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Zambian University Student Teachers' Conceptions of Algebraic Proofs

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Abstract
This paper reports on a case study that aimed to evaluate the perceptions of 73 student teachers at Mukuba University on the nature and purpose of mathematical proof. They were presented with a mathematical reasoning and proof arguments assessment tool followed by a 10-member focus group interview. Algebraic tasks presented to them involved (i) proof construction on simple number and set theory, (ii) assessment of given mathematical statements and arguments to validate them, and (iii) validating formally structured inductive proofs. More than 75% of the student teachers had limited understanding of the nature and purpose of mathematical proofs and they believed that proof construction and validation entailed inspection of a few examples and the testing of single extreme cases. Concrete examples of empirical forms of mathematical arguments which student teachers perceived as valid were found but these demonstrated limited forms of mathematical reasoning and comprehension. It is proposed that paying attention to analytical and deductive mathematical arguments in school based assessment will alleviate mathematical learning problems and improve the quality of mathematics education.
Keywords: student teachers, algebraic proof, conceptions of proof, mathematical reasoning, school based assessment.

1. Introduction

Mathematics is a science of logical reasoning and John Locke once stated that, "mathematics is a way to settle in the mind a habit of reasoning" (Sidhu, 1995; p. 222). Many researches in different educational and cultural contexts point out that to acquire the habit of reasoning, mathematical proofs or proving play an important role (Blanton& Stylianou, 2014; Ko& Knuth, 2013; Martinez, Brizuela, & Superfine, 2011; O'Halloran, 2015; Ovez& Ozdemir, 2013; Sengul& Guner, 2013; Wu, 1996; Yopp, 2011). Schoenfeld (1994) posited that proof or proving is not separable from mathematics but rather an essential component of doing, communicating, and recording mathematics pointing out "I believe it can be embedded in our curricula, at all levels" (p. 76). The mathematics forum hosted by the University of Drexel provides counsel on the relevance of reasoning and proof or proving worth heeding:

> A mathematics curriculum that does not support the development of understanding of these (reasoning and proof) is not providing students with access to the key distinguishing feature of the discipline. While the age and circumstances of students need to be taken into account appropriately, it is unacceptable for students to be denied appropriate opportunity and support to learn about reasoning and proof in mathematics over the course of their schooling (http://mathforum.org/pcmi/nature11.05.07Final.pdf).

In Zambia, the expectation in the Ordinary Level Mathematics syllabuses is for students to develop clear mathematical thinking and expression, and that they must reason logically and communicate mathematically in the course of their schooling (Curriculum Development Centre, 2013). Mathematical proof, including in the study of algebra where much logical deduction is required and developed, provides a context in which these learning outcomes and qualities of the individual can be developed (Blanton & Stylianou, 2014; Ko & Knuth, 2013; Martinez, Brizuela, & Superfine, 2011).

1.1. Statement of the problem

Generally speaking, students at many levels of education hold severely limited views of proof as well as limited confidence to engage in proof arguments. One of the potential factors could be that they lack proper understanding in handling problems involving algebra, the topic which gives a good approach to the study of abstract mathematical relationships through the use of new language and new symbolism Sidhu (2008:309).Unfortunately, literature is replete with studies showing that secondary school students, undergraduate students, and secondary school mathematics teachers have difficulties with proof and counterexamples especially in algebra (Ko & Knuth, 2013; O'Halloran, 2015; Ovez & Ozdemir, 2013; Sengul & Guner, 2013; Wu, 1996; Yopp, 2011). In Zambia, we take note of the Chief Examiner's (Examination Council of Zambia, 2012) report, for example, in which it was observed that grade 12 students failed to derive a quadratic equation from $(2x-1)(3x-2)=3$ but equated factors to 3, e.g., $(2x-1)=3$ and $(3x-2)=3$. Others failed to expand the expression on the left, e.g. $6x^2-7x+2=3$ or $6x^2-7-2=3$. Examinations Performance Report also reveals that a total of 181 779 candidates sat for mathematics examinations in 2014 where 12 076 (10.1%) candidates got a zero in paper 2 (ECZ, 2015). Again algebra was one of the topics that recorded low pass rate by the candidates. In addition, experience also shows that pupils have been having difficulties in recalling the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when solving equations of the form $ax^2 + bx + c = 0$. With that background, we conjectured that understanding student teachers' ability to construct proofs and their ability to validate the given proof arguments is necessary because these are the future teachers of mathematics in secondary schools who must transform the quality landscape that exists in mathematics education.

1.2. Scope of the case study

Engaging learners in mathematical proof or proving activities is an important if not the most central aspect of mathematics education. The case study presented here focused on a group of 73 student teachers in first term of their second year of studies at Mukuba University in Zambia. It was intended to explore three objectives:

(i) To find out student teachers' perceptions on the nature and purpose of algebraic proofs and arguments at Mukuba University.
(ii) To find out the forms of mathematical arguments that are accepted by student teachers at Mukuba University as valid proofs.
(iii) To find out the ways in which Mukuba University student teachers construct and validate algebraic proofs and arguments.

The above research objectives translated to the following research questions:

(i) What are student teachers' perceptions on the nature and purpose of algebraic proofs and arguments at Mukuba University?
(ii) What forms of arguments are accepted by student teachers at Mukuba University as valid proof of a mathematical statement?
(iii) What are the ways in which Mukuba University student teachers construct and validate mathematical proofs and arguments?

1.3. Conceptual, theoretical and methodological frameworks

As stated earlier, the notion of mathematical proof is important because it holds a central place in almost all university level mathematics courses and that it is an essential component of mathematics. Experience shows that it is quite difficult to teach any topic in mathematics effectively without engaging learners in some form of proving, and algebra, in particular, plays an important role. In the first half of the 17th century René Descartes proposed a program of problem solving based on the assumption that solving a problem in mathematics means essentially solving a problem in algebra, more precisely, solving a system of algebraic equations (Cipu, 2003). The conceptual framework within which student teachers' conceptions of algebraic proofs could be understood may have many domains but in this particular study, the conceptions were investigated using the following criteria:

- Firstly, respondents were asked to state the meaning and purpose of proof in the teaching and learning of mathematics
- Secondly, respondents were allowed to construct a simple proof based on their knowledge of odd and even numbers
- Several arguments on how a particular algebraic conjecture can be proved were given and then respondents were asked to choose the argument(s) which convinced them most or convincingly.
- After data analysis, the researcher understood the forms of arguments that student teachers perceived as valid proofs, the way student teachers constructed proofs and the way they validated the given proof arguments.

