Mairing, Budayasa, & Juniati, 2011, 2012). Two problems are called isomorphic if they have the same structure, but different contents (Sternberg, 2009).

Carrying out the plan is easier than devise it, what is needed are carefulness and patience (Polya, 1973). The Olympiad medallists are able to carry out the plans successfully because they have positive attitudes, and develop clear, step by step, and detailed plans. Moreover, the answers are calculated and obtained in their minds as they devise the plans (Mairing et al., 2011, 2012).

Students need to look back the solutions to make them have strong reasons as an assurance that the solutions are correct (Polya, 1973). The look back can be done in concurrent, or after carrying out the plan. A concurrent look back is conducted by checking the row of solutions as it is just written. The checking is conducted by comparing a row of solutions with the understanding of problems, existing knowledge in students' minds, or previous solution rows. The after look back is carried out by checking every row of the solutions after the students get the answers, or substituting the answers to the models/equations representing the conditions of the problems (Mairing et al., 2011, 2012).

Hence, the researcher conducted a research aiming to describe thinking processes of naive problem solvers to solve mathematical problems. The subjects were students with scores of 0 or 1 in solving mathematical problems. The scores were determined by using a holistic rubric. Several researches such as Carlson and Bloom (2005), and Mairing et al. (2011, 2012) have described the thinking process of good problem solvers. Furthermore, the results of this research will complete the theories of students' thinking process in problem solving. Moreover, the results can also be used by teachers to improve problem solving abilities of naive problem solvers.

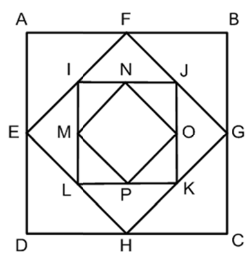
2. Method

The research subjects were two students of grade XI from a state high school in Palangka Raya city, Central Kalimantan, Indonesia. The subjects were classified as naive problem solvers. The procedures of subject selection are described as follows. First, the researcher gave two mathematical problems to 124 grade XI high school students. The problems were given just right after the school exam. The exam materials were geometric sequences and series. Therefore, the problems provided in this research were also in the same material. The second step, the researcher gave scores to each student's solution. The scores were determined by using a holistic rubric. Finally, the researcher decided two subjects who possessed the criteria of getting score of 0 or 1 for both problems. The researcher also considered the school exam scores, and suggestions from the mathematics teachers to decide the subjects. The chosen subjects were L0 and L1. Both subjects were female. Subject L0 and L1 got scores of 0 and 1 for both problems respectively.

Mathematical problems used in this research are mentioned as follows.

1) Given a geometric sequence with infinite sum of the terms is 1. If result of the second term minus the first one is -4/9, and the ratio is a positive number, determine the formula for nth-term of the geometric sequence!

2) Given a 4 cm × 4 cm square. Each midpoint of a side is connected to the midpoint of the adjacent sides to form a new square as shown below. If the process continues infinitely, determine the area of the squares formed!



A holistic rubric used in this research is presented as follows.

Table 1. A Holistic Rubric of Problem Solving (Charles et al., 1997; Bush & Greer, 1999)

Score	Description
0	a. The student does not write anything.
	b. The student writes the known, and the unknown/target, but does not seem understanding the problem.
1	a. The student writes the known, and the unknown/target correctly. There are steps of problem solution, but the means is used incorrectly.
	b. The student tries to achieve sub-target, but does not succeed it.
	c. The student answers correctly, but there is not the means.
2	a. The student uses inappropriate means, and the answer is wrong, but the solution shows some understanding of the problem.
	b. The correct answer is obtained, but the means cannot be understood or wrong.
3	a. The student applies correct means, but does not understand the part of the problem, or ignore certain condition of the problem.
	b. Appropriate means is applied, but the student answers the problem incorrectly without explanation, or does not write the answer.
	c. The correct answer is given by the student, and there are some evidences to suggest that the student chooses appropriate means of solution, but the implemented means is not entirely true.
4	a. Appropriate means is chosen by the student, and implement it correctly. The student writes the correct answer.
	b. The means is appropriate, the answer is correct, but there is little miscalculation.

After the subjects had chosen, the researcher collected data by conducting in depth interviews to each subject separately. The interviews are based on the students' solutions and problem solving heuristic of Polya. The interviews were semi-structured and recorded by using audio-recorder. There were questions have been designed by the researcher previously, and the other ones emerged during the interviews.

