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The Connection Competencies of Pre-service Mathematics Teachers about Geometric Concepts to Daily-life

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Abstract However, geometry is the area with the most concrete possibility of mathematical topics which contains more abstract concepts, students experience difficulties while understanding. Therefore, the connection of issues with daily life to concrete the subjects and the ability of connecting geometric concepts with daily life of the teachers and pre-service teachers who will teach these subjects become more important. In this study, the geometric concepts of pre-service mathematics teachers along with their ability of connection with daily life will be examined. 79 senior students studying at the Department of Elementary Mathematics Teaching at the Faculty of Education were asked to give examples from daily life of 21 geometric concepts in the Middle School Mathematics Curriculum. The received responses were analyzed with descriptive content analysis. Upon the examination of the answers given, it was observed that students mostly gave examples of what they had learned since primary school and of terms they used in the lessons frequently.

Keywords Geometric Concepts, Connecting Daily-life, Pre-service Mathematics Teacher

1. Introduction

We do not have the opportunity to directly see the majority of examples represented by mathematical concepts in our everyday lives, however there is the possibility to encounter the majority of concrete geometric concepts in the environment in which we live. Maybe for this reason, when first confronted with geometric concepts at school, students neither experience a lot of problems nor regard them as strange compared to mathematical concepts. Herein, because geometric concepts have a visual structure they are easier to be perceived and understood has a greater positive contribution. However, it can be intangible to the child to whom the connection with this information is trying to be taught at school, starting with knowledge learned unconsciously like his native language. For this purpose, teaching should be carried out by connecting information encountered in daily life and which has already been learned [20, 26].

Mathematical concepts can be formed in the period that named connection competencies in mathematics education, by following the steps of prior use of learning, observations of daily life, and connections of other concepts [18]. In that case, when teaching geometric concepts supporting these terms from daily life, knowledge of these concepts can be obtained from different situations of daily life. While geometric concepts are formed and learned, the use of observations of daily life can be considered the first step by while emphasizing on their meanings. Only giving examples related to the term should not be sufficient, geometric concepts can be made to be understood and their use can be ensured by giving importance to the use of mathematical language in order to form similar structures in the students' minds. The situations of daily life are transferred through the learning environments by means of two sub-components of connection with daily life, which is one of the four basic components of competencies of connection, which are "to address the concept within the context" and "to express its connection with daily life by means of oral examples" [3]. Connection with daily life can be defined as the connection between the mathematics taught at school and the outside world [21]. The connection of mathematics with daily life contributes to its being perceived as real, as it facilitates making sense of the concretization of mathematics, which is an abstract science [25]. Furthermore, the situations encountered in daily life prepare individuals for future professional and social life [21].

The competency of connection with daily life, which presents such importance, is repeatedly emphasized in national and international teaching programs and in standards and exams [15, 23]. In standards for school mathematics published by the American National Council of Teachers of Mathematics, which mentioned "the need to understand mathematics in daily life and to use mathematics in daily life", the importance of mathematical knowledge in the understanding of the world was brought to the forefront and connection was accepted as one of the standards set for school mathematics [23]. Connection with daily life, at different intensities, has been emphasized vigorously in all past and present primary school mathematics curriculums in Turkey. Nonetheless, the most significant emphases was on the updated 2005 primary school curriculum and, the connection of daily was mentioned for the first time. Besides, the skill-based learning environment has continued to be recommended in continuously revised programs [3]. Moreover, these innovations have been transferred to mathematics textbooks. On the other hand, in the scope of students' school curriculums, international exams such as TIMMS and PISA are conducted in order to measure how much they use information learned in the problems they encounter in daily life in the issues discussed. In these exams, due to Turkey's low achievement shown in the field of mathematics (e.g. in the PISA exam conducted between 2003-2012, the score of math literacy corresponds to the level 2 [27, 28]) and also the structure of the high school transition exam conducted on a national scale, these exams have been amended in line with their context and made suitable for the purposes of updating the curriculum. In this context, in the international exams conducted between 2009 and 2013, the rate of connection of the questions asked with daily life increased almost two-fold according to the years during 1998 and 2008 [22].

Connections with daily life in mathematics education literature are only predominantly discussed as a competency which students should acquire. However, whether connection is done within the classroom and how connection is done is also of great importance. Erturan (2007) [9] stated that students have inability to transfer mathematic concepts to daily life within the classroom and there is necessity of supporting the concepts by examples within daily life. However, in the conducted research, it was observed that very few studies on the subject of connection with daily life in a real classroom environment are available and teachers have not included the implementation of activities to improve the competencies of connection. Furthermore, during the teachers' teaching process, it is understood that they experience difficulties connecting mathematics with daily life [14]. Yet, it is necessary for teachers to design suitable learning environments and have the ability to implement them during the teaching period in which students need different learning environments. The conducted studies stated that teachers are not aware of the importance of this skill and those who are aware do not know how to carry it out. In classrooms, whether teachers who include connection with daily life are really qualified to mention the relationships is unknown [5]. Therefore, knowing the importance of connection with daily life, effective use and successful use in teaching is closely connected with teachers' competencies [3]. In this regard, the development of educational materials related to how to reflect it in teaching and its availability for teachers' use is important [3].

