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Pupils' error on the concept of reversibility in solving arithmetic problems

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The fact that there is no much study on reversibility is one of reason this study was conducted. Others, the importance of reversibility is also being researcher's motivation for focusing pupils' reversibility. On the other hand, the concern on pupils' reversibility is a major concern. The objective of this research is to identify errors done by the pupils in solving arithmetic problems related to reversibility concept. The result of this study can inspire teachers to consider the problem-solving in minimizing errors which must be done by the pupils in solving other arithmetic errors. The result of this study can be used as a reference in designing further learning and tasks for student's reversibility development. This research is qualitative with descriptive approach. The subjects of this research are fifth grade pupils of three Elementary Schools in Jombang, Indonesia. Researcher gave arithmetic task related to reversibility concept to the research subject. The pupils' worksheet was analyzed by calculating a number of pupils who did error for each arithmetic item. Then, it was classified to groups which were based on the error types done by the pupils. Furthermore, the researcher described error types done by the pupils related to Roberts, namely wrong operation, obvious computation error, defective algorithm, and random response. This case proved that there are some elementary school pupils who are still having difficulty in solving arithmetic problems related to reversibility concept.

Key words: Pupil's error, concept of reversibility, solving, arithmetic problems.

INTRODUCTION

Piaget's theory (Inhelder and Piaget, 1958) explained the levels of individual's cognition growth from newborn to adult into 4 stages:

1. Sensory-motoric stage (from the newborn to 2 years old)
2. Pre-operational stage (from 2 years old to 7 years old)
3. Concrete-operational stage (from 7 years old to 11 years old), and
4. Formal-operasional stage (from 11 years old to adult).

At sensory-motoric stage, infants learn about their surroundings by using their sensoric and motoric skills. They moved with reflexes. At pre-operational stage, their language conception were rapidly developed, but still in primitive manner. In developing their skills, they

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symbolized objects. At this phase, they had no figure on the nature of conservation for they were centering, they fully focused on one state only. Hence, their ideas were intuitive and not irreversible, they could not turn the ways of their thinking back into the initial state. At the concrete stage, their reversibility evolved. Reversibility is individual's mental ability to turn the way of thinking back into the original state. At formal-operational stage, they could readily have an abstract and logical construct.

In accordance to Piaget's theory on cognition growth as earlier discussed, it was suggested that the main characteristic of children at concrete stage was the development of reversibility. If reversibility was involved as the feature of an individual's cognition growth, it would be necessary and should be concerned since it evolved. Thus, this research is inspired by the theory of Piaget about reversibility. The researcher were also motivated by Lamon (2007), that there are few research about reversibility. Lamon requested researchers, especially in education field, to focus and investigate on students' reversibility.

Reversibility is defined as someone's capability to control their mentality in order to be able to return to the starting point (Slavin, 2006). For instance, the problem of conservation according to Piaget (Inhelder and Piaget, 1958), is given in two glasses which contained milk with comparable volume. When one of the milk is poured into a bowl (A container which is shorter and wider), then a question was asked, "Which is more, the milk in the glass or milk in the bowl?". When the children's reversibility has been properly developed, they will respond by saying that the milk in the glass is comparable with the milk in the bowl. Due to the way children think that milk in the bowl poured into a glass will have comparable volume, proving that the volume at both container are similar. It means that children's capability to control their mentality in order to be able to return to the starting point has been developed.

Krutetskii (1976) defines mathematical ability related to pupils' success in solving problems are reversibility and flexibility. Inhelder and Piaget (1958) said that reversibility can be considered a key requirement in a number of problems in mathematics. While Haciomeroglu and Presmeg (2009) stated that pupils' reversibility is really important in understanding mathematics topic related to the inverse. All those opinion prove that pupils' reversibility is important, because toward reversibility, pupils are able to solve a number of case related to mathematical problems, one of them is the topic about inverse.

