Obstacles in Investigating the Role of Restructuring in Insightful Problem Solving

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Abstract

In the present article, we articulate three assumptions underlying theories proposing that restructuring processes play a key role in insightful problem solving: representational difficulty, representational change, and discontinuity in solution processes. We argue that these assumptions need empirical validation to justify the proposition of restructuring mechanisms that are unique from those involved in classic information-processing theories of problem solving. To this end, we review some theoretical and methodological obstacles that are inherent in the investigation of the existence and nature of restructuring processes. We then offer some recommendations on how to overcome or avoid these obstacles in future studies. Finally, we discuss some questions to help motivate new research.

Keywords

insight, problem solving, creativity

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Introduction

In the present article, we review some of the important obstacles to the investigation of the representational change processes proposed by theories of insightful problem solving. Past theorizing has suggested a distinction between two qualitatively different classes of solution processes that might be involved in successful problem solving. For many problems, solution may be reached via rote, routine, incremental, or analytic solution processes. However, in some cases, progress is suggested to rely on more "insightful" means, involving intuitive, creative, non-incremental, or discontinuous solution processes.

This distinction in solution processes may be best understood via comparison. Analytic or incremental problem solving is often described as a systematic process of working one's way from a given problem state to a goal or solution state (Newell & Simon, 1972). Solvers' prior experience activates the relevant problem elements and appropriate operations for solving the problem based on the problem description. These elements and operators form and constrain the solvers' mental representations of the problem. Solving via this type of process involves navigating the problem space in an algorithmic, rule-based, or heuristically-guided manner in order to work toward the goal. The possible paths through a problem space are often represented as search trees in which the given state is at the top and all possible moves from that state and their consequences branch off below. Only certain branches in the tree lead to the goal state while others lead to dead ends. Problem solvers use operators to move from one state to another and heuristics serve to constrain the search and guide the solving process. If the path that is chosen does not contain the goal, then the solver either fails to solve or moves back to a previous state to explore another path (Ernst & Newell, 1969). Here successful problem solutions are obtained through deliberate, heuristic-based search processes such as means-ends analysis (Newell & Simon, 1972), planning (Hayes, 1978), and sub-goaling (Simon, 1975).

When a problem solver's initial problem representation is appropriate, these heuristic search processes can lead to the correct solution. However, it has been suggested that different solution processes are required when solvers form an initial problem representation that is *inappropriate* for the current situation (Duncker, 1945/1972; Maier, 1931). In these cases, prior experience elicits a problem representation that inappropriately constrains the search space or inappropriately combines problem elements. Given this inappropriate initial problem representation, classic insight theories suggest that no amount of systematic search can lead to the correct solution path. In these situations, solvers may exhaust the available paths in this problem space and come to a point where there are no further solution options for the solver to pursue. This has been referred to as "impasse." Further, many theories of insightful problem solving have proposed that at this state of impasse, problem solving processes must act to *restructure* or change the initial problem representation (Davidson, 1986; Duncker, 1945/1972; Kaplan & Simon, 1990; Ohlsson, 1992;

Wertheimer, 1954/1959). If the new problem representation elicited by the restructuring processes is appropriate, then the correct solution path becomes obvious and the problem can be solved.

While it is not the only potential cognitive process involved in creative problem solving, restructuring is a mechanism that cannot be accommodated by classic heuristic search accounts of problem solving, and requires an additional theoretical framework. As we detail below, what is needed to support theories of insightful restructuring is evidence directly supporting that the difficulty in some problem situations is due to solvers adopting an inappropriate representation of the problem, and that cognitive processes leading to new or different representations are involved in overcoming these obstacles. We argue that only once these issues are resolved can we start to disentangle other questions related to the mechanism and awareness of the restructuring process. To this end, we review some theoretical and methodological obstacles that are inherent in the investigation of the existence and nature of restructuring processes. In doing so we discuss a select set of example studies from the problem solving literature to provide concrete examples of the methodological issues that are articulated in this review. The primary goal of this paper is to summarize this set of important issues in order to give researchers a common language for discussing these design problems and present potentially fruitful directions for future research on this topic.

Representational Obstacles, Discontinuity and Representational Change

Since the early days of the Gestalt movement (e.g., Duncker, 1945/1972; Köhler, 1925; Wertheimer, 1954/1959), there have been common descriptive accounts of the sequence of creative problem solving. This view is illustrated by the following quotes from the writings of Max Wertheimer and Karl Duncker.

From Wertheimer's (1954/1959) *Productive Thinking*:

> It is this factor [reasonable reorganization, reorientation] that leads to or constitutes a discovery in a deeper sense. In such cases a discovery does not merely mean that a result is reached which was not known before, that a question is somehow answered, but rather that the situation is grasped in a new and deeper fashion—whereupon the field broadens and larger possibilities come into sight. These changes of the situation as a whole imply changes in the structural meaning of part items, changes in their place, role and function, which often lead to important consequences.

> Before the thought process takes place, or in its early stages, one often has a certain whole-view of the situation, and so of its parts, which is somehow

> unsuited to the problem, superficial, or one-sided. Such a first inadequate view often prevents a solution, a proper dealing with the task. If one sticks to this view, one will often be unable to solve the actual problem. On the other hand, when the change has occurred, and the problem has thereby been solved, one is sometimes astonished to see how blind one has been, how superficially one had viewed the situation (pp. 169-170).

From Duncker's (1945/1972) *On Problem Solving*:

> Every solution consists in some alteration of the given situation. But not only this or that in the situation is changed, i.e., not only such alterations take place as one would have to mention in a simple commonsense description; over and beyond this, the *psychological structure* of the situation as a *whole* or of certain significant parts is changed. Such alterations are called *restructurations*.
>
> In the course of a solution-process, the "emphasis-relief" of the situation, its "figure-ground" relief, for example, is restructured in this way. Parts and elements of the situation which, psychologically speaking, were either hardly in existence or remained in the background—unthematic—suddenly emerge, become the main point, the theme, the "figure." Of course, the reverse also happens.
>
> Aside from the emphasis, the material properties or "functions" of parts are changed as well. The newly emerging parts of the situation owe their prominence to certain relatively general functions: this one becomes an 'obstacle', a point of attack (conflict element), that other a 'tool,' etc…
>
> Especially radical restructurations tend to take place in the *nexus or context of the whole*. Parts of the situation which were formerly separated as parts of different wholes, or had no specific relation although parts of the same whole, may be united in *one* new whole….
>
> It has often been pointed out that such restructurations play an important role in thinking, in problem-solving. The decisive points in thought-processes, the moments of sudden comprehension, of the "Aha!," of the new, are always at the same time moments in which such a sudden restructuring of the thought-material takes place, in which something "tips over." (p. 29)

These quotes describe a problem solving sequence that was of particular interest to those in the Gestalt movement that consisted of: a) an initial attempt with an inappropriate conception of the problem situation, b) a failure to solve the problem with this initial understanding of the situation, and c) a sudden carrying out of the solution. To the Gestalt psychologists, the mental processes that bridged the gap between *failure* and *sudden solution* involve a fundamental and qualitative change of the initial mental problem representation. The idea that representational change processes are important in creative or insightful problem solving has been carried into modern times. Several different theories have been offered that propose restructuring or representational-change processes in terms of information-processing perspectives (e.g., chunk decomposition and constraint relaxation, Knoblich et al., 1999; selective combination, encoding, and comparison, Davidson & Sternberg, 1984; heuristic search of the problem space of problem representations, Kaplan & Simon, 1990).

However, over a quarter century ago, Weisberg and Alba (1981) pointed out that there was no empirical evidence for either the discontinuous problem-solving sequence or the restructuring process proposed by Gestalt psychologists. Regarding the discontinuous sequence, Weisberg and Alba claimed that, at that time, all evidence of this discontinuous type of solving sequence was merely anecdotal in nature. As for the existence of restructuring in problem solving, the authors pointed out a critical flaw in the logic of previous studies of insight, stemming from the basic design and assumptions of the previous work. In these studies researchers set up unfamiliar, odd, and purposefully misleading problem situations. Because these problem situations were designed to conflict with prior experience, it was assumed that anyone who solved the problem must have overcome this initial bias. The fact that the problems were seldom solved, or only solved with great effort, was taken as evidence of representational difficulty, while the fact that some people solved the problem was taken as evidence of restructuring. It is not difficult to see the problem with this type of argument, as solution failure or success serve as both the evidence for representational difficulty and the proof of restructuring processes.

Below, we borrow and expand the general framework of Weisberg and Alba's analysis in our discussion of the role of restructuring in insightful problem solving. We articulate three assumptions inherent in many representational change theories that still need empirical validation to implicate the role of these types of processes in problem solving.

1) *Assumption of Initial Representational Obstacles*: This assumption is that the primary difficulty in some problem situations stems from *representational obstacles* and *not* obstacles due to lack of knowledge or the size of the search space. A *knowledge obstacle* occurs when a solver does not have the information or skills necessary to find the solution to the problem. A *search space obstacle* occurs when the problem situation poses so many possible steps or operations that cognitive limitations prevent people from efficiently exhausting the problem search space. Representational obstacles, however,

are purely a result of the solver's initial perception of the problem. Knoblich et al. (1999) posed a question that illustrates the important theoretical difference between a representational obstacle and other types of problem difficulties. They asked, "If a problem is solved eventually, then the problem solver was, by definition, competent to solve it. If so, why does he or she encounter an impasse?" (p. 1534). Heuristic search theories (Newell & Simon, 1972) can explain solution in the face of knowledge or search-space difficulties. It is only surmounting representational difficulty that may require a unique explanatory process (Kaplan & Simon, 1990; Knoblich et al., 1999; Weisberg, 1995).