Furthermore, the audio recorded data was transcribed by the researcher. The transcript was encoded using six digits. The first and second digits were letters stating the research subjects (L0 or L1). Code L0 and L1 respectively stated the subjects with scores of 0 and 1 for both problems. The third digit was a number stating the problem 1 or 2. The fourth digit was letter U, P, C, or L stating **U**nderstanding the problem, making **P**lan, **C**arrying out the plan, or **L**ooking back respectively. Fifth and sixth digits stated code order for each problem solving phase. For example, L02U03 stated the sentence with the code come from the subject L0 on problem 2. The sentence showed that the subject L0 did activity to understand the problem (U). This code was third activity in understanding the problem.

There were two advantages of coding. First, the researcher could determine the subject, the used problem, and the performed activity in problem solving phases from the subject in a sentence with a certain code. The second advantage, a conclusion obtained from two different codes with the first, second, and fourth digits were the same, but the third digit was different. Then, this was based on the same subject, and the same activity in problem solving phases, but the problems were different. Thus, the conclusion employed time triangulation. For example, conclusion A came from code of L01U02 and L02U10. Thus, the conclusion had been triangulated based on two different problems. The subject still showed the same thinking process in different problems and different times.

The steps of making conclusion are described as follows. First, the data was presented based on the order of digits 5 and 6 on each code U, P, C, and L. Furthermore, the researcher gave meanings and explanations on the data presentation to obtain the conclusion. It was carried out by analyzing words/phrases/sentences. The analyzing steps are presented as follows. First, the researcher read the interview transcript, and focused on the words/phrases/sentences that are significantly attractive. Second, the researcher listed possible meanings of the words/phrases/sentences coming to mind. Third, the researcher read the transcript again to determine an appropriate meaning (Strauss & Corbin, 1998). Finally, the researcher drew a conclusion by looking for similarities of problem solving activities reflecting the students' thinking process.

Research credibility was satisfied by triangulations, member checking, and peer debriefing. Triangulations used in this research were methods and times. The method triangulation was conducted by comparing the students' written solutions with the data from interviews. The time triangulation was carried out by comparing the similarities of the students' responses during the interviews at two different times. Each subject was interviewed two times for each problem. If they gave some unclear responses, answers or conclusions, then the researcher asked the subject again. Thus, the researcher employed member checking. Peer debriefing was counducted by asking colleague to examine conclusions made by the researcher.

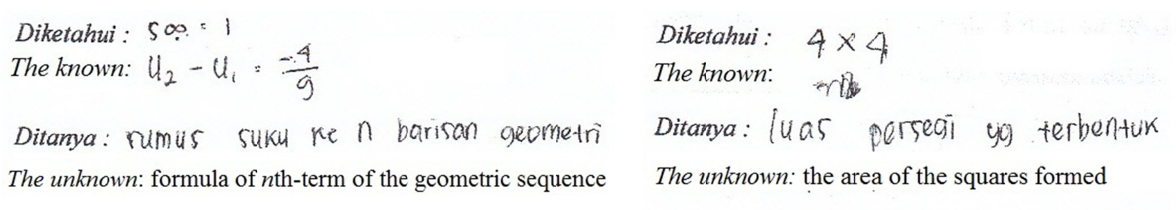
3. Results

The selected subjects were L0 and L1 who obtained score of 0 and 1 on both problems respectively.

3.1 The Thinking Process of Student with Score 0

3.1.1 Understanding Problem Phase

Subject L0 read problem 1 and 2 for 5-10 times because she had difficult times to understand the sentences in the problems. Although L0 did not understand the meaning, she could determine the known, and the unknown/target if they were written in the problems. In the first problem, L0 could determine two of the three knowns, but did not write r positive number as the known (Figure 1a). It was because L0 did not have knowledge of convergent series which the condition of ratio r was $-1 < r < 1$. The ratio r of convergent series could be negative or positive number. If L0 understood it, then she would realize that the words in problem "positive number r" was important information, and required to write as the known. Moreover, L0 misunderstood the concept of ratio. She thought ratio as $U_2 - U_1$, which U_1 and U_2 were the first and second terms of geometric sequences respectively. Actually, her understanding of ratio was contrary to the known. The understanding was $r = U_2 - U_1 = -4/9$ (negative number), whereas the known was r positive number. In addition, the formula remembered by L0 was S_∞, but she misunderstood the meaning. She thought S_∞ as total sum.