According to Carpenter and Moser (2008), one of the example about arithmetic problems related to reversibility is "Jim has 5 marbles. He has 8 marbles less than Connie's. How many marbles does Connie have?". If the pupils finish that exercise through involving reversibility, they should think "if Jim has 8 marbles **less than** Connie, so Connie has 8 marbles **more than** Jim. Due to Jim has 5 marbles, so the total of Connie's are $8 + 5 = 13$ marbles. Or pupils can think that Jim has 8 marbles". Or pupils can think that "Jim has 8 marbles less than Connie' so the **difference** between Jim's and Connie's is 8. Therefore, in arithmetic equation, it can be said $the\ number\ of\ Connie's\ marbles - the\ number\ of\ Jim's\ marbles = 8$ or $the\ number\ of\ Connie's\ marbles - 5 = 8$. So, the number of Connie's marbles are $8 + 5 = 13$".

According to Fuson (1992), reversibility is needed to deal with addition and subtraction problems that cannot be solved by direct modeling. This judgment implicates that one of the topic related to pupils' reversibility is arithmetic. According to Wong (1977), reversibility is important for the addition concept as "*If a child knows that* $3 + 2 = 5$ *, Is he able to answer* $5 = \square + 2$ *or* $3 + \square = 5$ *?" If he is able to answer it, then his reversibility has been developed, because he understand that* $3 + 2 = 5$ *similar with* $5 = 3 + 2$".

The explanation earlier mentioned shows that pupils' reversibility is important and needs to be noticed since the reversibility is being developed. As the first step in identifying pupils' reversibility, the researcher wants to reveal first condition of pupils' reversibility at the Elementary Scholl in solving arithmetic problems. Therefore, the researcher gave arithmetic task related to reversibility concept to the pupils, then the researcher can identify errors done by the pupils in solving arithmetic items related to reversibility concept.

The pupils of Elementary School was chosen as the research subject with the consideration that reversibility is being developed at the concrete level, that is when a child is up to 7 to 11 years old (Piaget and Inhelder, 1958), this means that the pupils are all in Elementary School. Moreover, reversibility is related to arithmetic. Otherwise, arithmetic for the first time was given to the pupils at the Elementary School.

Therefore, the objective of this research is to identify errors done by the pupils at the Elementary School in solving arithmetic problems related to reversibility concept. If the teacher knows the description of pupils' reversibility based on the errors done by them, the researcher's expectation is the teachers are able to think the problem-solving in minimizing errors must be done by the pupils for the next. Furthermore, the research result can be as orientation to compose the next learning and duties for the pupils' reversibility development.

REVIEW OF LITERATURE

Reversibility

Reversibility is a term adopted from Piaget's theory that one of children's characteristic at the concrete

operational level starts at the phase when reversibility is being developed. Furthermore, Inhelder and Piaget (1958) said that "reversibility is defined as the permanent possibility of returning to the starting point of the operation in question".

For instance, two glasses containing milk with comparable volume. When one of the milk is poured into a bowl (A container which is shorter and wider), then a question was raised "Which is more, the milk in the glass or milk in the bowl?". Children at the pre-operational will answer that the milk in the glass is comparable with the milk in the bowl. This is because the children's mentality is "centrally" and *irreversible.* Children only focused on one aspect, that is the milk volume, and ignoring the other aspect. While children at the concrete operational level will answer that 'milk in the glass' is comparable with 'milk in the bowl'. Because children at this level has the mentality that if milk in the bowl is poured into the glass, the volume will be as same as that in the bowl, which shows that the volume for the both container are comparable. It means at that at this level, children's ability to control their mentality return to the starting point where it has been developed. In this case, *starting point* means *two glasses that contained milk with comparable volume*. While, *change their mindset to the starting point* is *when children pour the milk in the bowl to the glass*. So the milk volume will be as same as the condition before it is poured.

According to Kang and Lee (1999), "reversibility enables the recognition of problems in various ways". For instance, the pupils of Elementary School were given an arithmetic problem, that is "$43 - \square = 24$", then they were asked to determine the value at the box. So, through the reversibility, the pupils are able to investigate $43 - \square = 24$ through some ways, that is:

1. $-\square = 24 - 43$,
Since children would think that the two parts of aritmethical equation $43 - \square = 24$ was added by -43, the equaition would be $(-43) + 43 - \square = 24 + (-43)$, hence the result found $-\square = 24 - 43$.

