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ADDRESSING CHALLENGES IN URBAN TEACHING, LEARNING AND MATH USING MODEL-STRATEGY-APPLICATION WITH REASONING APPROACH IN LINGUSTICALLY AND CULTURALLY DIVERSE CLASSROOMS

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Abstract

This study examined the effects of using the Model-Strategy-Application with Reasoning Approach (MSAR) in teaching and learning mathematics in linguistically and culturally diverse elementary classrooms. Through learning mathematics via the MSAR, students from different language ability groups gained an understanding of mathematics from creating visual models, developing procedural fluency from using various strategies, and building competence in problem solving from real world application. The findings demonstrate that the MSAR results in improved diverse student mathematics learning in urban areas.

Keywords: model, strategy, application, reasoning, ELL learners, mathematics problem solving

Introduction

International assessments from TIMSS (1999 & 2007) and PISA (2010 & 2012) revealed that the disparity between U.S. students' mathematics, science, and reading achievement and those from other countries has not significantly improved. In the recent PISA (2010 & 2012), U.S. students' mathematics, science, and reading average scores were still below average among the 34 OECD nations, as in previous years (OECD, 2013). Locally, mathematics, science, and reading achievement levels of California's public school students in grades 4 and 8 on the *National Assessment of Educational Progress* (NAEP) have been below the national level in recent years (National Center for Education Statistics, 2013). The results of the recently released 2015 California Assessment of Student Performance and Progress (CAASPP) show that more than 40% of elementary students in grades 3-5 scored below standards in the areas of concepts and procedures, followed by more than 38% below standards in problem solving/Modeling and

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Data Analysis; in the area of Communicating Reasoning, more than 39% of students did not meet standards at grades 4 and 5 (California Department of Education, 2015). The recent international, national, and local assessment results also show that a significant achievement gap continues to exist for African American, Hispanic/Latino, low-income, and English-learner students in urban areas, compared to their peers (Orfield, Losen, Wald, & Swanson, 2004). Struggling students have a limited understanding of basic math concepts and they are notably deficient in their ability to apply mathematical skills to solve even simple problems (National Research Council, 2001).

Recent developments in the new Common Core State Standards for Mathematics (CCSSM) have heightened one of the key shifts toward "rigor" in mathematics education: pursuit of conceptual understanding, procedural skills and fluency, and application with equal intensity (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). The shift towards rigor provides a clear direction in mathematics instruction as well as assessment that requires students to achieve a balance in mathematics learning in the aspects of conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems (An & Wu, 2014; Wu, 2008). However, this standard has posed a challenge to classroom teachers on how to balance mathematics instruction to meet the needs of the rigor requirements of the CCSSM. Although many professional development programs have sought to enhance teaching practice, little progress has been made in terms of supporting diverse student mathematics learning in urban areas.

The purpose of this research was to conduct an empirical study that addressed the challenges in urban teaching and learning by engaging classroom teachers in applying the Model-Strategy-Application with Reasoning (MSAR) in teaching mathematics and in assessing the effects of the MSAR model on their student learning in linguistically and culturally diverse classrooms in urban areas. The MSAR is defined as developing students' conceptual understanding using various visual **models,** building procedural fluency with different computational **strategies**, and developing problem solving skills with real world **applications** (An & Wu, 2014), while focusing on fostering **reasoning** skills throughout the three components of models, strategies and applications.

This project aimed at investigating the following research questions:

1. What are the effects of using the MSAR approach on improving ELLs' conceptual understanding, procedural fluency, application, and reasoning in mathematics?
2. What are the differences in student performance in MSAR tasks between diverse groups?

Theoretical Framework

The MSAR of Model-Strategy-Application with Reasoning Approach

Various studies suggest using rich mathematical tasks to develop students' capacity in reasoning and argument (Mok & Kaur, 2006; Shimizu, Kaur, Huang, & Clarke, 2010). Mathematical tasks are important vehicles for classroom instruction and aims to enhance students' learning. To achieve quality mathematics instruction, then, the role of mathematical tasks to stimulate students' cognitive processes is crucial (Hiebert & Wearne, 1993). One example of rich mathematical tasks demonstrated by Wu and An (2006) is employment of the Model-Strategy-Application (MSA) in developing teachers' pedagogical content knowledge and assessing their progress, and its effects of children learning in diverse classrooms. This approach had the three aspects of Model-Strategy-Application originally, followed by explanations of reasoning in each aspect. The MSA approach has been taught in a mathematics education graduate program and used by classroom teachers in urban areas in Southern California. The positive effects of the MSA were demonstrated in mathematics classrooms in urban areas through various studies (e.g., An & Burson; 2010Wu & An, 2007). In recent years, this model has evolved through explicitly addressing the reasoning component, resulting in the Model-Strategy-Application with Reasoning approach in An and Wu's study (2014), in alignment with the needs of implementation of the CCSSM.