In order to clarify what is meant by *representational obstacles*, it is useful to explicitly consider what we mean by "problem representation." The traditional information processing approach to problem solving defines four main attributes of a mental problem representation: 1) the solver's understanding of the initial state of the problem, including the solver's prior experience with the problem elements, 2) the solver's understanding of the goal state, 3) the set of allowable operators, i.e., what manipulations can be applied to the initial state so as to move toward the goal, and 4) the set of constraints, i.e., what manipulations are not allowable and what elements cannot be manipulated in pursuit of the goal (Newell & Simon, 1972). It is important to differentiate the mental representation of the problem from the "objective" presentation of the problem. A solver's mental representation embodies not only the information given in the problem, but also all of the solver's prior knowledge that is activated by the situation (e.g., Chi, Feltovich, & Glaser, 1981; Hinsley, Hayes, & Simon, 1978). Therefore, the allowable operators or constraints imposed by the mental representation may come either directly from the problem stimuli, or may be imposed by the solver's understanding of the situation. These four aspects of mental problem representations are used by solvers to develop their solving strategy and monitor progress toward their goal (Newell & Simon, 1972).

To fully explain the difference between those difficulties due to the size of the search space and those due to inappropriate representation, it is helpful to consider examples of the two different types of situations. The Tower of Hanoi problem (see Figure 1) has long been used in studies investigating the effects of the size of the problem space on problem difficulty and strategy use in problem solving (Anzai & Simon, 1979; Egan & Greeno, 1974; VanLehn, 1991). It has also been used in the development and testing of computational models of human problem solving (Gunzelmann & Anderson, 2001; Karat, 1982; Newell & Simon, 1972). Figure 2 presents a proposed mental representation of this problem that includes the initial representation, goal state, operators, and constraints. The branching network below the initial state represents the *problem space* that follows from this representation. Suppose Solver A has this representation and adopts a means-ends, or "hill climbing," strategy to solve this problem. This solver may choose a path to solution where he attempts to use each move to make the current state look more like the goal state. This may send him down the search path on the far left of the diagram, where, in two moves, he is able to get two of the disks over to the right-hand post. However, this solver would soon find himself stuck in that configuration and having to backtrack to make any further progress. Now, consider if Solver B decided on a sub-goaling strategy where she first wants to free the bottom disk and move it to the end location, since it cannot be stacked on any other disks. This may lead her to follow the far-right path in the diagram, which leads more directly to the goal state. Clearly, Solver A's strategy would force him to make many more moves and lead to more backtracking than Solver B's strategy. However, both of these solution paths are available under the exact same mental representation. Therefore, the differential difficulty that these solvers experienced during problem solving had to do with how effectively they navigated the search space and was not due to the nature of their problem representations.

Figure 1. The Tower of Hanoi Problem (directions from Cushen & Wiley, 2008).

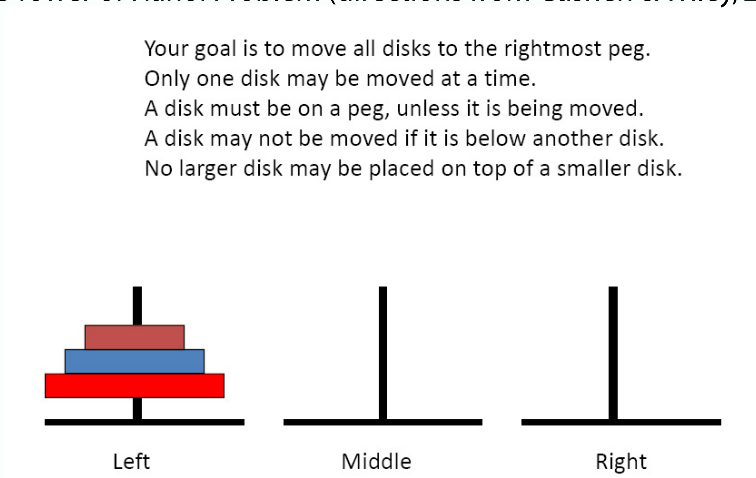
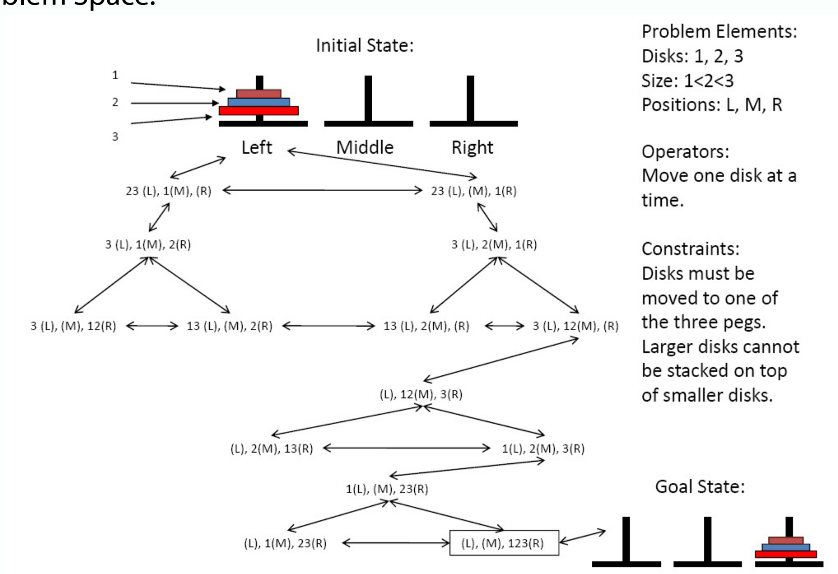


Figure 2. Schematic of a Mental Problem Representation of the Tower of Hanoi and Resulting Problem Space.



Next, consider the problem presented in Figure 3 (from Ash & Wiley, 2006; adapted from Ashcraft, 1994). If you have not encountered this problem before, we encourage you to attempt it now before moving on. Suppose that Solver A forms the mental problem representation described in Figure 4. In this representation, he conceives of the six glasses as the elements of the problem that can be manipulated in order to reach the goal state. Furthermore, the goal state in this representation is more vaguely defined than that in the Tower of Hanoi, as it does not specifically determine the order of full and empty glasses or the number of coaster spaces in between the glasses. A means-ends or a hill-climbing strategy may direct him to first target glass B, since this is the glass that is preventing no two full glasses from being next to each other. However, given the solver's representation of the glasses as the movable objects and the empty coasters as viable destinations, no possible movement of this glass will satisfy the goal state. In fact, inspecting the diagram of the search space, one can see that no possible movement in this problem representation will lead to the solution.

Figure 3. The Glasses and Coasters Problem (Ash & Wiley, 2006).

The picture below is of six glasses and thirteen coasters. The first three glasses contain liquid. Describe how you could make it so no two glasses containing liquid are next to each other, and no two empty glasses are next to each other, while keeping three of the six glasses full. To do this, you are only allowed to move one glass and all glasses must end up resting on coasters.

Now, suppose that Solver B represented the problem as depicted in Figure 5. This solver represents the liquid in the glasses as an element that can be manipulated and the empty glasses as the possible destinations for the liquid. This representation allows for a different operator: pouring liquid from a full glass to an empty glass and returning the glass to its original position. Once again, a means-ends or hill-climbing strategy would target the liquid B as the object separating the initial state from the goal state. This would lead to her quickly, and with little effort, finding the appropriate solution to the problem.

Figure 4. Schematic of Inappropriate Mental Problem Representation of the Glasses and Coasters Problem, and Resulting Problem Space.