During an introductory undergraduate course in a typical mathematics sequence, students are assumed not to have received substantial formal instruction in the creation or evaluation of mathematical proofs beyond what they might have received in secondary school. In this study, data collection involved students in the first term of their second year. They had completed an introductory mathematics course (MAT 110) the previous academic term.

The theory guiding this study is the constructivist theory. Brooks and Brooks (1993) posit that many people learn better when they do something themselves rather than when someone does something for them. This is a tenet of constructivist learning theory. This theory states that learning is an active process of creating meaning from different experiences. Constructivism views each learner as a unique individual with unique needs and background. The learner is also seen as complex and multidimensional. The theory not only acknowledges the uniqueness and complexity of the learner, but actually encourages, utilizes and rewards it as an integral part of the learning process (Wertsch, 1997).

Informed by constructivist thinking and through a descriptive survey research design, the study examined the ways in which students construct and validate proof arguments. It used data from a written questionnaire as well as interview data (from the focus group discussion). In an assessment tool that was administered, students were tasked to describe the nature and purposes of proof in mathematics, construct a simple proof and validate several purported proof arguments. To verify those responses, a follow up was made during a focus group discussion in order to get richer and detailed perceptions on algebraic proofs from the respondents. Data from questionnaires (assessment tool) were used primarily to quantify the frequency with which students constructed and accepted various forms of arguments, while interview data have been used to provide insights into the reasoning behind their constructions and validations. The philosophical foundation or paradigm on which the data were collected and analyzed was the critical theory. ***While the study is conducted in an African setting, the results may be relevant cross-culturally and internationally, since proofing is a universal mathematical competency***. This is why it is much researched, for example, how students learn and solve proofs (Herbst, 2002; Hazzan and Zazkis, 2003; Kuchemann and Hoyles, 1999; Balacheff, 1988), teaching techniques of proving (Marty, 1986; Lamport, 2012; Hanna and de Villiers, 2008), how proofs are validated (Selden and Selden, 2003; Weber, 2008; Weber and Alcock, 2005), how students and teachers perceive proofs (Patkin, 2011; Knuth, 2002; Varghese, 2009; Jones, 1997; Healy and Hoyles, 2000), how proofs relate to convincing and refutation (Stylianides, 2009; Stylianides and Al-Murani, 2010), difficulties in the transition of high school to undergraduate mathematics (Moore, 1994; Almeida, 2000; Blanton, 2003; Raman, 2002; Tall, 2008), and the extent to which proofs are important in educational settings (de Villiers, 1990; Hanna, 1995; Volminik, 1990; Tucker, 1999; Pfeiffer, 2010).

2. Literature Review

The precise definition of mathematical proof and its role vary by context and scholar (Reid, 2002; 2005), but the general purpose of proving is to verify, explain, communicate, and systematize statements into deductive systems (Almeida, 2000; Hersh, 1993). The World view is that proofs are more than instruments to establish that a mathematical statement is true. Indeed, they embody mathematical knowledge in the form of methods, tools, strategies, and concepts. Some previous studies have advocated for the need to educate teachers and preservice teachers about the importance of proof in learning any and all mathematics (Martin & Harel,1989; Knuth, 2002). This has been attributed to the misconceptions that undergraduate students have regarding proofs and proving in mathematical learning. A study carried out by Stavrou (2014) on common errors and misconceptions in mathematical proving by education undergraduates reveals that the most common error made was proving general statements using specific examples. The literature refers to this as using empirical evidence in place of a valid proof (Stylianides, 2009). Furthermore, Stavrou also recommended that more research needs to be carried out in this domain to understand more on student teachers' perceptions regarding the nature and importance of proof by focusing on their proof construction and validation practices. This is why the present study investigated how second year mathematics and science education students view the nature and purpose of proof, how they construct simple proofs and how they validate the given proof arguments.

As earlier alluded to, the expectation in the Zambian O Level Mathematics syllabus is for students to develop clear mathematical thinking and expression, and that they must reason logically and communicate mathematically in the course of their schooling. However, the present situation at Mukuba University (a Zambian University) is that second year student teachers expressed limited views on what constitutes proof and the purpose that it serves in the teaching and learning of mathematics. Their understanding was limited to proof as a means of verification, communication and evidence. They also believed that proof construction and validation entailed inspection of a few examples and the testing of single extreme cases.

3. Methodology and procedures

Data to explore the three research objectives were collected via a 'Mathematical Reasoning and Proof Argument Questionnaire' which comprised of six (6) mathematical proof tasks. These tasks were adapted and/or developed as informed by previous work on mathematical proof conceptions among students and teachers (Harel & Sowder, 1989; Janelle, 2010; Knuth, 2002; Martin & Harel, 1989; Vargese, 2009). The first two tasks, respectively, asked respondents for the definition and the purpose of proof. The third task required respondents to construct an algebraic proof. The last three tasks were proof validation items in which respondents studied an algebraic statement and chose among examples of empirical and analytic arguments to validate the algebraic statement. These items are reported together with their results in a later section.

A week or so after the administration of the questionnaire, a 10-member focus group discussion was conducted to probe on their thoughts concerning algebraic proof and proving as well as to clarify insights in written explanations to the questionnaire. The data were analysed into categories of meaning as informed by previous research that studied student proof conceptions with special stress being given to empirical proof schemes and analytical proof schemes (e.g., Harel & Sowder, 1998; Martin & Harel, 1989). Empirical proof schemes are inductive whereby proof is based on giving numerical values to expressions or explore similar cases while analytical proof schemes are based on reasoning and logical deductions to arrive at valid general statements reflecting mathematical relationships in the proof confirmation process. This permitted comparisons of results to prior studies in order to further evaluate their significance.

4. Results and findings

We present here a sample of the results and findings.

4.1. Perceptions on nature and purpose of proof

Table 1 and Figure 1 compare the results of analyzing their written definitions (n = 76) and written statements on the purpose (n = 78) of mathematical proof. These responses are scattered around five categorical definitions or purpose of proof. Only three of these categories carrying 20% or more of the responses appeared important to be ascribed as reflecting the views of the student teachers as a group, i.e., proof as verification, as evidence, and proof as communication. In terms of purpose, a substantial number of students tended to express the view of mathematical proof as for communication (34.6%) and for verification (41.0%).