2. $\square = 43 - 24$,
Since children would think that aritmethical equation $43 - \square = 24$ implied that 43 minus particular number (symbolized with $\square$) equaled to 24. If 43 minus particular number (symbolized with $\square$) equaled to 24,then, 43 minus 24 should be that particular number (symbolized with $\square$). Indicating that $43 - 24 = \square$ atau $\square = 43 - 24$.

Such ideas described the notion of reversibility. Firstly, children involved reversibility with reciprosity, operating the two parts of equation with similar element. At the second manner, they involved reversibility with negation, thinking if 43 minus particular number (*symbolized with* $\square$) *equaled to 24, then, 43 minus 24 should be that particual number* (*symbolized with* $\square$).

According to Piaget and Inhelder (1998) they stated that there are two reversibilities concept, which are negation and reciprocity. Here, negation includes understanding which is a way one could be delayed by other way. In this case, reversibility shows the idea which is in every operation has invers which is used for canceling the operation. In the example earlier given, subtraction is simply the reversal of addition while multiplication which is canceled by dividing operation. This means that addition negation is subtraction and multiplication is dividing. While the reciprocity concepts are related to the equivalent relation. The other example of negation and reciprocity in algebra case which is explained by Ardi (2009) are:

"In mathematics education, Adi (1978) used the concept of negation and compensation to study the relationship between college students' developmental level and their performance on equation solving. She provides the equation $14 - \frac{15}{7-x} = 9$ to illustrate her interpretation of negation and compensation. In solving this algebraic equation, *negation* is involved when one is asked to make the following inferences: 'Fourteen minus what equals nine?', 'Fifteen divided by what equals five?', and 'Seven minus what equals three?'. On the other hand, *compensation* is involved when one multiply both sides of the equation by $7 - x$ to obtain $98 - 14x - 15 = 63 - 9x$."

Based on these explanation, the researcher conclude that if the reversibility is being developed optimally, so the children are able to solve the arithmetic problems correctly. To acquire it, the children's reversibility need to be practiced through giving problems related to reversibility concept.

Krutetskii (1976) explained that one of the mathematical ability related to pupils' success in solving problem is reversibility. Reversibility refers to the ability of establishing two-way reversible relations as opposed to one-way relations which function only in one direction. This view implied that reversibility had two process within:

1. A process that started from the initial state moving into the end point as the goal and
2. A process that started from the end point moving back into the initial one, however, it was fine to use another path as its way.

Furthermore, he also explained on reversibility of the mental process , thinking in a reverse direction from the result or the product to the initial data. For instance, the pupils of Elementary School are asked to answer arithmetic problem "$29 + \cdots = 46$", if the pupils involve reversibility in answering the task, so the pupils will think

Table 1. Indicators of error classification of the reversibility concept in solving arithmetic problem by Roberts (1968).

Error classification of the reversibility concept in solving arithmetic problem	Indicators
Wrong operation	The problems are solved using the operator other than the one specified in the problem. Children were considered in conducting a wrong operation when they completed an arithmetical task by changing the operation presented. Given a task 23 + … = 10, they completed the task by changing the addition operation into the subtraction, which changed the task into $23 - \ldots = 10$, with 13 as the result. This error was classified as wrong operation
Obvious computation error	In this form of error, the pupil uses the correct algorithm but due to carelessness in recalling number facts, the wrong answer is given. Given a task 3 + . . . = 10, the pupil complete the task with correct algoritm, (. . . = 10 – 3). The result was supposed to be 7, however, the pupil miscalculated the equation into 10 minus 3, which result in 6. This errors was classified as *obvious computation error*
Defective algorithm	The pupil uses the wrong algorithm in the problem-solving process. Given a task: $\ldots - 4 = 3$, the pupil completed the task by subtracting 4 with 3, which result was 1. This was absolutely false due to the wrong algoritm
Random response	These are errors in which no general pattern is detected. Students' errors were not clearly detected

"if 29 plus a particular number was 46, then, 46 minus 29 should be that particular number. This was due to the fact that the result of 29 plus the particular number was 46". Thus, to fill the blank they need to apply this "$46 - 29 = \cdots$", and the result is 17. After getting the result, the next mentality activity done by them is to return to the result to the previous data. In this case, the previous data is its problem (that is $29 + \cdots = 46$). Then it can be acquired $29 + 17 = 46$. So it is right that the problem-solving is 17.