The new CCSSM not only challenges teachers to offer more relevant, practical and rigorous instruction with a set of focused content standards, but also requires students to solve math problems and think critically with a set of Mathematical Practice Standards (CCSS-MP). The key shift of "rigor" asks teachers to pursue conceptual understanding, procedural skills and fluency, and application with equal intensity. This pursuit of a balance within mathematics instruction is supported by research showing that effective classroom teachers always use multiple forms of instructional strategies and assessments that are meaningful and applicable toward the goal of supporting student learning (An & Wu, 2014; McMillan, 2000).

According to An and Wu (2014), the MSAR of Model-Strategy-Application with Reasoning approach includes four connected components aligned with the four categories of CCSS-MP standards in the teaching and learning process: 1) Learn and create various visual **models** to build conceptual understanding, 2) Develop procedural fluency to master multiple **strategies** of basic and complex computation skills in an accurate, efficient, and flexible manner, 3) Build strategic competence to **apply** knowledge in word problem solving, and 4) Focus on fostering **reasoning** skills throughout the other components. The MSAR is also aligned with the essential components of the five indicators of student math proficiency, as provided by RAND (2003) and the National Research Council (2001). In addition, they are supported by the NCTM Process Standards (2000) and the Guiding Principles of California Mathematics Conceptual Framework (2006 & 2013), which calls for a balance within mathematics by focusing on the three key components -- conceptual understanding, computational skills, and problem solving, for more effective math programs. Therefore, the MSAR model can be viewed as a fundamental framework for teachers to teach CCSSM and for students to learn CCSSM effectively and proficiently. This project focuses on engaging teachers in applying the MSAR approach in teaching mathematics, and also investigates the effects of the MSAR on student learning, especially on diverse students in urban areas. Such studies have become increasingly important in meeting the needs for CCSSM implementation.

English Language Learners and Role of the MSAR Approach

Balfanz and Byrnes (2006) called for closing the mathematics achievement gap in high-poverty schools in urban areas by focusing on shifting classroom practices, and providing relevant teacher training, which had a significant impact on raising student scores in their study. To support diverse students, such as English Language Learners, teachers need to provide them with an opportunity to learn the attendant English vocabulary words, in order to discuss and study the concepts they are learning in the second language (Cummins, 1981); this is especially important in order to support their learning in word problems because their difficulties are primarily on account of encountering word problems in an unfamiliar second language

(Bernardo, 2005). There are two types of language proficiency: 1) Basic interpersonal communication skills, and 2) Cognitive academic language proficiency (Cummins, 1981). The MSAR components provide students an opportunity to enhance their cognitive academic language proficiency because they are required to explain their reasoning in their models, strategies, and applications in problem solving. By engaging diverse students to work in groups to discuss and solve the MSAR tasks, the students participated and persevered in solving word problems while working in cooperative groups (Kroesbergen & Van Luit, 2003).

One of the NCTM Process Standards (2000) is representation. The model component of the MSAR encourages students to construct their own visual representations, such as pictures, tables, charts, figures, and symbols to demonstrate their conceptual understanding in a meaningful manner (Wu, 2008). The most notable part of the MSAR is that it provides and allows students to use multiple ways to show their mathematics proficiency. According to the NCTM Assessment Standards (1995), it is important to use multiple indicators for student assessment. “One assessment or one type of assessment should not be the sole measure of a student's achievement, because it is not likely to give an adequate picture of that student's learning. Nor should any one assessment be used to make decisions of any consequence about a student's educational future" (Koelsch, Estrin, & Farr, 1995, p.11). The MSAR assessment assists classroom teachers in gauging their students’ learning in the four aspects of conceptual understanding, procedural fluency, and problem solving in real world application, along with an emphasis on reasoning throughout.

Methods

Site and Subjects

This study was conducted at three schools from three school districts, situated in urban areas, low-income neighborhoods in Southern California in fall 2013 and spring 2014. Table 1 shows the demographic information of student participants at three grade levels in the three schools.