Besides, teachers establish connection of daily life orally or through connection, but it has been observed that they do not enter into much distinction [5]. In the scope of the context, especially in relation to RME (Realistic Mathematics Education) principles or in studies often using more problems in the context of daily living, connection with daily life is often understood in the context discussed [3]. Therefore, there is a need to provide studies of a limited number of verbal examples in literature related to this subject. On the other hand, geometry is the most unsuccessful subject for pre-service teachers among the mathematics subjects, the most trouble experienced when taught by teachers and the least connected field with daily life [26]. The teaching of concepts in this field is ensured by showing examples of concepts instead of providing the definition. Students looking at these examples should explain what they understand from the concepts, and other examples of these concepts should be shown. For example, when teaching the concept of a rectangle, after giving the concept mathematically, examples from the classroom or from daily life of the concept's connection should only be given orally. From this point of view, it is said that when teaching a concept, especially geometric concepts, there is need for studies of sub-components referring to "only the use of daily life contexts verbally" including "giving verbal examples from real life" [3].

As a result, the inadequacy of studies done in the context of the subject of this study in mathematics education should not be overlooked. Moreover it should be drawn attention with following studies to the connection competencies from the students as well as the teachers' deficiencies and the causes. From this viewpoint, in this study, it was aimed to research the competencies of connecting geometric concepts with daily life of mathematics pre-service teachers, who are studying in teacher education programs.

2. Method

In this study, which was carried out with the objective of examining mathematics pre-service teachers' success of connecting geometric concepts with daily life, a case study was adopted as a research method.

The research study group consists of 79 senior pre-service teachers studying at the Department of Elementary Mathematics Teaching at the Faculty of Education located in the Turkey's northeast. In the undergraduate program in which this research study group has studied, lessons providing to improve pre-service teachers' knowledge and skills in the field of geometry are "Geometry", "Special Teaching Methods I-II" and "Geometry Teaching" classes. Pre-service teachers who will be the future middle school teachers (5th to 8th grade with an age range of 9 to 13) will teach 4 hours of mathematics classes to students per week. In the area of geometry learning, which makes up 35% of these lessons curriculum, it is aimed to give descriptions of geometric concepts with their unique geometric terminology, introduced intuitively with concrete and finite models in primary school [15]. It is expected from students in the 5th grade to name the description of basic geometric concepts such as straight line, segment of a line and ray, polygons. At this level, a place is given to achieving an understanding of the basic properties of rectangle, parallelogram, rhombus and trapezoid. In the 6th grade, a place is given to acquiring an understanding of the concepts of angle and circumference. It is expected from students in the 7th grade to calculate the length of a circumference and parts of a circumference and the area of a circle and section of a circle is expected when examining a rectangle, parallelogram, rhombus and trapezoid in the classroom. It is expected from students in the 8th grade to calculate the area and volume of geometric objects in the classroom [19].

In the study, a questionnaire consisting of open-ended questions was used as a tool for collecting data. In the questionnaire, which was developed by the researcher, the connection of these concepts with examples from daily life was only expected from the students to whom the mathematical concepts were given.

It was required from students to give examples from daily life to geometric terms like point, line, line segment, ray, angles, triangles, triangular region, rectangle, rectangular region, square, square region, parallelogram, parallelogram region, rhombus, rhombus region, trapezoid, trapezoidal region, circle, circular arc, disk, sector…etc. The questionnaire was carried out on pre-service teachers in the final year at the end of the Geometry Teaching course. Pre-service teachers were given 30 minute to respond to the questionnaire.

Responses received from students were examined with descriptive content analysis. Which terms the students were most able to give as examples and the accuracy of the examples given was questioned, and the frequency of the number of examples given was calculated. This calculation, consisting of 4 tables, is presented in the results section. At the classification stage, coding is done by two mathematics educators which one of them is the researcher and between the two coders the kappa coefficient, indicating that the strength of agreement between the two raters was substantial [29] and should be bigger than %70 [30], was obtained % 83.

3. Results

Results have been organized as four tables including the terms in the subjects of basic concepts (point, line, line segment, ray and angle), planar shapes I (triangle, square, rectangular), the planar shapes II (parallelogram, rhombus, trapezoid) and circle & disk, and the daily life examples pre-service teachers gave regarding this terms.