The classification of the errors which is done by the pupils for arithmetic solving which is related to the reversibility concept

The reversibility of pupils could be practiced through giving the task which is related to the reversibility concept. One of material which could be used for practicing the reversibility is arithmetic. Ramful (2008) stated that, in mathematics, the reversibility is related to the operation of arithmetic, decimal, ratio, algebra, and other cases. According to Wong (1977), the educators' assumes that reversible thought is related to children's performance at arithmetic equations. Secondly, according to Maf'ulah (2015), he stated that reversibility is having strengthened the relation with decimal and arithmetic.

In this study, the researcher has focused on the arithmetic material. The researcher would like to identify the errors which have been done by the pupils in solving the problem of arithmetic which is related to the reversibility concept. Through this study, hopefully this could be used as previous study of the other arranging the teaching and learning for developing the reversibility of the pupils. In this study, the errors of pupils in solving of arithmetic case which is related to the reversibility concept which is describe is based on the classifications of the error according to Roberts (1968) as shown in Table 1.

RESEARCH METHOD

Research design

The research design of the study is qualitative design with descriptive approach. This study met the characteristics of qualitatif research, as Bpgdan and Biklen (1998) stated which are:

1. It was naturalistic because the data sources was real with researchers as the primary instrument
2. The data was descriptive due to its qualitative nature, in the form of essay. In this case, the data was derived from the result of subjects' works
3. It was inductive, which had no intention to test a hypothesis, but merely describing a phenomena.

Research subject

This study involved 96 pupils of the fifth graders in jombang with 55 males and 41 females as the research subject. Elementary students were selected with consideration that reversibility began to

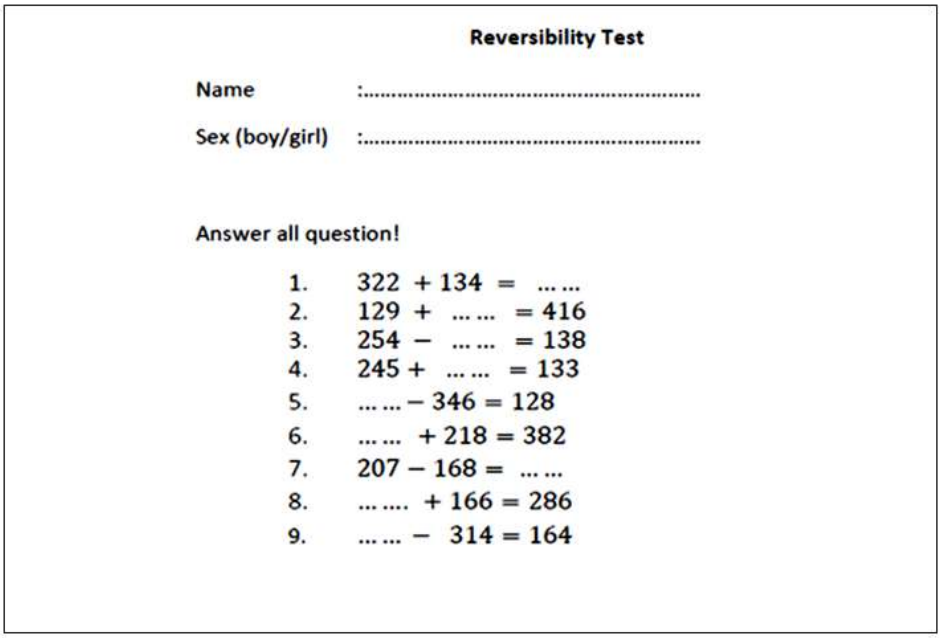

Figure 1. Arithmetic task.

evolve in the age-range between 7 to 11 years old, indicating elementary graders. Besides, the fifith graders were chosen due to the fact that they had already learned arithmetic. However, the researchers took one sample in each category of errors for data analysis and research findings.

Research instrument

The objective of the study is to identify the errors of arithmetic which is related to the reversibility concept. For reaching up the objective of the study, the researcher made arithmetic task which is related to reversibility concept as this instrument has been validated by expert validator. There are 9 items of arithmetic task as shown in Figure 1.

