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The Failure to Construct Proof Based on Assimilation and Accommodation Framework from Piaget

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Abstract

The purpose of this article is to describe the process of a proof construction. It is more specific on the failure of the process. Piaget's frameworks, assimilation and accommodation, were used to analyze it. Method of this research was qualitative method. Data were collected by asking five students working on problems of proof using think aloud strategy. Student's works were compared with the problem structures. Based on the data analysis, it shows that there were three causes of failure of proof construction. The failure occurs on constructing proof because (1) Incomplete schemes on assimilation, (2) Incomplete schemes on accomodation process, (3) Complete schemes but unrelated on assimilation and accomodation process.

Keywords: failure, construct proof, assimilation, accommodation

1. Introduction

Proof has a significantly important role in mathematics or mathematics education. It is known as the core of mathematics and deductive thought (Hanna et al., 2009). Proof is a mathematical tool. If someone would like to learn mathematics, he/she has to learn how to construct proof or at least he understands it (Wu, 1996). It can be inferred that if students do not have sufficient competence in mathematical proof, they will have difficulty in learning mathematics.

In fact, some researchers (Dreyfus, 1999; Cadwallder, 2009; Lee, 2009; Azrov, 2013) have found out that mostly student meet difficulties to construct proof. Therefore, there were some studies concerning with this. J. Selden and A. Selden (1987) had categorized kinds of errors resulted by students when they tried to prove theorems. Pinto (1999) found that a different path when the student failed to apply concept of definition in constructing formal proof, i.e. giving meaning unsuccessfully and extracting meaning unsuccessfully. Dreyfus (1999) concluded that students met difficulties to work on proof problem, it was due to their inability to use proper term to deliver their ideas. Gholamazad, Liljedahl, and Zazkis (2003) analyzed why a preservice teacher for elementary school could fail to construct proof. Furinghetti and Morselli (2010) analyzed the invalid proof resulted by the student was caused by affective and cognitive factors.

Study about the processes that undergraduates use when they attempt to construct proofs isnot much. Alcock and Weber (2010, p. 6) said while there has been considerable research on undergraduates' difficulties with proof, some researchers have noted that there has been comparatively little work about the processes that undergraduates use when they attempt to construct proofs, and that more research of this type is needed. To find out what made the students failed to constructproofwould of course be useful as guide to design the most effective way to teach proof.This article examines examples of wrong proof or invalid proof produced by the students. However, the study focuses more on identifying the failure of students in constructing the proof by analyzing the mistakes of the child's thinking process based on Piaget's framework of assimilation and accommodation

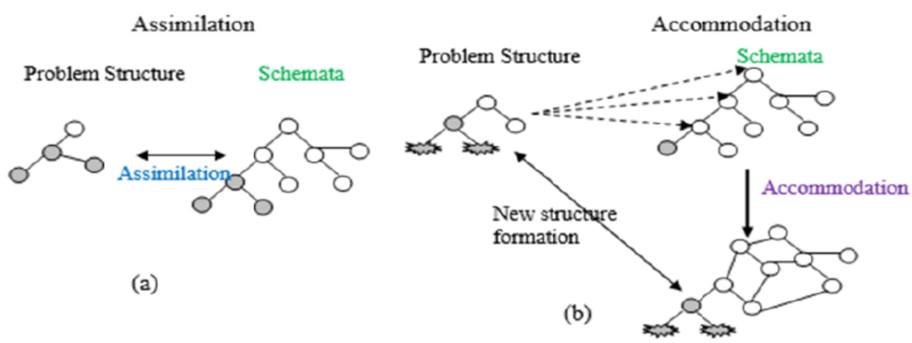
2. Assimilation and Accommodation Framework

Piaget (Kaasila, Pehkonen &Hellinen, 2009) had confidence that individuals have to adapt with environments. For this Piaget discribed two processes of adaptation that represents the ability of organisms to adjust with their environment. They are assimilation and accommodation.

