INTRODUCING MULTIPLICATION AND DIVISION CONTEXTS IN JUNIOR PRIMARY CLASSES

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Abstract

This paper shares research from a pilot study in which young children were introduced to multiplication and division problems in their first year of school. The focus was on building children's conceptual understanding of the idea of "repeated groups" as a fundamental aspect of multiplication and its relation to division. The particular mathematics lessons in this study began with simple word problems involving groups of two, using familiar contexts such as pairs of shoes and socks and progressed to groups of five. Children worked with materials to solve problems, often using addition (and subtraction) as they solved multiplication (and division) problems.

Keywords

Mathematics, Numeracy, Multiplication, Division, Primary.

Introduction

New Zealand, like many Western countries, responded to concerns about achievement levels by making mathematics a high priority in schools (Ministry of Education, 2009). The Numeracy Development Projects (NDP), a major initiative in mathematics pedagogy, were aimed at raising expectations for students' achievement in mathematics, and enhancing the professional capability of teachers (Bobis et al., 2005; Ministry of Education, 2001). Key features characterizing the NDP include a research-based framework (The Number Framework) to describe progressions in mathematics learning (Bobis et al., 2005), an individual diagnostic interview to assess students' thinking and reasoning, and a programme of intensive school-wide professional development for teachers. The Number Framework consists of a sequence of stages describing the mental processes students use to solve problems (Strategy), as well as key pieces of knowledge needed to be able to use strategies effectively (Knowledge). The three Strategy domains are Additive (Addition & Subtraction), Multiplicative (Multiplication & Division), and Proportional (Proportions & Ratios). Knowledge domains include Basic Facts, Grouping/Place Value, Fractions, Number Sequence, and Numeral Identification.

The initial stages (0 to 4) on the Number Framework focus on counting, with each stage involving increasingly sophisticated *counting* skills. The final stages involve the use of increasingly complex *part-whole* strategies, based on knowledge of number properties to split numbers apart (partitioning) and recombine them in ways that make the solution easier (e.g., by making a group of ten). A key component of the NDP is the Numeracy Project Assessment (NumPA). This individual diagnostic interview aligns with the Number Framework (Ministry of Education, 2008a, 2008b) and includes a series of questions that are designed to give information about a student's number knowledge and mental strategies. This information is used to help teachers make decisions about the learning experiences necessary for students, both individually and in groups. There are three different levels of interview schedules—Form A, B, and C.

Questions based on the additive strategy domain are asked to decide which difficulty level should be used to assess the remaining strategy and knowledge domains (Ministry of Education, 2008a). If children show no evidence of *counting on* to solve a single-digit addition problem, then Form A is used for the remainder of the assessment. Form A does not include tasks for the other two strategy domains (Multiplicative and Division, Proportions and Ratio). Evidence shows that virtually all

Year 1, more than half of Year 2, and nearly a quarter of Year 3 students are assessed on Form A (Young-Loveridge, 2010). It could be perceived that the absence of multiplication and division tasks in Form A has sent an implicit message to teachers working at this level that their mathematics programme should focus exclusively on addition and subtraction, leaving multiplication, division, and fractional quantities to teachers in the later years.

Although multiplication and division are not mentioned in the New Zealand mathematics curriculum (Ministry of Education, 2007) at Level One, they are referred to implicitly in objectives where "grouping" and "equal-sharing" are mentioned. According to the New Zealand Curriculum:

> [Students at Level One] will solve problems and model situations that require them to:
>
> - use a range of counting, grouping, and equal-sharing strategies with whole numbers and fractions;
> - know groupings with five, within ten, and with ten;
> - communicate and explain counting, grouping, and equal-sharing strategies, using words, numbers, and pictures. (Ministry of Education, 2007)

Despite this *implicit* presence of multiplication and division in New Zealand's mathematics curriculum at Level One, anecdotal evidence suggests that few teachers in junior classes (Years 0–2) use multiplication and division contexts with their students in mathematics lessons.