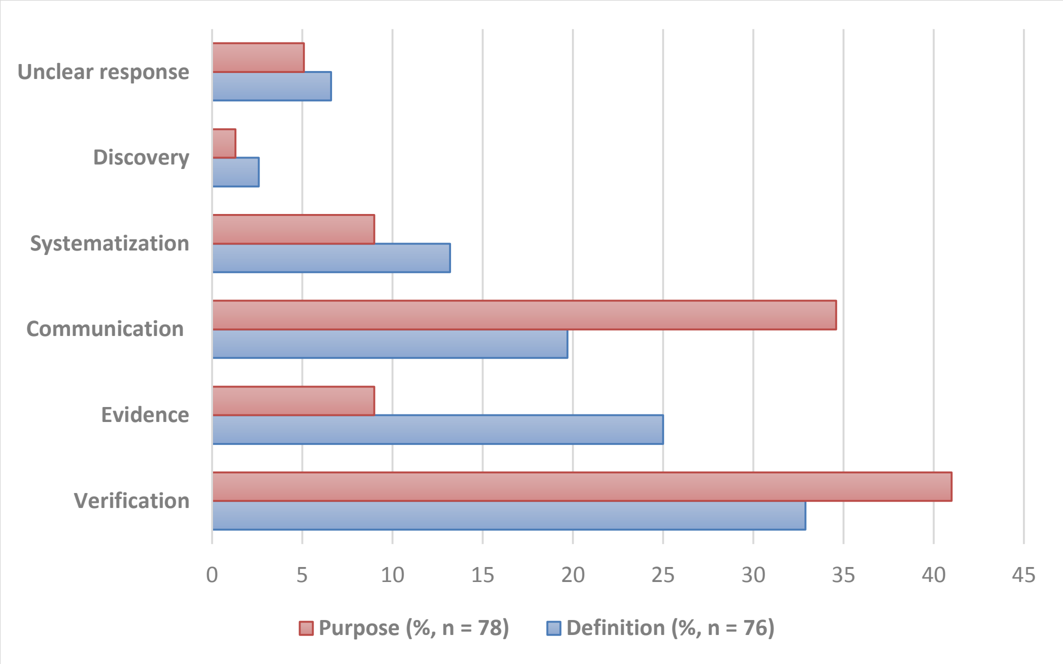
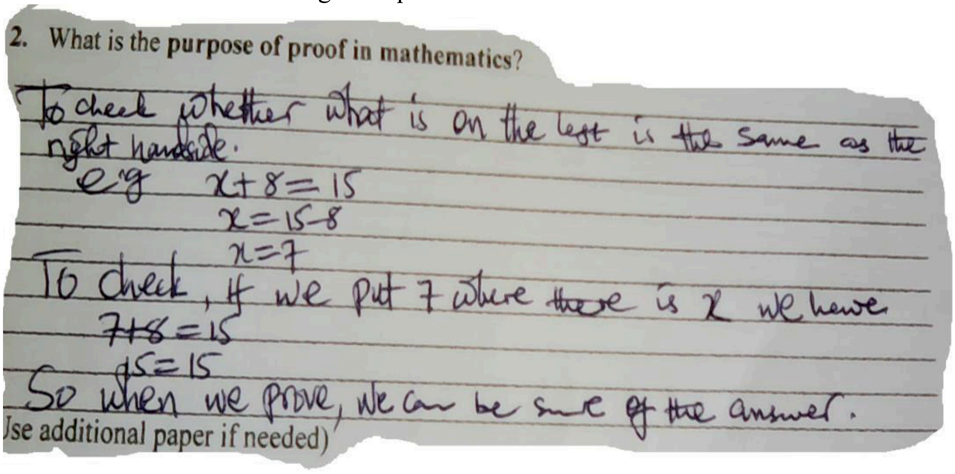


Figure 1: Students' definitions and perceptions of purpose of mathematical proof

Table 1: Categories of meaning student teachers associated with definition and purpose of mathematical proof.

Category	Description	Definition of proof (n= 76)	Purpose of proof (n=78)
1. Verification	The process of ensuring that a mathematical statement or procedure is true, accurate or justified.	25 (32.9%)	32 (41.0%)
2. Evidence	Proof as a way of generating information about the truth or validity of a mathematical statement or concept	19 (25.0%)	7 (9.0%)
3. Communication	Proof as a method of teaching or explaining mathematical concepts to others or a way of convincing others about the truth of a mathematical statement or concept.	15 (19.7%)	27 (34.6%)
4. Systematization	Validation of theories so that they can be incorporated into, or rejected from the existing body of mathematical knowledge.	10 (13.2%)	7 (9.0%)
5. Discovery	Proof as an action or practice that makes someone to be aware of a new or existing mathematical concept	2 (2.6%)	1 (1.3%)
6. Not Clear	Unclear or ambiguous explanations that could not be interpreted	5 (6.6%)	4 (5.1%)

One illustrative case of the statement and explanation of purpose of proof as verification which has been found to be the major category in their definitions (32.9%) and the expression of purpose (41%) is Figure 2. It shows the belief that after solving an equation, you have to verify whether the answer is correct or wrong by substituting the numerical value in the original equation.


2. What is the purpose of proof in mathematics?

To check whether what is on the left is the same as the right handside.

e.g $x+8=15$

$x=15-8$

$x=7$

To check, if we put 7 where there is x we have

$7+8=15$

$15=15$

So when we prove, we can be sure of the answer.

Jse additional paper if needed)

Figure 2: Example of explanation of purpose of proof in mathematics as verification

Both Figure 1 and Table 1 show that the student teachers did not express the full range of definitions or purposes served by mathematical proof. Their responses do not coalesce around any particular view as reflected by the fact that significantly less than 50% were expressing a view depicted in each category in Figure 1 and Table 1. This suggests inadequate understanding which is supported by the view concerning verification illustrated in Figure 2.