Process of assimilation is an interpretation process of an event by using term of their existing cognitive structure. Piaget (Kaasila, Pehkonen, & Hellinen, 2009) claimed that assimilation involves the interpretation of event in term of existing cognitive structure.

Accommodation is a process of increasing knowledge by modifying the existing knowledge or cognitive structure to account for new experience. Piaget (Kaasila, Pehkonen, & Hellinen, 2009), *Accommodation increases knowledge by modifying structure to account for new experience.* Futhermore, the discussion is about the differences between the process of accommodation and assimilation. In the process of assimilation it absorbs the new stimuli directly and integrate it into the existing knowledge scheme. While in the process of accommodation the existing konwledge structure can not directly absorb the new stimuli for it needs a phase to modify the structure to cope with the new stimuli and then it will be integrated (Subanji, 2011)

Futhermore these two processes were illustrated by Subanji (2007, 2011, 2016) using diagram so that it will help to understand what do the two terms assimilation and accommodation really mean. It would be as follows:



⟷ means the suitability of a problem struktur with the structure of thinking

- - - -▸ means the mismatch of structure of a with the structure of thinking

Figure 1. Assimilation and accommodation process (adapted Subanji & Nusantara, 2016)

3. Methods

This qualitative study would describe the phenomenon of failure to construct proof by students. Therefore, the subjects of this research were five students of mathematics department at Brawijaya University Malang who had passed real analysis subject. They were given two task proof problems (TPP) that they had to solve. During working on the proof problem they were asked to think aloud. Observer recorded this process audiovisually and than it would be followed by interviewing in order to clarify the think aloud process by subject.

There following are the TPP given to subjects:

1. Let A, B and C is sets, if $A \subseteq B \cup C$ and $A \cap B = \emptyset$, show that $A \subseteq C$.
2. Let Q is rational number, $Q = \left\{\frac{p}{q}, p, q \in \mathbb{Z} \; and \; q \neq 0\right\}$, and f is a function from cross product integer numbers to Q, $f : \mathbb{Z}x\mathbb{Z} \rightarrow Q$ by $f(n,m) = \frac{2n}{m}$. does f is injektif? proof your answer.

Focus of the study is to analyze the failures done by the subject (student), then results analyzed that consists of an incompatibility or an error, five student's error works were analyzed. Creswell (2012) explained that the research term used for qualitative sampling is purposeful sampling. In purposeful sampling, researchers intentionally select individuals and sites to learn or understand the central phenomenon.

Three students' works were analyzed by comparing student's thinking structure by Assimilation and accommodation Framework. There are several steps in this framework, namely (1) developing problem structure of TPP; (2) developing transcription of think-aloud process and conversation with research subject; (3) making scheme of subject's thinking structure; (4) identifying the completeness of subject's thinking scheme, there are two categories of the completeness thinking scheme such as (a) *incomplete scheme*, it happens if subject's thinking structure is not as complete as problem structure and it does not appear in interviewing session, (b) *complete-uncorrelated scheme*, it happens if subject has scheme which doesn't appear in thinking structure; (5) Identifying assimilation process; (6) identifying accommodation process.

The problem structure can be described as an ideal sequence of the process of constructing proof done by researchers (Subanji, 2011). The problem structure for the first TPP are as follows (1) read and understand TPP, (2) write the meaning of A ⊆ B U C, namely if ∀x∈ A, then $x \epsilon B$ or $x \epsilon C$, (3) write the meaning of A∩B =∅, namelyif$x \epsilon A$, than $x \notin B$, (4)note the meaning of the second premise of the intersection to be taken in order to obtain conditions that meet both, namely$\forall x \epsilon A$, then$x \epsilon B$, $x \epsilon C$,and$x \notin B$, (5) establish suitable conditions, namely $\forall x \epsilon A$, maka $x \epsilon C$, (6) Establish the conclusion that A ⊆ C. A subject is said to fail if the structure of thinking is not equal to the structure of matter. Incompatibility will be explored if the assimilation and accommodations process were made, which is known through interviews and think aloud.