As seen in Table 1, the pre-service teachers have exemplified the term of Point 23 models; term of Line 16 models; term of Line segment 22 models; term of Ray 30 models; term of Angle 16 models.

Point models have mostly been preferred from very tiny sizes (trace, particles, pinpoint) but also models from a variety of areas (polka-dot button, top view of the soccer ball in a soccer field, right middle of a soccer field etc.) and volumes (chickpeas, sun, planets, star, etc.) are taken. Another aspect of the examples given is that the view of the objects from a far distance forms the point model (star, planet, sun, the location of Mehmet's house in the district A, a fly on the white curtain, the view of people from a plane, a small stain on the dress).

When Line models are examined, it is seen that either so long models whose ends cannot be seen (highways, solid lines on the highway, rail tracks, horizon, power line etc.) or models representing arrows in each end (pencil with both ends sharpened, where your fingers reach when you open your arms 180^0 parallel to the ground) were chosen to give examples. Pre-service teachers didn't emphasize linearity in their examples. A pre-service teacher expressed an idea of not being able to model the concept of infinity with the following statement: "We cannot give examples from daily life since it doesn't have any beginning or end".

When the Line segments models are examined, it is understood that pre-service teachers prefer objects that are linear and that have measurable limits. Some pre-service teachers gave examples for Line segments with models representing ray such as knitting needles, arrow, sunlight, and laser.

When Ray models are examined, mostly examples that are thought to start at a constant point and that go to infinity (laser, solar rays) are preferred. Furthermore, models representing objects with one end constant and the other end having an arrow such as nail, pencil, and matches have been given as examples.

When Angle models are examined, it is seen that as well as the examples representing the angle correctly, angle's internal part has been modeled. The angle model of the "area between the long hand and short hand of a clock" in fact models the internal part of an angle.

Overall, when examples of pre-service teachers for all concepts are considered, almost half of the angle concept is wrong while almost all others are right. It is also seen that examples given for some concepts were the same (pencil, road etc.). For example, unsharpened pencil has been shown as a model to Line, a pencil that is sharpened at one end has been shown as the Line segment and the pencil whose both sides are sharpened has been shown as a model to Ray. Also, models that are expressed as area or volume in the daily life (such as rivers) have been thought as too big and they were perceived as length.

Table 1. Examples given for some geometrical concepts in the subject of basic concepts (point, line, line segment, ray and angle)

Geometric concept	Examples	f
Point	Trace (the pen / chalk / nail leaves on the board, the rod leaves on the ground)	38
	Star	7
	Dirt / dust / sand / salt particle	5
	Pinpoint, Polka-dot	3
	Spot on the body, Rain drop, Top view of the nail that holds the long and short hands of the clock together, The mark used at the end of the completed sentences in Turkish	2
	Acne, Red spots on the body during measles disease, Top view of the soccer ball in a soccer field, The middle of soccer field (substations), Looking at the ferrule of the umbrella from a top, Sun, Planet, The knot on the rope, The location of Mehmet's house in the district A, Chickpea, A fly on the white curtain, The view of people from a plane, Button, A small stain on the dress	1
Line	Rail tracks	20
	Highways, Solid lines on the highway	16
	Rubber string	5
	Horizon	4
	Power line, Double-sided laser	3
	Pencil sharpened at both sides	2
	Where our fingers reach when you open your arms 180^0 parallel to the ground, The image on the mirror when you stick a string on two mirrors facing each other, The trace planes leave in the sky, The path followed by a bird flying in the sky in a linear direction, The connection line of the garden wall with the ground, ***Light, Rainbow, The beams that are formed by fireworks explosion***	1
Line segment	Broken lines on the highway	18
	Pencil	10
	Rod (shovel / ax handle, walking stick), spaghetti	8
	Rope of a certain length	7
	Length of the board/table/desk	6
	Ruler	5
	Crosswalk	4
	Electric pole, The beams of a room	3
	Cracker, Pencil tip	2
	A piece of matches, Column, Beads lined on a tense rope, The distance between two houses on a linear path, Stairs, Rolling pin, ***Knitting sticks, Arrow, Sunlight, Laser***	1
Ray	Laser	16
	Solar rays	13
	Arrow	6
	Light coming out of the lamp	5
	The path of the space shuttle, Light coming out of the flashlight	4
	Knitting ball, Nail, Road	3
	Light coming out of the car headlights, Railway, Horizon	2
	Rubber cord with one stable end, Pencil, Straight path with one end closed, Water flowing from the tap, Shovel, Lane arrows, Matchstick, The direction of the bullet fired between the gun and the destination, Our finger pointing forever when we lift our right arm in parallel to the ground, Cable extending from the beginning of the lift, Kite, Tree, River, Power line, Tape measure, Size chart, Starting from the district A and moving further, ***The distance between two streets***	1
Angle	***The area between the long and short hands of the clock***	17
	The open state of door	15
	The state of two fingers / two arms / the arm and body relative to each other	10
	The open state of the scissors	8
	The gap between the intersection of two walls, ***The part left in the middle as a result of the intersection of main road and side road***	5
	The open state of the window	4
	The wheel spokes, The open state of the book, Situation formed by the short and long hands of the clock, The place where two edges connect at the top surface of the table, ***The part left in the middle between the house's roof and wall***	2
	The open state of the fan, the open state of the compass, The distance between the tree branches, ***The arc clock draws as time passes***	1