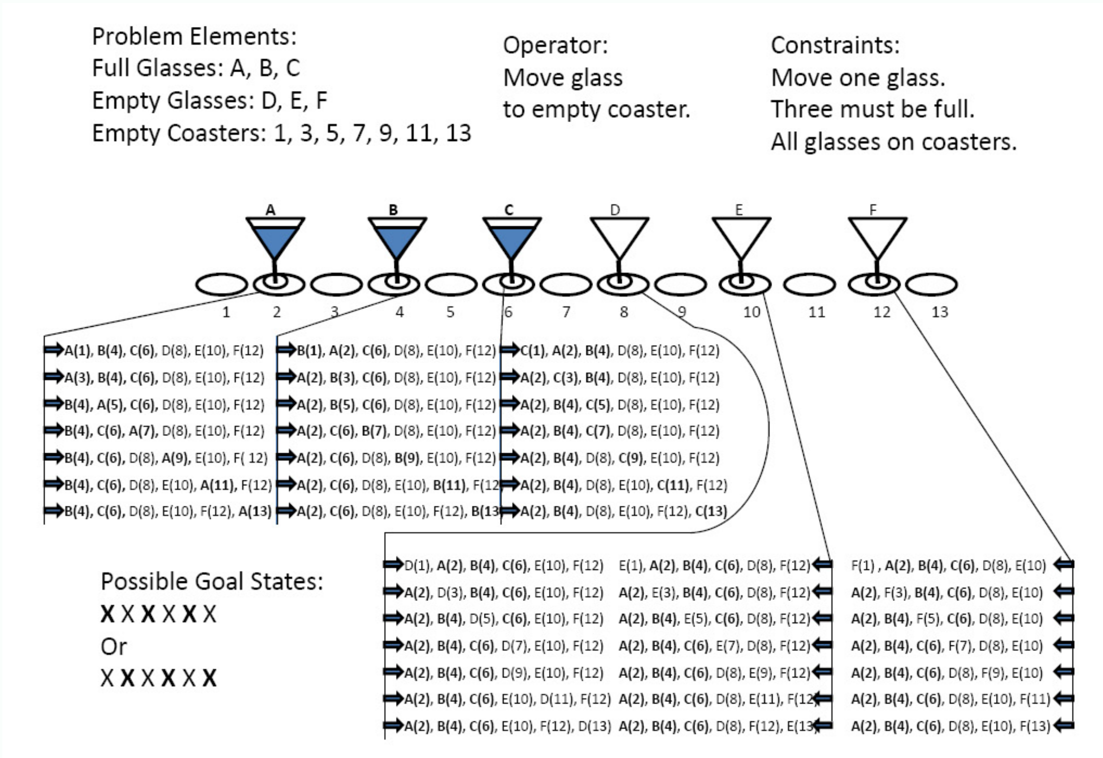
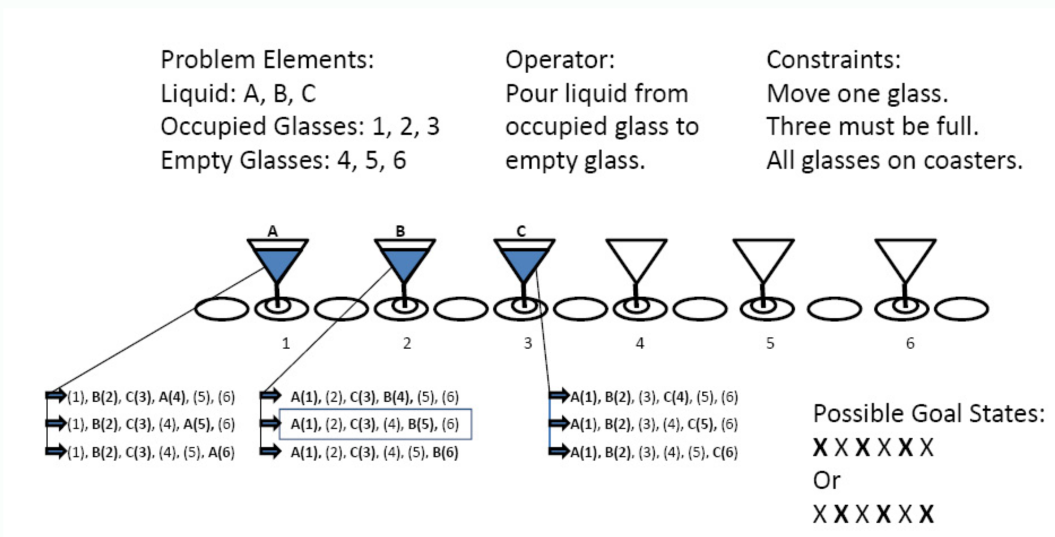


Figure 5. Schematic of Appropriate Mental Problem Representation of the Glasses and Coasters Problem, and Resulting Problem Space.



This second example illustrates what we mean by a *representational obstacle*. Notice that in this example, both solvers employed the same search heuristic. The difference in their success was not in the effectiveness of their search process, but instead in the appropriateness of their problem representation. Solver A's representation could not possibly lead to the goal state, regardless of the effectiveness of his search. Yet, Solver B's representation led directly to the goal state without much need for a systematic search. Solver A's difficulty is what we are referring to when we use the term "representational obstacle." A representational obstacle occurs when a solver's understanding of the initial state, conception of the goal state, available set of operators, or set of imposed constraints prevents the solution of a problem.

2) *Assumption of Internally-Generated Representational Change*: Of particular importance to Gestalt psychologists was the distinction between restructuring that occurs due to the introduction of new information from an external source versus that which occurs as a result of internal mental processes (e.g., Duncker, 1945/1972; Wertheimer, 1954/1959). This distinction can also be seen today in modern theories of creative or insightful problem solving. For example, opportunistic assimilation theory (Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995) proposes that the sudden resolution of representational difficulties often occurs when a person, having reached impasse on a particular problem, happens to encounter a new situation that offers an analogous solution. We propose that classic heuristic search theories are adequate to explain how solvers overcome representational obstacles when new information is introduced.

However, other researchers have proposed mechanisms for representational change that do not depend on the introduction of new information from external or environmental sources. In other words, these theories propose problem solving processes that can lead to internally-driven or *spontaneous* restructuring. By spontaneous, we do not mean to imply this type of restructuring is *magical* or *unexplainable*. Instead, we are simply trying to differentiate between changes in representation due to the assimilation of new information, acquisition of new skills, or the cueing of different memory traces in response to environmental events, from changes in representation due to internal mental or perceptual processes that do not rely on changes to the given problem stimuli or the addition of new information.

An example of a modern problem solving theory that proposes processes that lead to spontaneous restructuring is the Representational Change Theory of Knoblich et al. (1999). This theory proposes that, when faced with an impasse, the activation of problem elements and operators in the initial problem representation stop being reinforced by continued search processes. This allows activation to automatically spread to other memory items that are below threshold in activation. Knoblich et al. proposed two possible results of this automatic spread of activation that could lead to representational change. The first is *chuck decomposition*, where activation spreads to the components of problem elements that had been perceptually or conceptually grouped in the initial representation. The second is *constraint relaxation*, where activation to operator constraints imposed by the initial representation dissipates and enables the activation of new operators. According to this theory, when activation of the deconstructed components reaches threshold, or the initial constraints fall below threshold, a qualitatively different problem representation can appear in consciousness in a sudden and holistic manner.

In another example, Davidson and Sternberg (1984; Davidson, 1986) proposed three processes that can lead to internally-generated representational changes in problem solving: selective combination, selective encoding, and selective comparison. Selective combination is a process by which solvers attempt to chunk or group elements of the problem in new ways. Selective encoding is a process by which solvers systematically focus attention on different problem elements, intentionally ignoring others. Selective comparison is a process by which solvers search long-term memory for different, previously-experienced, problem situations and attempt to find common features with the current problem. According to this theory, restructuring occurs when one of these processes activates problem elements, groupings, or analogous situations that qualitatively change the solver's conception of a problem in a way that reveals the path to the solution.

While Knoblich et al.'s Representational Change Theory and Davidson and Sternberg's three-process theory may differ in the automaticity and conscious control of the proposed processes (Ash & Wiley, 2008; Sio & Ormerod, 2009), they both suggest *internally-generated* cognitive processes that can lead to the restructuring of a solver's representation of a problem situation when faced with a representational difficulty. Furthermore, they both propose that restructuring processes play a key role in creative or insightful problem solving.

In the current article, we will focus on this type of restructuring that implies spontaneous representational change because, we argue, this is the type of restructuring that so-called "special process" accounts of insightful problem solving are designed to explain. Models of problem solving based in heuristic search processes (Newell & Simon, 1972) have achieved success in explaining problem solving in well-defined problem spaces. However, it has proven more difficult for these types of models to simulate representational change to any significant degree without being provided with new information (e.g., Langley & Jones, 1988). Such difficulties highlight the need to evaluate evidence for spontaneous representational change in order to assess whether the exclusion of such a process constitutes a deficiency in heuristic search theories.

3) *Assumption of Discontinuity in Solution Process*: A final element in many theories of insightful problem solving is reference to the discontinuity of the solution process, the suddenness or surprising nature of the solution, or the lack of metacognitive awareness of solution progress. This assumption has been tied to the restructuring process by suggesting that, when the new problem representation elicited by the restructuring processes is

appropriate, the correct solution path becomes instantly obvious. In the problem-solving literature, the sudden and surprising nature of solutions (i.e., the *Aha!* experience) has often been cited as the primary evidence for a difference between the processes involved in insightful and non-insightful problem solving (Davidson, 1995; Metcalfe & Wiebe, 1987; Seifert et al., 1995). This has also been suggested as evidence of discontinuity in solution processes.

In the present paper we focus specifically on the need to support the first two representation-based assumptions. This is especially critical if our ultimate goal is to justify theories that propose *different* solution processes from those involved in heuristic search accounts. As such, the phenomenological experience of "insightful" solution is a topic that we think may be best approached once the first two issues have been addressed. Thus, we leave the intriguing questions about the possible differences in perception of the solution process in different problem types (and what they may imply about the nature of solution processes) aside for the time being. In the next section we outline several key theoretical and methodological issues facing the empirical investigation of restructuring processes in problem solving.