The structure of the problem can be illustrated by the chart in Figure 2 as follows:

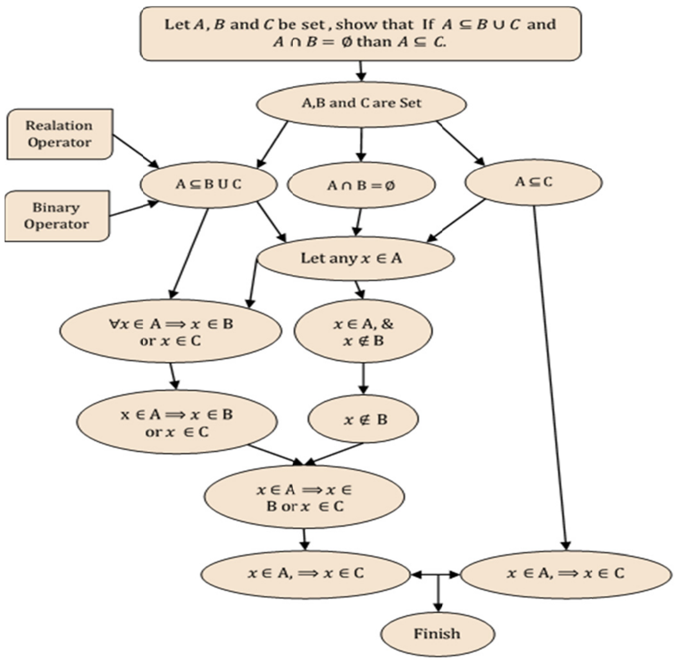


Figure 2. Structure problem of first TPP

Problem structure for the second TPP are (1) to read and understanding the problem, (2) to emerge the formal forms of rational numbers, namely $\frac{a}{b}$ with $a, b \in \mathbb{Z}$ and $b \neq 0$, (3) to emerge the definition of injection, namely $\forall x, y \in \mathbb{Z}, f(x) = f(y)$ then x = y, (4) to emerge the understanding of function value $f(n, m) = \frac{2n}{m}$, (5) to understand ordered pairs $\mathbb{Z}x\mathbb{Z}$, (6) to write $f(n_1, m_1) = f(n_2, m_2)$ such that$\frac{2n_1}{m_1} = \frac{2n_2}{m_2}$, (7) to apply the properties of equivalent fractions to get an equality $\frac{n_1}{m_1} = \frac{n_2}{m_2}$, (8) to apply the properties of equivalent fractionsand deductive proof to conclude that if $\frac{n_1}{m_1} = \frac{n_2}{m_2}$, then there is no guaranties that n1 = n2 and m1 = m2, (8) to conclude that function is not injection. The problem structure is visualized by the following Figure 3.