Data collected procedure

The researcher gave instrument of arithmetic task which is related to reversibility concept to the pupils. Then the pupils completed arithmetic task individually.

Data analysis

Students' works were analysed by counting the students with errors for each number of mathematics task given. Then, they were classified into groups based on their errors. The researchers dercribed the kinds of students' errors for each group based on Roberts' error classification (1968) including *wrong operation, obvious computation error, defective algorithm* and dan *random response*. The researchers selected one subject with errors in each group. Analysis was conducted within some procedures which are:

1. Data reduction that aims at assert, select, focus, abstract, and transform all raw data into meaningful ones.

2. Data presentation that included classifying and identifying data, which transcripted the organized and categorized data that enabled one to make the conclusion; and (3) conclusion making.

RESULTS

The item number 1 is not relating item to the reversibility concept. As what Wong (1977) stated that, "the form of $x + y = \cdots$" was not included in Piaget's abstract concept of reversible thought even though it constitutes on form of arithmetic equations". The item 1 is only to check the subject's understanding concerning to sum operation. Thus, for number 1 is not paid more attention in error analyzing. Based on pupils' answers, the data was gotten as sshown in Table 2. Based on Table 2, information concerning the types of errors which were committed by the pupils in solving arithmetic problem related to the reversibility concept is gotten and they are presented below:

Wrong operation error

Data of the pupils who committed an error due to this type of wrong operation in completing arithmetic problem related to reversibility concept is presented in Table 3. Based on Table 3, there are 3 items where the pupils have committed error due to the type of wrong operation, those are number 4, 7, and 9. For number 4, all pupils answered 112 by changing "$254 + \cdots = 138$" to "$254 - \cdots = 138$". Due to the problem number 7, all pupils

Table 2. Summary of the number of pupils who committed errors in solving arithmetic problem related to the reversibility concept.

Number	Arithmetic problem	The number of pupils who commited error for each type of error				The number of pupils who committed error
		Wrong operation	Obvious computation error	Defective algorithm	Random response	
1	$322 + 134 =$ ...	-	-	-	-	-
2	$129 +$ ... $= 416$	0	3	3	16	25
3	$254 - \cdots = 138$	0	5	15	16	36
4	$245 +$ ... $= 133$	7	2	56	18	84
5	$\ldots - 346 = 128$	0	2	12	22	36
6	... $+ 218 = 382$	0	3	11	12	26
7	$207 - 168 =$ ...	3	39	2	24	68
8	... $+ 166 = 286$	0	1	9	16	26
9	$\ldots -\ 314 = 164$	2	2	12	17	33

Table 3. Data which present the number of pupils who completed *wrong operation* error.

Number	Arithmetic problems	The number of pupils who committed wrong operation error	
		Quantity	Percentage (%)
1	$322 + 134 =$ ...	-	-
2	$129 +$ ... $= 416$	0	0
3	$254 - \cdots = 138$	0	0
4	$245 +$ ... $= 133$	7	6.72
5	$\ldots - 346 = 128$	0	0
6	... $+ 218 = 382$	0	0
7	$207 - 168 =$ ...	3	2.88
8	... $+ 166 = 286$	0	0
9	$\ldots -\ 314 = 164$	2	1.92
-	Total	12	-

answered 375 by changing "$207 - 168 =$ ..." became "$207 + 168 =$ ...". In solving the problem, pupils change the operation given on the task. It means that the pupils commit *wrong operation*in solving arithmetic problem related to the reversibility concept. Figure 2 presents one of the examples of pupil's error. Figure 2 shows the example of error at the type of wrong operation which was committed by the initial subject AI. The problem was $245 + \cdots = 133$, however AI changed the sum operation on $245 + \cdots = 133$ which became minus operation $245 - \cdots = 133$. Thus, the answer which was gotten was wrong.