Issues Facing the Empirical Investigation of Restructuring

1) *Selection of Problem Stimuli*. This issue involves difficulties in finding relevant problems on which to test theories of representational change. In order to investigate restructuring in problem solving, researchers need to agree what problems elicit representational difficulties (Chronicle, MacGregor, & Ormerod, 2004; Gilhooly & Murphy, 2005; Weisberg, 1995). The investigation of the role of restructuring processes in problem solving has been frequently been conceptualized as the search for "insight." The variety of problems that have been classified as "insight" problems in different studies is quite wide and includes Remote Associate problems, graphical puzzle problems, and word riddles. Having cast such a wide net, it is difficult to identify the common psychological construct or process shared by all of these problems. Without a clear operational definition of the construct being investigated, any researcher's evidence for or against restructuring theories can be dismissed by arguing that the problems used were not appropriate problems.

2) *Appropriate Controls or Comparison Conditions.* In order to provide convincing evidence for or against the additional problem-solving processes proposed by representational-change theories, researchers need to be able to provide control conditions by which to compare their evidence of representational difficulty, representational change, or discontinuity. Several comparison strategies have been employed in the literature. However, many of these strategies introduce their own difficulties and confounds.

One regularly employed strategy is to compare those who successfully solved a target problem with those who failed to solve, and then attempt to find evidence for different final representations between the two groups. The main problem with this design is that there is no evidence for representational change *during* the solution attempt. Therefore, any evidence of representational differences between groups at solution could as easily be attributed to different *initial* representations (or other individual differences between groups), as opposed to representational change processes during successful solution. We will refer to this issue as the *Solver vs. Non-Solver Confound*.

Another problem present in many studies is that they fail to test their proposed restructuring measures on problems that are presumed to be solved without requiring restructuring. Measures providing evidence for representational change must show expected patterns of results only on problems which pose representational obstacles and should not demonstrate the same patterns on problems posing search-space or knowledge obstacles. In essence, this design confound relates to the divergent validity of a study's measurement of representational change. We will refer to this as the *Lack of Comparison Condition Confound*.

For studies that do involve a set of comparison problems, a further confound arises from simultaneously varying the baseline problem difficulty with the proposed representational difficulty. Restructuring theories do not propose that problems that require representational change are necessarily more difficult than those that do not; they are simply difficult for different reasons. In situations where target and control problems differ in their baseline solution rates, any difference in the solution process between problem types could just as easily be attributed to problem difficulty as to the need for representational change. We will refer to this as the *Problem Difficulty Confound*.

3) *Spontaneous Representational Change vs. New Information Confound*: Many studies fail to differentiate between those who successfully solved a problem, and those who failed to solve and were then shown the solution. While being shown the solution may lead an unsuccessful solver to better comprehend the steps involved in solving the problem, their experience clearly deviates from that of those individuals who independently reached the solution. In fact, there is evidence that solving a problem and being shown the solution are not equivalent in terms of their effects on participants' solution behaviors and final problem representations (Ash & Wiley, 2008; Dominowski & Buyer, 2000; Durso, Rea, & Dayton, 1994). A related issue occurs when hints are given during the solution attempt. When solvers are exposed to hints, then the study can no longer be used to support theories that propose processes of spontaneous and internally-driven representational change, as classic heuristic search models of problem solving can explain solution following the presentation of new information. Experiments that are attempting to provide evidence for the role of an internally-driven representational change process, or attempting to investigate correlates or mechanisms of that process, must be very careful to control for the influence of external cues, or the introduction of new information, during the problem solving process.

Evidence for Representational Obstacles During Problem Solving

There is evidence that some problem obstacles are due to solvers' forming initial problem representations that overly constrain or inappropriately construe the problem situation. One source of evidence for the role of representational obstacles in problem solving has come from studies that give hints designed to help correct a solver's inappropriate representation of the problem or by constructing problem isomorphs that are designed to lead to appropriate representations. For example, Weisberg and Alba (1981) investigated the role of representational difficulty in solving the nine-dot problem, which asks solvers to connect nine dots arranged in a 3 x 3 square array, without lifting their pencil from the paper, and by drawing exactly four straight, connected lines that will go through all nine dots, but through each dot only once. It has been proposed that one of the difficulties of this problem is that solvers constrain their solution search to trying to find a series of lines within the shape of the square array. Weisberg and Alba improved solution success on this problem by telling participants that, in order to solve the problem, one must make lines that go beyond the square shape formed by the array of dots. This suggests that incorrectly representing the nine dots as a box plays some role in the problem's difficulty. However, the modest improvement gains that resulted from this intervention clearly show that this is not the only determinant of difficulty in the problem. More recent investigations have shown that practice with 13- to 17-dot versions, requiring drawing lines outside the central 9-dot area, improves performance (MacGregor, Ormerod, & Chronicle, 2001). Combining several hint types, Kershaw and Ohlsson (2004) were able to substantially improve performance on the nine-dot problem. They took this as evidence that the problem may pose *multiple* procedural and representational difficulties.

In another example of evidence for representational difficulty in problem solving, Kaplan and Simon (1990) had participants solve a difficult problem called the "mutilated checkerboard" problem, which asks solvers to prove whether one can completely cover a checkerboard that is missing two opposite-corner squares using dominoes. They found that manipulating the presentation of the problem so as to emphasize the parity of the different colored squares, and giving hints alluding to this parity, led to more solvers realizing the solution: that each domino must cover one square of each color and, because two squares of the same color had been removed, a complete covering was impossible.

In a third example, Ormerod, MacGregor, and Chronicle (2002) presented participants with a novel eight-coin problem that required solvers to move coins so that all coins were touching three other coins. The solution required moving the coins from a single, flat array into a three-dimensional array with two groups of four coins each, in which one coin was stacked above the other three. They found that hints telling the solvers that the solution required making two groups, and that the solution was three-dimensional, improved performance on this problem. Without these hints, very few people solved. Again, this shows that the problem, as originally presented, involves some specific representational difficulties.

We suggest that all of these hint studies demonstrate that a failure to solve some problems seems to be the result of specific representational difficulties, as presenting hints designed to bring attention to the frequently-misrepresented elements in these problems has been shown to improve solvers' performance. This suggests that the representational obstacles which the hints, problem manipulations, or training procedures are designed to counteract are major sources of problem difficulty. However, these examples cannot serve as evidence that people actually *could* spontaneously change their own faulty problem representations in the absence of these externally-provided hints. In fact, in all of the above examples, there were no or very few instances of participants completing the problems without the hints, training, or other problem manipulations. Investigations that use hints or training cannot be used as evidence for a spontaneous and internal restructuring process because they all suffer from the *Spontaneous Representational Change vs. New Information Confound*. Hints or problem manipulations such as the ones used in the above studies serve to either change the problem situation so that it is less likely to be misrepresented, change the solvers' prior experiences so that they do not misinterpret the problem, or give external cues that can activate different information in long-term memory. Therefore, we propose that the multitude of studies that involve hints or training provide convincing evidence that representational difficulty is an important obstacle in the type of problem situations studied in the insight problem solving literature. Furthermore, hint and training paradigms are extremely useful methods for identifying, and operationally defining, the particular representational obstacles in different problem stimuli. However, researchers should be wary of interpreting the results of these studies as evidence that spontaneous restructuring processes are involved in overcoming these representational obstacles.

Evidence for Representational Change During Problem Solving

Below, we review three empirical studies investigating representational change in creative problem solving and use these studies to illustrate the importance of resolving these issues and confounds. Our goal is not to offer a complete review of the literature on restructuring. Instead, our goal is to present some of the most compelling evidence for the role of restructuring in creative problem solving and demonstrate how the methodological issues we discussed earlier prevent the refutation of alternative explanations. We do not mean to suggest that these studies have not made significant contributions to the literature. Quite to the contrary, we have chosen these studies because we feel that they represent some of the most convincing findings in the field of insightful problem solving and that, in order for the field to advance, researchers will need to focus on overcoming these very difficult methodological obstacles. Also, we acknowledge that these studies were designed to test the nature of restructuring, instead of providing evidence for its existence. However, we believe that re-examining the methods of these studies without making the *a priori* assumption that restructuring processes play a role in problem solving provides a useful demonstration of the thorny and pervasive nature of the design issues that we are discussing.

Manipulating Representational Difficulty. In one line of research often taken as support for representational change theories, Knoblich et al. (1999) constructed a series of matchstick arithmetic problems that were proposed to differ in terms of their severity of representational difficulty. These stimuli involve matchsticks set up to form incorrect equations that include both roman numerals and operators (e.g., X + III = XI or X - II = XI). The object of each problem is to move one matchstick in the incorrect equation to make the expression into a true arithmetic statement. The representational difficulties proposed in the problem stem from solvers' inappropriately applying the operations of mathematics to this situation, and from the automatic representation of operators (i.e., +, -, =) and roman numerals (X, V) as chunks, predisposing solvers to only move the single sticks representing ones (I).

Solution rates were found to vary as a function of the proposed representational difficulty. In particular, problems that involved the breaking down of "loose" chunks (e.g., moving the "I" in "IV," which was not thought to require chunk decomposition) were easier to solve than problems that involved the breaking down of "tight" chunks (e.g., changing a V to an X). The authors further proposed that evidence for restructuring could be obtained by giving participants an initial set of problems to solve and then looking at performance on a set of isomorphic problems that differed superficially but involved the same representational difficulties. They found improvement on the isomorphic problems, with the greatest improvement for the problem types proposed to have the greatest representational difficulties. This was interpreted as evidence to support the role of restructuring in solving these problems. Initial difficulty on the first set of problems was offered as evidence of incorrect initial representations. The improvement of performance on the "tight chunk" isomorphs was offered as evidence that the solvers were coming to the second set of problems with more appropriate problem representations. A lack of improvement on the "loose chunk" isomorphs was taken as evidence that restructuring was unnecessary and thus similar problem representations were used on both occasions.