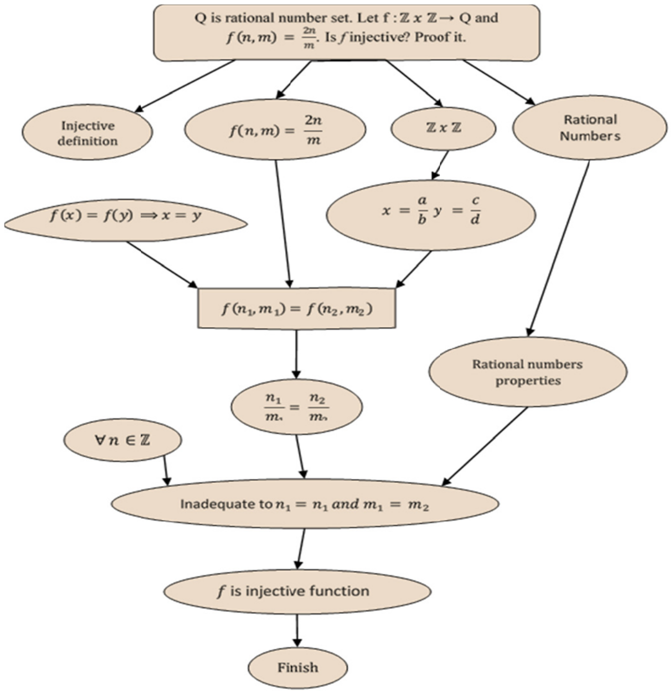


Figure 3. Structure problem of second TPP

Based on the result of the subject work, then the observer created the subject thinking process that would be the sequence of the step the subject passed to construct proof that may reflect the subject thingking process based on data on thinking aloud and interview. From the result of comparison then it was obtained the charracteristic of the failure student met, which took place during the process assimilation and accommodation to cause error.

4. Result

Based on the result of observation, it can be inferred that the failure could occure because of incompleteness and uncorelated of thinking scheme during the process of assimilation and accommodation. This study also found that both conditions occured in one TPP . Table 1 shows the number of failure had been done, at sub-schemes by the subject, as follows.

Table 1. Mental activities on constructing proof

Thinking process	Mental activities on constructing proof	Frequency					
		S1.1	S1.2	S2.1	S2.2	S3.1	S3.2
Assimilation	Incompleted scheme	2	0	1	2	1	2
	Completed scheme but uncorelated	1	1	1	1	1	1
Accomodation	Incompleted scheme	3	0	3	1	3	1
	Completed scheme but uncorelated	0	1	0	1	1	0

Information: $S_{i,j}$:i-th subject and j-th proof.

Table 1 shows that data of three subjects have similar characteristics, even in different situation and TPP, every subject experienced (1) Incomplete schemes on assimilation, (2) Incomplete schemes on accomodation process, and Complete schemes but unrelated on assimilation and (3) accomodation process. So as example, this part will explain the thinking process of one subject, namely S_1, that leading to a failure can be described qualitatively in more detail. Description of subject's thinking process on how he/she failed to construct the proof would be as follows.

1) Description of constructing process on first TPP

The failure of constructing proof couldoccur when the subject thinking structure was not compatible with the structure of problem. It can be described by Subject's thinking process in solving the first TPP. Look at the following problem structure and Subject's thinking structure in solving the first problem.