Table 1

Demographic Information of Student Participants

Class	# of Students	Girl	Boy	ELL	Hispanic American	African American	Other
2nd	25	11	14	17	23	1	1
3rd	29	16	13	17	23	2	
5th	31	15	16	21	21		4

Table 1 shows that close to 60% of students were ELL learners, with close to 70% Hispanic Americans in each class. One hundred percent of students at these schools were considered socioeconomically disadvantaged and therefore qualify for free breakfast and lunch. The teacher participants were three classroom teachers at grades 2, 3 and 5, one teacher at each grade level. The three participant teachers were graduate students in the math education graduate program. They learned about the MSAR approach in the graduate study, and started to apply the MSAR in fall 2013 and spring 2014 in their classrooms. The participating teachers were selected based on the following criteria: 1) they volunteered to use the MSAR in their teaching, and 2) they had needs for learning CCSSM implementation and agreed to use the MSAR in their mathematics instruction and to provide the data relevant to the reliability and validity of the

MSAR study. The teachers' roles were to provide the MSAR interventions, collect and analyze the data for their action research projects with different areas of focus based on their interests. This combined the individual data sets to study the effects of the MSAR in diverse groups at different grade levels.

Procedure and Data Collection

The classroom teachers developed the MSAR student worksheets based on the CCSSM standards in five content standard areas. Figure 1 shows an example of the MSAR student worksheet for grade 3.

Table 2 shows the intervention of the MSAR and data collection by each teacher. The 2nd grade teacher used model, group, individual, and sharing strategies in applying the MSAR. She also used the language structure to guide her 2nd graders to learn how to reason and think in a structured manner. The third grade teacher used individual work, focusing on Cognitive Guided Instruction (CGI) lessons. The 5th grade teacher used model, group, and sharing approaches, focusing on peer support in groups. Both the 2nd and 5th grade teachers used the time series design, and the 3rd grade teacher used the pre- and post-test design in data collection

MSAR

Jill made 5 rows of blocks, with each row containing 7 blocks. How many blocks did Jill have altogether?

Modeling	Strategies of Computation	Creating and solving a similar word problem and solve it
Explain why:	Explain why:	Explain why:

Figure 1. MSAR example at grade 3.

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Table 2
Intervention and Data Collection

Class	Intervention	Design	Data Collection
2nd	Model, Group, Share; Individual Language Structure	Time series design	Baseline 5; Intervention 7; Post Intervention 1
3rd	Individual work CGI Lessons	Pre & Post Design	Pre 5 & Post 5
5th	Model, Group, Share; Peer Support	Time series design	Pre 5, Intervention 7

Instruments

The MSAR structure worksheets were used to measure the effects of the MSAR interventions. Each MSAR task has a different content area. For example, for the 3rd graders, MSARs 1, 2, 3, 4, and 6 all focus on Number Sense: i.e., understanding of place value, number understanding, and simple concepts of multiplication and division; MSAR 5 focuses on Measurement and Geometry; MSAR 7 focuses on Statistics, Data Analysis and Probability (a sample MSAR task can be found in Appendix 1). The student MSAR worksheets were evaluated by the teachers using the author-developed MSAR rubrics, which have four levels (See Appendix 2). Content validity was ensured, as all MSAR tasks were created based on Common Core Standards for Mathematics at each grade level. The MSAR tasks were also tested for reliability in this study. Cronbach's alpha for the 2nd Grade MSAR Items was .958, for the 3rd Grade MSAR Items was .898, and for the 5th Grade MSAR Items was .939, which indicates a very high level of internal consistency (Cronbach, 1951) for the MSAR items for all three grade levels in this study.

Data Analysis

Data was analyzed quantitatively in this study. To answer research question 1, Paired t-tests were used for comparing differences in model, strategy, application, and reasoning between baseline and intervention for grades 2 and 5, and between pre- and post-tests for grade 3. To answer research question 2, One-Within-One-Between Subject ANOVA tests were used for comparing differences between different language groups that include the English Language Learners (ELL) and the English Only (EO) groups for grades 2 and 3, and included an additional group of the Redesignated Fluent English Proficient (RDFEP) group for grade 5. Student MSAR worksheets were evaluated and scored using the MSAR rubrics in this study.

Results

The findings from this study show that the MSAR approach had significant positive effects on students' conceptual understanding, procedural fluency, application, and reasoning.

Student Improvement in Model, Strategy, Application, and Reasoning

The results of a Paired t-test demonstrate the significant differences in the 2nd graders' mean scores in four areas of Model (*t* (22) = -3.084, *p* = .005 < .05), Strategy (*t* (22) = -4.348, *p* = .000 < .05), Application (*t* (22) = -4.522, *p* = .000 < .05), and Reasoning (*t* (22) = -5.311, *p* = .000 < .05) between the baseline and the intervention. Figure 2 confirms the 2nd graders' growth in mean scores in the four aspects of Model, Strategy, Application, and Reasoning between the baseline and the intervention.