Although transfer performance can be used as a measure of representational change, there are three issues preventing this study from providing clear support for the role of spontaneous restructuring processes in problem solving. First, this study represents an example of the *Difficulty Confound*. The problems that were designed to be most likely to pose representational obstacles (i.e., the ones with multiple or tighter constraints posed by prior experience) were also the most difficult on the first attempt. Therefore, one cannot take the larger amount of transfer improvement on these problems as evidence for restructuring. The other problems showed less or no transfer improvement because performance on these problems was already close to or at ceiling. Furthermore, this study did not include comparison problems to show that the pattern of transfer improvement, which was taken as evidence of representational change, would not have also been observed on problems posing obstacles unrelated to the participants' representation of the problem (*Lack of Comparison Condition Confound*). Finally, in this study, the transfer effects for those who failed to solve and were shown the solutions and those who correctly solved the problems were analyzed together. This *Spontaneous Representational Change vs. New Information Confound* makes it impossible to identify whether those who actually solved the problems showed these transfer effects, or if those who received new knowledge via the experience of being shown the solution drove the finding. These issues preclude the interpretation of these results as evidence for the type of internal and spontaneous representational change proposed by restructuring theories of insightful problem solving.

Examining Repeated Solution Attempts. In another example, Dominowski and Buyer (2000) designed an investigation to test for representational change that was intended to correct for the *Spontaneous Representational Change vs. New Information Confound* found in many studies. Similar to the logic of using performance on transfer problems, they had solvers attempt a number of classic "insight" problems on two occasions. If the participants did not successfully solve the problems on the first attempt, they were shown how to solve the problem by the experimenter. Subjects returned one week later and attempted the same problems again. Solution times for prior solvers and non-solvers on both attempts were then analyzed separately. Prior solvers showed considerable decreases in solving time from the first to the second attempt. Furthermore, the solving times on the second attempt for those who were simply shown the solution were slower than the prior solvers' second-attempt times, and more closely resembled the prior solvers' initial times. That successful solvers showed decreased solving times on the second attempt was seen as evidence for representational change occurring during their solving process. The observation that those who did not solve, and were merely shown the solution, did not show similar time-savings suggested that acquiring information about the solution procedure from an external source did not lead to the same type of representational change.

However, this study also suffers from several issues that prevent it from providing unambiguous support for the type of internally-driven representational change proposed by restructuring theories. First, in order to avoid the *Spontaneous Representational Change vs. New Information Confound,* this study falls prey to the *Solver vs. Non-Solver Confound*. Comparing solvers to non-solvers can show that their solving behaviors are different. However, measures like solving time cannot pinpoint the cause of these solving differences. Although the pattern of solution times is consistent with the predictions of a representational change theory, one could just as easily interpret these results in several alternative ways. For example, one could propose that all of the successful solvers simply began with a more appropriate problem representation than the non-solvers, and that no representational change was necessary in their solutions. Therefore, the time savings may not have been due to having a restructured problem representation, but to other aspects of re-solution such as decreased reading and problem-comprehension time, decreased time in forming solving strategies, decreased solution-checking time, or from simply retrieving the solution from long-term memory. All of these time-saving aspects of re-solving problems have nothing to do with representational change processes, and may not be expected in people who were simply shown the solution and did not explore the correct solution path themselves. Finally, this study also did not compare the re-solution effects found on the classic insight problems to re-solution effects on a set of problems designed to pose search-space obstacles (*Lack of Comparison Condition Confound*). It seems very reasonable to expect the very same pattern of re-solution time savings on problems solved via heuristic search processes for all the same reasons as proposed above. Therefore, without re-solution time data on control problems that do not pose representational difficulties, it is not clear whether these re-solution effects are truly diagnostic indicators of representational change.

Solution Progress while Solving a Riddle. In what could be considered the most convincing evidence for representational change during problem solving, Durso, Rea, and Dayton (1994) attempted to model successful and unsuccessful solvers' problem representations during a solution attempt. The stimulus for this study was a riddle. Subjects were asked to come up with an explanation of the following situation: "A man walks into a bar and asks the bartender for a glass of water. The bartender points a shotgun at the man. The man says, 'Thank you', and walks out" (p. 95). The correct solution is that the man had the hiccups. In a first study, after attempting to solve the puzzle, participants made relatedness judgments on all possible pair-wise comparisons of 14 concepts. Some of the concepts were in the problem (e.g., bartender, man), some were things related to the situation but not mentioned in the problem (e.g., TV, pretzels), and others were concepts related to the solution but not mentioned in the problem (e.g., remedy, relieved). The researchers used the 91 relatedness judgments to model the representations of solvers and non-solvers, finding that non-solvers' representations centered around the concepts of "bartender" and "man," while solvers' representations centered around "remedy" and "relieved." This was taken as evidence that those who successfully solved had different mental representations of the problem situation than those subjects who did not solve the problem. Obviously, this study's results do not clearly implicate the role of representational change because of the *Solver vs. Non-Solver Confound*. Since the structure of solvers' and non-solvers' representations was only assessed after their solution attempt, this particular result does not provide evidence as to whether the differences were due to the solution process or due to differences in their initial representations.

In a second study, however, solvers made multiple ratings *throughout* the solving process. Similarity ratings on concepts that were central to the solution were used to track the appropriateness of the solvers' representations at multiple points during the solving process: before seeing the problem, after hearing the problem, 10 minutes before solving the problem, directly before solving the problem, and after solving. Using this data, Durso et al. found evidence that everyone began by rating the key concepts as very dissimilar, thereby showing that even successful solvers began with an inappropriate problem representation.

As their solution progressed, the key concepts were rated as increasingly similar to each other. Key concepts were rated as most similar immediately after participants reached the solution. This experiment serves as a landmark example of an attempt to actually map the change in problem representation within single solution attempt. The repeated measurement of a subject's problem representation within a solution attempt controls for many of the major confounds found in other investigations in the literature, and provides some of the most convincing evidence that representational change processes may play a role in problem solving.

However, there are still several design issues that prevent these results from being interpreted as conclusive evidence for the types of representational change processes proposed by restructuring theories. First, this study suffers from the *Lack of Comparison Condition Confound*, as it did not perform a similar test on problems expected to be solved without representational change. Furthermore, on closer examination, the methods of the study may also have led to a *Spontaneous Restructuring vs. New Information Confound*. While solving, participants were allowed to ask the experimenter any yes/no questions they wished, in order to help them find the correct solution. The content and frequency of these questions was not reported. Through asking the experimenter questions, solvers could access new information that was not presented in the original problem situation. This gradual acquisition of new knowledge would be consistent with the observed gradual change in key-concept similarity ratings. Further, by giving the solvers pairs of concepts to rate, with a large proportion of the pair-words being related to solution, the rating procedure itself may have provided hints toward solution. This could also be responsible for the change to a more appropriate representation (Cushen & Wiley, 2007). Thus, because this design provided solvers with new information throughout the problem solving process, it also does not offer clear evidence that spontaneous representational change processes were responsible for problem solving success.

In summary, several theories of insightful problem solving have proposed mechanisms of representational change to account for creative solutions to problems. While there is evidence that some problems are difficult to solve because of inappropriate initial representations, there is little conclusive evidence that spontaneous representational change processes play a role in solving these problems. A plausible alternative hypothesis, which cannot be discounted, is that *only* those who approach these problems with appropriate representations, or who receive new information, ever reach solution. If this is indeed the case, then no unique restructuring processes are necessary to explain performance on these types of creative problems.

Recommendations

Below we will consider the design issues that we have introduced in this paper and propose possible ways of dealing with each issue. Many of our recommendations are conceptual or theoretical in nature, not practical plans of action. We hope that this exercise can help give interested researchers a common language and theoretical framework that might aid in eventually overcoming these difficult methodological issues.

Selection of Problem Stimuli. The first issue pertains to what sorts of problems can be used to test theories of restructuring. We are by no means the first ones to tackle this problem, and will surely not be the last. Weisberg (1995), for example, proposed that a taxonomy of problems needed to be developed where problems would be submitted to a task analysis in order to determine whether restructuring was a theoretical necessity for problem solution. Based on his task analysis, he categorized problems as *non-insight* problems (those that do not involve restructuring), *hybrid* problems (those that could be solved via restructuring or other methods), or *pure* problems (those that could only be solved through restructuring). He argued that only *pure* insight problems should be used in the investigation of representational change because only these problems are internally-valid laboratory models of the problem solving situations that restructuring theories are designed to explain. However, in advocating for a *problem-focused* categorization scheme, this suggestion ignores one of the most general claims of restructuring theories: that it is the interaction of the solver's prior experience and the current problem situation that leads to representational obstacles and sets the stage for the occurrence of representational change processes. Therefore, *any* problem may lead to a representation obstacle for one solver and not for another, depending on the assumptions the solver brings to the problem (Davidson, 1995).