4.2. Perceptions of validity of arguments on algebraic statements

Student teachers were provided with the proof construction task: '*The sum of an odd number and an even number is always an odd number*'to decide whether or not the algebraic statement was true or false and construct an explanation. Seventy (70) student teachers representing 95.9% concurred that this statement was true and only 3 (4.1%) indicated that the statement was false. Having obtained this result, a follow up analysis was conducted on 69 submissions representing 94.5% of the group of student teachers with the results in Figure 3. The submissions showed that the majority (n = 60 (86.9%)) had proof constructions that could be classified as the inductive proof scheme (empirical evidence, n = 55 or 80%) and the deductive proof scheme (analytic proof, n = 5, i.e. 7%). The remainder (13%), justified their choice with explanations outside the domain of odd and even numbers and therefore were (out of context (n = 9).

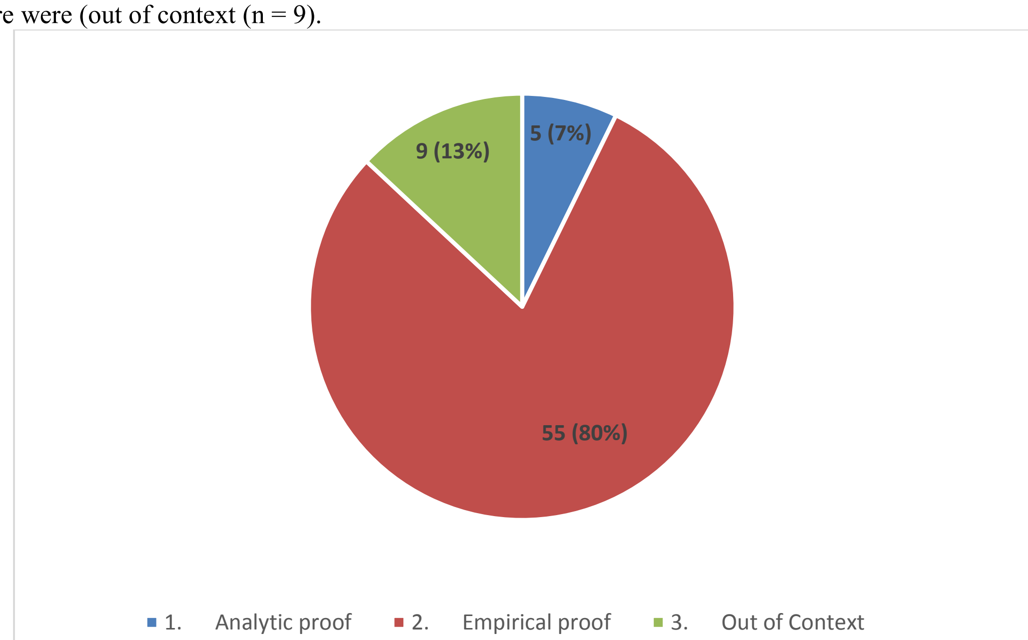


Figure 3: Categories of meaning ascribed to explanations of arguments for algebraic statement (n = 69)

A further analysis of the 80% (n = 55) responses on the empirical proof scheme showed that 22 responses or 40% provided the single case inductive proof, i.e., tested only one case in the domain to inform the conclusion and 60% (n = 33) provided an explanation based on testing several cases from the domain, i.e., the multiple case inductive proof. Overall, the majority explained that proof of an algebraic statement could be justified inductively

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using specified numerical cases in the specific domain. This empirical proof scheme is a limited view of algebraic proof. Two illustrative cases are shown in Figure 4.

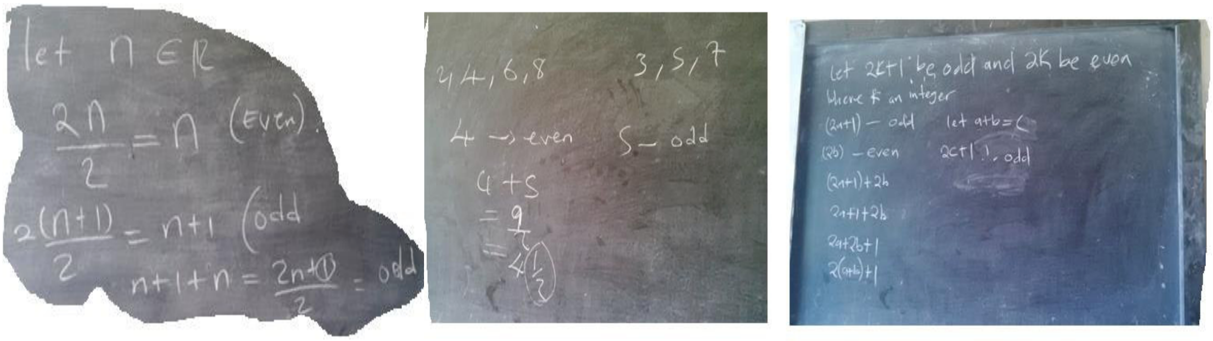

Figure 4: Sample of student teachers'proofs for the sum of an odd and an even number

The first proof is not justifiable in that it lacks an understanding of the general form of an even number and that of an odd number, fails to provide a systematic flow of information, and fails to demonstrate the general form of an odd and an even number. The second constructs proofs by giving examples of even numbers and some odd numbers from which she picked 4 from even numbers and then 5 from odd numbers. When added the result was 9 which is an odd number and then concluded that the statement was true. However, the respondent disagreed with another participant who claimed that when an odd number is divided by 2, the remainder is 1. She indicated that when 9 is divided by 2, the remainder is half. That is why half was circled in her presentation in Figure 4. The student teacher had some misunderstandings of certain mathematical concepts, axioms were not used in her proof, and the mode of argumentation was empirical by using specific numbers to prove the truth of the statement. The student seems to believe that non-deductive arguments constitute a proof.

The third example in Figure 4, is an example of a valid deductive proof that is systematically presented. Even if he started with k and later on switched on to a and b, the argument can be generalised to all the odd and even numbers. As shown above, this reflects the analytic proof scheme or deductive understanding of only a minority of the student teachers (n = 5; 7.2%) who can be said to have produced valid proofs.

4.3. Forms of arguments accepted as valid proofs; Example 1

Three questionnaire items evaluated the forms of arguments that were accepted by student teachers as valid proofs. These items focused on integer addition and division. Student teachers were presented with different proof arguments and then asked to rate how well each argument convinced them on the scale:
(1) Not a convincing mathematical proof; (2) Slightly convincing mathematical proof; (3) Mostly convincing mathematical proof and (4) Completely convincing mathematical proof.