As seen in Table 2, the pre-service teachers illustrated the term of Triangle over 13 models; term of Triangular Region over 9 models; term of Square over 13 models; term of Quadratic over 9 models, term of Rectangular over 25 models, and term of Rectangular Region over 17 models. In fact, the examples given to these concepts are much more than the ones stated in this table. However, since these examples include special occasions such as "earrings in triangle shapes", they were not included in the coding.

Table 2. Examples given for some of the geometric concepts included in the subject of planar shapes and regions I (triangle, square, rectangular)

Geometric concept	Examples	f
Triangle	Roof (as in shape of a triangle)	17
	Roadsign	15
	Rack(billiards), Triangle (musical instrument), ***Triangle cream cheese***	4
	Tent	3
	Cookie cutter	2
	Bottom part of a stool, The top view of the burner stand, Construction bevel, ***Bell,*** Sail, ***Pine tree***	1
Triangular region	Roof (as in shape of a triangle)	15
	Roadsign	14
	Triangle cream cheese	9
	The surface of the tent	5
	The surfaces of the Egyptian pyramids	4
	Chips (triangular)	2
	Sail, Laz (ethnic) pastry, Ice cream (a picture of)	1
Square	***Tile***	23
	Photo frame	16
	Window	7
	Table	6
	Base edges of the cubic sugar, Sides of any surface of a dice, Monitor, Wall clock (square)	2
	One surface of a brick, Electric Outlet, Crochet made on the scarf, Peripheral of the piece of Baklava	1
Square region	Tile	19
	Field, The room's ceiling / floor / wall	7
	One side of the cubic sugar	6
	A surface of a Dice, Table surface (quadratic)	5
	Glass window	4
	Piece of Baklava, Chessboard,	2
	Scarf, Billboard, Slice of a cake	1
Rectangle	***Writing board***	27
	Window	13
	Photo frame	12
	Table, Door	8
	A4 size paper	3
	Lines of a soccer field, Soccer net, ***Home, Desk,*** ID card, ***Television***	2
	Carpet weaving looms, ***Turkish flag, Banknotes, Book, Notebook, Eraser, Field, Room, Box of pen points, Billboard***	1
Rectangular region	Writing board	18
	Table	9
	Soccer field, Door	7
	Field	6
	Carpet	5
	Desk, Flag	4
	Window, Classroom floor/ceiling, Photo frame, Tile	3
	The front of the building	2
	Book, Notebook, Laptop Screen, Billboard, A4 size paper, Cardboard, Faces of a matchbox	1

Examples given for triangle and triangular region represent three-dimensional objects such as pyramid, triangle pyramid (such as tents) or area (road sign etc.). It is seen that triangle was illustrated with rack, triangle (musical instrument), cookie cutter, and top view of the burner stand, bell and construction protractor. However, in other examples given (such as road sign, the surfaces of the Egyptian pyramids, chips), it is understood that no distinction was made between triangle and triangular region. A few pre-service teachers expressing these examples mentioned some details as "skeleton of the roof, the red tape surrounding the road sign", which shows that they could distinguish between triangle and triangular region. On the other hand, while roof and tent are generally given as example for pyramid, in this study they were illustrated as triangle. Most of the pre-service teachers expressed the roof as “the roof of the house” as an example of triangle while some of them tried to emphasize that they are different from pyramid by adding more details like "front view of the house's roof" or "one-dimensional view of the houses’ roofs”. Also, despite not being stated, by saying “the roof of a house”, house picture might have been interpreted as examples like ice-cream, tent and Egyptian pyramids where 3-D objects were drawn were given as examples of different triangle models by pre-service teachers.

For square and quadratic region, examples that are 3-dimensional like cube or tetragonal prism in daily life (floor/ceiling/wall of a room etc.) or examples representing area (tiles etc.) were given. In some 3 dimensional examples, square was indicated as the surface of some objects (one side of a cubic sugar, one side of a dice, table surface) while it wasn’t mentioned in others (slice of a cake). Pre-service teachers exemplified square with window and quadratic area with the window frame. A few pre-service teachers expressed window as “window with no glass in”. This tells us that pre-service teachers’ expressive skills are not sufficient. The table also demonstrates that the examples pre-service teachers gave for the quadratic area were all correct while there were wrong models in their examples for squares. This may result from the pre-service teachers’ lack of expressive skills or knowledge on geometry subject.