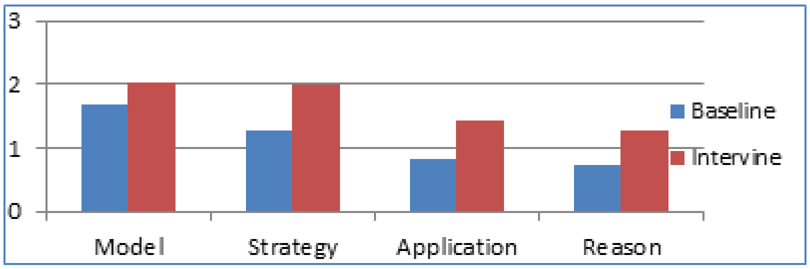


Figure 2. The 2nd graders' growth in mean scores in the four MSAR areas.

The results of a Paired t-test revealed the significant differences in the 3rd graders' mean scores in the four areas of Model (*t* (28) = -22.854, *p* = .005 < .05), Strategy (*t* (28) = -23.616, *p* = .000 < .05), Application (*t* (28) = -14.893, *p* = .000 < .05), and Reasoning (*t* (28) = -5.090, *p* = .000 < .05) between the baseline and the intervention. Figure 3 confirms the 3rd graders' growth in mean scores in the four areas of Model, Strategy, Application, and Reasoning between the pre- and post-MSAR assessments.

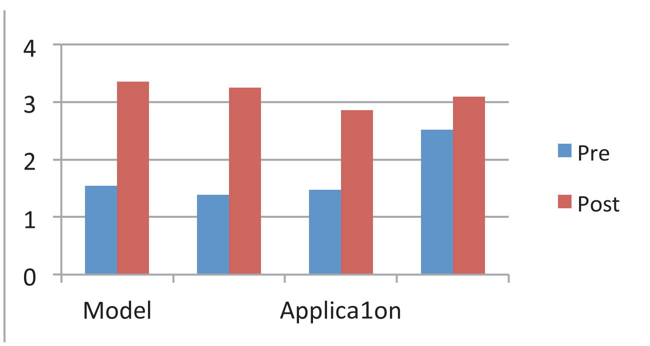


Figure 3. The 3rd graders' growth in mean scores in the four MSAR areas.

The results of a Paired t-test demonstrated the significant differences in the 5th graders' mean scores in the four areas of Model (*t* (27) = -4.861, *p* = .005 < .05), Strategy (*t* (27) = -4.170, *p* = .000 < .05), Application (*t* (27) = -2.921, *p* = .007 < .05), and Reasoning (*t* (27) = -4.233, *p* = .000 < .05) between the baseline and the intervention. Figure 4 confirms the 5th graders' growth in mean scores in the four areas of Model, Strategy, Application, and Reasoning between the pre- and post-MSAR assessments.

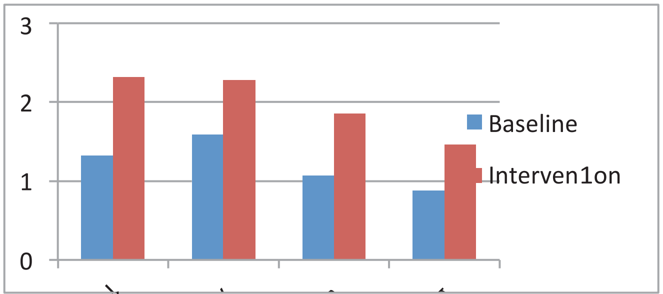
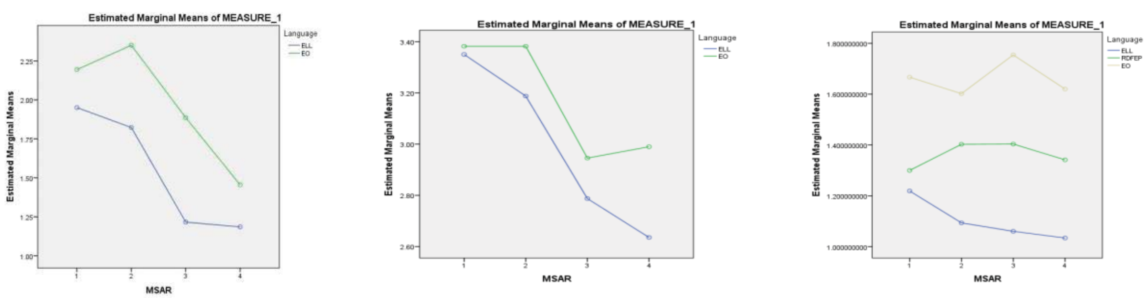


Figure 4. The 5th graders' growth in mean scores in the four MSAR areas.