We suggest that categorizing stimuli based on a hypothetical cognitive task analysis is a fundamentally flawed way to operationally define experimental stimuli. The problem with categorizing a problem as an "insight" problem based on whether restructuring is theoretically necessary to solve the problem is that it assumes (1) that all participants will represent the problem in the same way and (2) that representational change is the only mechanism by which people can overcome representational obstacles. We argue the first assumption is surely not true for any problem and that the second assumption should actually be taken by psychologists as an open and important research question. As long as problem stimuli are operationally defined by their proposed solving process, then using these problems to investigate the existence, frequency, or nature of these processes is bound to lead to inconclusive inferences. For example, if a study shows no differences between solving behavior on sets of "insight" and "non-insight" problems, then it could be argued that this evidence stands in contrast to the predictions of restructuring theories. However, someone else may argue that this just means that your "insight" problems were not problems that required restructuring, or that your "non-insight" problems required restructuring in some way. One can never provide a manipulation check that supports the construct validity of a problem-type distinction when the manipulation is defined by the behavior being investigated.

One solution to the issue of how to select appropriate problems is to continue in the spirit of the isomorph and hint studies that have been used to identify the difficulties that occur in problem solving. What makes any task a "problem" is the fact that the solution is not immediately obvious and that there is an obstacle between the current problem state and the goal state (Newell & Simon, 1972). As such, a prudent approach may be to identify problems specifically by their obstacles. In this approach, instead of calling a matchstick arithmetic problem an "insight" problem, these problems should be operationally defined as "representational obstacle" problems or, even more specifically, "chunking obstacle" problems. Other studies have used the Remote Associate Task (RAT) as their experimental model for insightful problem solving (Beeman & Bowden, 2000; Bowden & Jung-Beeman, 2003; Smith & Blankenship, 1991; Wiley, 1998). In the RAT, solvers are given three words (such as cottage, blue, and knife) and must find a fourth word that forms a meaningful two-word phrase with each of the three words. The type of difficulty involved in RAT problems seems very different from the type of representational obstacles involved in matchstick arithmetic. Perhaps these problems could be referred to as "retrieval obstacle" problems or "semantic obstacle" problems. Moreover, either of these types of problems could be contrasted with problems where the size of the search space is the primary source of difficulty, which could be referred to as "search-space obstacle" problems.

The difference made by operationally defining problem stimuli based on the proposed problem obstacle, instead of the theoretical solving process, is more than a mere change in nomenclature. The first important benefit of this type of system is that it allows for manipulation checks that are independent from the behavior of interest. Problem isomorph, hint, and training studies have proven to be effective ways of identifying potential difficulties among various problems. These types of studies, used in convergence with other techniques that can demonstrate that solvers are experiencing fixation on a particular solution, or are initially focused on irrelevant features of the problem, could independently verify that such problems evoke an initially-inappropriate representation. However, we do not think that mere identification of the sources of problem difficulty via these types of "obstacle-lesion" approaches is a particularly useful end in and of itself. As we argued earlier, merely identifying an obstacle does not imply what cognitive processes, if any, are involved in overcoming that obstacle. Instead, we think that identifying the primary obstacle in a problem is a useful first step in operationally defining and selecting problem stimuli that can provide internally-valid experimental materials for investigating the nature of the cognitive processes by which such obstacles may be overcome.

Adopting a system of obstacle identification and verification in the development of experimental stimuli would allow researchers to design studies that are more resistant to challenges concerning the construct validity of their problem stimuli. Furthermore, this system provides a language with which to ask new questions that are difficult to conceptualize in an "insight" vs. "non-insight" framework. For example, the difference between matchstick arithmetic and RAT problems can be investigated by asking, "Are chunking obstacles and semantic obstacles overcome by the same problem solving processes?" This language also focuses the psychologist on what is important to our field: the differences between psychological problem solving processes, not the differences between problems. By focusing the debate on how people overcome specific types of obstacles and not on what it means to be an "insight" problem, researchers can avoid arguments that may make for interesting debate in enigmatology, but are not of central interest in cognitive psychology.

Appropriate Controls or Comparison Conditions. At the heart of both the *Lack of Comparison Condition* and the *Problem Difficulty* confound is the question of how researchers should develop control conditions to show the validity of their experiment's measure of representational change. We think that operationally defining problems based on the obstacle and not the hypothetical solution process can help researchers frame the issues of problem difficulty and appropriate comparison conditions in a more tractable manner. In order to make sure that their evidence of representational change has divergent validity, researchers need to be able to compare problems that pose representational obstacles to problems *without* representational obstacles. However, manipulating problem obstacles independently of problem difficulty is a tricky business. For example, Knoblich et al.'s (1999) matchstick arithmetic problems represent a clever way to manipulate the type of obstacle within a single problem-solving task. However, as we mentioned, when all representational obstacles are removed from a matchstick arithmetic problem, solution rates are at ceiling. Both "type of obstacle" and "existence of an obstacle" are confounded with this manipulation. It may be unfair to point this out as a weakness in their study, as this was an *a priori* prediction of the experiment. But, any study that manipulates the difficulty of the problem along with the proposed solving process will have the same issue.

Several studies have attempted to control for difficulty by comparing measures taken from "insight" problems to those taken from a set of "non-insight" problems that have been matched for overall solution rates (Fleck, 2008; Gilhooly & Murphy, 2005; Jausovec & Baracevic, 1995; Jung-Beeman et al., 2004; Lavric, Forstmeier, & Rippon, 2000; Lipincki & Byrne, 2005; Metcalfe & Wiebe, 1987; Schooler & Melcher, 1995; Schooler, Ohlsson & Brooks, 1993). In Ash and Wiley (2008), we followed this tradition by using a set of multi-step mathematical word problems as comparisons. However, this method was not without its own issues. First, even with extensive pilot testing, solving rates were somewhat volatile and did not necessarily generalize across samples, making control of difficulty across problem types approximate at best. Second, the type of behavior that was described as an "incorrect" or "unsuccessful" solution was very different between the two sets of problems. On the mathematical word problems, many incorrect answers were due to solvers who were executing correct solving strategies but making simple calculation errors. On the classic insight problems, however, most incorrect responses were either due to a participant giving up on the problem or accepting an incorrect solving strategy. As such, this method only loosely controlled for difficulty and may present some confounds of its own.

Another interesting technique for providing control conditions in creative problem solving investigations was developed by Ormerod, MacGregor, and Chronicle (2002). In this study, the researchers developed insight problems (multiple variations of the eight-coin problem mentioned earlier) in which the number of steps available in the faulty solution space could be manipulated without changing the proposed representational difficulty of the problem. Ormerod et al. suggested that people initially try to solve these problems by moving coins around on the two-dimensional plane. They proposed that the primary insight in this problem involves switching from an inappropriate representation, where movement is restricted to a two-dimensional change in position, to a representation that allows for three-dimensional moves such as stacking. In one variant of the problem, the coins were arranged so that no possible two-dimensional move made any coin touch three other coins. In the other variant of the problem, there were 20 available two-dimensional moves in which a coin could be moved to touch three other coins. In both problems, no two-dimensional moves led to the solution, and only stacking the coins could lead to the correct configuration. We propose that this method of manipulating the initial problem search-space can be used to isolate processes involved in overcoming representational difficulties from search-space difficulties in creative problem solving research.

In Ash and Wiley (2006), we used this method of creating comparison problems to test whether the different stages of creative problem solving involved attention-demanding or automatic processes. To test this, we correlated measures of attentional control (working memory span tasks, see Kane, Bleckley, Conway, & Engle, 2001) with problem solving success and solving times on variations of classic insight problems that allowed for either many or few moves in a faulty initial problem space. Results revealed that working memory span predicted performance on problems that allowed for a large initial search-space. However, working memory span did not correlate with performance on problems that constrained the initial search-space (see Fleck, 2008, for a similar result comparing insight vs. analytic problems). These interesting initial results suggest that this method of creating control problems may be a useful way to isolate the different stages of creative problem solving and create stimuli designed to isolate the processes involved in overcoming representational difficulties.

Another potential method for providing appropriate control conditions is to not rely on pre-existing biases in participants as the source of representational difficulty, but instead attempt to *evoke* appropriate and inappropriate problem representations via experimental manipulations. This method was utilized by Duncker (1945/1972, Chapter 7) in his investigations of functional fixedness. In these studies, Duncker set up problem situations where participants needed to use a familiar object (e.g., cork, pliers, pendulum, paperclip, etc.) in a different manner than usual. In one condition, he first had solvers use the object in its normal fashion and then attempt to solve the target problem. In the control condition, he had participants attempt the target problem without pre-utilization. He found that the pre-utilization of the common object negatively affected solution rates and led to more failed attempts during solving. Although Duncker's original studies do not necessarily stand up to the experimental rigor and statistical precision of modern psychological research, the underlying methodological considerations could still aid researchers in developing experimental manipulations that use the same problem stimuli in both "representational obstacle" and control conditions. Although, the method of actively inducing fixation has been occasionally used in studies involving RAT problems, there are very few modern studies that use this type of manipulation with more complex verbal or visual problem solving tasks (see Sio & Ormerod, 2009). We believe that this method for creating comparison conditions, when combined with modern psychological "trace" techniques (which are discussed below), holds great potential in helping researchers to systematically investigate the role of both fixation and restructuring processes in problem solving.