Examples given for rectangle and rectangular region are also divided into two groups as three-dimensional ones (room etc.) and the ones representing area (paper etc.). It is seen that most of the examples given for rectangle such as writing board or window and door represented rectangular region and in that aspect, they were wrong models. One-third of the pre-service teachers emphasized the details like “the bar part of the writing board” or “window without glass” and tried to show the difference between rectangle and rectangular regions. This ratio indicates that pre-service teachers were aware of the shape-region distinction and this is not a result of the lack of geometry knowledge but because of the insufficiency in expressive skills.

Overall, it is understood that specific examples were emphasized by the majority in the subject of the planar shapes (triangle, square, and rectangle) and other examples were given by a few pre-service teachers so that variety of models were provided. It is noteworthy that there are wrong illustrations resulting from not paying attention to the shape-region distinction.

As seen in Table 3, the pre-service teachers illustrated the term of Parallelogram over 26 models; the term of Parallelogram Region over 27 models; the term of Rhombus over10 models; the term of Rhombus Region over 10 models, the term of Trapezoid over 11 models, the term of Trapezoidal Region over 12 models.

Examples given by the pre-service teachers for parallelogram can be divided into three groups. The first group includes right examples such as parking lines, the shape of the edges formed by the cross section of the timber, stairs / window railings, the carpet motifs. The second group includes structural examples of rectangle but in terms of their remote appearance they are examples of parallelogram. Examples such as table with stretched legs, iron bars that are parallel to each other in laundry rack, window frame are examples of rectangle structurally but in terms of their remote appearance, they are examples of a parallelogram. The expression of pre-service teachers as “legs of a folding table”, which indicates the table with stretched legs, confirms this thought. Similarly, examples given for parallelogram region, such as solar panel, door, wall, writing board, tile, and desk are structural rectangular region; however, their remote appearance (perspective) or images are parallelogram regions. This situation should be emphasized while presenting these examples in the classroom in order to prevent misconceptions (confusing parallelogram with rectangle). The third group is about perceiving objects in upright prism structure model as parallelogram surfaces in order to gain dimension although they have rectangular surfaces. Examples given such as roof, one side of the tent, rows, floor / ceiling / wall, bookcases, cabinets, stairs represent upright prism itself or its one side and they are drawn as parallelogram. Here, however, the eraser example (a particular brand) should be distinguished from the other prism examples. Eraser is structurally an inclined prism model and its side surfaces are really parallelogram regions. On the other hand, another difficulty pre-service teachers had as they did in other planar shapes (triangle, square, rectangle) is that they couldn’t tell any examples that distinguish parallelogram and parallelogram regions. Examples such as piece of Baklava, paving stone, or writing board were given for parallelogram. It is understood from some of pre-service teachers’ expressions like “the strip on the edge of the writing board” that they were aware of this situation.

All of the examples given in rhombus (except for the roof and Rubik's Cube) represent areas. In addition, among these examples there are the ones like "honey frame, electric outlet, Rubik's Cube, which represent square, the special case of a rhombus. Besides, when examples were examined, it is seen that examples given for rhombus and rhombus region are similar.

Table 3. Examples given for some of the geometric concepts included in the subject of planer shapes and regions II (parallelogram, rhombus, trapezoid)

Geometric concept	Examples	f
Parallelogram	Table with legs stretched	9
	Roof (3D home image)	6
	Window frame, ***Piece of Baklava***	3
	Photo frame, Stairs / window rails, ***Table (3D image), Solar panel, Door, Desk, Floor / Ceiling / Wall, Paving stone, Writing board***	2
	Park lines, The edges of the shape formed with the cross cut of timber, The carpet motifs, Door casing, Iron bars in parallel to each other on laundry racks, Stairs, ***A4 size paper, Bookshelf, Cabinet, Painting, Road sign, Tile, One side of the tent***	1
Parallelogram region	Garden	10
	Piece of Baklava, Roof	7
	Paving stone, Eraser (a particular brand)	3
	The carpet motifs, A4 size paper, Window, Door, Table surface	2
	The edges of the shape formed with the cross cut of timber, Curved path, Car headlight, Park space between lines, Matchbox surface, Desk surface, Kitchen counter tops, Solar panel, Stairs, Plate, Projection board, Metal sheet, Painting, Sliced cheese, Banner, One side of the tent, Floor / Ceiling	1
Rhombus	***Piece of Baklava***	25
	Kite	4
	Road sign	3
	The carpet motifs, ***Electric Outlet***	2
	Lace pillow, Stairs rails, ***Honey frame, Cabinet cover, Tile***	1
Rhombus region	Piece of Baklava	22
	Kite	6
	The carpet motifs, Sweater Patterns, Tile, Roof	2
	Paving stone, Lace, ***Electric Outlet, Rubik's cubes***	1
Trapezoid	Car glass (picture), Roofing (2D home image)	5
	Paving stone	4
	Window / Balcony railings	3
	Ship (picture)	2
	The hood's edges, Tool bars that make up the tents face, Stripes edge of the hood of a car, Skirt (image), ***Side view of the top part of the fedora, Side surface of the disabled ramp***	1
Trapezoidal region	Roofing (2D home image)	9
	Car glass (picture)	8
	Paving stone	3
	Skirt (picture)	2
	Ataturk bust, Ship, Tent's surfaces, View of disabled ramp from a side, View of stairs from a side, Car headlight, Chair, Road with perspective drawing (with pictures)	1