Table 4 and Table 5, respectively, summarise the results in which the student teachers chose arguments to support two algebraic statements. Figure 4 and Figure 5, respectively, display the data sets for the two algebraic statements and proof schemes. The responses are summarised as percent of student teachers who responded to each of the items and then in the last column the aggregate percentage is given for those "mostly" and "completely" convinced with the argument for an algebraic proof shown.

Table 4 and Figure 4 are based on the statement: *If the sum of the digits of any whole number is divisible by* 3, *then the number itself is divisible by* 3. Table 4 shows that between 55% and 81% chose that inductive arguments and their associated proof scheme in support of the validity of the algebraic statements shown. In fact, more than three quarters agree with inductive argument A (76.9%) and inductive argument D (81.3%). Less than 8% completely rejected the argument A and B and their associated proof schemes. This may be compared to only 57.6% choosing deductive argument E which explains the formal structure of mathematical proof. As many as 20.3% completely rejected the argument D and its associated proof scheme convincing mathematical proof. Figure 5 displays these data and shows that the students chose inductive arguments for proof of the algebraic statements more than they did the deductive proof argument.

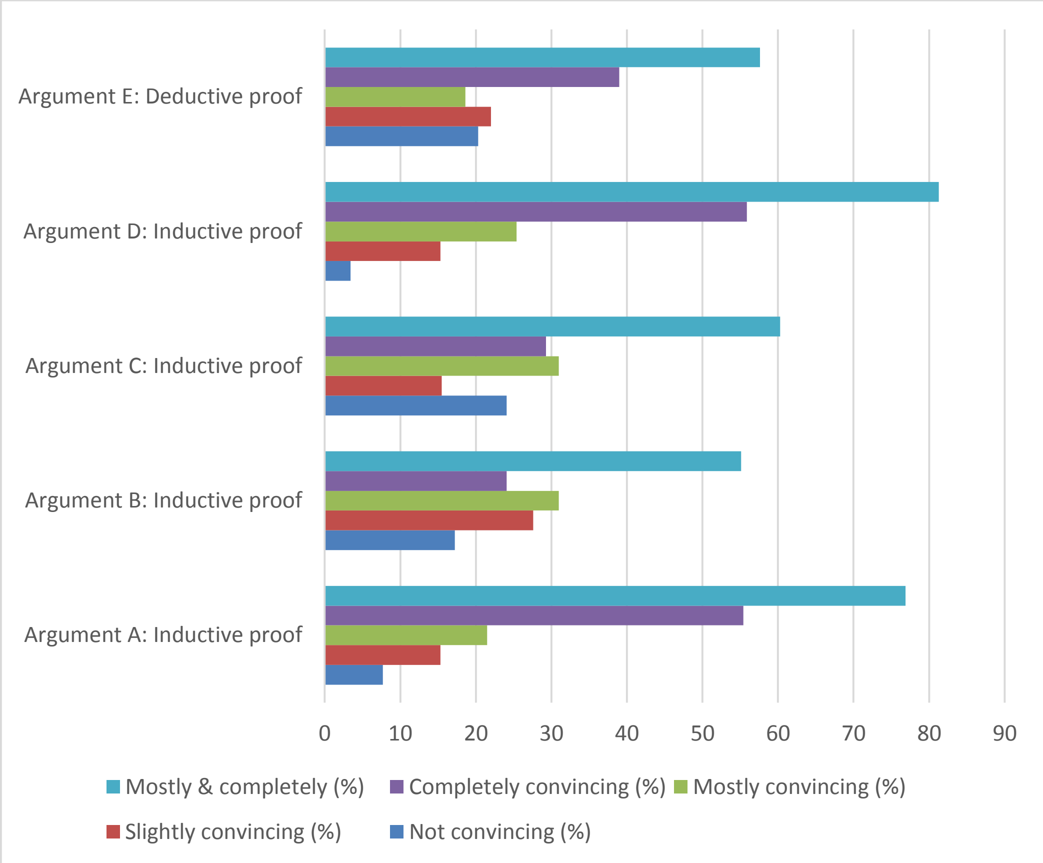

Figure 5: Student teachers' choices of arguments for algebraic proofs

When student teachers were asked to justify the motivations behind their ratings during a focus group interview, the majority of them indicated that a proof should not be too complicated. So they chose those arguments that involved testing of discrete cases because they are easy to understand. The statement below is an example of those who expressed that opinion during a focus group interview.

Moderator: Let's have someone to rate the arguments in question 4 using the given criteria.

That is; Not a convincing mathematical proof (1), Slightly convincing mathematical proof (2), Mostly convincing mathematical proof (3) and Completely convincing mathematical proof (4).

KN: I am rating argument A at 4, argument B at 3, argument C at 2, argument D at 4 and argument E at 3.

Moderator: Okay, are you able to give a comment on any of those ratings?

KN: Yes, I will comment on A, why I had to choose 4 is that if you pick any natural numbers that are consecutive in ascending order, it may not be the very numbers that we have here. You can pick 4, 5 and 6 which are ascending and they are consecutive. If you add those numbers 4 + 5 is 9, 9+ 6 is 15 and that 15 is divisible by 3. Again if you pick 6, 7 and 8 if you add those numbers will be divisible by 3 and it is easy to follow.

Table 4: Student teachers choices of convincing arguments for an algebraic statement (item 4).

Algebraic Statement and Argument	*Frequency*					
If the sum of the digits of any whole number is divisible by 3, *then the number itself is divisible by* 3.	*Resp*	*Not convi*	*Slightly*	*Mostly*	*Completel*	*Mostly &*
A. The sum of the digits 1, 2, 3 is 6 which is divisible by3. The number 123 is also divisible by3.	65	7.7	15.3	21.5	55.4	76.9
B. We can pick any number so that the sum of its digits is divisible by 3, say526845. The sum of the digits is 30 which is divisible by 3 and the number itself is divisible by 3.	58	17.2	27.6	31.0	24.1	55.1
C. The number 41 is not divisible by3, and the sum of its digits is5, which is not divisible by 3. On the other hand 42 is divisible by 3 and the sum of its digits is6, which is divisible by3.	58	24.1	15.5	31.0	29.3	60.3
D. If we list several numbers that we know are divisible by 3 say 3, 6, 12, 15, 18, 24, 36, 48, 1002, 1008 and so on. We can see that the sums of the digits in each of these cases are divisible by3.	59	3.4	15.3	25.4	55.9	81.3
E. Let a be a whole number such that the sum of its digits is divisible by 3. Assuming its digits are $x, y and z$, then $a = xyz$. Since $x + y + z$ is divisible by 3 it follows that xyz is divisible by 3.Therefore, a is divisible by 3.	59	20.3	22.0	18.6	39.0	57.6

On the other hand, there were few students who believed that giving examples is not the best way of proving because you cannot generalise by giving examples. This is further shown in the results in Table 5 and Figure 6 in response to a task: '*Show that the sum of any three consecutive even numbers is divisible by 6'* and then to choose among three arguments A, B, and C. Argument A and argument B involve testing of several discrete cases (empirical proof scheme) and then conclude whereas argument C is a formally structured way of proving (analytic proof scheme).