Obvious computation error

Data of the pupils who committed an error due to the type of obvious computation error in completing arithmetic problem related to reversibility concept is presented in Table 4. The information presented in Table 4 shows that for each item there are some pupils who are definitely committed to such error. But the obvious computation error was committed mostly by the pupils when they solved arithmetic problem number 7, more than 39 (or 37.44%) pupils committed the obvious computation error. Figure 3 shows the different types of error. The answer presented in Figure 3 should be 39. However, subject LA answered 239 because he committed an error in accounting.

Defective algorithm error

Data of the pupils who commit an error at this type of defective algorithm in completing arithmetic problem related to reversibility concept is presented in Table 5. The information presented in Table 5 shows that for each item there were some pupils who definitely committed the type of error.

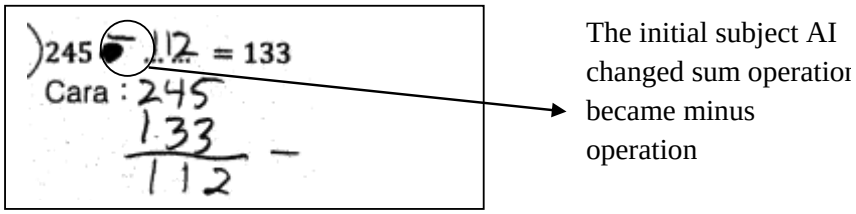

Figure 2. The example of pupil's error due to the type of *wrong operation.*

Table 4. Data which present the number of pupils who completed *obvious computation error.*

Number	Arithmetic problems	The number of pupils who committed"obvious computation error"	
		Quantity	Percentage (%)
1	$322 + 134 =$ ...	-	-
2	$129 +$ ... $= 416$	3	2.88
3	$254 - \cdots = 138$	5	4.8
4	$245 +$ ... $= 133$	2	1.92
5	$\ldots - 346 = 128$	2	1.92
6	... $+ 218 = 382$	3	2.88
7	$207 - 168 =$ ...	39	37.44
8	... $+ 166 = 286$	1	0.96
9	$\ldots -$ $314 = 164$	2	1.92
-	Total	57	-

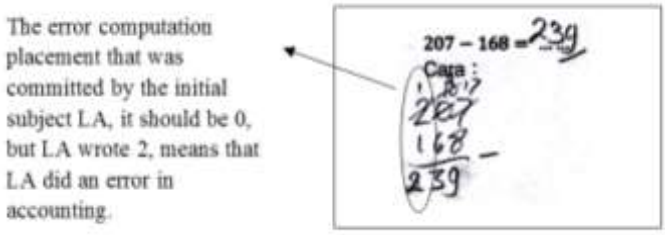

Figure 3. The example of pupils' error in obvious computation error type.

Table 5. Data which presentthe number of pupils who commit an error due to the defective algorithm type.

Number	Arithmetic problems	The number of pupils who committed"defective algorithm" error	
		Quantity	Percentage (%)
1	$322 + 134 =$ ...	-	-
2	$129 +$ ... $= 416$	3	2.88
3	$254 - \cdots = 138$	15	14.4
4	$245 +$ ... $= 133$	56	53.76
5	$\ldots - 346 = 128$	12	11.52
6	... $+ 218 = 382$	11	10.52
7	$207 - 168 =$ ...	2	1.92
8	... $+ 166 = 286$	9	8.64
9	$\ldots -$ $314 = 164$	12	11.52
-	Total	120	-

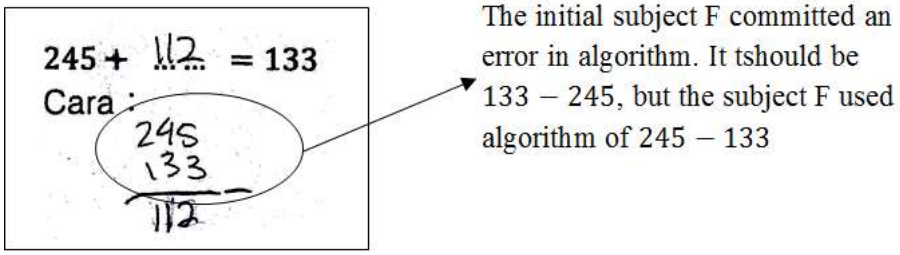

Figure 4. The example of pupil commited on error due to the defective algorithm type

Table 6. Data which presents the number of pupils who commitan error due to *random response* type.