It is possible to examine the examples given for trapezoid in three groups. The first group includes mainly object pictures such as car windows, roof, ship, skirt, road with a perspective drawing, car headlights, etc. The second group is composed of the edges of 3 dimensional objects such as of hoods, tents, and chairs. There are some area representations in the last group such as paving stone, window / balcony rails. On the other hand, among the examples given for trapezoid, there are wrong examples modeling trapezoid region.

In general, fewer examples were given for planar shapes (parallelogram, rhombus and trapezoid) than spatial shapes (triangular, square, and rectangular). Also, wrong examples were seen for parallelogram, rhombus, and trapezoid while almost all of the examples given for parallelogram region, rhombus region, and trapezoid region were correct. The reason for this can be explained by the lack of attention to the distinction between shape and region.

As seen in Table 4, the pre-service teachers illustrated the term of Circle over 18 models, the term of Disk over 20 models and the term of Circular Arc over 16 models, and the term of Sector over 6 models.

It is seen that pre-service teachers have given examples distinguishing circle and disk. When the examples given for circle are examined, examples like peripheral of the clock or coin or the mouth of the bucket, which represent the limits of the disk or the base of the cylinder, were given as well as the models that represent circle directly. When examples given for disk are examined, it is seen that sphere models such as ball, sun or moon were given. However, it is thought that 2 dimensional pictures of these were mentioned. Similarly, it is thought that in the circle models given such as cake or wall clock, surfaces of these objects were mentioned.

Examples given for a circular arc have been divided into two groups as concrete objects seen in daily life such as a piece of bagel and virtual models thought as arc that the swing makes or the clock. Considering the examples given for sector, it is understood that watermelon slice, which is 3 dimensional in daily life, was visualized by sphere models like half-moon and a slice of a cake was visualized by cylinder models like triangle cheese or 2 dimensional visuals.

Overall, all of the examples cited by pre-service teachers for the four geometric concepts are correct. The number and kind of the examples given to circle & disc and circular arc & sector is remarkable on the contrary of the other geometric concepts.

Table 4. Examples given for some geometric concepts on the issue of circle & disc

Geometric concept	Examples	f
Circle	Bagel	19
	Hula-hoop, wedding ring	16
	Wheel	13
	Basket	9
	Bracelet	7
	Soccer mid-line, Pulley, Ring	3
	Circle of Time, The circumference of the coin	2
	Hive's mouth, Princess crown, anklet, Ferris Wheel, Hoop earring, Band, ring formed by 5-6 students by holding each other's hands	1
Disk	Wall clock (round)	18
	Coin	15
	Cake (round)	12
	Lid (trash, manhole, bottle, jar), Fruit section (watermelon, orange, banana)	8
	Ball	5
	CD	4
	Pizza, Plate, Tray	3
	Sun, Moon, Area formed by a center line, ***Wheel***	2
	Wafer, City roundabouts, Road sign, Pan, Helicopter pad, The area where wrestlers are competing	1
Circular Arc	A piece of bagel	17
	Bow	11
	Between the two rims of the bicycle wheel	8
	Crown	5
	The arc formed by short and long hands of the clock, Twisted iron wire piece	4
	A part of Bracelet, Hair clip	3
	Cut hula-hoop piece	2
	Nose ring, Opening band of the packaging of the triangle cheese, The view of iron bars holding the overpass, Virtual arc formed by the swing, Virtual arc formed by the clock pendulum, Boat's steering, Wheel, Hook	1
Sector	A slice of cake (round)	50
	Pizza slice	18
	Watermelon slice	4
	Half Moon, Triangle cream cheese slice	2
	Fan	1

4. Discussion and Conclusion

Geometric concepts are taught by showing examples of concepts instead of giving a definition, and students, looking at these examples, describe what they have understood from the concepts and present other examples for these concepts [18]. Therefore, it can be said that the examples of concepts given by the teachers form the basis of learning these concepts. With this study, the ability of pre-service teachers to give examples of geometric concepts from daily life was examined.