In Table 5, 88% of the student teachers chose inductive argument A and 82.9% chose inductive argument B, respectively, as 'mostly' or 'completely' convincing. None see argument A and 4.7% view argument B as not convincing. On the other hand, 61.3% viewed the deductive argument C as 'mostly' (14.5%) or 'completely' (46.8%) convincing while 17.7% viewed it as not convincing. Figure 6 displays the percentages of student teachers finding the three algebraic proof arguments as 'mostly' or 'completely' convincing. The two inductive proof arguments A and B are rated as 'mostly' or 'completely' convincing compared to the deductive proof argument.

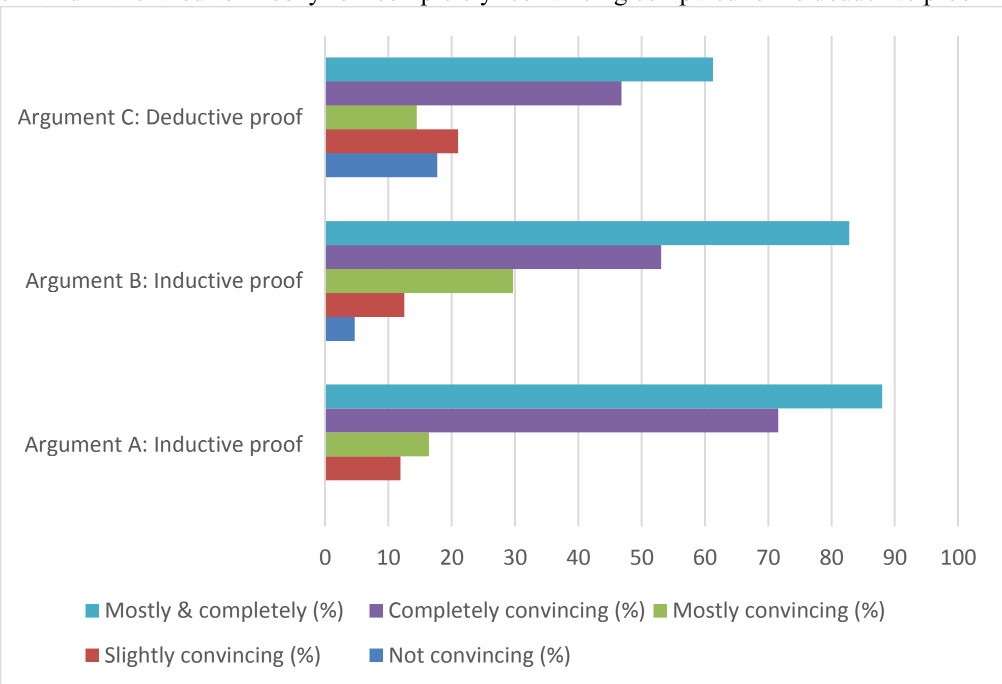

Figure 6: Student teachers' choices of arguments for second algebraic statement (Item 5)

Table 5: Student teachers choices of convincing arguments for the second algebraic statement (item 5).

Algebraic Task and Argument	*Frequency*					
Show that the sum of any three consecutive even numbers is divisible by 6	*R*	*N o*	*S l*	*N o*	*C o*	*N o*
A. We begin by checking the first few sets of three consecutive even numbers as follows: $2+4+6=12$ $4+6+8=18$ $6+8+10=24$ $8+10+12=30$ $10+12+14=36$ $12+14+16=42$ $14+16+18=48$ $16+18+20=54$ $18+20+22=60$ From these first few cases we can conclude that the sum of any three consecutive even numbers is a multiple of 6 and we could go on checking even larger numbers e.g. $1024+1026+1028=3078$ which is also a multiple of 6.	67	0.0	11.9	16.4	71.6	88.0
B. Let's check the first three cases and observe the pattern. $2+4+6=12$ $4+6+8=18$ $6+8+10=24$ From these cases, the sums are forming the pattern $12, 18, 24, \ldots$.. Here our first sum is 12, a multiple of 6 and every time the total goes up by 6, so every other total is a multiple of 6.	64	4.7	12.5	29.7	53.1	82.8
C. Let $a, b and c$ be any three consecutive even numbers, then $a=2n, b=2n+2\ and c=2n+4$ where n is a whole number. $Thus, a+b+c=2n+2n+2+2n+4=2n+2n+2n+2+4$ $=6n+6$ $=6(n+1)$Since $6(n+1)$ is a multiple of 6 then we can conclude that the sum of any three consecutive even numbers is always divisible by 6.	62	17.7	21.0	14.5	46.8	61.3

The results from focus group discussions illustrated that those who accepted inductive proof schemes were more concerned with the accuracy of individual calculations than with the generality of mathematical relationships conveyed by the arguments. They did not raise concerns about the possibility of a single example not being representative of all possible cases, and did not discuss the need for greater generality. Those who correctly opted for the analytic proof scheme believed that those arguments effectively worked because of their generality as shown in the excerpt below:

> **JS:** I think the way we should be rating these arguments is in such a way that we will be able to convince someone easily not like just picking numbers for you to convince someone, it should convince permanently for instance when we rate C to be 4 yeah! This will convince anyone because that person will think of any number and when you pick that number he would still be convinced that the statement is true than when we first pick maybe 3 simple numbers. The person may think that when maybe I pick big numbers this statement won't be fine.