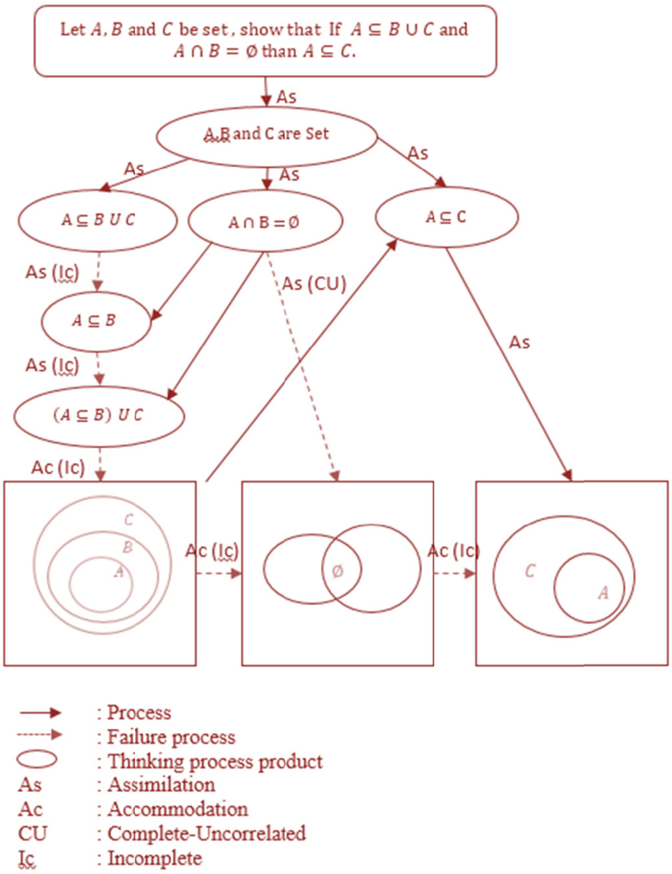


Figure 4. Student's structure thinking on first TPP

Figure 4 shows that the Subject's thinking structure is not compatible with the problem of structure of first TPP 1 in Figure 2. Based on the Figure 2 and 4, it can be interpreted that she/he had concept scheme of subsets as relations operator and union as a binary operator partially. Subjects understand the meaning of symbol⊆as a subset meaning. Subjects do not assimilate the symbol ⊆ completely with the symbol function as operator relations. But they see it as binery operator. So he/she operates it with the stage (A⊆B)∪C. In this regard it can be stated that she/he applied assimilation process by using thinking scheme incompletely. The following Subject's answer sheet (Figure 5) shows how Subject solved the first problem

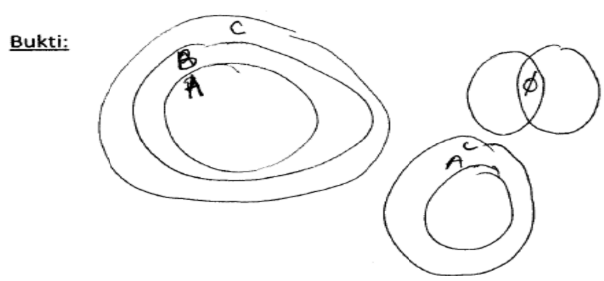


Figure 5. Subject's anwer sheet

Based on Her answer, Subject drawn representation of $A \subseteq B$, then she turned the page around for several times to read the information/premis and also the conclusion. After that she drawn outer circle as represantation of set C. At that time, observer confirmed to Subject about her thinking process. Consider the following transcript of conversation between observer and Subject:

Researcher : Please, explain the meaning of your draw (pointed to the Subject's draw)

Subject : I tried to visualize the meaning of $A \subseteq B \cup C$

Researcher : It means that each figure shows one statement on the problem. Isn't it?

Subject : yes, it is.

Researcher : What is the first figure (pointed to picture in figure 5) compatible with $A \subseteq B$ and it is equal to empty set?

Subject : No….(smile)

Researcher : why did you so?

Subject : I tried to draw every premise, then I tried to connect them each other but I did not know the way. Firstly, A is subset of B, then it was unitized with set C.

Researcher : did you read it gradually?

Subject : yes…

Researcher : Why?

Subject : Because there is no parenthesis in here (added parenthesis on the text of problem)

Researcher : Ok, would you please five is equal to two plus three.

Subject : (wrote down directly $5 = 2 + 3$)

Researcher : then, wil you read it as $5 = 2$ and then plus 3?

Subject : eh...yes I will (smile)

Based on the Subject's says, it could be concluded that she had concept scheme of subset as relation operator and union as binary operator partially. Consequently, she applied assimilation process by using thinking scheme incompletely

Subject is thinking structure is not compatible with problem structure resulted by her incomplete thinking schemes when doing the process of assimilation and accommodation,. So that she was stuck on that phase and could not progress to proof process. Therefore it can be said that Subject failed to construct proof because of incompleteness thinking structure during the process of assimilation and accommodation on the first premis. This failure will then couse for E4 error to exist or the error on applying theorems (J. Selden & A. Selden, 1987) called this case an inability to read the problem as well a tendency to have a quick problem solution.

The failureto construct proof on the second premis can described as follows,the second premis is $A \cap B = \emptyset$, Subject also has the knowledge scheme about intersection concept as in the statement on the premis. When she could read it well, that "there is no set intersection between A and B. However, what she say is not same by what she drew. She drew two circles intersected, although drew the symbol of empety set ($\emptyset$) at the intersection area. As we know, the diagram pict should it have been intersected. It showed that Subjecthave the knowledge of intersection but unrelated.