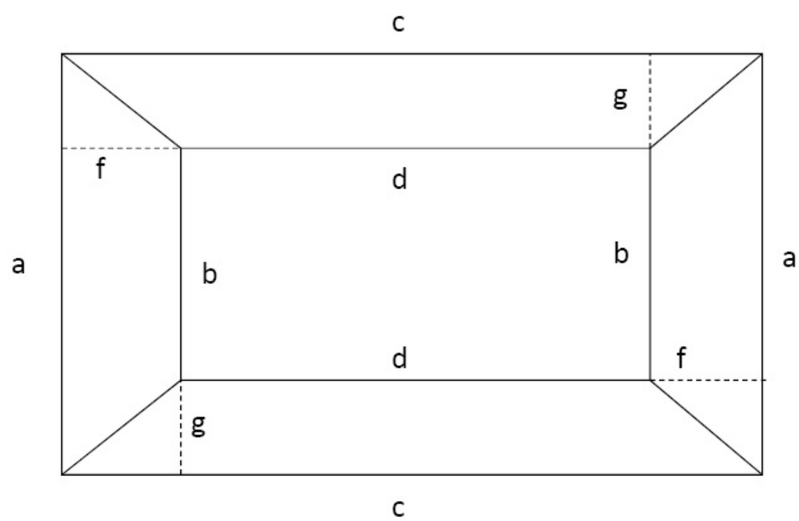
The *Solver vs. Non-Solver Confound* poses a particularly tricky methodological conundrum. Of course, it is of great importance to understand the difference between successful and unsuccessful problem solvers. But, how is it possible to isolate differences due to problem solving process from those due to pre-existing individual differences? One strategy is to have solvers attempt multiple problems and select participants whose performance allows you to control for solution success within-subject (Ash & Wiley, 2008; Jung–Beeman et al., 2004; Metcalfe & Wiebe, 1987). However, in order to do this type of quasi-experimental control, a researcher must select participants who both solve and fail to solve at least one of each type of problem. This requires a sizable number of different experimental and control problems to be given to each participant, increasing the time and difficulty associated with completing the study. It also necessarily excludes those participants who solve all the problems, and those who fail on all problems, from the analyses. Finally, this design requires careful consideration of the problem difficulty confound. If certain problems are overrepresented in the different within-participant comparison conditions, then one simply trades the *Solver vs. Non-Solver Confound* for the *Problem Difficulty Confound*, by comparing hard to easy problems instead of comparing successful to unsuccessful attempts. These concerns are a large part of the rationale behind using stimuli such as RAT problems, anagrams, and (more recently) rebus puzzles (MacGregor & Cunningham, 2008a; 2008b), as such stimuli allow for many trials and increase the likelihood that participants will both solve and fail to solve a number of puzzles.

However, controlling for problem success within-subjects does not necessarily solve this issue. As previously stated, representational difficulty is a person-by-situation interaction. Even if the problem solving task is held constant (i.e., the solver is given a set of all RAT problems) each problem represents a new situation that an individual solver could represent either appropriately or inappropriately. Therefore, even when comparing solution success within-subjects, researchers are still making the assumption that underlies the *Solver vs. Non-solver Confound*: that a successful solution is, in and of itself, evidence of restructuring. Within-subject comparisons do not exclude the possibility that one is still simply comparing correctly-represented problem solutions to incorrectly-represented problem failures.

A possible strategy for dealing with the Solver vs. Non-solver Confound is to focus data collection and analysis on the solving *process* instead of on the result of the process (i.e., solution). This type of focus was preferred by the Gestalt psychologists. As an example, Wertheimer (1954/1959) employed a process-based method of analysis that he referred to as the A/B method, where solving behaviors indicative of an appropriate understanding of the problem situation were referred to as A-responses, and B-responses were behaviors involving the inappropriate application of previously-learned operations (indicating an inappropriate representation). In his research, Wertheimer would identify these different solving patterns in participants and then attempt to systematically map out the solution processes involved in these different responses to a problem. Although his original research can be criticized for its unsystematic nature and the anecdotal presentation of results, his focus on making comparisons based on the solving process, instead of the result, circumvents several methodological confounds associated with using solution success as the main criteria by which to compare solution processes. Indeed, protocol analysis and the detailing of individual solvers' behaviors played a key role in the early development of information processing theories of problem solving (Newell & Simon, 1972). Furthermore, modern advancements in the collection, coding, and analysis of protocol data can overcome many of the methodological issues apparent in Wertheimer's earlier work (Ericsson & Simon, 1993).

An example of Wertheimer's (1954/1959) A/B method can be illustrated by his story of students asked to calculate the area of a picture frame after learning how to calculate the area of a trapezoid. Figure 6 shows a diagram of a picture surrounded by a frame made up of four trapezoids. In Wertheimer's A/B method, a student who is blindly applying the previously-learned material would represent the problem as finding the area of the four trapezoids. This student's solving behavior would take the form of "area = 2f[(a+b)/2]+2g[(c+d)/2]," which would constitute a B-response. However, if a student represented the problem as a rectangular picture within a rectangular frame, then this student may complete the much simpler procedure of "area = ac-bd" (an A-response). While this is clearly a simplistic example, it illustrates how problem situations can be developed where the structure of a solver's representation can be inferred from observable behaviors other than solution success. Instead of forming comparison groups based on solution success, researchers could form their comparison conditions based on the solving behaviors that are indicative of different problem representations. Even more importantly, researchers could target their investigation of the existence and nature of restructuring methods on those instances that show changes in solving behavior, which would potentially indicate changing representations. This type of detailed protocol would provide the type of step-by-step data needed to model possible restructuring mechanisms in cognitive architectures, such as ACT-R (Anderson, Bothell, Byrne, Douglass, Lebiere, & Qin, 2004).

Figure 6. Area of a Picture Frame Problem (adapted from Wertheimer, 1954/1959).



There are, however, some pragmatic issues that may arise when forming comparison groups on the basis of solving behavior instead of solution success. First, the problem stimuli used must lend themselves to having multiple, clearly-defined and observable solution strategies. This will require more complex problem stimuli than are often used in insight research. Although coming up with new problem materials poses a clear challenge for researchers, we believe that our suggestions to operationally define problems by their obstacles and to make use of fixation manipulations can help aid in this daunting task by providing a guiding framework for problem creation and manipulation. Second, collecting, coding, and analyzing written or verbal trace data clearly requires much more effort than simply categorizing successful and unsuccessful solutions.

There also may be some concern that verbalizing solving behavior may interfere with the very restructuring processes under investigation. For example, Schooler, Ohlsson, and Brooks (1993) found that asking solvers to verbalize their thought processes while solving a set of insight problems led to decreased solution success, which was dubbed the "verbal overshadowing" effect. While this is clearly a concern, there are several reasons why this finding does not necessarily prohibit the use of verbal protocols in the investigation of restructuring. First, the verbalization directions used by Schooler et al. were elaborate and specifically designed to interfere with the types of automatic restructuring processes proposed by the authors. As such, their results do not preclude the possibility that minimally invasive directions would avoid these effects (see Ericsson & Simon, 1993; Fleck, 2008; Fleck & Weisberg, 2004). Furthermore, other types of trace data could be collected without adding any extra cognitive load to the problem solving task. For example, in the picture frame example from above, progress could be tracked based on the written calculations of the solvers. Or, on those problems that involve objects, the physical movement of objects or hand gestures could be recorded and used to code for solving strategy. Some studies have made use of eyetracking methodology (Grant & Spivey, 2003; Jones, 2003; Knoblich, Ohlsson & Raney, 2001). Somewhat similarly, new neuroscientific methods may be able to offer converging measures that can help to identify different problem solving processes at the neural level while subjects are engaged in solving (Bowden et al., 2005; Knoblich, 2008; Luo & Knoblich, 2007). In summary, though research designs that focus on the process instead of the solution may require greater ingenuity and dedication on the part of the researcher, we believe that the potential benefits of these methods will be invaluable in answering the open questions about the role of representational change in problem solving.

Spontaneous Restructuring vs. New Information. Research has shown that there are qualitative differences between those who come to problem solutions on their own and those who are shown the correct solution (Ash & Wiley, 2008; Dominowski & Buyer, 2000; Durso, Rea, & Dayton, 1994). As such, research paradigms that collapse data across solvers and non-solvers who are shown the solution cannot inform the question of how people overcome representational obstacles and cannot be used as evidence for or against any particular problem solving theory. We encourage researchers to go back to archived data that was reported using such designs to see if solvers and non-solvers display diverging behaviors.

Similarly, as discussed earlier, we propose that "hint" paradigms may be extremely useful for identifying the types of obstacles posed by different problems, or for investigating the cognitive processes associated with the integration and utilization of new information (as in *opportunistic assimilation*). However, these methodologies are not as obviously useful in investigating the type of internally-generated and spontaneous restructuring processes proposed to be important in overcoming representational obstacles.