However, it must be observed too that mathematical language was proving an impediment for explanations of proof statements among the student teachers. The following example suffices:

> **JK:** Yes, maybe I will rate the first one at 3, the second one at 2 and the last one at 4.
> Moderator: Why?
> **JK:** The reason for rating like this is that even if you are likely to make an error on A, the explanation is very sufficient for A.
> Now if you go to B, its true because as you are explaining someone is supposed to be convinced and then looking at examples, they are quite…. But C is a complete summary of everything added up together. So if the intended quite alright ……probably 6 as we are seeing 6 is the one that is intended to be divisible…..ah! by, if 6 as you can see is the intended number, if we multiply by any integer okay, and then divided by itself, you see that the multiple is still going to come out as itself probably as 6…… I don't know if I am explaining myself

4.4. Forms of arguments accepted as valid proofs; Example 2

The final task presented to the student teachers required them to '*Prove that for every positive integer n, the expression* $2^{2n}-1$ *is divisible by* 3and the two arguments A and B' basing on an example from their foundational course Mathematics (MAT 110). Three student teachers did not respond to this item leaving the response rate at 95.9% (or n = 70). Basing on this response rate, 41 student teachers chose inductive argument A (58.6%) that was as follows:

> Pick any odd integer (E.g. $n=5$), then $2^{2n}-1=2^{2(5)}-1=2^{10}-1=1024-1=1023$ which is divisible by3.
> Pick any even integer (E.g. $n=8$), then $2^{2(8)}-1=2^{16}-1=65536-1=65535$which is divisible by 3.
> Since the statement is true for both odd and even integers, it follows that the statement is true for all positive integers.

The remaining 29 student teachers representing 41.4% chose the deductive argument B thus acknowledging that a proof must cover more than a single case:

(i) Let $n = 1$ then $2^{2n} - 1 = 2^{2(1)} - 1 = 4 - 1 = 3$ which is divisible by3. So the statement is true.

(ii) If $n = k$, a positive integer, then $2^{2k} - 1$ is divisible by 3.

(iii) If $n = k + 1$, then $2^{2(k+1)} - 1$ is divisible by 3.Here we need to deduce (iii) from (ii).
Since $2^{2k} - 1$ is divisible by 3. Then$2^{2k} - 1 = 3x$.Multiplying throughout by 2^2 we obtain $2^2.2^{2k} - 2^2 = 2^2.3x \Rightarrow 2^{2k+2} - 4 = 12x$

$$2^{2k+2} - 1 - 3 = 12x$$
$$2^{2k+2} - 1 = 12x + 3$$
$$2^{2(k+1)} - 1 = 3\,(1 + 4x)$$

Since the right hand side is divisible by 3 so is the left hand side.Therefore, the expression $2^{2n} - 1$ is divisible by 3 for every positive integer n.

In the focus group interviews, the explanation for the choice of each argument was evaluated. The inductive argument A was considered quite simple, and that it covered the case of both the odd and even number making it a valid proof. A typical rationale was as follows:

> ***BN:*** I think am going for A because it could be easily understood by someone. But for B, you can make some mistakes as you are proving, trying to convince some one. I would go for A.

The majority perceived that deductive argument B was too demanding. However, the formally structured proof B was acknowledged as a valid proof as explained by one student teacher:

> **FN**: As for me I am going for argument B to be the best because for argument A, it's first by trial and error. Just trying any number, you feel like. But for B, there is some truth about mathematics that is being used. You need to induce something, you have to do something like induction and that is what we know its mathematics. You are using mathematics in order to solve the problem.

4.5. Student teachers' views about the integration of proofs into secondary school mathematics education

Towards the end of a focus group discussion, a question was posed on whether it is necessary to teach or incorporate proofs in the teaching and learning of mathematics in secondary schools. The following is the excerpt:

Moderator: Do you think it is necessary to teach proofs to secondary school pupils?

JS: Yeah! It is very necessary because for instance when you first give a formula to students, they will not appreciate it, unless you prove to them how that formula came about then they will be able to appreciate even when they are working and make mistakes they will refer to the proof.

FN: I would say, it is very necessary to teach mathematical proofs in secondary schools because a secondary school student is a student who needs to have abstract thinking. So with the introduction of proofs in mathematics, students will be able to think in line of mathematics as a science. They will think of mathematics as something that is real and of substance.

WN: Also pupils can have confidence in what they are doing.

LN: Yes, and a bit of no reason being; when you look at the curriculum especially at grade 12 level, it's quite difficult to infuse the concept of proving especially if the curriculum is not revisited. Because you need somebody who has gone through a lot of these working outs, like they know the basic things at that level. Now when you look at grade 12, they don't know certain aspects of mathematics, they are on basic level. Now how can you start proving when you don't know some other things, so I feel someone should revise the curriculum if proofs are to be introduced at that level.

BT: I think we have to teach proofs to pupils because they are going to understand what you are talking about and also when they are solving something, they are going to refer to a proof when they make a mistake.

JK: Yeah! It's important. Let's think of a quadratic equation, how they jump from a three-term equation to the formula $\left[\text{Meaning if } ax^2 + bx + c = 0, \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$ they confuse the pupils to say how did they come up with the formula. So if then you prove to them you say okay from this quadratic equation, we come up with this formula, they will be able to connect well.

KN: I will support JS as well because I am talking from experience. I once taught at Kasama skills, everything I presented to the pupils on the board, pupils used to ask, ''Sir how are you coming up with that formula'? Even a quadratic thing….., someone asked **"Nikwisa Cafuma cilya ci bus?"[1].** So due to that question, I was eager, every time I tried to prove to them just to build confidence[1].

CM: Proofs should be reserved for tertially because pupils cannot understand them. So it is very important but not necessary.

JS: I think mathematical proofs are very important to teach. One example I would give for instance you give a formula to the class, they will just memorise that formula and once something changes in that formula they will

[1] This is an expression in the Bemba language, an African language spoken widely in southern and central Africa in reference to the quadratic formula which shapes up like a bus.

have no direction. You find that maybe the quadratic formula someone will just divide square root by 2 leaving b alone which is a mistake. But when they understand how that formula came about, they will also avoid making unnecessary mistakes.