The Level Two curriculum achievement objectives make no reference to multiplication and division, although students are expected to understand place value, knowing "how many ones, tens, and hundreds are in whole numbers to at least 1000" and "know simple fractions in everyday use" (Ministry of Education, 2007). This is somewhat paradoxical given the fact that place value is inherently multiplicative in its use of groups and powers of ten (see Ross, 1989), and fractions are based on division processes. Yang and Cobb (1995) point out that most place-value instruction involves a sudden switch from a counting-based to a collections-based approach to number. It is not until Level Three of the New Zealand curriculum that there is explicit reference to multiplication and division strategies, facts, and properties.

Both counting-based and collections-based (groups) approaches to working with numbers are important (Yackel, 2001). Children's mathematics learning could be assisted in the long term by providing them with experiences of units other than one, from the beginning of primary schooling (e.g., Behr, Harel, Post, & Lesh, 1994; Sophian, 2007; Thomas, 1996). It has been suggested that working with composite units (i.e., groups) in the early school years provides an important foundation for understanding ideas such as place value, multiplication, division, and proportions/ratios.

At the simplest level, multiplication and division involve three values: the *number* of equal groups, the *size* of these groups, and the overall *total.* If the first two values are known and the *total* is not, the process is multiplication (as in: 2 x 5 = ? means two groups of five equals ten). If what is known is the *total* and the *size* of the group and the unknown is the *number* of groups, the process is quotitive or measurement division (10 ÷ 5 = ?). When the *total* and the *number* of groups is known but not the *size* of the groups, the process is partitive or fair-sharing division, the form of division that is most familiar (10 ÷ 2 = ?).

Evidence shows clearly that children prior to school age can work with equal-group multiplication and fair-shares division (e.g., Blote, Lieffering, & Ouwehand, 2006; Matalliotaki, 2012; Park & Nunes, 2001; Squire & Bryant, 2003). Hence, it makes sense to capitalize on that prior knowledge in the mathematics classroom.

The introduction of multiplication and division word problems provide the opportunity to work with units other than one (equal groups), but still leaves open the possibility that children can solve these problems using their preferred strategy, whether it be counting all, counting on, repeated addition, repeated subtraction, or some form of multiplication or division. Even those using less sophisticated strategies may still learn something important about units greater than one. Our study set out to explore the impact on students' mathematics learning of using multiplication and division contexts familiar to children in their first year at school.

The study

The participants in the study were 18 five-year-olds (10 girls & 8 boys) in a Year 1 classroom (average age = 5.5 years; range 5.2 to 5.7 years) in an urban setting (a decile 5 school). The children came from a diverse range of ethnic backgrounds, with approximately one third of European ancestry, one third Māori, and other ethnicities including Asian, African, and Pasifika. One third of the children had been identified as English Language Learners (ELL). At the start of the study, the children were assessed individually using a diagnostic task-based interview designed to explore their number knowledge and problem-solving strategies. The assessment interview was completed before the focused lessons began, and after the two four-week teaching blocks in Terms 3 and 4. Tasks included forming groups (2, 5, 9), addition (4+3, 8+7), simple multiplication (5x2, 4x5, 3x10), division (half of 4, 8, 10÷2, 8÷4), subitising (3, 4, 5, 6, 8), basic facts (doubles, combinations for 10), incrementing in tens, counting sequences (ones forwards, backwards from 20, and multiples of 2, 5, 10).

Teaching using multiplication and division contexts

Two series of 12 lessons were taught at the beginning of Terms 3 (July–August) and 4 (October–November). In these lessons the children were introduced to groups of two, using familiar contexts such as pairs of socks, shoes, gumboots, jandals, and mittens. Multiplication was introduced using simple word problems, such as:

> Kiri, Sam, and Len each get 2 socks from the bag. How many socks do the 3 children have altogether?

We focused on two different types of division problems. The first was *quotitive* (measurement) division and this type of problem involved finding how many groups of a particular size could be made from a collection. This was introduced using problems such as:

> We have 6 gumboots. How many pairs do we have?

The other type of division problem was *partitive* (sharing) division in which the size of each group must be found. In the following problem there were two equal groups and the unknown was the size of each group:

> Mrs L shared 10 lollipops with Miss R. How many lollipops did each teacher get?