Number	Arithmetic problem	The number of pupils who commit "randomresponse" error	
		Quantity	Percentage (%)
1	$322 + 134 = \ldots$	-	-
2	$129 + \ldots = 416$	16	15.36
3	$254 - \cdots = 138$	16	15.36
4	$245 + \ldots = 133$	18	17.28
5	$\ldots - 346 = 128$	22	21.12
6	$\ldots + 218 = 382$	12	11.52
7	$207 - 168 = \ldots$	24	23.04
8	$\ldots + 166 = 286$	16	15.36
9	$\ldots - 314 = 164$	17	16.32
-	Total	141	-

1. For solving the problem number 2, there are 2.88% pupils who used $416 + 129$ method.
2. For solving the problem number 3, there are 14.4% pupils who used $254 + 138$ method.
3. For solving the problemnumber 4, there are 10.52% pupils who used $245 - 133$ method, and there are 43.24% pupils solving the problem who used $245 + 133$ method.
4. For solving the problem number 5, there are 11.52% pupils who used $346 - 128$ method.
5. For solving the problem number 6, there are 10.52% pupils who used $382 + 218$ method
6. For solving the problem number 8, there are 8.64% pupils who used $166 + 286$ method.
7. For solving the problem number 9, there are 11.52% pupils who used $314 - 164$ method.

The information earlier mentioned explains that there were still many pupils who committed an error when applying method or strategy in solving arithmetic problem. It means that there were many pupils who committed *defective algorithm*in solving arithmetic problem related to the reversibility concept thus consist an error. Figure 4 shows an example of this type of error.

Random response error

Data of the pupils who committed an error at this type of random response in completing arithmetic problem related to reversibility concept is presented in Table 6. Data on the Table 6 shows that for each item there were some pupils definitely committed to the type of random response error. Figure 5 shows the example of this type of error. The answer of the problem on Figure 5 should be 287. However, the initial subject ALA answered 136. ALA's anwer was wrong. Moreover, the solving process was unclear. Thus, the error which was completed by ALA was not detected clearly. So, the solving problem of Figure 5 was categorized into random response.

DISCUSSION

The objective of this research is to identify the Elementary Schoolpupils' error in solving arithmetic problem which is related to the reversibility concept. The research result goes with Roberts (1986) finding which mentioned, the type of error, namely wrong operation, obvious computation error, defective algorithm, and random response. The error due to this type of random response

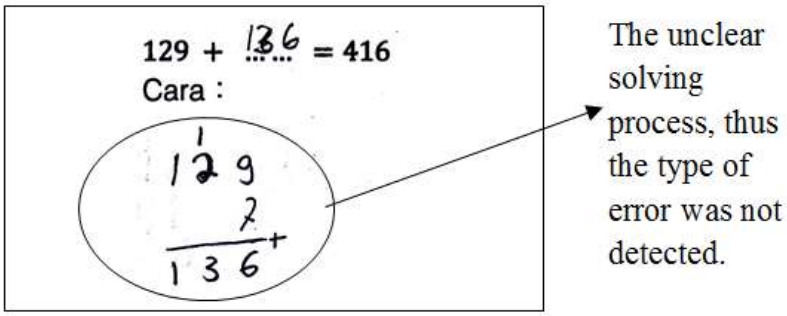

Figure 5. The example of pupils' error due to the type of *random response.*

occured when the pupils did not understand what they should complete in solving the item thus, they carried out an unclear completion. This means that the students did not understand arithmetic concept which is related to the reversibility.

According to Krutetskii (1976), "reversibility of the mental process, is the thinking in a reverse direction from the result or the product to the initial data". If it is related to the solving process of arithmetic problem which is concerned with the reversibility concept, then better for the pupils check their work which they completed back to the first data. With this, in solving arithmetic problem which is given by the researcher, most of the pupils did not check their work according to the first data, thus they did not understand that their obtained work was wrong.