Hint paradigms seem less useful for investigating these types of cognitive processes, as research using hints cannot be taken as evidence for or against any theory attempting to explain spontaneous restructuring. As such, we encourage researchers to attempt to control or explicitly manipulate the effects of external information in future research (see Luo & Knoblich, 2007, for a similar suggestion).

The "Aha!" Experience and Sudden Restructuring

Until now, we have not directly tackled the third assumption of many theories of representational change: the "Aha!" or "Eureka!" feeling that is thought to accompany this particular type of problem solving. The reason for this delay, we hope, is clear. The first and second assumptions should take precedence, as the existence of both representational obstacles and of internally-generated representational change needs to be established before questions regarding the *nature* of that change should be considered. Nevertheless, the abrupt and unanticipated emergence of a new way of thinking about a problem, a new direction for solution, or the solution itself in consciousness is a common element in many accounts of insightful problem solving (Duncker, 1945/1972; Köhler, 1925; Maier, 1931; Ohlsson, 1992; Poincaré, 1952; Wertheimer, 1954/1959). Insight has been viewed as the sudden awareness of a solution in consciousness, with little or no conscious access to the processing leading up to that solution (Bowden & Jung-Beeman, 2003; Kounios, et al., 2006; Smith & Kounios, 1996). Further, for many researchers, the subjective "Aha!" experience is taken as the defining feature of insightful solution (Bowden & Jung-Beeman, 2003; Kaplan & Simon, 1990), and considered to be a marker that reorganization or restructuring occurred in a discontinuous and abrupt fashion.

Several studies have provided evidence consistent with the assumption of discontinuity in solution processes. For example, Metcalfe and Wiebe (1987) had participants make feeling-of-warmth ratings while solving either algebra or "insight" problems. Participants were asked to indicate how close they felt they were to the solution at 15-second intervals. Results indicated that subjects solving algebra problems demonstrated steadily increasing warmth ratings as they neared the solution. Subjects solving "insight" problems, however, demonstrated little variation in their warmth ratings (with assessments often staying near floor) up to and even immediately prior to their solving the problem. This suggests that, to the solver, the sequence of solving the "insight" problem was more likely to appear sudden and discontinuous. (However, see Weisberg, 1992, for several alternative explanations for these results.)

In another example, Knoblich, Ohlsson, and Raney (2001) tracked subjects' eye-movements during the solution of matchstick arithmetic problems. They found that solvers began by fixating on the inappropriate elements of the problems. This initial difficulty was often followed by a period of extremely long fixations, suggesting a state of impasse (imagine the participants as staring blankly at the problem). Finally, successful solvers began to fixate on the elements of the problem that were useful in the solution. This shift between short and long fixations, and between attention to inappropriate versus appropriate features, seems to correspond to the expected discontinuous "insight" sequence. Similar long-fixation patterns immediately prior to solution have also been observed by Jones (2003) on a different multi-step insight problem.

However, when discussing the subjective experience associated with creative solution, it seems people often make the tacit assumption that phenomenology recapitulates process. That is to say, it is taken for granted that the discontinuous *experience* that occurs during insightful solution is the result of a discontinuous restructuring *process*. This assumption can be called into question for a simple reason: solvers' awareness does not seem to track particularly well to their solution progress. Take, for example, Maier's classic studies using the two-string problem (Maier, 1931). The experimenter was able to introduce a hint (brushing against one string to set it into motion) that subjects could not identify as having played any role in their solution, but that nonetheless influenced their likelihood of reaching the pendulum solution. Interestingly, subjects were more likely to identify a second hint (that of twirling a string around with a weight attached to the end) as having played a role in their solution, when that hint actually did not show any signs of influencing the likelihood of subjects' generating the pendulum solution. This suggests subjects' awareness of their solution process and what factors actually played a role in determining their solution were remarkably misaligned.

A number of other researchers have also produced evidence of progress toward solution in the absence of awareness of any such progress on the part of the solver. For example, Bowers, Farvolden, and Mermigis (1995) report an experiment in which participants engaged in a task called the Accumulated Cues Test. Subjects were given a series of words sequentially, with each word being related to one target word. Subjects were tasked with identifying the associated target word, but had to respond with a *possible* solution after each new word was presented (despite the fact that participants tended not to get hunches as to the solution until having seen approximately 10 words from the list). The researchers found that, when independent coders were given the incorrect words generated by the participants, a linear effect emerged whereby later solution attempts (those immediately prior to solution) were rated as more associatively similar to the target word than were earlier solution attempts.

Bowers, Farvolden, and Mermigis also report a separate experiment by Mermigis that employed the same task but additionally had subjects completing feeling-of-warmth ratings. While a number of the feeling-of-warmth patterns indicated a linear increase in warmth as subjects approached solution, many were found to correspond to the classic "insight" pattern as seen by Metcalfe and Wiebe (1987). Analyzing those solution attempts in which subjects generated an insight-like pattern, Mermigis nevertheless found evidence that latter solution attempts were more associatively related to the solution words than were earlier attempts. It was argued that, for this subgroup of problems, subjects had clearly made progress on the problem without any explicit knowledge of that progress.

In a final example of the dissociation between awareness of solution progress and actual solution progress, we can look to the priming work of Bowden and Jung-Beeman. Across a series of studies using RAT problems, the critical condition of interest has been when solvers work on problems, but do not reach correct solutions. Yet, following the solution attempt, these unsuccessful solvers show priming for solution words even though the correct solution had not been reached (Beeman & Bowden, 2000). Further, a later study (Bowden & Jung-Beeman, 2003) found that the strength of activation (as measured by the magnitude of the priming effect), particularly in the right hemisphere, was related to degree of "Aha!" reported when subjects were shown the solution words. Subjects who reported the strongest "Aha!" experience to the presented solution were those who showed the strongest semantic priming for that word. Bowden and Jung-Beeman take these results to indicate that insightful recognition of a solution is preceded by processes that operate beneath a threshold of awareness and thereby lead to unreportable, but measurable activation of the correct solution.

These examples suggest that solvers do not have reliable access to their solution progress during the creative problem solving process. Based on the literature reviewed above, it seems that the process of "insightful" restructuring may not be a sudden affair after all, but rather the result of a gradual, though unconscious, process. At present, it is too early to assume a direct relationship between the "Aha!" experience and representational change. Recently, there has been increasing research on how attentional states may affect the likelihood of Aha!-type solutions. This work is suggesting that creative solutions are more likely to occur when people are in diffuse attentional states (Kounios et al., 2008; Sio & Ormerod, 2009; Subramaniam et al., 2008). This unconscious, or non-goal-directed, processing may also be necessary for restructuring to occur. As such, the link between Aha! and insightful solution may actually be one of common cause. Although this work on attentional states is exciting, we need more studies that evaluate both changes in representation and the phenomenological experience of solutions in order to understand the link between the Aha! experience and problem solving processes.

Conclusion

Although the focus of our paper has been on restructuring processes, we do not mean to equate "insight" with only this type of solution process. There are other routes to creative solution that can be accounted for within a heuristic search framework. These explanations include systematic searches of memory and the incremental accumulation of knowledge from failure (Weisberg & Alba, 1981), systematic manipulations to problem representations

(Kaplan & Simon, 1990), and switching navigation strategies within the initial search-space (Ormerod, MacGregor, & Chronicle, 2002). In the Gestalt tradition, "insight" was used as a term to describe the phenomenon of discovery, deeper comprehension, or understanding; "restructuring" was a proposed psychological process by which "insight" occurred. We think that it is important to revive this differentiation between the psychological or empirical phenomenon of "insight" and the proposed cognitive mechanisms that might lead to it. We believe that equating "insight" and "restructuring" has led to an underlying assumption in the literature that different theories of "insight" are in some way competing or mutually exclusive, when in fact there may be multiple processes which can lead to new discoveries or "Aha!" experiences.

John R. Anderson (1985) argued that "all cognitive activities are fundamentally problem solving in nature. The basic argument...is that human cognition is always purposeful, directed to achieving goals and to removing obstacles to those goals" (pp. 199-200). This would imply that, by studying how people solve difficult problems, psychological researchers are attempting to understand the very nature and purpose of human intelligence. We have admittedly taken a "devil's advocate" position in this article by challenging the very assumption that restructuring plays *any* role in human problem solving. We did not take this stance to suggest that restructuring processes are unable to be empirically studied or to trivialize the potential importance of these types of processes in everyday life. Rather, we suspect creative problem solving processes may be at least as important as the heuristic search processes that have received so much more attention in the literature. As function of our limited capacity to maintain and process information, the source of our species success is *clearly* not our ability to systematically search large problem spaces and complete multi-step procedures with perfect accuracy. We suggest that the types of processes proposed by restructuring theories provide a powerful mechanism by which people avoid cognitive quagmires through the *re*-representation of problem situations. We propose that by documenting and studying the types of cognitive processes that allow people to overcome representation obstacles, researchers are investigating that which lends particular distinction to human cognition.

Past research on insight and creative problem solving presents several important and interesting questions that are yet to be completely answered. The goal of this article was to point out some of these fundamental questions and open a discussion of some of the methodological issues involved in answering them. Studying higher-order behavior like creative problem solving is a messy, laborious, and frequently perplexing endeavor. We hope our analysis of some of these issues will help inspire new researchers to put their minds to work on these methodological obstacles and provide a potential direction and vocabulary for future research in this area.

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