a. The First Problem b. The Second Problem

Figure 1. L0's understanding of the first and the second problems

A similar thing occurred in the second problem, L0 could not determine the known because it was not written in the problem. The information in the problem needed to process to form a mental image. L0 said: "this is it (pointed 4 cm x 4 cm), but I do not know what it is". She did not know that it was a measure of square ABCD. She could not process the information because of a lack of understanding on the words in the problems, i.e: "the process continues infinitely". L0 thought the area of the formed squares was the area of 4 squares like the picture in the second problem. Moreover, L0 could determine the unknown because it was written in the problem (Figure 1b).

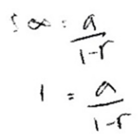
3.1.2 Devising Plan Phase

L0 was not able to find some appropriate plans for the two problems. This situation occured since L0 had limited mathematical knowledge, and did not understand the problems. L0's knowledge was only limited on mathematical formulas without making sense of the formulas. For the first problem, L0 devised a plan (though it was inappropriate) based on the remembered formula, that was $S_\infty = a/(1 - r)$. L0 chose the formula based on her knowledge of the relationship between the known ($S_\infty = 1$) and the remembered formula, but L0 could not use the known $U_2 - U_1 = -4/9$. L0 told that she was so confused of (pointing the known), what it meant by $-4/9$, and what the function was, she could not understand it. The circumstance occured since L0's previous experiences were limited to find a value using certain formula, such as determining the value S_∞ if the values a and r were given.

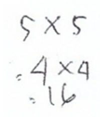
In the second problem, L0 made a plan (though it was inappropriate) on limited understanding of the meaning of S_∞ as total sum, so the area of squares formed was total area of four squares as shown in the figure of the problem. The plan was calculated from the sum of area of ABCD, EFGH, IJKL and MNOP squares. Nevertheles, L0 did not understand that 4 cm × 4 cm written on the problem was the measure of a ABCD square. Therefore, L0 did not understand the measures of the three other squares either.

3.1.3 Carrying Out the Plan Phase

L0 made the solutions based on the remembered formulas, or the known of the problems. In the first problem, L0 wrote the formula S_∞ only with the known $S_\infty = 1$ (Figure 2a). In the second problem, L0 wrote the known only and calculated it (Figure 2b).



a. The First Problem



b. The Second Problem

Figure 2. L0's solution of the first and the second problems

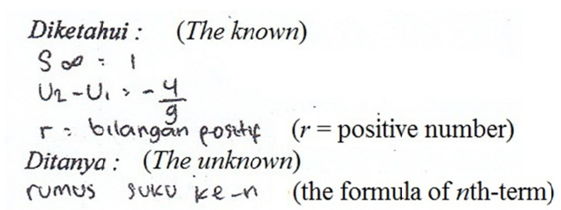
3.1.4 Looking Back Phase

Based on the process of solving both problems, L0 did not look back to the solutions because L0 did not understand the problems. Therefore, L0 did not know how to solve them.

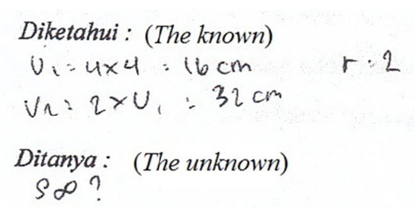
3.2 Thinking Process of Student with Score of 1

3.2.1 Understanding Problem Phase

Based on the process of solving both problems, L1 read the problems for several times to get the understanding. She could get the understanding for the first problem, which was shown by her ability to determine the known and the unknown/target. Furthermore, the known was expressed in mathematical symbols (Figure 3a).



a. The First Problem



b. The Second Problem

Figure 3. L1's undertanding of the first and the second problems

In the second problem, L1 could not understand the problem (Figure 3b). It differed from the first problem which the known was written. L1 told that as usual, for the problem, the known is written, while in the second problem, L1 was confused which square was the first term of a geometric sequence (U_1) because it was not written. She should process the information, and determine the meaning of the sentence in the problem, i.e. "each midpoint of a side is connected to the midpoint of the adjacent sides to form a new square ...". L1 said "oh, I do not know about this one". L1 had a mistake in determining U_1, and chose the smallest square as U_1. She said that the adjacent sides seemed to be these sides (pointing points M, N, O, P). L1 used word "seemed to be" that can be meant as doubtfulness.