The above student teachers' responses tend to indicate three lines of thinking. One student teacher said there would be no need to teach proofs to secondary school pupils because proofs are too abstract. A second group of students, and in the majority, said that proofs should be taught in secondary schools and then offered reasons for their belief. A third group consisting of one student only believed that proof should be part of the curriculum; however, he also expressed reservations about emphasizing it in secondary schools. The perception that was reflected from the first and third groups of students may imply that mathematical proofs should only be introduced to select groups of students, especially those who plan to study advanced mathematics. However, those in the majority emphasised and reiterated the need to incorporate proofs in the teaching and learning of secondary mathematics as that would improve the quality of learning mathematics at that level.

5. Discussion and implications

Overall, these results demonstrate that student teachers had varied views of the definition of proof and on purpose served by mathematical proof with several implications. First, their responses were scattered around five categorical definitions or explanations of the purpose served by mathematical proof. Only three of these categories carrying 20% or more of the responses appeared important to be ascribed as reflecting the views of the student teachers as a group, i.e., proof as verification, proof as evidence, and proof as communication. This reflects a narrow range of awareness of the functions of proof in mathematics of which a comprehensive list may include the following: (i) verification or justification (establishing truth), (ii) illumination or explanation (providing insight into why it is true), (iii) systematization (organizing into axioms and major results), (iv) discovery (the discovery or invention of new results), (v) communication (the transmission of mathematical knowledge), and (ii) intellectual challenge (the self-realization/fulfilment derived from constructing a proof) (Yopp, 2011, basing his analysis on Bell (1976) and de Villiers (1999).If prospective teachers do not have this wide range of understanding of the functions and purpose of mathematical proof, how likely are they to incorporate them in teaching, learning, and assessment activities? Wu (1996) posited that "logical deduction — proof — is the backbone of mathematics. If we are serious about mathematics education, we should aspire to making every high school student learn what a proof is".

Second, in this study, student teachers associated mathematical proof more with inductive empirical proof schemes than with analytical proof schemes that are based on deductive reasoning. Empirical inductive schemes use specific examples or specific numerical cases are the grounds of the argument (Blanton & Sylianou, 2014). Student teachers chose inductive arguments supported by examples as valid proofs of algebraic statements; this reflects an inadequate view of nature and function of mathematical proof and proving. On the other hand, fewer student teachers chose analytical proof schemes that demand logical analysis and deductive reasoning. Blanton and Sylianou (2014) point out that analytical proof schemes use logical and deductive arguments to arrive at generalizable aspects of a mathematical statement. Empirical proving is considered 'primitive' in comparison to analytical proving that requires logical reasoning rather than mere recognition of patterns.

Third, in their choice of proof or proving arguments associated with algebraic statements, student teachers did not seem to recognise the qualitative difference in arguments, from the empirical to the analytic proof scheme. Given two arguments A and E below, for example, 76.9% of student teachers chose argument A as opposed to 57.6% who chose argument E as the 'mostly' or 'completely' convincing argument. An analytical argument is a more powerful and mathematically reasoned argument.

Argument A: The sum of the digits 1, 2, 3 is 6 which is divisible by3. The number 123 is also divisible by3.

Argument E: Let a be a whole number such that the sum of its digits is divisible by 3. Assuming its digits are x, y and z, then $a = xyz$. Since $x + y + z$ is divisible by 3 it follows that xyz is divisible by 3.Therefore, a is divisible by 3.

To many student teachers, an argument was just an argument, as long as the examples fitted the mathematical statement. It raises the question about the extent to which these student teachers, later as qualified teachers of school mathematics, would be able to focus on the qualitative aspects of mathematical thinking and deductive reasoning.

A fourth and final observation to make on the student teachers in this study concerns the difficulty they encountered in explaining their choices of argument or demonstrating verbally their proof arguments. This may suggest a difficulty in communicating mathematically. It is therefore prudent to observe that mathematical proving provides opportunities for communicating mathematically, i.e., in its grammar, notation, symbolism, and logic, and acquiring the mathematics register (O'Halloran, 2015). It is therefore important to pay attention to assessing the written and the spoken words as students engage in mathematical proving in the classroom.

6. Limitations of the study and future directions

We conceded that there are more empirical arguments than analytical proof arguments and thus we urge caution in interpreting proof validation items 4 and 5.

More research is also needed to investigate the connection between proof construction and validation. The present study provided some evidence for a connection between these practices, but there was insufficient data to fully explore this connection. Since only one construction task was used, it is possible that features of the particular prompt used led to the discovery of a connection that does not exist in general. To account for this possibility, a greater number and variety of construction tasks should be considered during future investigations into the connection between proof construction and validation.

7. Conclusion

The results of this case study echo those in other parts of the world suggesting that prospective teachers of secondary school mathematics have some deficiencies with proof skills and argumentation (Ovez& Ozdemir, 2013). These results point to the need for higher profiling and careful assessment of proof or proving activities in the teaching and learning of mathematics. It is important to base this assessment on the evaluation of how students think and reason, and on how they communicate their thinking and reasoning. In Zambia the expectation is that students studying school mathematics must develop clear mathematical thinking and expression, and that they must reason logically and communicate mathematically (Curriculum Development Centre, 2013).

Taking cognisance of the research connecting mathematics teacher knowledge and beliefs and the way they design or choose and implement proof-related tasks and activities in their classrooms (Ko & Knuth, 2013), it is important to suggest an important role for school based assessments that focus on proof or proving. It is important for teachers of mathematics to increase the range of opportunities and activities in which their students engage in proving or refuting and assessing the quality of the proof arguments systematically. College and university mathematics teacher educators should ensure that student teachers receive a substantial treatment of what consititutes proof in mathematics and why proof occupies a central position in the teaching and learning of mathematics. The content students have opportunities to learn is largely dictated by the content knowledge, both pedagogical and mathematical, of their teachers (Ball et. al, 2008). Therefore, steps must be taken within teacher preparation programs to ensure that prospective teachers are well versed in the functions and characteristics of proof and how deductive reasoning can be systematically integrated into all levels of school mathematics instruction. A great deal of additional study is needed to determine how teacher preparation programs can improve the proof conceptions and proof instruction practices of prospective teachers.

Above all, we strongly suggest that school based assessment should be formative, providing feedback and feed forward on the following: i) how students construct and justify their proofs, ii) how they express and communicate proofs verbally, iii) how they express and present proofs in writing, and iv) how they judge, criticize, justify, or elaborate their proofs or those presented by others.

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