Once children were familiar with working with groups of two, groups of five were introduced using contexts such as gloves focusing on the number of fingers on each glove, and five candles on a cake (particularly salient, given the children had all recently celebrated their fifth birthdays). Quotitive division problems involved splitting up the collection, putting a particular number into each group, and determining how many groups could be made:

> Rabbit has 15 lollies. He wants to put 5 lollies into each bag. How many bags will he need?

Lesson structure

The lesson began with all students completing a problem together on the mat and using materials to support the modelling process. The teacher recorded the collective modelling and discussion in a modelling book, a large scrapbook. The problem for the day was already written in the book and both drawings and number sentences were also recorded, acknowledging individual children's contributions. At this time in the lesson children were introduced to the idea of "working like a mathematician" and recording the appropriate multiplication and/or division equation. The children then completed a similar problem in their own project books, choosing to work with a similar number or a larger number. Materials were still available and children were encouraged to show their thinking using representations and to record a matching equation.

Assessment results

After the conclusion of the series of 12 focused lessons, the children were re-assessed using the same individual interview schedule halfway through Term 3 and again in Term 4. More children (83%) could solve a multiplication problem (5 pairs of mittens in a line; 5 x 2 = 10) than could solve an addition problem (4 and 3; 4 + 3 =7; 50%). Half the children skip counted the mittens in pairs or knew the basic fact, whereas only three counted on or used a basic fact to solve the addition problem (see Appendix A).

All children could rote count by ones at least to 10, and many could skip count by twos to 12 (61%). Skip counting by fives and tens was still quite a challenge that only a couple of children had mastered to any extent. Thirteen children knew at least one basic fact. The easiest basic facts included the small doubles, such as 1 + 1 (73%), 5 + 5 (56%), and 2 + 2 (44%). It was interesting to note that more children knew 5 + 5 = 10 than knew 1 + 4 = 5 or 2 + 3 = 5 (only one child knew these) even though the total number was 10 rather than 5 or smaller (as specified in the progressions outlined in the Framework document: Ministry of Education, 2008a). This finding indicates that teachers need to be aware that certain key number facts are easier even though the sums are greater than five. There is a strong case to be made for children being encouraged to learn the easiest facts, regardless of the number size.

Discussion

The assessment tasks provided the children with familiar contexts in which to think about addition and subtraction, and multiplication and division. The findings from the analysis of responses to strategy tasks and knowledge tasks (Appendix A) are consistent with research showing the hierarchical progression of strategies from counting all (Stages 2–3), through counting on/skip counting (Stage 4) to early additive part-whole thinking (Stage 5) (Young-Loveridge & Wright, 2002).

The findings of this small study are consistent with the idea that Knowledge provides the foundation for Strategy, a major principle underpinning the New Zealand Number Framework (Ministry of Education, 2008a). This data also provides further support for the idea that having some minimal level of knowledge about numbers is a prerequisite for the development of part-whole strategies (Ministry of Education, 2008a; Young-Loveridge & Wright, 2002).

Most of the literature on place value is based on an assumption that children will learn about the decade-based structure of the number system through working with addition and subtraction (e.g., Baroody, 1990; Fuson, 1990; Fuson, Smith, & Cicero, 1997). The emphasis is on the positional property, the base-ten property, and the additive property of numbers (see Ross, 1989), but the multiplicative property seems to have been overlooked. If children were introduced to multiplication and division prior to place-value instruction, the idea of "groups of ten" might make much better sense to students, not just in understanding multiplication and division, but also to consolidate place-value understanding. Yang and Cobb's (1995) idea about the sudden switch from a counting-based to a collections-based approach to number that is typical for most place-value instruction suggests that providing children with experience of different units (e.g., twos, fives, tens) could be immensely helpful in supporting place-value understanding.

We found that introducing groups of two was a good way to start talking about multiplication and division because of the familiarity children had with the idea of "groups of two" shoes, socks, gumboots, jandals, etcetera. However, the meaning of the word "pair" to refer to a group of two was difficult for some children to learn. This contrasts with Sullivan and McDuffie's (2009) investigation in which students focused on the use of collective nouns to describe groups in multiplication and to connect their mathematical problems with real-life contexts. However, this may have been because the children in this study were approximately two years younger than the Grade 3 students in Sullivan and McDuffie's project. It was in response to the enthusiasm of the teacher and the student responses that the lessons moved quite quickly from a focus on groups of two to groups of five. In a subsequent study we plan to explore children's understanding of fives, extending that to groups of ten, and making links to place value.