Considering the numerical data, it can be said that usually each of the pre-service teachers indicated approximately one example for each concept. Taking into account the fact that pre-service teachers teach these concepts at middle-schools and want these types of examples to be given by the students, it can be said that the number of examples for per concept is insufficient. This situation suggests that pre-service teachers do not possess sufficient knowledge in the field of geometry. In related literature, that pre-service teachers' abilities of connection with daily life remain low is encountered in similar studies [24]. In the study of Eli (2009), it was stated that the knowledge of pre-service mathematics teachers for geometry instruction was at a low level and that the mathematical connection carried out was a lot more operational rather than being conceptual. It is also emphasized that the abilities of connection with daily life of pre-service teachers who give more importance to operational knowledge than conceptual knowledge should not be expected to be high [24]. The study by Gainsburg (2008) [10], which examines high school mathematics teachers' abilities of connection with daily life, also lends support to this idea. Although a large number of the teachers' real life examples can be counted, it has been determined that the connections done through these examples were minimal and summarized, and that there was not an ability to motivate and canalize the students to think.

Another remarkable piece of numerical data is the fall in the number of examples given for the trapezoid (approximately one-third of the examples), and rhombus (approximately half of the examples) types. Among the examples given for the rhombus, it was suggested that in a quite large number of examples representing the square, which is the special case of the rhombus, the pre-service teachers experienced difficulty specifying models which represent the rhombus directly. Moreover, the scarcity for examples given for the triangle and square, which have been learned about since pre-school, is noteworthy. In the study of Dane (2008) [7], which examined the examples about geometric concepts given by students, it was stated that the number of analogies done was very low, and the conducted study indicates that about half of the students and one in four students achieved connection with correct analogy and parallel results for the concept of a point and the concept of a line, etc., respectively. The results obtained support the idea of a deficiency in pre-service teachers' content knowledge of geometry. In related literature, the number of studies which reveal that there are difficulties in the basic geometric concepts of pre-service teachers and students is very high [4, 6, 8, 13]. In these studies, when students used the features of geometric shapes such as the square, rectangle, triangle, rhombus, in spite of not experiencing problems, they did not exactly know the features of some geometric shapes such as the parallelogram [6]; the geometric concept related to the parallelogram was not fully clarified in the minds of the majority of students [13]. On the other hand, all the pre-service teachers' examples given for a circle, disk, circular arc, and segment, were correct. Unlike other geometric concepts, the differences and diversity of the examples given for circle-disk and circular arc-segment are noteworthy. The circle & disk is one of the areas of sub-learning in geometry in which pre-service teachers are successful. In addition, while other geometric shapes and regions as a whole have been discussed since primary school, the inclusion of the circumference and the circle as two different concepts in the curriculum may have led to the emergence of these results.

In this study it was observed that in spite of the given examples represented correct models, the majority of them also included the models that will be caused misconceptions and scarcely any of them included wrong models. In the examples given for basic concepts in geometry, students were faced with two problems such as size and infinity, which could cause misconceptions. The geometric concept known as dimensionless (point) includes two and three-dimensional objects among the examples given. Similarly, among the examples given for a line, line segment and ray known as one-dimensional, two or three-dimensional examples are available. These can create a contradiction in terms among the students. In related literature, research is available on the manner of students having these types of misconceptions. For example, by saying "A solid angle has a thickness" Kesici (2005) [12] has demonstrated that high school students have misconceptions. To avoid these types of errors, the creation of correct models should be emphasized if the above-mentioned examples are considered too small or too distant. Moreover, it should be noted that the acquisition of these models in various sizes, lengths, areas and volumes does not mean that these geometric concepts have length, area and volume [15]. On the other hand, abstract concepts such as infinity are expressed with objects of a particular length, such as infinity represented with an arrow sign which indicates a very long length or that which can be extended indefinitely.

With problems about size continuing in the examples given for geometrical shape and geometrical area in basic concepts, it was observed that problems with perspective also arose. Generally, the examples given were divided into two groups: those that were 3-dimensional and those representing area. Some of the 3-dimensional examples which form the first group were considered direct objects and upon encountering the problem with size, some of these were expressed as the surface of these objects. In the examples which were indicated as an object, the object itself exemplified was also considered a valid example. In examples indicated as a picture, the object's two-dimensional or three-dimensional drawings were taken into consideration and this also gave rise to the problem of perspective (despite objects, which are in a perpendicular prism structure having a rectangular surface, they were perceived as parallelogramic surfaces in order to achieve the portrayed size). As for examples which were indicated as the surface of an object, sometimes they were directly specified in the expressions/statements of the pre-service teachers, sometimes they were not specified. As for the area that constitutes the second group, there were some examples by their appearances from afar rather than their actual structures (the rectangle as a structure was the example of a parallelogram by their appearances from afar and objects in right prism structure were the examples of parallelogramic surfaces by their appearances from afar although they have rectangular surfaces). Moreover, there were very few examples of models expressed virtually (part of a circumference – swing) and they were involved in the group that constitutes the examples by their appearances from afar.