Student Growth in MSAR between Diverse Groups

Student overall growth in MSAR between diverse language groups. The results of a One-Within-One-Between Subject ANOVA tests demonstrate that overall there was no statistically significant difference in mean scores in the four areas of MSAR between the English Language Learner and the English Only groups during the intervention at the 2nd grade level (F (1, 21) = 2.422, p = .135 > .05) and in the post tests at the 3rd grade level (F (1, 25) = 1.447, p = .240 > .05). The grade 5 group also had the same result among the three Language groups during the intervention (F (1, 24) = .496, p = .615 > .05).

Figure 5 confirms the results in the four areas of MSAR between diverse language groups at all three grade levels, but it indicates that the English Language Learner group is strong in the Modeling component compared to the other three MSAR components at all grade levels. They had almost the same mean scores in Modeling as in the English Only groups in grades 3 and 5. The largest gap between these two groups was Application at grade 2 (see Figure 5 (a)), Reasoning at grade 3 (See Figure 5 (b)), Application at grade 5 (See Figure 5 (c)).



Grade 2 (a) Grade 3 (b) Grade 5 (c)

Figure 5. Comparison of MSAR mean scores during intervention (2nd & 5th) and in the post-tests (3rd grade) in the four areas between language groups.
Note: ELL = English Language Learner; RDFEP = Redesignated fluent English proficient; EO = English Only

Student growth in each MSAR component between diverse language groups. To further investigate what progress the English Language Learner group made in each MSAR area during the intervention or in the post-tests, this study used a One-Within-One-Between Subject ANOVA test to examine each area of the MSAR in diverse language groups in each grade level. The results produced in SPSS output tables were confirmed by Figures 6, 7, and 8.

The 2nd graders' growth in each MSAR area between diverse language groups. The results of a One-Within-One-Between Subject ANOVA test show that there is no statistically significant difference in mean scores in Model (F (1, 10) = 1.304, p = .280 > .05), Strategy (F (1, 10) = .387, p = .548 > .05), Application (F (1, 10) = .449, p = .518 > .05), and Reasoning (F (1, 21) = .020, p = .891 > .05) between the English Language Learner and the English Only groups during the intervention in the 2nd grade group.

Figure 6 shows that the 2nd grade English Language Learner group had higher mean scores in the Model component than the English Only group in MSARs 2 and 4 (see Figure 6 (a)); they also had higher mean scores in Strategy in MSARs 2 and 6 than the English Only group (see Figure 6 (b)); in addition, they had higher mean scores in Application in MSARs 2 and 3 than the English Only group (see Figure 6 (c)); finally, they had higher mean scores in Reasoning in MSARs 1, 3, 4, and 7 than the English Only group (see Figure 6 (d)).