Although this was a small pilot project, it provides interesting insights into the challenges young children face when solving problems using multiplication and division contexts. This also challenged the teacher, who had previously focused on the operations of addition and subtraction only. The writing of appropriate word problems proved to be a more difficult and time-consuming process than initially anticipated. We noticed that children had difficulties in making connections between their sequence knowledge (e.g., of twos) and the operation of adding two each time. It is important not to underestimate the need for many opportunities to be reminded that the purpose of skip counting is the iterative adding of a constant group in order to determine the total. It is not easy for children to recognise the relationship between repeated addition and multiplication. The children sometimes confused the role of the number of groups (multiplier) with the size of each group (multiplicand). This is consistent with the work of Sullivan and McDuffie (2009). Despite the added challenge for children of distinguishing between the multiplier and the multiplicand (language we used with the teacher but not the children) we observed that they seemed very excited about working with multiplication and division contexts and the related material. If teachers of young children introduce multiplication earlier, this could help children develop a better understanding of higher-order units and provide a stronger foundation in multiplication, division, proportion/ratio, and algebra. As the research progresses, we will be interested to look at how the children's thinking develops, not only in the domain of multiplication and division but in the knowledge domains as well.

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Appendix A

Numbers and percentages of students according to gender and ethnicity, and who could do particular tasks

	(n=18)	%			(n=18)	%
Gender				**Multiplication Strategy**		
Males	8	***44***		**5 x 2 mittens**	15	***83***
Females	10	***56***		5 x 2 by Counting All	6	***33***
				5 x 2 by Skip Counting	7	***39***
Ethnicity				5 x 2 by Basic Facts or Derived Facts	2	***11***
African	1	***6***				
Asian	3	***17***		**4x5 monkeys with bananas**	3	***17***
European	6	***33***		4x5 by Counting All	2	***11***
Maori	6	***33***		by Skip Counting &/or Counting On	1	***6***
Pasifika	2	***11***				
				3x10 rows of cupcakes	7	***39***
Language				3x10 Counting All	7	***39***
First Language English	15	***83***				
English Language Learners	3	***17***		**Basic Facts - Doubles**		
				1+1	13	***72***

	(n=18)	%			(n=18)	%
Forming Groups				5+5	10	***56***
Get 2	17	***94***		2+2	8	***44***
Get 5	17	***94***		3+3	6	***33***
Get 9	14	***78***		4+4	5	***28***
9 and one more	14	***78***		10+10	4	***22***
9 and ten more	1	***6***		6+6	1	***6***
Subitising				**Number-word Sequences**		
3	15	***83***		Counting by **ones** to 10	18	***100***
4 (dice)	13	***72***		by ones to 20	12	***67***
4 (line)	8	***44***		by ones to 50	6	***33***
5 (dice)	13	***72***		by ones to 100 or higher	2	***11***
6 (dice)	10	***56***				
8 (ten-frame)	9	***50***		Counting by **twos** to 10	15	***83***
				by twos to 12	11	***61***
Addition & Subtraction Strategy				by twos to 20	6	***33***
3+4	9	***50***		by twos to 100	2	***11***
3+4 by Counting All	6	***33***				
by Counting On or Skip Counting	2	***11***		Counting by **tens** to 50	3	***17***
by Basic Facts or Derived Facts	1	***6***		by tens to 100	2	***11***
Division Strategy				Counting by **fives** to 20	7	***39***
half 4 beans	13	***72***		by fives to 50	3	***17***
half 8 beans	11	***61***		by fives to 100	2	***11***
10 ÷ 2 (quotitive: pairs of socks)	4	***22***				
8 ÷ 4 (partitive: block of chocolate)	9	***50***		**Backwards** Counting from 10	14	***78***
				backwards from 20	2	***11***