L1 could determine the unknown for both problems because it was written. She said: "it is written in the problem, determine nth-term formula". In addition, previous knowldedge and experiences helped L1 to determine the unknown. L1 said: "it is because I ever learned about it, if the question looks like that, it means s infinite".

3.2.2 Devising Plan Phase

L1 already devised the plans for both problems. In the first problem, the plan was looking for values of a and r in order to determine the formula of *n*th-term. L1 knew that the formula required the values. L1 said: "the initial plan is derived from previous examples that was ever learned in the classroom". There were similarities in the known and the unknown between the previous examples and the first problem. The plan was also based on the meaning of U_1, U_2, r, and the formulas of r and the infinite sum.

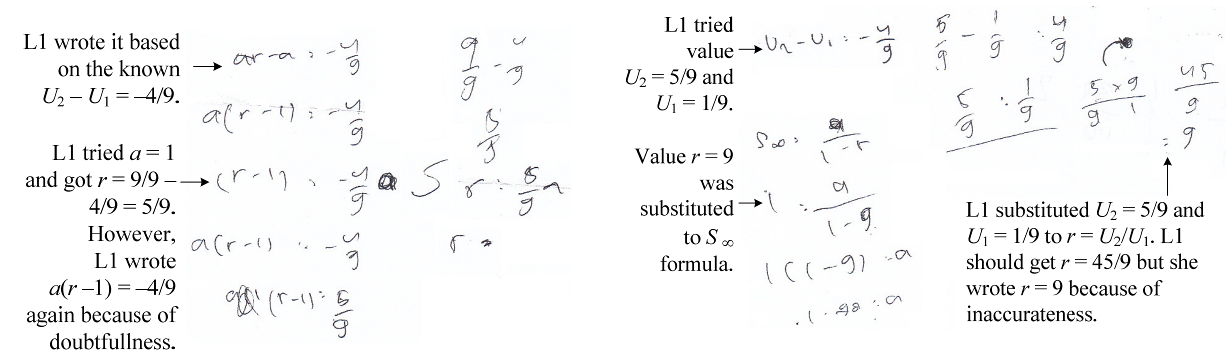
Based on the initial plan, L1 tried to find the values of a and r by using the known $U_2 - U_1 = -4/9$. However, L1 was unable to obtain the values because of the confusion of negative fraction in the equation. She thought to solve $a(r-1) = -4/9$ by finding a formula. L1 did not occupy her previous knowledge to find the values of a and r. It was the concept of linear equations system of two variables. She should search another equation, and use the concept to find the values. Therefore, L1 did not elaborate the previous knowledge, and the known to create an appropriate plan for the problem.

L1 read the problem for several times because the initial plan did not succeed in solving the problem. Therefore, L1 occupied an alternative plan to find the values of a and r by using the known $S_\infty = 1$, and tried some numbers of U_1 and U_2. She did not think to use the known $U_2 - U_1 = -4/9$, and $S_\infty = 1$ simultaneously so she failed to find the values. Thus, L1 did not elaborate the relevant concepts, and did not have knowledge of problem solving strategies especially intellegent trial and error. The plans were not able to bridge the gap between the known and the unknown yet.

In the second problem, L1 chose MNOP as U_1 based on the previous examples learned in the class. L1 planned to determine r by looking for the value of U_2 as the larger square than MNOP, i.e. U_2 were area of IJKL. L1 determined the area of IJKL (U_2) as twice bigger than the area of MNOP (U_1) without performing some calculations. L1 determined it based on the previous examples. L1 also knew that $a = U_1$, and formula of S_∞. Furthermore, the values of a and r were substituted into S_∞ formula to get the answer.

3.2.3 Carrying Out the Plan Phase

In the first problem, L1 carried out the initial plan by finding the values of a and r by using the known $U_2 - U_1 = -4/9$ (Figure 4a). In the alternative plan, L1 used the known S_∞ to find the values (Figure 4b).



b. The Alternative Plan

a. The Initial Plan

Figure 4. L1's solution of the first problem

In the second problem, L1 substituted $a = 16$ and $r = 2$ to S_∞ formula based on her understanding of the problem (Figure 5). Value $r = 2$ was obtained from $U_2 = 2 \times U_1$. L1 determined the value based on the previous examples learned in the classroom. It was on the contrary because the examples showed that geometric series converges if $-1 < r < 1$. The knowledge changing was caused by L1's understanding in geometric series on the formula only (procedural knowledge) without its meaning (conceptual knowledge). L1 got the answer of $S_\infty = -16$ after substituting the values.