That sort of knowledge level disabled her to solve the proof problem to construct proof, so that she failed to construct proof. Based on the type of error made by Subject to proof according to J. Selden and A. Selden (1987) Subject had an error E5 that is circularity, an error to the statement and error of E6 that is the proof can not be understood locally.It has been seen here, the incomplete and unrelated thinking scheme made her failed to construct proof. It occure on both given premis. Consequently, she failed to relate the statements on premis to the conclusion. Herewith, Subject could be said as a whole failure to construct proof.

2) Second task proof problem

The answer sheet of Subject for the second TPP, as in the following Figure 4 berikut.

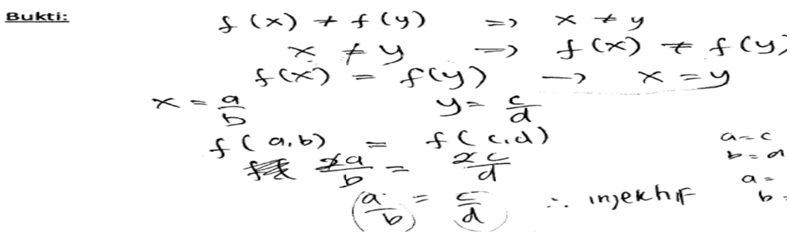


Figure 6. Subject's work sheet for the second TPP

At first glance, the subject's work looks right, the structure of thinking is similar to the structure of the existing problems in Figure 3. The difference is only in the final conclusion, the subject concluded that f is a injective function while the structure of the problem is concluded that the function f is not injective.

Subject's thinking structure is compared to the problem structure in Figure 3 as in the following Figure 7.

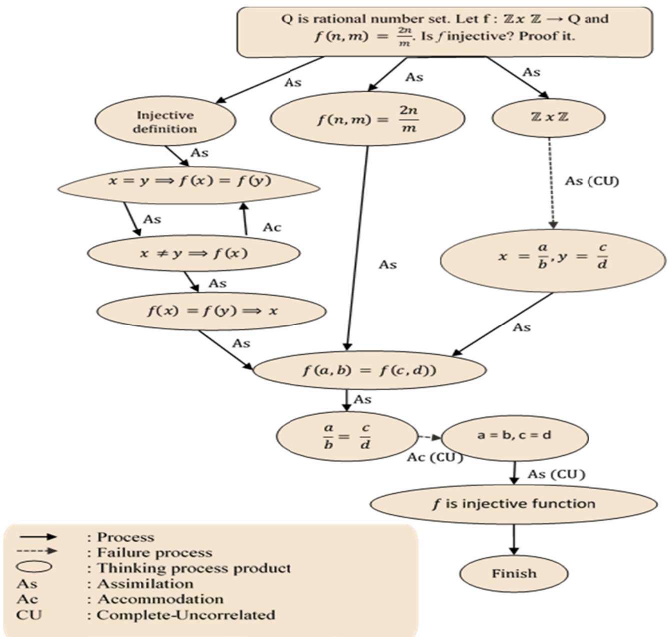


Figure 7. Subject's Strukture thinking in second TPP

By comparing Figure 3 and 7, it is clearly seen that structure of thinking Subject on second TPP is quite similar to the structure of TPP. Even though, there is an incorrect result. The causes of the failure will elaborate in the following paragraph.

First, in the part of definition for injective function, she couldn't make assimilation directly with proper scheme. When she wrote down the first, she wass in doubt, she had a sort of disequilibrium, then she followed with doing accommodation so that she was able to do assimilation about definition of injective function properly.

Second, when she wrote down element of cross product of integer number, then it was found incompatible between what she already wrote down to what she uttered. She was uttering "take any element of cross product of interger number ($\mathbb{Z}\, x\, \mathbb{Z}$), in fact she wrote two element rational numbers. It can be said in this case that Subject had done the assimilation with complete scheme but unrelated.