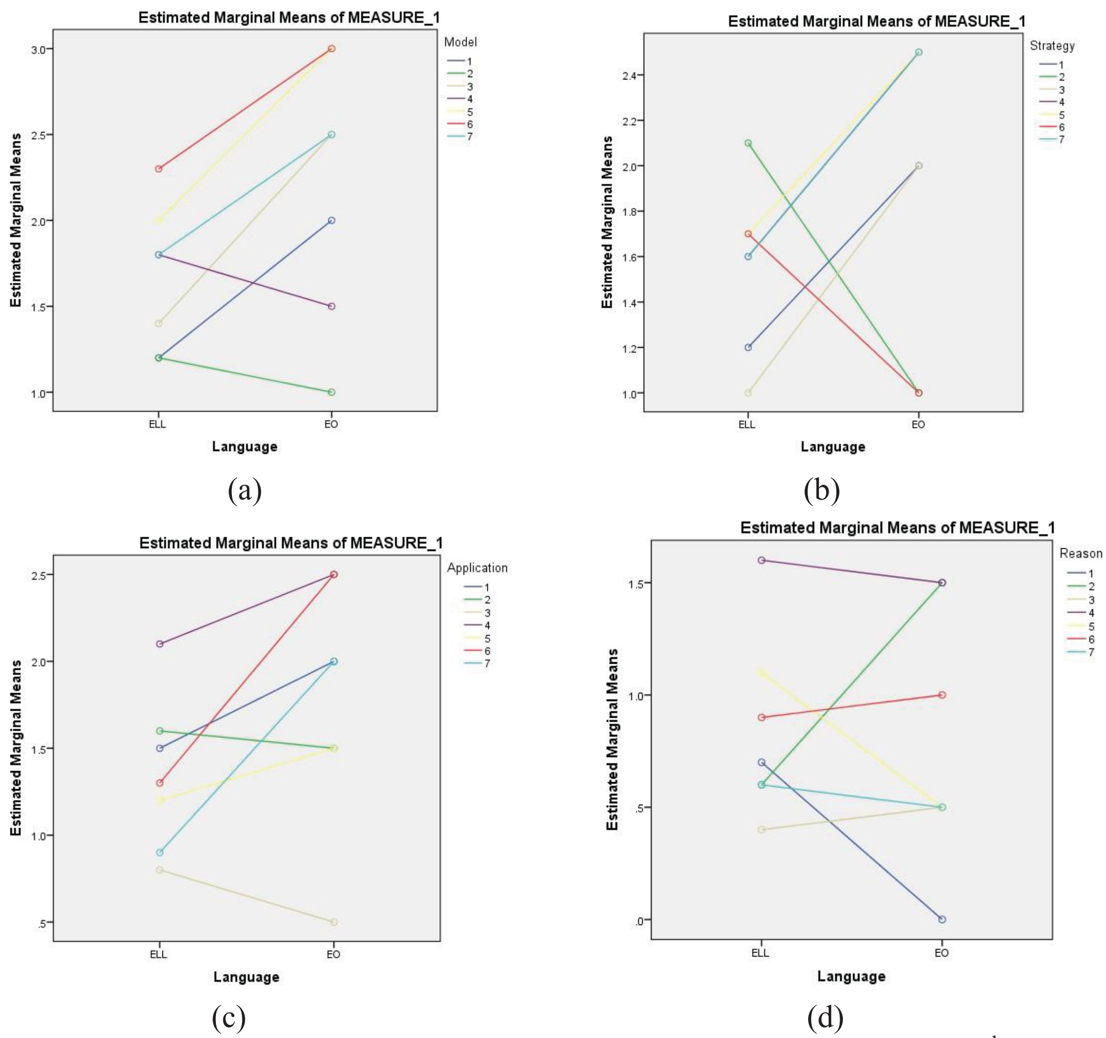
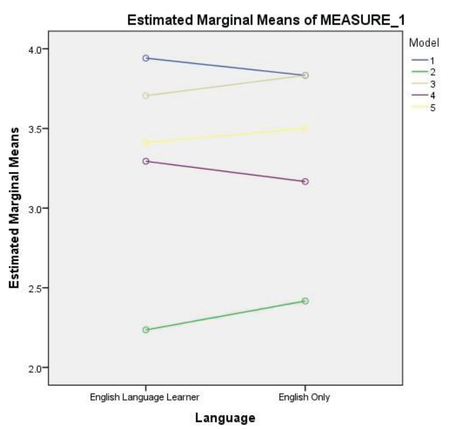


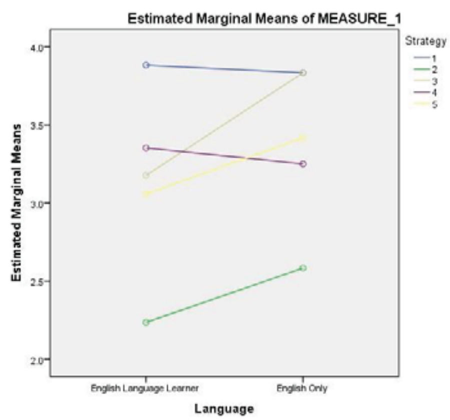
Figure 6. Comparison of each MSAR area between language groups at the 2nd grade level.
Note: ELL = English Language Learner; RDFEP = Redesignated fluent English proficient; EO = English Only

The 3rd graders' growth in each MSAR area between diverse language groups. Results from a One-Within-One-Between Subject ANOVA test show that there is no statistically significant difference in mean scores in Model (F (1, 27) = .051, p = .824 > .05), Strategy (F (1, 27) = 2.237, p = .146 > .05), Application (F (1, 27) = .717, p = .404 > .05), and Reasoning (F (1, 27) = 3.511, p =.074 > .05) between the English Language Learner and the English Only groups during the post-tests at the 3rd grade level.

Figure 7 shows that the 3rd grade English Language Learner group had higher mean scores in the Model component than the English Only group in MSARs 1 and 4 (see Figure 7 (a)); they also had higher mean scores in Strategy in MSARs 1 and 4 than the English Only group (see Figure 7 (b)); in addition, they had a higher mean score in MSAR 1 than the English Only group (see Figure 7 (c)); finally, they had a higher mean score in Reasoning in MSAR 2 than the English Only group (see Figure 7 (d)).