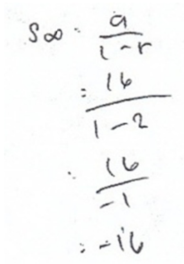


Figure 5. L1's solution of the second problem

3.2.4 Looking Back Phase

In the first problem, L1 looked back to the solution because she did not obtain the answer. She checked whether the formula is right or wrong. L1 felt unconfident of the answer because of some mistakes and failures to find the answer.

In the second problem, L1 was not confident either because the answer was $S_{\infty} = -16$ (negative number). The answer should be greater than the area of first square = 16. L1 checked the calculation, and decided it had already been true. Moreover, L1 was not confident whether the second term should be twice of the first term of geometric sequence, but she did not know about the mistake and how to correct it.

4. Discussion

This research aimed to describe the thinking process of naive problem solvers in solving mathematical problems. The naive problem solvers were students with score of 0 or 1 (scale 0 – 4). The score was determined by using the holistic rubric. The thinking process was described by using heuristic of Polya. It is understanding problems, devising plans, carrying out the plans, and looking back. L0 could not solve the problems because of the difficulty in reading problems, understanding the problems, and determining the solutions means. L0 copied the known of the problems without processing the information to form an appropriate mental images. L0 also did not have an appropriate understanding of the concepts related to the problems. Consequently, L0 read the problems for several times, and was unable to move toward the answers.

L1 could not solve both problems and found some difficulties in understanding the problems because of the need to process information to determine the known. Instead, L1 could determine the known which was written in the problem. L1 also found some difficulties in devising some plans because she did not elaborate her prior knowledge, or made the plans based on trial and error strategy without its understanding. Consequently, L1 was also not able to move toward the answers.

In general, the solution of subjects of scores of 0 or 1 based on one or two strategies, i.e. substituted some values to remembered formula, or limited trial and error strategy. The metacognitive thinking did not appear in verbal, or written communication. The written communication was usually inadequate. Mistakes occured begining in understanding problem or creating plan phases. The problem solving process showed that both subjects can be classified as naive problem solvers (Muir et al., 2008). Although L1's solving process was different from the research results, L1 could identify similar problem that was learned previously. However, the leraning experience was not meaningful, and was not elaborated with other knowledge to form an appropriate problem solving schema.

Moreover, the naive problem solvers had some difficulties in processing information in order to understand the problem. They were lack of language and information skills. These weaknesses emerged uncertainty, confusion, and inaccuracy for them in making connection among informations. They also misunderstood some mathematical concepts. The weaknesses and the misunderstanding inhibited students to solve mathematical problems (Tambychik & Meerah, 2010).

The naive problem solvers behaviour could be classified as Direct Translation Approach (DTA)-not proficient. Pape (2004) divides the reading and problem solving behavior into: (a) Direct Translation Approach-proficient (DTA-proficient), (b) Direct Translation Approach-not proficient (DTA-not proficient), (c) Direct Translation Approach-limited context (DTA-limited context), (d) Meaning-Based Approach-full context (MBA-full context), and (e) Meaning-Based Approach-justification (MBA-justification). The main behaviour of DTA-not proficient is the subjects have not ability, show difficulties in reading and understanding the problem, can not devise an appropriate plan, and make calculation mistakes. The research showed that student of MBA-justification behaviour more capable in solving mathematical problem than the other behaviours. Furthermore, student with MBA-justification can be classified as good problem solver (Mairing et al., 2011).

The thinking proccess of naive problem solvers were different from good problem solvers on all phases of heuristic of Polya. In understanding problem phase, the naive problem solvers could not form the mental images. Whereas, the good problem solvers can effectively make mental images in their mind, or represent the known in some pictures as they constructed the meaning. They can also identify some relevant concepts in the problems (Carlson & Bloom, 2005; Mairing et al., 2011, 2012). In this case, understanding problem is a very important factor to solve. Students cannot solve mathematical problem if they did not understand it (Polya, 1973).