Third, Subject did have scheme about the property of similarity of rational number but she didn't realize on how it was important to consider the character of similarity of rational number, that she had done assimilation directly and concluded that $a = c$ and $b = d$.

In this case she had done assimilation with complete scheme about similarity of rational number. It was there when she was asked about equivalent fraction she could answer well and properly. The following conversation ensued:

Researcher : What type of number do you think the last number you gained (pointed of $\frac{a}{b} = \frac{c}{d}$)

Subject : it manifest ational number

Researcher : well, right, so how do you say, when two of rational number are said similar

Subject : excuse me, I didn't yet what did you mean.

Researcher : Lets see this equation (point to equation *a/b=c/d* written by student) it shows the similarityof two rational number, do you think it willguarentce $a = c$ dan $b = d$.

Subject : It's no certainly that way.

Researcher : What do you mean by not certainly?

Subject : Well … I think, A can be... (silent) it can be any number..... so it is similar when $a = c$ and $b = d$.

Researcher : It might be inpossible for $a \neq c$ and $b \neq d$, however the value of these rational Number are similar.

Subject : It can be.., It is when $a = 2c$, $b = 2d$ or $\frac{1}{2} = \frac{2}{4}$ (firmly)

Researcher : That's good, so what does it mean, then? Is it f an injective function

Subject :No, it's not.

Researcher : So, you didn't realize it just now

Subject :I consider the form first. So, I spontaneosly thought that $a = c$ and$b = d$.

It turned to be worse, as Subject was in a hurry to decide that $a = c$ and$b = d$, as the result. She did an error E4, using the wrong theorems because J. Selden and A. Selden (1987) claimed that error in applying theorems may lead to misconception or neglacting hypothesis or came into wrong conclusion. Other error such as E8, that is, to ignore and expand the quantifier ($\forall$) that made Subject failed to do proper assimilation was because misunderstood the word "any" for a, b, c andd as the member of $\mathbb{Z}$. Subject understood learn, that the universal quantifier is not as it used to be. Therefore, Subject assimilation a possibility for $a = c$ and $b = d$. Actually, I should have been said Subject failed to come to right conclusion. As compared to first TPP, at second TPP, she failed partially to construct proof

5. Discussion

Selden, Benkhalti, and Selden (2014) have identified the causes of students' difficulties in constructing proof, namely (1) not using framework of proof to construct proof, (2) not unpacking conclusion, (3) not using definition correctly. Study in this research focuses on causes of failure of proof constructing based on scheme of student's knowledge using framework of assimilation and accommodation. From the findings of this study, it can be seen that this framework of this study can describe the process of proof construction done by the students in more detail. So how failure can also be explained in more detail. Thus it can be identified that the failure to construct proof might occur on three conditions.

a. Incomplete schemeson assimilation and accomodation process

An example of incomplete thinking schemes on assimilation cases is when the students assimilate the symbol $\subseteq$, they only have one scheme on the meaning of the symbols of a subset $\subseteq$, students do not have a schematic of a symbol $\subseteq$ as a relation sign. In other words, in general, it can be said that the students only had a single scheme of a mathematical object, they do not have other schemes of the elements associated with the object. So that the student can not create or manipulate the object into another form or associate the object with other objects. It clearly supports to what J. Selden and A. Selden (1995, 2014) said that one of the difficulties students in megonstruksi evidence is students can not unload and unpack mathematical statement of intent from a conclusion on the theorem. So the student's inability to unload mathematical statements is due to the incompleteness of knowledge scheme had students about the statement.

b. Incomplete schemes on accomodation process

Explanation of incomplete schemes on accommodation proces thinking is quite similar with the problem of incomplete thinking schemes on assimilation process, it is about the incompleteness scheme. The difference is the process of assimilation occurs just when they face the problem, while the accommodation process takes place after they face the problem and had done several thinking processes such as remembering, recalling long-term memory or associating with matters related to the problems encountered. Such efforts will emerge the scheme needed but it can not be. If it is not, then the condition is called as Incomplete thinking schemes on accomodation proces. As experienced by the subject when they try to understand the statement A ⊆B ∪C. Various attempts have been made by the subject to understand the statement like reading the questions repeatedly, make a Venn diagram. But the subject remains understand the statement as (A ⊆B) ∪C.

c. Complete schemes but unrelated on assimilation and accomodation process.