While the meaning of examples given by pre-service teachers for geometric region was generally correct, it was observed that their examples given for geometric shape were confused with geometric region and contained incorrect models. Generally, planary shapes, such as triangular, square, rectangular, etc., are mixed up with their specified planary regions or upon mentioning the name of a planary shape it is understood along with its specified planary region [2]. In the studies carried out, while this case is clearly observed in the examples given for square-square region, rectangle-rectangular region, parallelogram-parallelogram region and, trapezoid-trapezoidal region, distinctive examples given for triangle-triangular region and circle-disk are observed. In spite of triangle-triangular region and circle-disk distinction included in the curriculum since primary school, other geometric shapes and areas have begun to be emphasized with the 2005 curriculum. Therefore, pre-service teachers might not have content knowledge noticing the shape-region distinction with the effect of the education they received in pre-undergraduate. They might not pay attention in spite of being aware of this distinction, or they might not be able to express it in spite of forming an example of the distinction in their minds. Studies on the presence of these types of misconceptions are included in related literature. Gülkılık (2008) [11] shows the angle as a shaded (or colored in) region indicated in the corner point of the two rays, which are the common starting points in drawings of pre-service teachers. As for Kesici (2005) [12], in a study conducted on misconceptions of high school students, it was stated that they have misconceptions such as "the points located on the interior region separating the plane of a triangle belong to the triangle" and "The points on the inside separating the angle in the plane belong to the angle".

As it is apparent from the above, at the root of the problems experienced when pre-service teachers gave examples for geometric concepts there was a lack of content knowledge of geometry. As for another cause of the problems experienced, the pre-service teachers had difficulty expressing themselves. When pre-service teachers expressed their thoughts verbally, that they experienced difficulties was obviously felt particularly in the analysis of data. Considering the factors that lead to this problem, it is understood that the most common encountered one of the examples shown by pre-service teachers was not referring to the objects' details. For example, in some examples given for the concept of a line, in spite of considering infinite length to be linear (e.g., like accepting that the highway is linear), this detail was not included in the expression. In this case, since linearity emphasis is not made, the examples given by students are likely to be perceived as a line. Another factor which could lead to the withholding of details is the use of the same object to model different geometric concepts. For example, pre-service teachers who gave the model of a pen have demonstrated the model of part of a line, meaning the pen nib in its unopened state, a ray, meaning its opened state, and a line, meaning two opened ends. Another factor which leads to difficulty in expression is the lack of expression. For example, the meaning expressed in examples given relating to the concept of angle such as "the distance between tree branches" or "the arch drawn when the time on a clock moves forward", contains uncertainty. Until this problem (difficulty expressing oneself) is corrected, the pre-service teacher will continue to provide similar examples in the classroom in the future, and since the imagined model and the model formed in the student's mind do not overlap, misunderstanding will occur.

In addition, the discourse of pre-service teachers while giving examples for geometric concepts using problems they encounter in daily life is different from geometric terminology. For example, concept confusion arose among some of the pre-service teachers when a type of cheese was sold as "sector" in daily life, but called "triangle cream cheese" by the general public. Pre-service teachers who are aware of this distinction use expressions such as "cheese cut into triangles".

As a result, pre-service mathematics teachers should be trained in this respect in order to have an adequate level of knowledge and experience in the content knowledge of geometry. The pre-service teachers should be involved in the learning environments that developed their ability of connection with daily life (examples for concepts related to subjects in the classroom, school and close environment can be enriched with models) either as a student or as a teacher when they receive their pre-service training. Teaching staff in regulated environments should be experienced in practice so as to indicate where the student should pay attention while showing a model for concepts, and how a conceptual education should be carried out without falling into concept confusion by taking advantage of these models. On the other hand, those who found the opportunity to encounter middle-school students before becoming teachers in courses such as School Experience and Teaching Practice obtained experience such as providing examples of concepts suitable for the students' level, helping students to give examples and giving feedback on these examples, and assisting in the realization of conceptual learning with the help of the students' abilities of connection. These types of learning environments (connected with daily life) are designed and in these environments the abilities of connection with daily life can be improved in practice and how to improve them can be learned. As clarifying the examples given in these environments, giving importance to using mathematical language must be regarded in order to prevent deduction about concept complexity, realize conceptual understanding and provide interaction while teaching. Likewise, with making connection with daily life, the content and pedagogical content knowledge are supported and learning by experience can be ensured in the pre-service training. As a result, those who are familiar with the learning environments connected with daily life and recognize the contribution of learning these competencies will consciously achieve their inclusion in education in the future.

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