In the phase of devising plan, the naive problem solvers could not make an appropriate plan due to their limited understanding of the problem, and their limited mathematical knowledge. On the contrary, the good problem solvers can devise some appropriate plans based on the mental images, an indepth knowledge of mathematical concepts, and some solution means of previous solved problems. Moreover, they can think some possible solution means and how it worked while considering to use some tools or strategies (Carlson & Bloom, 2005; Mairing et al., 2011, 2012).

In the phase of carrying out the plan, the naive problem solvers could not move toward the answer, and the solution depended only on substituting some values to remembered formula, or they only tried some values. Whereas, the good problem solvers can move toward the answer because they have thought how the plans worked in their mind. They also show metacognitive thinking skills during carrying out the plan. They logically write rows of solution, and can give the reason for each row (Mairing et al., 2011, 2012). They use factual and conceptual knowledges, implemented strategies and procedurals, and make some calculations (Carlson & Bloom, 2005).

In the phase of looking back, the naive problem solvers did not look back to the solution, they only checked the formulas or some calculations. Whereas, the good problem solvers look back to the solution as implementing the plan by checking the solution row as it was just written. If the solution row meet the agreement with the previous rows, the internal representation of the problem, or the matematical knowldege, the good problem solvers move to next row or step. In addition, they also look back to the solution after the answer obtained by checking each row of the solution, or substituting the answer to the formula that represents condition of the problem (Carlson & Bloom, 2005; Mairing et al., 2011, 2012).

5. Conclusions

In this research, the two subjects, L0 and L1, were classified as naive problem solvers. L0 was student who got a score of 0 in first and second problems. Meanwhile, L1 got a score of 1 in both problems. The problems were related to the concept of infinite geometric series. In the phase of understanding problem, the naive problem solvers ignored words that were not understood, or translated directly the words of the problems into mathematical symbols without processing the information to form an appropriate mental image. Furthermore, the naive problem solver had limited understanding of relevant concepts. It indicated that the naive problem solvers did not understand the concepts involved, they only knew the concept but it was limited to finding some values, e.g. determining infinite sum of geometric series if the value of a and r were given, or they did not elaborate the understood concept with other concepts. The naive problems solvers' limited understanding made them could not identify important information in the problems, and they did not use relevant concepts in making the plans.

The naive problem solvers devised some limited plans. The plans were used to substitute the known numbers into certain formula. The naive problem solvers tried to use previous learning experiences to achieve the target. However, the unmeaningful experiences made them unrealized the mistakes, and they were difficult to move toward the target. Moreover, the plan which was not elaborated other knowledges caused the naive problem solvers could not see the solution means. The alternative plan was to substitute some values to a remembered formula or known equations of the problem without the understanding of intelligent trial and error strategy.

The naive problem solvers made some mistakes in carrying out the plan because the plan was limited. They read the problem repeatedly, and tried to make a new plan. However, the new plan also failed to move toward the target. As a result, the custom of reading repeatedly became habit, and it was actually not intended to understand the problem. One strategy which was attempted was trial and error. However, this strategy also failed because of lack of understanding of problem solving strategies. In addition, the calculation error occurred in the implementation of the plan caused the naive problem solvers could not get the answer.

The naive problem solvers did not look back to the solution. They just check the formulas or the calculations. It was because of their inability to understand the problem, to make a plan and to implement it, or they found some doubts on the soluliton means. In general, the solutions made by them were not meaningful, and it was done just to finish the tasks.

Students' abilities to solve problems can be classified as naive, routine and good problem solvers. The characteristics of each problem solver had described by Muir et al. (2008). Some researches have described good problem solver in the process of finding solution (Carlson & Bloom, 2005; Mairing et al., 2011, 2012). This research described the thinking process of naive problem solvers. The future research needs to conduct to complement theories of students' thinking process in solving mathematical problems. It is a research that is aimed to describe the thinking process of routine problem solvers.

Moreover, the result of this research can be used by other researchers to develop teaching plans, or as a learning resource that can improve students' problem solving ability. Some research has indicated that students' ability to solve the problems was affected by learning experiences given by teacher in the class (Ho & Hedberg, 2005; Pimta et al., 2009). In addition, Krulik et al. (2003) have given keys of teaching priciples to improve students' ability to solve mathematical problems. In fact, there were students as naive problem solvers nowadays, so future research is needed to improve the students' ability with various conditions.

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