(a)



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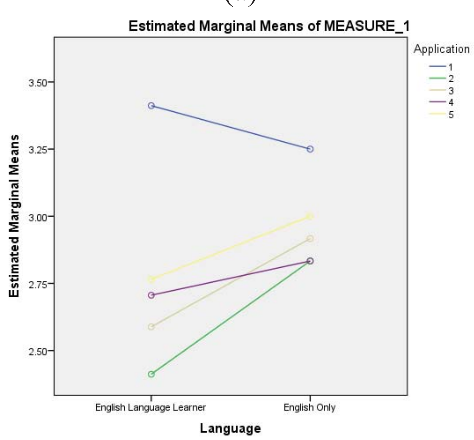

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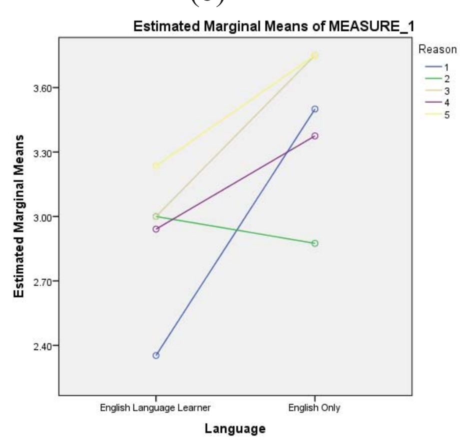


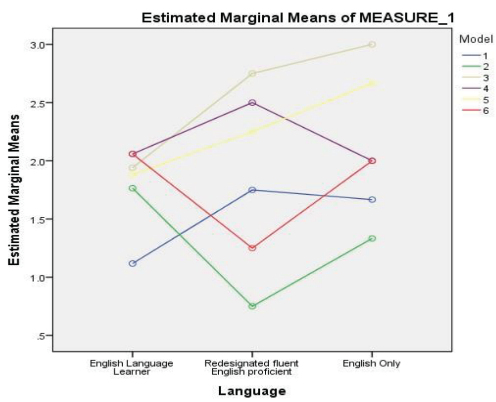
(d)

Figure 7. Comparison of each MSAR area between language groups at the 3rd grade level.
Note: ELL = English Language Learner; RDFEP = Redesignated fluent English proficient; EO = English Only

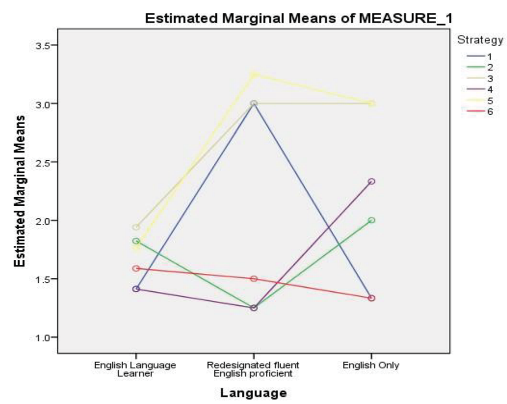
The 5th graders' growth in each MSAR area between diverse language groups. Results from a One-Within-One-Between Subject ANOVA test show that there is no statistically significant difference in mean scores in Model (F (2, 21) = .376, p = .691 > .05), Strategy (F (2, 21) = 1.556, p = .234 > .05), Application ((F (2, 21) = 3.399, p = .053 > .05), and Reasoning (F

(2, 21) =1.197, $p = .318 > .05$) between the English Language Learner group, the Redesignated fluent English proficient group, and the English Only group during the intervention at the 5th grade level.

Figure 8 shows that the English Language Learner group had higher mean scores in the Model component than the other two groups in MSARs 2 and 6 (see Figure 8 (a)); they also had a higher mean score in Strategy in MSAR 6 than the other two groups and higher mean scores in Strategy in MSARs 2, 4, and 6 than the Redesignated Fluent English Proficient group (see Figure 8 (b)); For Application, the ELL group had only one higher score in MSAR2 than the other two groups (see Figure 8 (c)); the same result was exhibited in Reasoning in Figure 8 (d).