Condition of complete thinking schemes butun related to assimilation process also appears when they faced with statement A ⊆ B ∪ C. It was initially expected that the subject did not have a schematic of the formal definition of a subset. In fact, he/she has a union set definition. It was known when the interviews were conducted after the completion of work on the TPP. As the following dialogue:

Researcher : Do you know definition of subset $A \subseteq B$?

Subject : yes, I know. Every element of set A is in set B.

Researcher : Could you write the symbol?

Subject : (with hesitation he wrote) ∀ a∈A → a ∈B

Researcher : Why do you hesitate to write it? aren't you sure with what you wrote?

Subject : yes

Researcher : Why don't you convince?

Subject : I think definition is only a statement that it is not like this.

Researcher : you write the definition of subsets and it can be used for proof of this.

Subject : oo yes.

This differs with the incomplete scheme. In incomplete scheme in the assimilation and accommodation, the students really do not have scheme which has been given an intervention.

In constructing proof. If one of the four condition above occurs or if all those conditions occurs it may cause impasse that student does not know how to continue. Weber (2001, p. 2) Said "Students often fail to construct a proof because they reach an impasse where they simply do not know what to do".

There are some effects of incompleteness and unrelated knowledge schemes. Incompleteness or unrelated of knowledge schemes will make students relate difficult to manipulate a mathematical expression that is equivalent to other forms. They are unable to write a concept image held in the form of formal symbols and otherwise unable to make a concept image of a known object. Students also do not have warrant to make conclusion and it was often found out that students have verbal knowledge but they were unable to write in the formal symbol. Such conditions cause the impasse or incompatibility in proving that leads to failure to construct proof.

The study related to schemes constructed by students in problem solving has been done by Subanji (2007, 2011). The study found that there were three factors causing pseudo thinking processes, namely (1) Incomplete substructures in the assimilation process, (2) Incomplete substructures in the accommodation, (3) mismatching on the use of substructures in the process of assimilation and accommodation. Thinking pseudo is a result of a problem solving process and not the output of real mental activity. Remembering proof is a special form of problem solving (Weber, 2001; Furinghetti & Morselli, 2010) then pseudo thinking process can appear when students construct proofs. It can be further investigated in subsequent research .

Discovering how the construction process of proof done by the students is very essential, because it can give an idea of how success and failure proof construction going on. It is also useful to get guidance on how proof should be taught and in general how a concept should be constructed in the student's mind. Incompleteness scheme does not mean the students have not studied the scheme, they might have learned but the concept constructed were not meaningful so that the concepts were difficult to remember.

Subanji (2015) said that another cause is wrong construction of scheme. He found that there were many mathematical concepts constructed by junior high school students mistakenly. It would affect to the outcome of next assimilation and accommodation in solving problems and/or constructing a new scheme. Scheme has two main functions, namely (1) to integrat existing knowledge and (2) as a mental tool for the reception of new knowledge (Skemp, 1982).

The conclusion that can be drawn from this research is that the failure of the proof construction not only as presented by Selden, Benkhalti, and Selden (2014), but also occur due to the incompleteness of the scheme knowledge of students when assimilation and accommodation and or as a complete scheme but not connected in the process asimolasi and accommodation.

This failure factors could be more than what has been found in this study. What if student has a complete scheme but he/she can not use these schemes in constructing proofs. This will be further investigated in subsequent research.

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