The number of errors which was completed by the pupils also shows pupils who lack understanding to arithmetic due to the fact that they did not used the reversibility properly. Due to the fact that the reversibility has a role in understanding the mathematical material related to the inverse, while the arithmetic is part of the mathematical material related to invers. According to the study of Haciomeroglu and Presmeg (2009), the reversibility of the pupils is very important in understanding the material relating to the inverse mathematical, and Fuson (1992) who said that reversibility is needed to deal with addition and subtraction problems. In addition, the research finding by Wong (1977) explained that when he gave reversibility assignment which contain 20 arithmetic equation, the result indicated significant correlation between reversibility and Arithmetic Performance which was found on female subject.

If related to the meaning of the equal sign "=" for pupils, Mc. Neil et al. (2006) said, "equal signs were often presented in standard operations-equals-answer contexts (for example, 3 + 4 = 7) and were rarely presented in nonstandard operations on both sides contexts (for example, 3 + 4 = 5 + 2)". The equal sign "=" is often given meaning by the pupils as the context of the answer. And rarely interpreted as connecting both sides contexts of the equal sign "=", (that is, the right side is the same as the left side). If reversibility pupils are involved in meaning the equal sign "=", then the pupil should think if x = y then y = x, nor vice versa. Which imply that the equal sign means "both side are the same or equal right side to the left side".

Arithmetic are basic materials for studying algebra and the other materials. According to what was explained by Greenes (2004), algebra is sometimes referred to as generalized arithmetic because it formalizes arithmetic relationships. Its power lies in the ways it allows us to represent relationships among quantities, to describe properties of operations (such as commutative and distributive), and to describe patterns. Algebra provides rules for manipulating symbols, such as simplifying an expression and then solving for an unknown. Therefore, by detecting the mistakes of the pupils in solving arithmetic problems, is expected to minimize the error.

CONCLUSION

The research conclusions are:

1. There were many pupils who committed errors in solving the second problem, as many as 23% pupils with the following detail: there are 3 pupils who committed error due to the type of obvious computation error, 3 pupils committed due to the type of defective algorithm error, and 16 committed error due to the type of random response.
2. There were many pupils committed errors in solving the third problem, as many as 36 pupils with the following detail: 5 pupils committed error due to the type of obvious computation error, 15 pupils committed error due to the type of defective algorithm and 16 pupils committed error due to the type of random response.
3. There were many pupils committed errors in solving the fourth problem, as many as 84 pupils with the following detail: 7 pupils committed error due to the type

of wrong operation, 2 pupils committed error due to the type of obvious computation error, 56 pupils committed error due to the type of defective algorithm and 18 pupils committed error due to the type of random response.
4. There were many pupils who committed errors in solving the second problem, as many as 36 pupils with the following detail: 2 pupils committed errordue tothe type of obvious computation error, 12 pupils committed error due to the type of defective algorithm and as many as 22 pupils committed error due to the type of random response.
5. There were many pupils who committed errors in solving the second problem, as many as 26 pupils with the following detail: 3 pupils committed error due to the type of obvious computation error, as many as 11 pupils committed error due to the type of defective algorithm and 12 pupils committed error due to the type of random response.
6. There were many pupils who committed errors in solving the second problem, as many as 68 pupils with the following detail: 3 pupils committed error due to the type of wrong operation, as many as 39 pupils committed error due to the type of obvious computation error, 2 committed error due to the type of defective algorithm and 24 pupils committed error due to the type of random response.
7. There were many pupils who committed errors in solving the second problem, as many as 26 pupils with the following detail: 1 pupil committed error due to the type of obvious computation error, 9 pupils committed error due to the type of defective algorithm and 16 pupils committed error due to the type of random response.
8. There were many pupils who committed errors in solving the second problem, as many as 33 pupils with the following detail: 2 pupils committed error due to the type of wrong operation, 2 pupils committed error due to the type of obvious computation error, 12 pupils committed error due to the type of defective algorithm and 17 pupils committed error due to the type of random response.

The explanations earlier given put up the fact that there are still many Elementary School pupils who experience such difficulties in solving arithmmetic problem which are related to their reversibility. The researcher expects that this reseach result can inspire the teachers especially who teach at Elementary School grade in order to pay attention more to their pupils' reversibility earlier, and draft up the solution to minimize the errors which are probably committed by the pupils in solving the certain arithmetic problems later.

Conflict of interests

The authors have not declared any conflict of interests.

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