(a)



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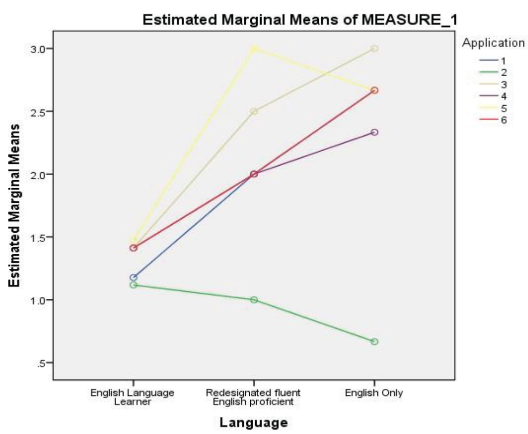

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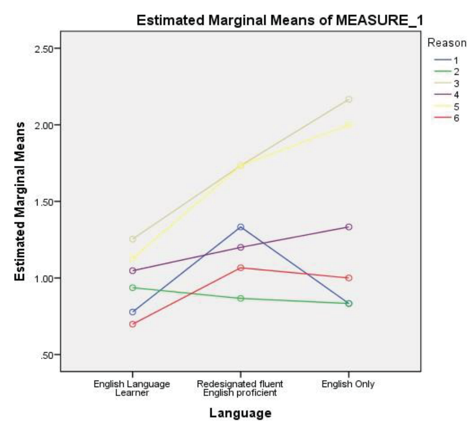

(d)

Figure 8. Comparison of each MSAR area between language groups at the 5th grade level.
Note: ELL = English Language Learner; RDFEP = Redesignated fluent English proficient; EO = English Only

Discussion and Conclusion

The results of this study show that the MSAR approach is a powerful instructional and assessment approach for achieving a balance within mathematics and for developing mathematics proficiency for diverse students. Overall, the students from three grade levels made progress in Model, Strategy, Application, and Reasoning, which is evident from Figures 2 thru 8.

The most important finding in this study was that the MSAR has improved diverse student mathematics learning. The results demonstrate that there was no significant difference in mean scores in all four areas of the MSAR components between diverse language groups at three grade levels due to the MSAR instruction; meaning that ELL learners performed as well in the MSAR areas as the other groups – English Only and Redesignated Fluent English Proficient groups. The most interesting finding was that the English Language Learner groups were stronger in Model compared to Strategy, Application, and Reasoning areas in this study, indicating that using visual representations may provide a meaningful opportunity to the English Language Learner groups to demonstrate their conceptual understanding despite their language difficulties. Therefore, multiple forms of assessment are necessary for accommodating diverse student needs (NCTM, 1995).

What is surprising is that the English Language Learner groups even outperformed the English Only groups in some MSAR tasks in Model, Strategy, Application, and Reasoning in this study, and at all three grade levels. In fact, the most predominant challenge was having students providing a written explanation to their results, as indicated by research (Cummins, 1981). For this reason different strategies were implemented by the 2nd grade teacher to help the students feel more confident and successful when solving mathematical word problems. The teacher used math sentence starters, math vocabulary charts, manipulatives, and different models to support ELL learners to represent their conceptual understanding, which is supported by Wu's study (2008) that various visual representations are important in supporting ELL learners' mathematics learning.

The 5th grade students also improved in their use of academic language from the MSAR intervention. This was most evident in the results of the MSAR tasks. Prior to the intervention, the students struggled with explaining their models during the baseline test. By the end of data collection, students were able to make connections between math academic language and its meaning, and their scores in the four areas showed significant improvements over their baseline tests. The 5th grade teacher also used cooperative groups in her MSAR intervention. In accordance with the cooperative group format (Kagan, 1985), students were required to discuss the math word problems together. This helped the 5th graders, especially ELL learners to use math academic language in dialogue, in spite of their language obstacles. Students were able to participate and persevere in solving word problems presented in the MSAR format while working in cooperative groups (Kroesbergen & Van Luit, 2003).

One underlying objective of the MSAR is the application of mathematics to real life scenarios. The word problems that were used were selected to reflect students' knowledge of real world experiences. Students are required to recreate the word problem to reflect the original word problem with the MSAR task. This allowed students to explore the word problems in a manner that encouraged more in depth understanding. In addition, it encourages students to create word problems according to their own experience (Wiest, 2000).

The results of this study imply that with a well-developed instructional tool such as the MSAR approach, diverse students in urban areas can improve their math learning. The MSAR approach requires students to receive instruction on how to analyze a problem, learn multiple visual representations to explain their understanding, and be given the opportunity and instructional strategies they may use to solve any given problems (An & Wu, 2014). The findings in this study indicate that it is possible to see struggling students, especially English Language Learners have a better outcome in representing their conceptual understanding through models and using appropriate strategies to solve real world related math word problems with appropriate reasoning.

The findings in this study further support the idea of teacher knowledge growth (An & Wu, 2014) through learning and teaching mathematics by using the multiple components of MSAR. Teachers not only will gain rich mathematics content knowledge but also increase their pedagogical content knowledge from the inherent structure of the MSAR. The study indicates that applying the MSAR approach in teaching mathematics helps diverse students understand mathematics from visual models, develop procedural fluency from using various strategies, and build competence in problem solving from real world application.

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